# Estimating dilepton rates and electrical conductivity from vector current correlation functions in quenched QCD

Frithjof Karsch, BNL and Bielefeld University

- Introduction:

  Dilepton rates, Euclidean correlators and spectral functions

- Vector correlation function on the lattice

  volume and cut-off dependence
  thermal moments of the spectral function
  continuum extrapolation

- Dilepton rate and electrical conductivity

  using an ansatz for the spectral function
  using MEM

- Conclusions

# Estimating dilepton rates and electrical conductivity from vector current correlation functions in quenched QCD

Frithjof Karsch, BNL and Bielefeld University

- in collaboration with:

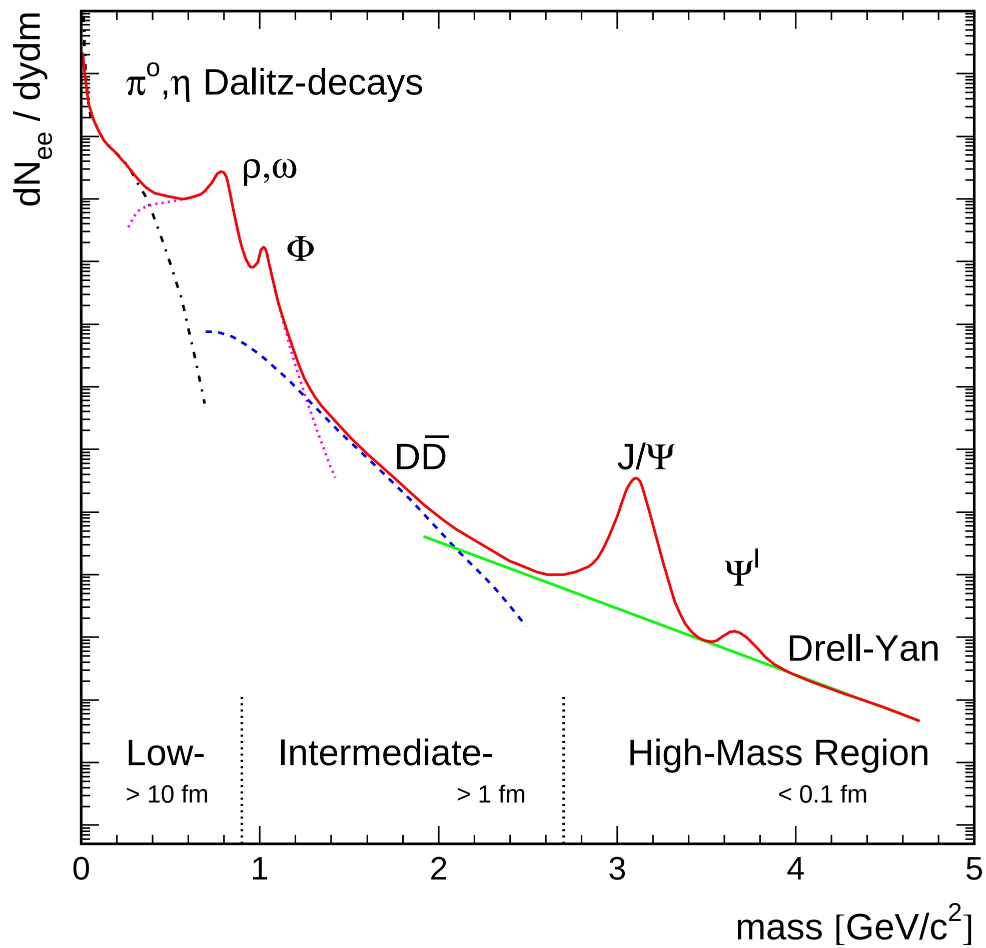
  HengTong Ding, Anthony Francis, Olaf Kaczmarek (Bielefeld) and Wolfgang Söldner (Frankfurt/GSI)

- see also poster by A. Francis

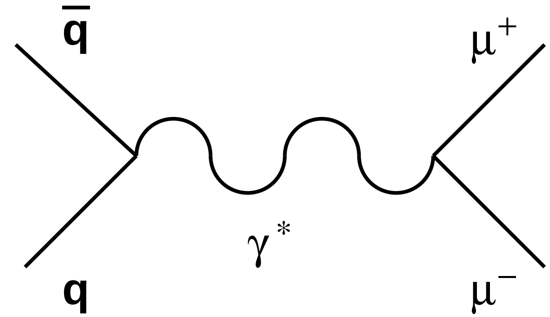
  The continuum limit of hadronic correlation functions in the deconfined phase of an SU(3) gauge theory

# Thermal vector meson properties from dilepton rates in heavy ion collisions

dilepton rate vs. invariant mass of $l^+l^-$ pair

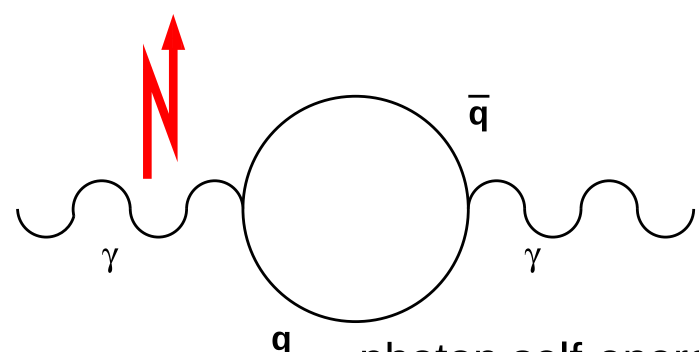



differential cross-section for $l^+l^-$ pair production




dilepton pair ($e^+e^-$, $\mu^+\mu^-$) production through annihilation of "thermal" $\bar{q}q$-pairs in hot and dense matter

rate $\sim |q\bar{q} \rightarrow \gamma^*|^2 \cdot |l^+l^- \rightarrow \gamma^*|^2$

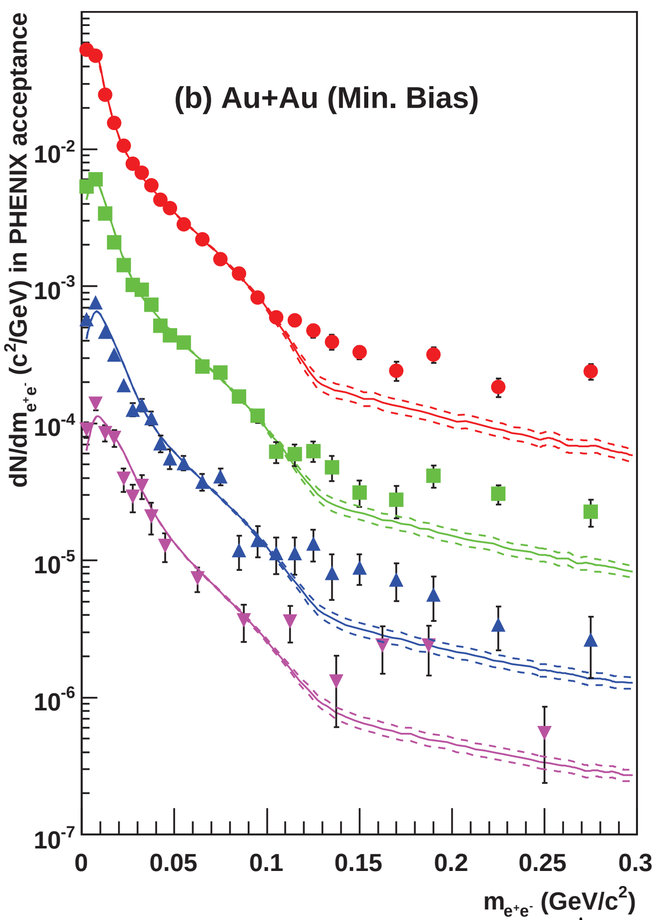


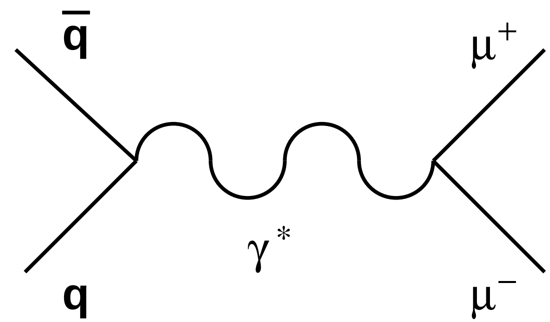

photon self-energy

# Thermal vector meson properties from dilepton rates in heavy ion collisions

low-mass $e^+e^-$-pairs:

