Lattice Study of Trapped Fermions at Unitarity

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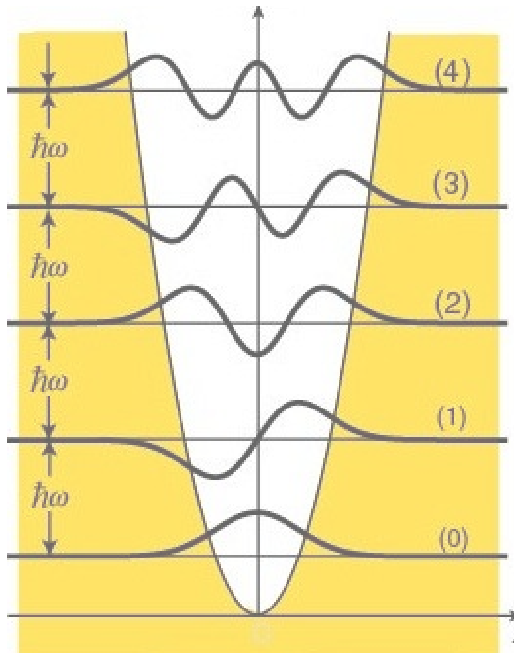
Our Goal

- High-precision studies of large, strongly interacting systems of fermions
  - → Unitary fermions - ideal laboratory
    - Universal physics
    - Relevant for all systems with $r_0 \ll n^{-1/3} \ll a$
      - Nuclear matter
      - Cold atom experiments
  - → Harmonic trap
- Up to N=20 fermions, <1% errors