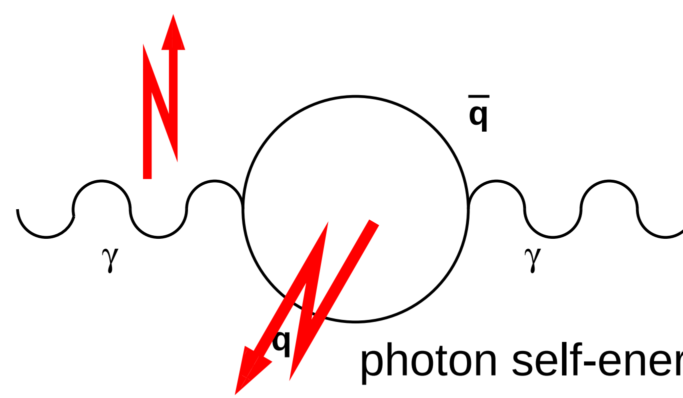
A. Adare et al. (PHENIX Collaboration), PRL 104, 132301 (2010)




dilepton pair ($e^+e^-$, $\mu^+\mu^-$) production through annihilation of "thermal" $\bar{q}q$-pairs in hot and dense matter

rate $\sim |q\bar{q} \rightarrow \gamma^*|^2 \cdot |l^+l^- \rightarrow \gamma^*|^2$



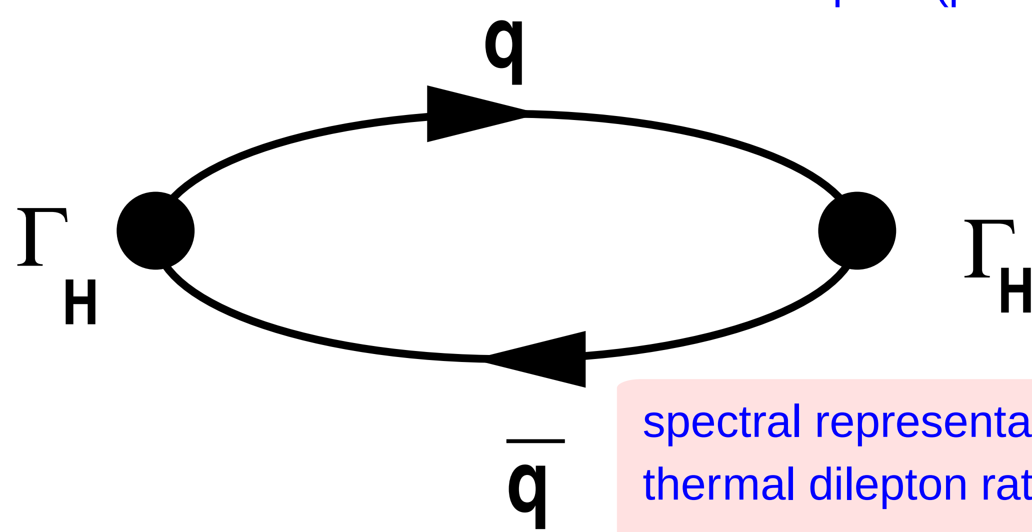

photon self-energy

differential cross-section for $l^+l^-$ pair production $\Rightarrow$ thermal meson correlation function

# Thermal 'meson' correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$-pair

spectral representation of correlator $\Rightarrow$ in-medium properties of hadrons; thermal dilepton (photon) rates




spectral representation of thermal dilepton rate (2-flavor)

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{\rho_V(\omega,\vec{p},T)}{\omega^2(e^{\omega/T}-1)}$$

spectral representation of Euclidean correlation functions

$$G_V^\beta(\tau,\vec{r}) = \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3\vec{p}}{(2\pi)^3}\, \rho_V(\omega,\vec{p},T)\, e^{i\vec{p}\vec{r}}\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

# Vector correlation functions at high temperature

- time-like ($G_{00}$) and space-like ($G_{ii}$) correlator (at $\vec{p}=0$) of local, non-conserved current: $J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\gamma_\mu \psi(\tau,\vec{x}))$

$$
\begin{aligned}
G_{\mu\nu}(\tau,\vec{x}) &= \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle \\
G_{\mu\nu}(\tau,\vec{p}) &= \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})\, \mathrm{e}^{i\vec{p}\cdot\vec{x}} \\
G_V(\tau,\vec{p}) &= -G_{00}(\tau,\vec{p}) + G_{ii}(\tau,\vec{p})
\end{aligned}
$$

- conserved current, $J_0 \Rightarrow \tau$-independent correlator $G_{00}$

  $\sim$ quark number susceptibility $\chi_q$:

$$G_{00}(\tau,\vec{p}=0)) \equiv \chi_q T + \mathcal{O}(a^2)$$

- ratios are free of renormalization ambiguities, e.g.

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} \quad ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau) G_V^{free}(\tau T)}$$

# Spectral functions at high temperature

- free vector spectral function (infinite temperature limit)

$$\rho_{00}^{\text{free}}(\omega) = 2\pi T^2 \omega\delta(\omega)$$

$$\rho_{ii}^{\text{free}}(\omega) = 2\pi T^2 \omega\delta(\omega) + \frac{3}{2\pi}\,\omega^2\ \tanh(\omega/4T)$$

- $\delta$-functions cancel in $\rho_V(\omega) \equiv -\rho_{00}(\omega) + \rho_{ii}(\omega)$
- $T < \infty$: $\delta$-function in $\rho_{00}$ protected; $\delta$-function in $\rho_{ii}$ gets smeared out:

ansatz:

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\ \omega^2\ \tanh(\omega/4T)$$

3-4 parameter: $(\chi_q)$, $c_{BW}$, $\Gamma$, $\kappa$

# Electrical Conductivity

- electrical conductivity $\Leftrightarrow$ slope of spectral function at $\omega = 0$

$$\frac{\sigma}{T} = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

- using our ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW} \cdot C_{em}$$

with $C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2$, *i.e.* $\frac{5}{9} e^2$ for $n_f = 2$, or $\frac{6}{9} e^2$ for $n_f = 3$

previous studies using staggered fermions

S. Gupta, PL B597 (2004) 57: $N_\tau = 8 - 14$, $N_\sigma \leq 44$

G. Aarts et al., PRL 99 (2007) 022002: $N_\tau = 16,\ 24$, $N_\sigma = 64$

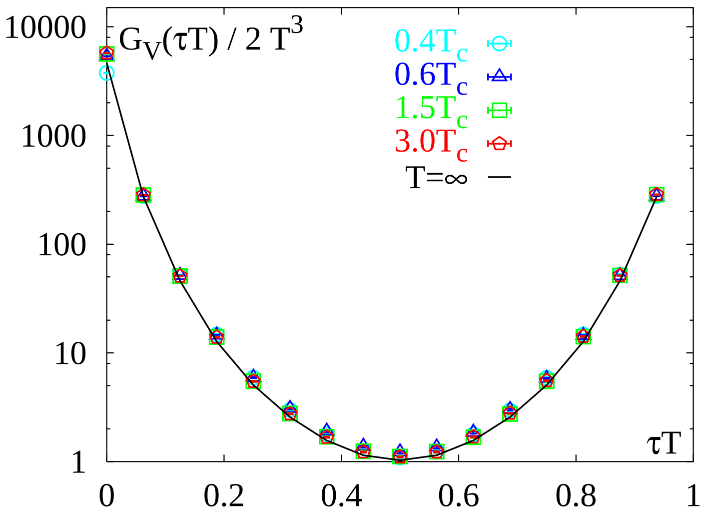
(need to distinguish $\rho_{even}(\omega)$, $\rho_{odd}(\omega)$)

OLD MEM analysis

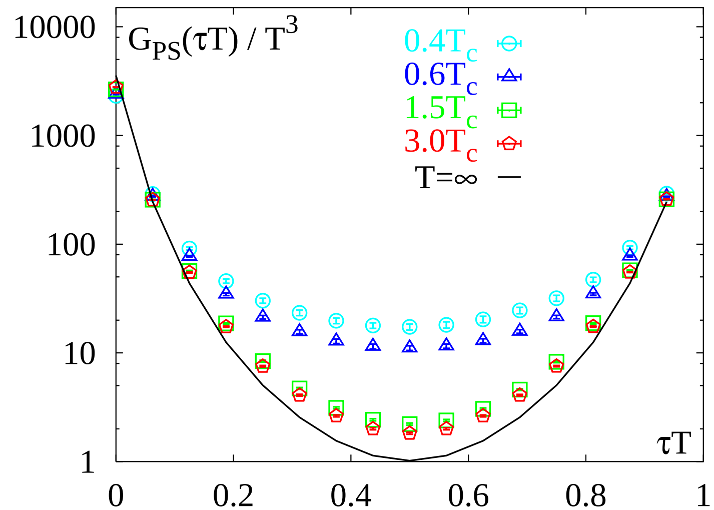
# Light quark correlation functions and spectral functions

FK et al., PLB530 (2002) 147

vector correlator




pseudo-scalar correlator



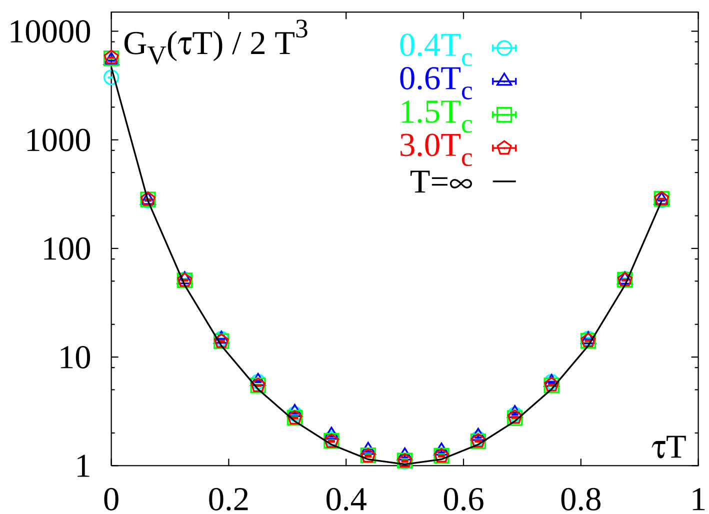

$\mathbf{64^3 \times 16}$ lattices, clover fermions

OLD MEM analysis

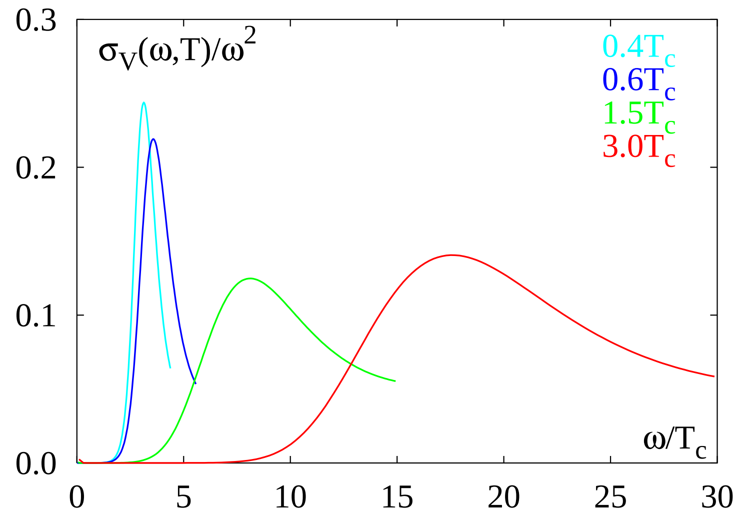
# Light quark correlation functions and spectral functions

FK et al., PLB530 (2002) 147

## vector correlator




## vector spectral functions

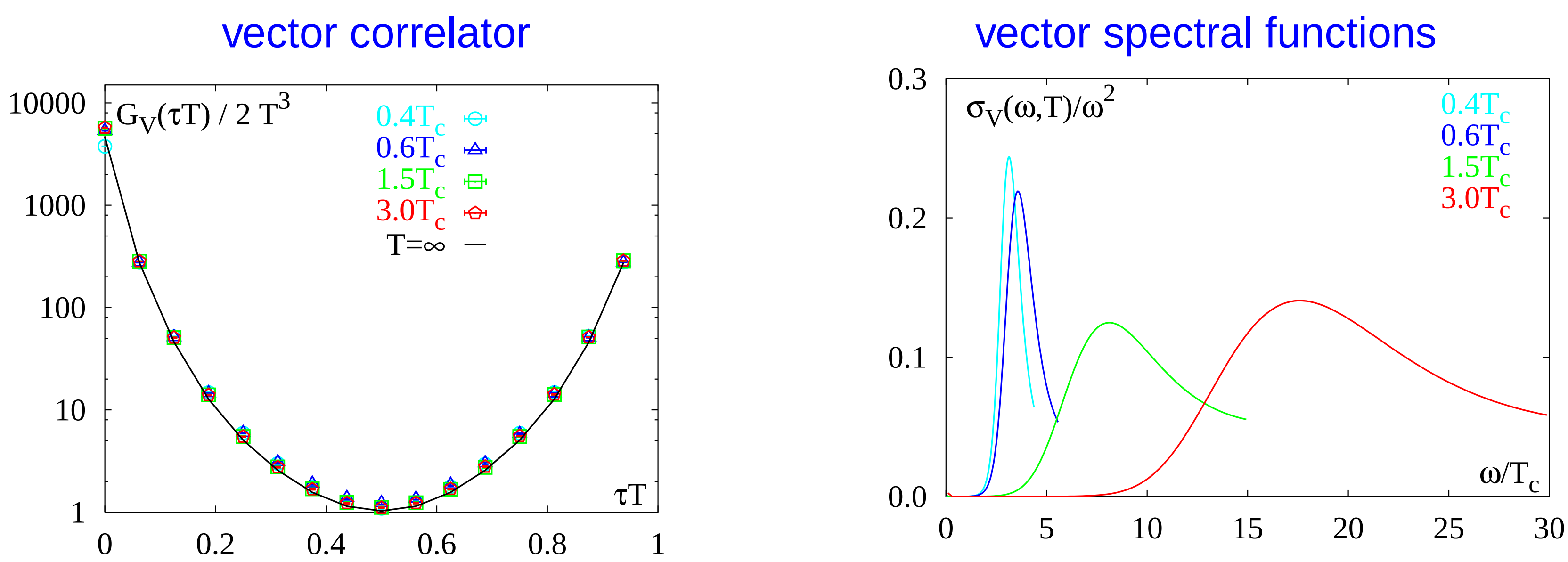



$\mathbf{64^3 \times 16}$ lattices, clover fermions

OLD MEM analysis

# Light quark correlation functions and spectral functions

FK et al., PLB530 (2002) 147




$64^3 \times 16$ lattices, clover fermions

- $N_\sigma / N_\tau = 4$: finite volume effects?
- $64^3 \times 16$: cut-off effects?
- expect $\rho_V(\omega) \sim \omega$ for $\omega/T \ll 1$;
  not captured by MEM because default model did not allow for it;
  redefinition of kernel helps (G. Aarts et al., PRL99 (2007) 022002)

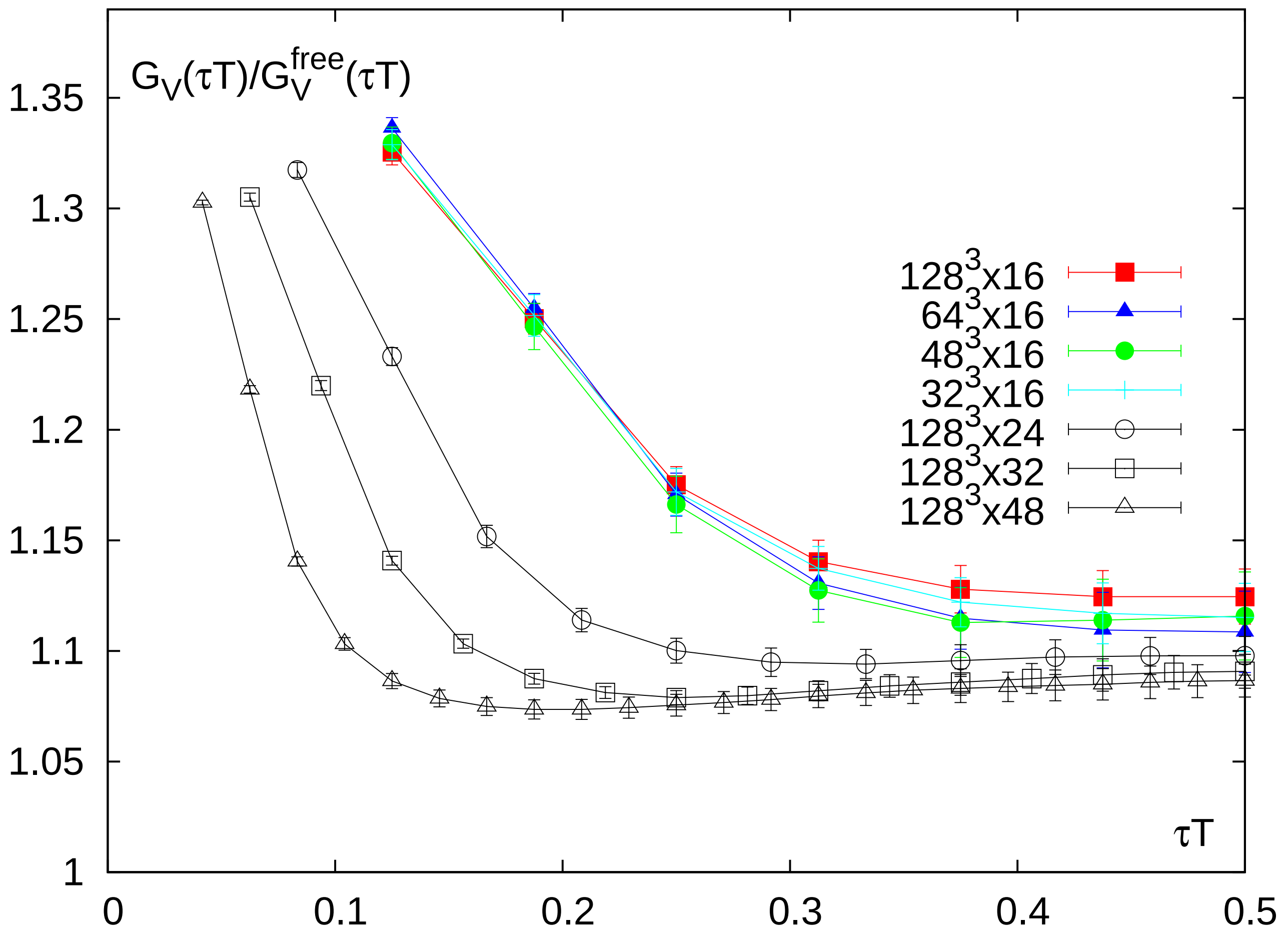
# New Analysis: Vector correlation function on large & fine lattices

- SU(3) gauge configurations at $T/T_c = 1.5$
- lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 32 - 128$,
  $$N_\tau = 16,\ 24,\ 32,\ 48$$
- vector correlation functions at $\kappa \simeq \kappa_c$ using non-perturbatively improved clover fermions & (non-perturbative renormalization constants)

| $N_\tau$ | $N_\sigma$ | $\beta$ | $c_{SW}$ | $\kappa$ | $Z_V$ | # conf† |
|---|---|---|---|---|---|---|
| 16 | 32 | 6.872 | 1.4125 | 0.13495 | 0.829 | 60 |
| | 48 | 6.872 | 1.4125 | 0.13495 | 0.829 | 62 |
| | 64 | 6.872 | 1.4125 | 0.13495 | 0.829 | 77 |
| | 128 | 6.872 | 1.4125 | 0.13495 | 0.829 | 191 |
| 24 | 128 | 7.192 | 1.3673 | 0.13440 | 0.842 | 156 |
| 32 | 128 | 7.457 | 1.3389 | 0.13390 | 0.851 | 255 |
| 48 | 128 | 7.793 | 1.3104 | 0.13340 | 0.861 | 431 |

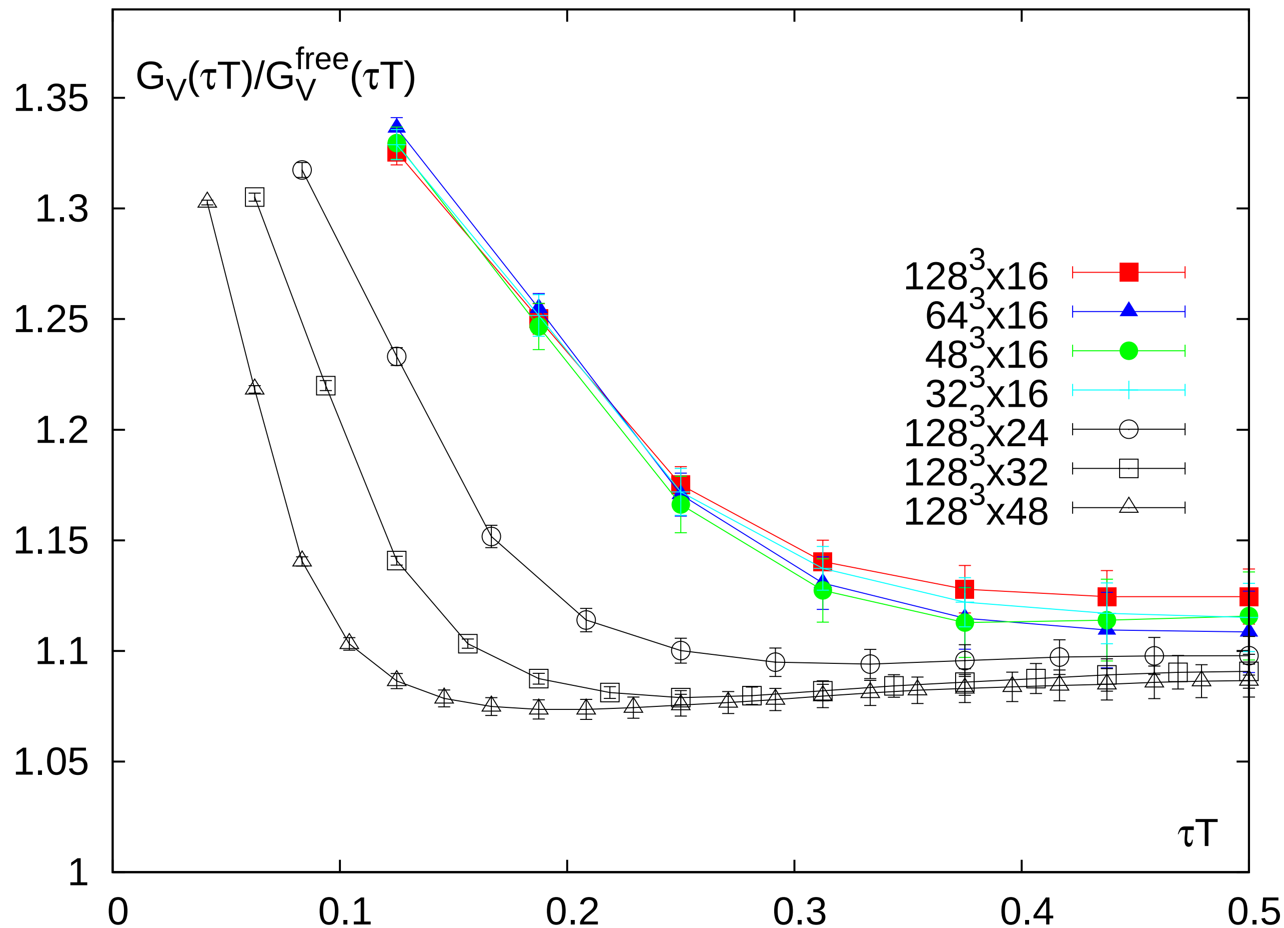
† separated by 500 updates

# Vector correlation function
# volume & cut-off dependence




- cut-off effects more severe than finite volume effects

# Vector correlation function
# volume & cut-off dependence

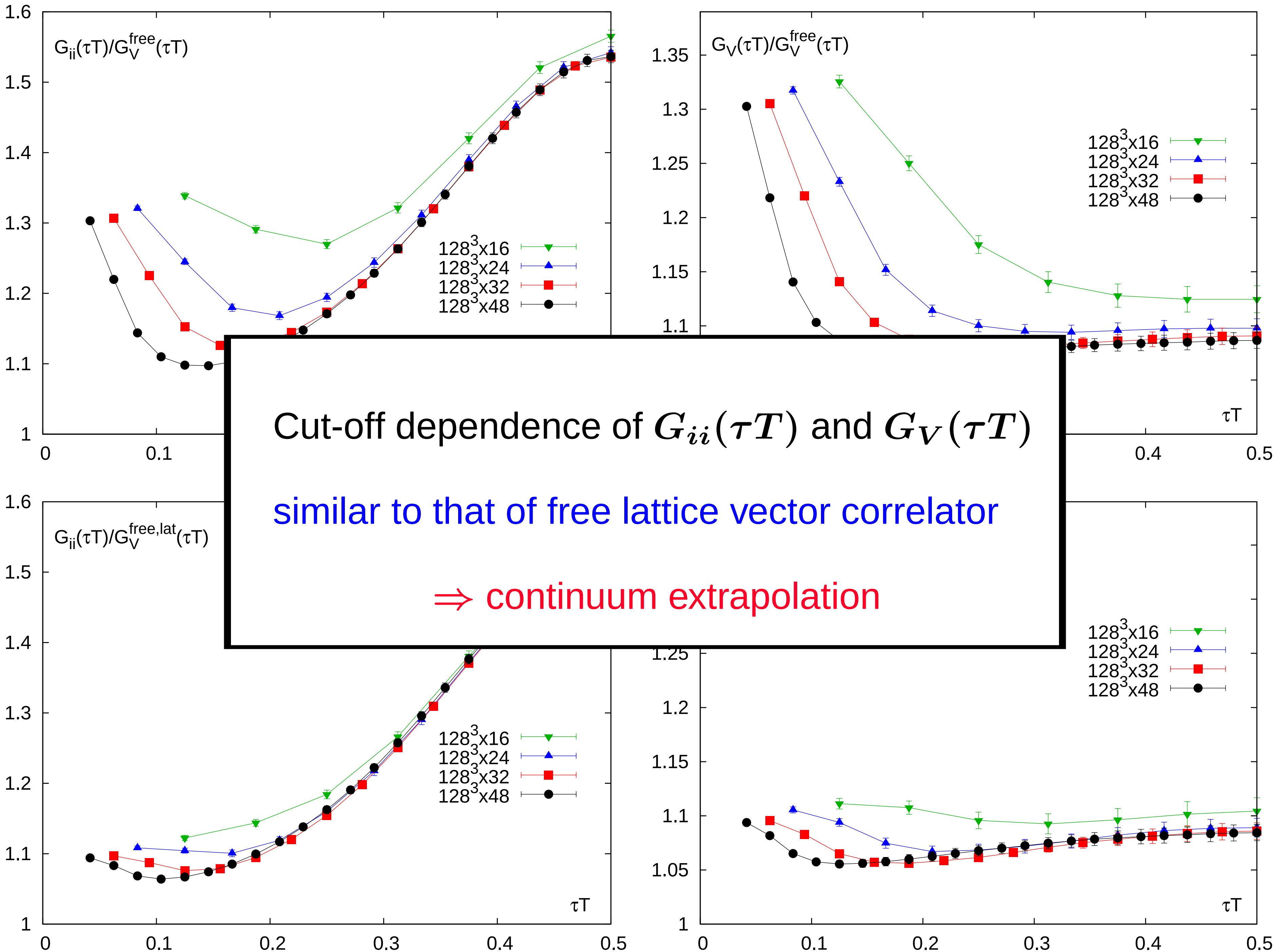



weak $\tau$-dependence:

almost like $G_V^{free}(\tau T)$

but...incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$ ?

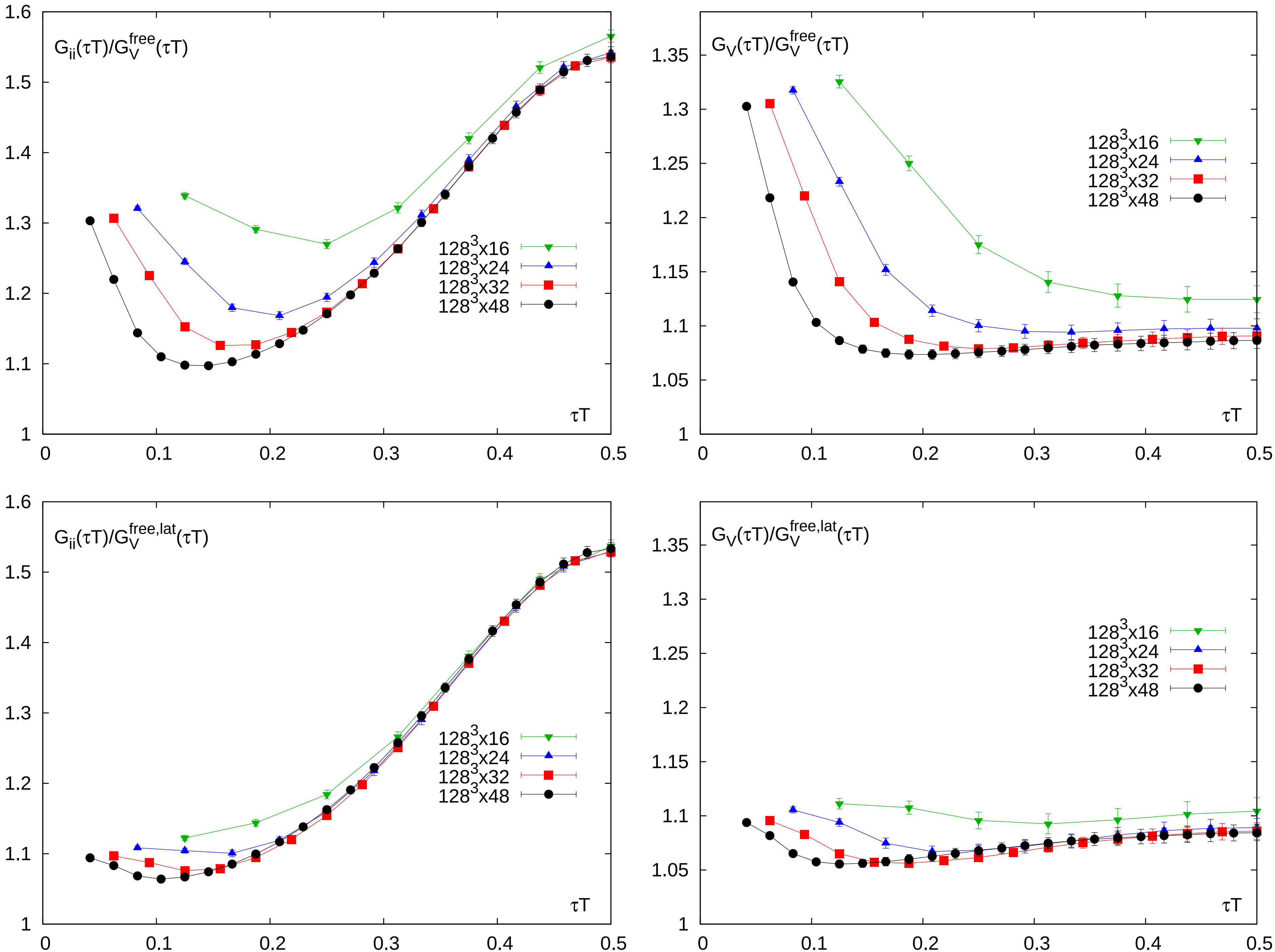
- cut-off effects more severe than finite volume effects




Cut-off dependence of $G_{ii}(\tau T)$ and $G_V(\tau T)$

similar to that of free lattice vector correlator

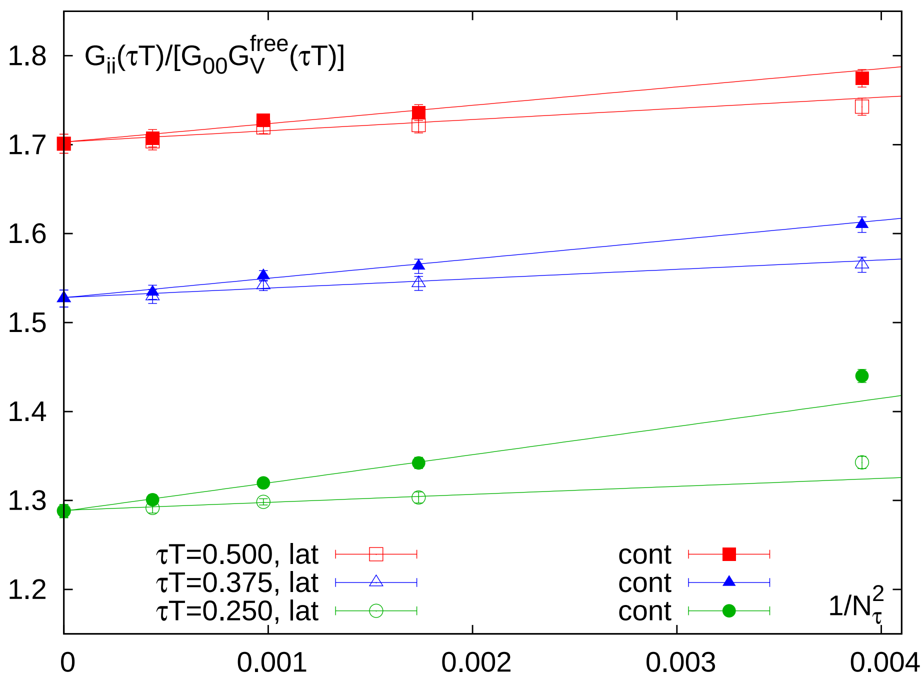
$\Rightarrow$ continuum extrapolation



$G_{ii}(\tau T)/G_V^{free}(\tau T)$
$G_V(\tau T)/G_V^{free}(\tau T)$
$G_{ii}(\tau T)/G_V^{free,lat}(\tau T)$
$G_V(\tau T)/G_V^{free,lat}(\tau T)$
$128^3$x16
$128^3$x24
$128^3$x32
$128^3$x48
τT

# Vector correlation function: Continuum extrapolation of $G_{ii}(\tau T)$

- extrapolation in $(aT)^2 = 1/N_\tau^2$: $\bar{G}_{00} \equiv \chi_q/T^2$




$$\frac{G_{ii}(\tau, T)}{\bar{G}_{00} G_V^{free}(\tau, T)}$$

$$1.701(11)\ ,\ \tau T = 0.5$$

$$1.527(9)\ ,\ \tau T = 0.375$$

$$1.288(7)\ ,\ \tau T = 0.25$$

- extrapolation at other values of $\tau T$ use spline interpolation on data at fixed cut-off
- extrapolation under control for $\tau T \gtrsim 0.2$

# Vector correlation function: Curvature at $\tau T = 1/2$

- thermal moments of the spectral function ⇔ Taylor expansion coefficients

$$G_V^{(n)} = \frac{1}{n!} \frac{\mathrm{d}G_V(\tau T)}{\mathrm{d}(\tau T)^n}\bigg|_{\tau T = 1/2} = \frac{1}{n!} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho_V(\omega)}{\sinh(\omega/2T)} ,$$

$$G_V(\tau T) = G_V^{(0)} \sum_{n=0}^{\infty} \frac{G_V^{(2n)}}{G_V^{(0)}} \left(\frac{1}{2} - \tau T\right)^{2n}$$

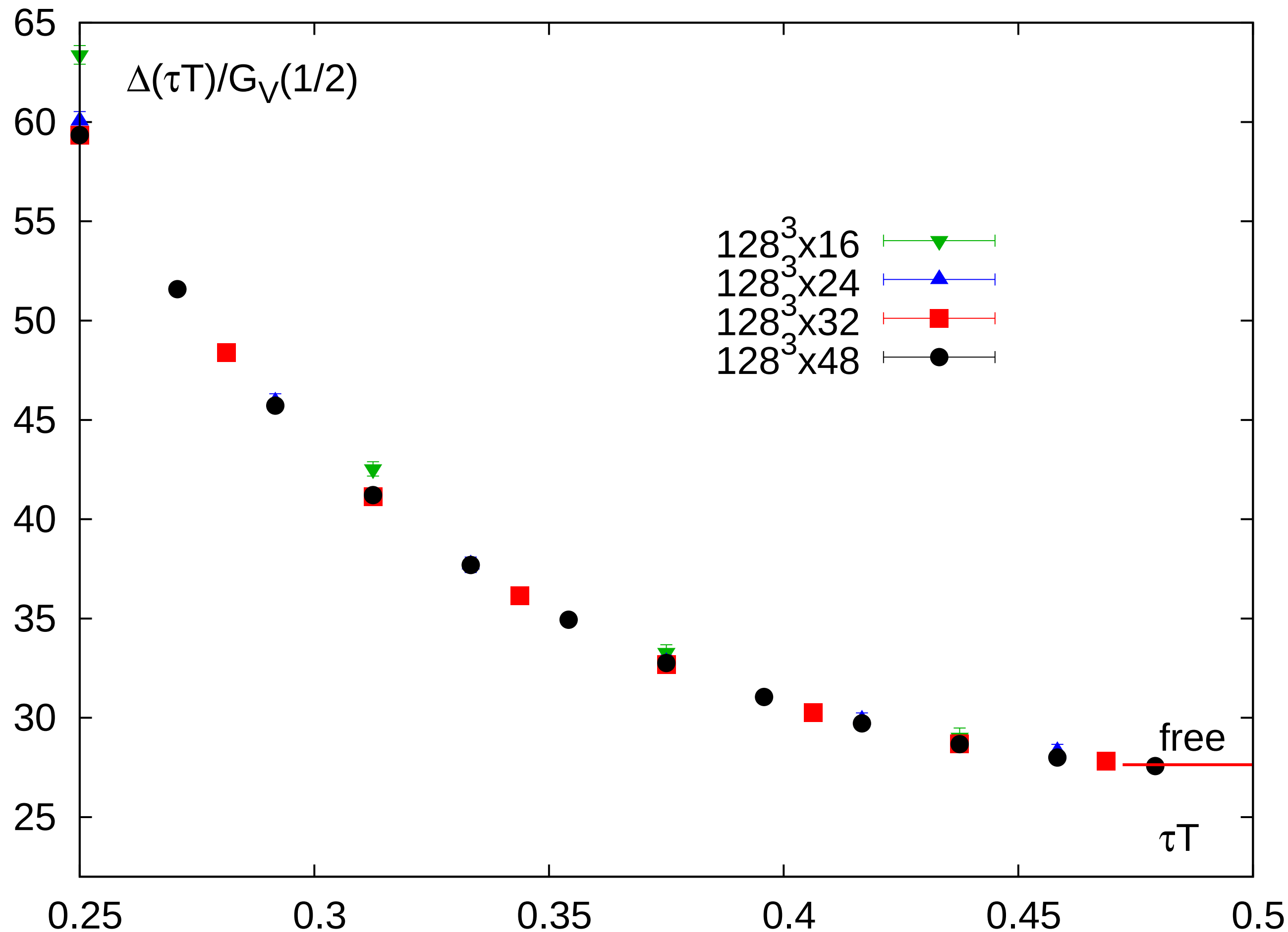
- finite difference approximants

$$\Delta(\tau T) = \frac{G_V(\tau T) - G_V(1/2)}{(1/2 - \tau T)^2}$$

$$\frac{G_V^{(2)}}{G_V^{(0)}} = \lim_{\tau T \to 1/2} \frac{\Delta(\tau T)}{G_V(1/2)}$$

(correspondingly for $G_{ii}(\tau T)$)

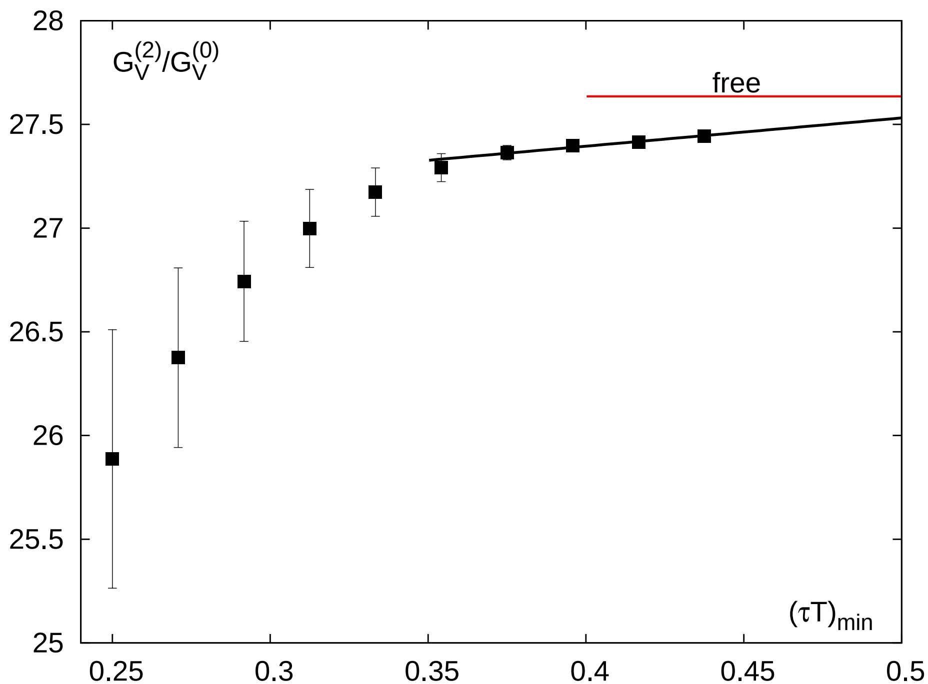
# Vector correlation function: Curvature at $\tau T = 1/2$




- normalized curvature close to that of the free vector correlator

# Vector correlation function: Fits to curvature approximants at $\tau T = 1/2$

- quadratic fits to $\frac{\Delta(\tau T)}{G_V(1/2)}$ with varying lower fit range $[(\tau T)_{min}, 1/2]$



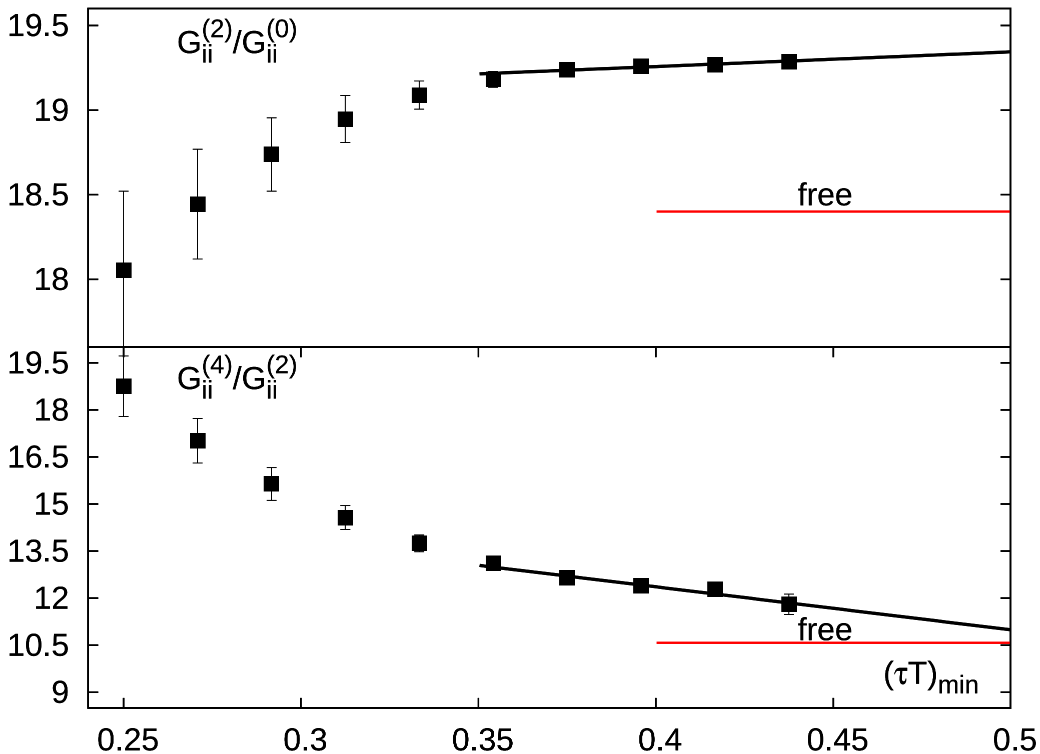

$$\frac{G_V^{(2)}}{G_V^{(0)}} = 27.53 \pm 0.08$$

$$\frac{G_V^{(4)}}{G_V^{(2)}} = 10.9 \pm 1.4$$

$$\frac{G_V^{(2),free}}{G_V^{(0),free}} = \frac{14\pi^2}{5} \simeq 27.635 \quad ; \quad \frac{G_V^{(4),free}}{G_V^{(2),free}} = \frac{155\pi^2}{147} \simeq 10.407$$

# $G_{ii}(\tau T)$ correlation function: Fits to curvature approximants at $\tau T = 1/2$

- quadratic fits to $\dfrac{\Delta(\tau T)}{G_{ii}(1/2)}$ with varying lower fit range $[(\tau T)_{min}, 1/2]$




$$\frac{G_{ii}^{(2)}}{G_{ii}^{(0)}} = 19.34 \pm 0.13$$

$$\frac{G_{ii}^{(4)}}{G_{ii}^{(2)}} = 10.9 \pm 1.4$$

$$\frac{G_{ii}^{(2),free}}{G_{ii}^{(0),free}} = \frac{28\pi^2}{15} \simeq 18.423 \quad ; \quad \frac{G_{ii}^{(4),free}}{G_{ii}^{(2),free}} = \frac{155\pi^2}{147} \simeq 10.407$$

# Analysis of continuum correlator

- Ansatz for spectral function:

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 4\frac{\chi_q c_{BW}}{\Gamma}\frac{\omega}{(2\omega/\Gamma)^2+1} + \frac{3}{2\pi}(1+\kappa)\ \omega^2\ \tanh(\omega/4T)$$

$\Rightarrow$ correlation function:

$$R_{ii}(\tau T) = \frac{G_{ii}(\tau T)}{G_{ii}^{free}(\tau T)} = \chi_q c_{BW} F(\tau T, \Gamma) + (1+\kappa)$$

- differences:

$$R_{ii}(\tau T) - R_{ii}(1/2) = \chi_q c_{BW}\left(F(\tau T, \Gamma) - F(1/2, \Gamma)\right)$$

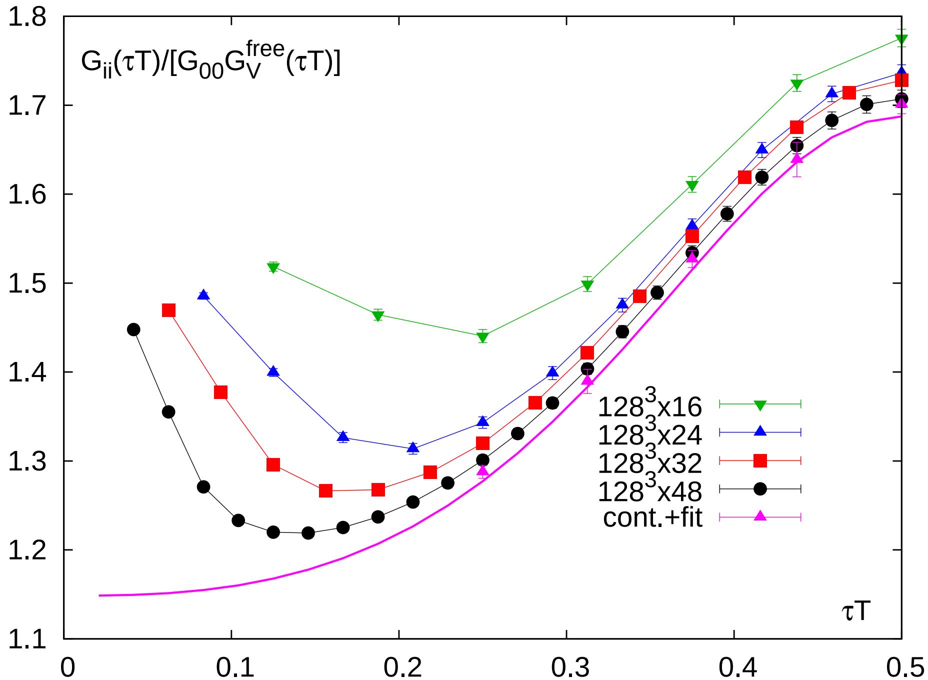
- ratios:

$$\frac{R_{ii}(\tau_1 T) - R_{ii}(1/2)}{R_{ii}(\tau_2 T) - R_{ii}(1/2)} = \frac{F(\tau_1 T, \Gamma) - F(1/2, \Gamma)}{F(\tau_2 T, \Gamma) - F(1/2, \Gamma)}$$

# Vector correlation function: Continuum extrapolation of $G_{ii}(\tau T)$

- extrapolation in $(aT)^2 = 1/N_\tau^2$:

$$\bar{G}_{00} \equiv \chi_q/T^2$$




fit to...

$$\frac{G_{ii}(\tau,T)}{\bar{G}_{00}G_V^{free}(\tau,T)} \quad \& \quad \frac{G^{(2n)}}{G^{(2m)}}$$

$$c_{BW}/\Gamma = 0.631(44)$$

$$\Gamma = 2.30(36)$$

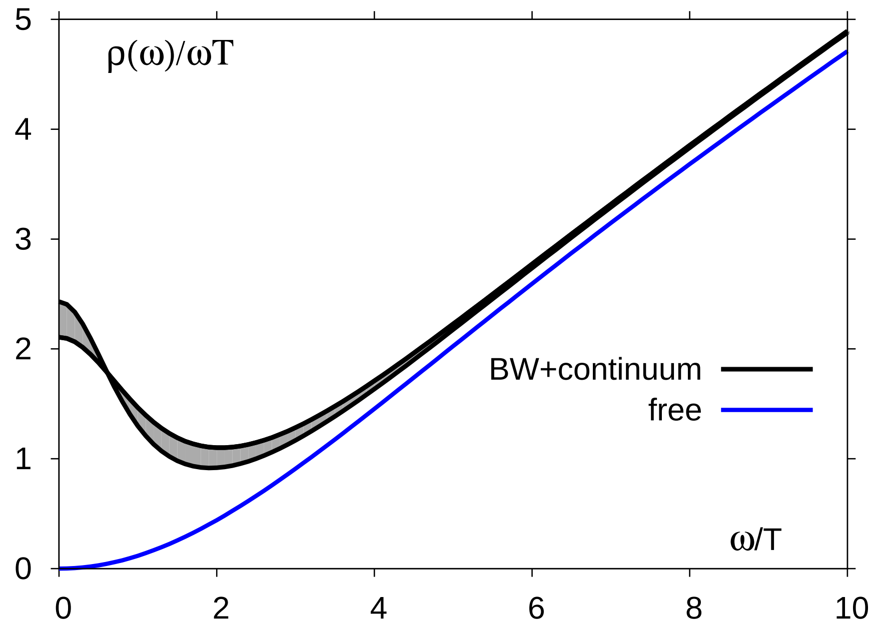
$$\kappa = 0.031(9)$$

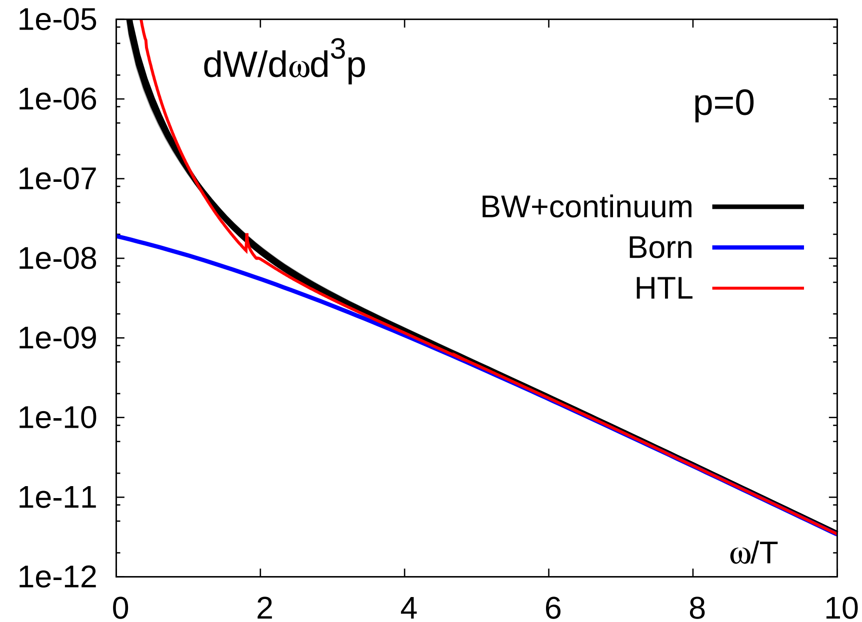
- extrapolation at other values of $\tau T$ use spline interpolation on data at fixed cut-off
- extrapolation under control for $\tau T \gtrsim 0.2$

# Vector spectral function:
# dilepton rate & electrical conductuvity

- fit to correlation function and curvature at the midpoint

hard thermal loop (HTL):
E. Braaten, R.D. Pisarski, NP B337 (1990) 569.



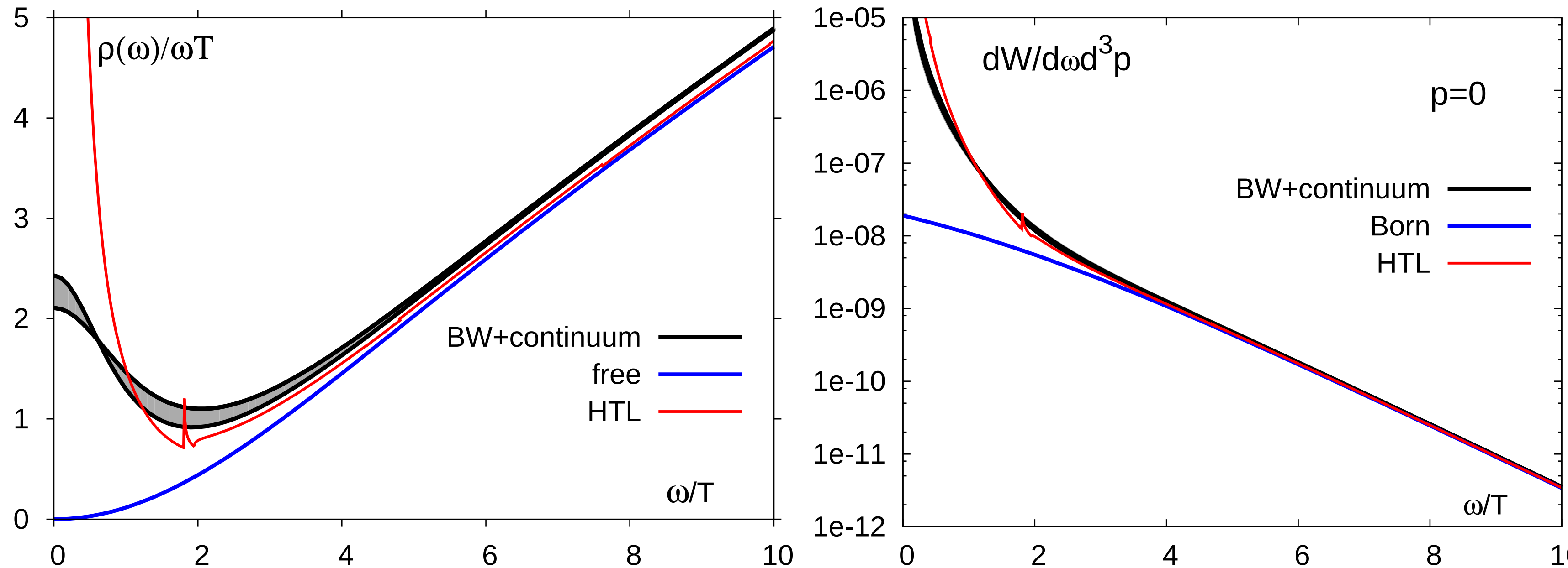

- shaded area corresponds to $\mathbf{0.587 \le c_{BW} T/\Gamma \le 0.675}$, $\mathbf{\Gamma/T = 2.30(36)}$

  $\Rightarrow$ electrical conductivity: $$\frac{\boldsymbol{\sigma}}{\boldsymbol{T}} = \mathbf{(0.38 \pm 0.03)} \boldsymbol{C_{em}}$$

# Vector spectral function:
# dilepton rate & electrical conductuvity

- fit to correlation function and curvature at the midpoint

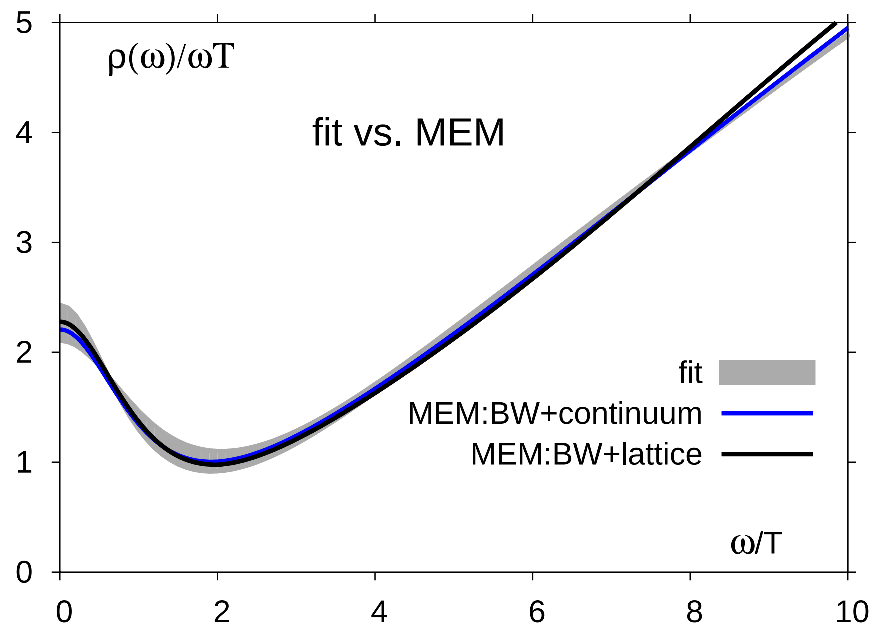
hard thermal loop (HTL):
E. Braaten, R.D. Pisarski, NP B337 (1990) 569.




- shaded area corresponds to $0.587 \leq c_{BW} T/\Gamma \leq 0.675$,
  $\Gamma/T = 2.30(36)$
  $\Rightarrow$ electrical conductivity: $$\frac{\sigma}{T} = (0.38 \pm 0.03) C_{em}$$

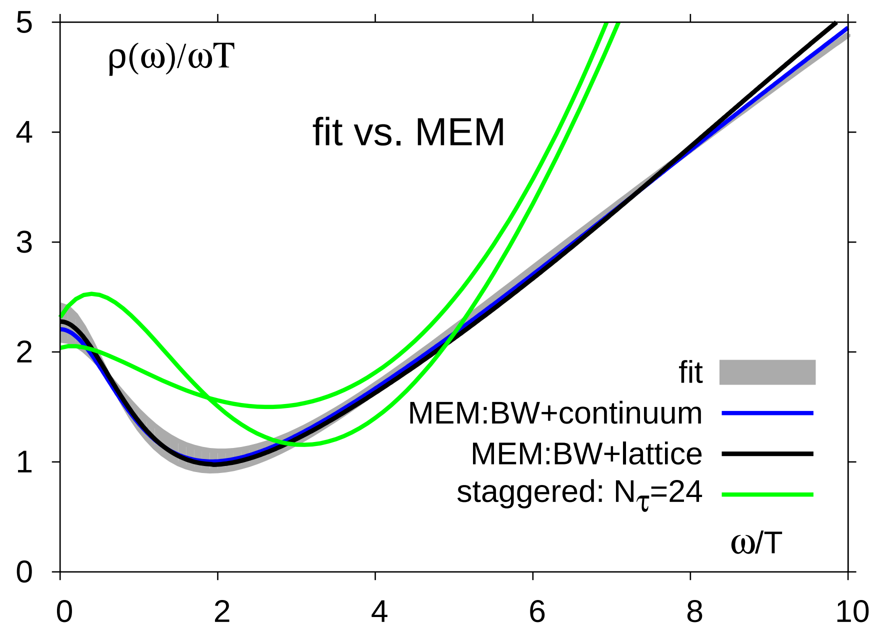
# Maximum Entropy Method

- did not make use of MEM so far; may, however, use result of our fit as default model in a MEM analysis
  $\Rightarrow$ consistency check: influence of poorly fitted short distance (large $\omega/T$) part on large distance (small $\omega/T$) analysis
- MEM analysis of $N_\tau = 48$ data set using (i) fitted BW and continuum parameter and (ii) replace continuum by lattice free spectral function



# Maximum Entropy Method

- did not make use of MEM so far; may, however, use result of our fit as default model in a MEM analysis
  $\Rightarrow$ consistency check: influence of poorly fitted short distance (large $\omega/T$) part on large distance (small $\omega/T$) analysis
- MEM analysis of $N_\tau = 48$ data set using (i) fitted BW and continuum parameter and (ii) replace continuum by lattice free spectral function




G. Aarts et al., PRL99 (2007) 022002

# Conclusions

- We calculated the vector correlation function at $T/T_c = 1.5$ in quenched QCD and performed a continuum extrapolation.
- $G_V(\tau T)$ is well reproduced using a Breit-Wigner plus continuum ansatz for the vector spectral function
- electrical conductivity: $\frac{\sigma}{T} = (0.38 \pm 0.03) C_{em}$ (preliminary)
- dilepton rate: approaches leading order Born rate for $\omega/T \gtrsim 4$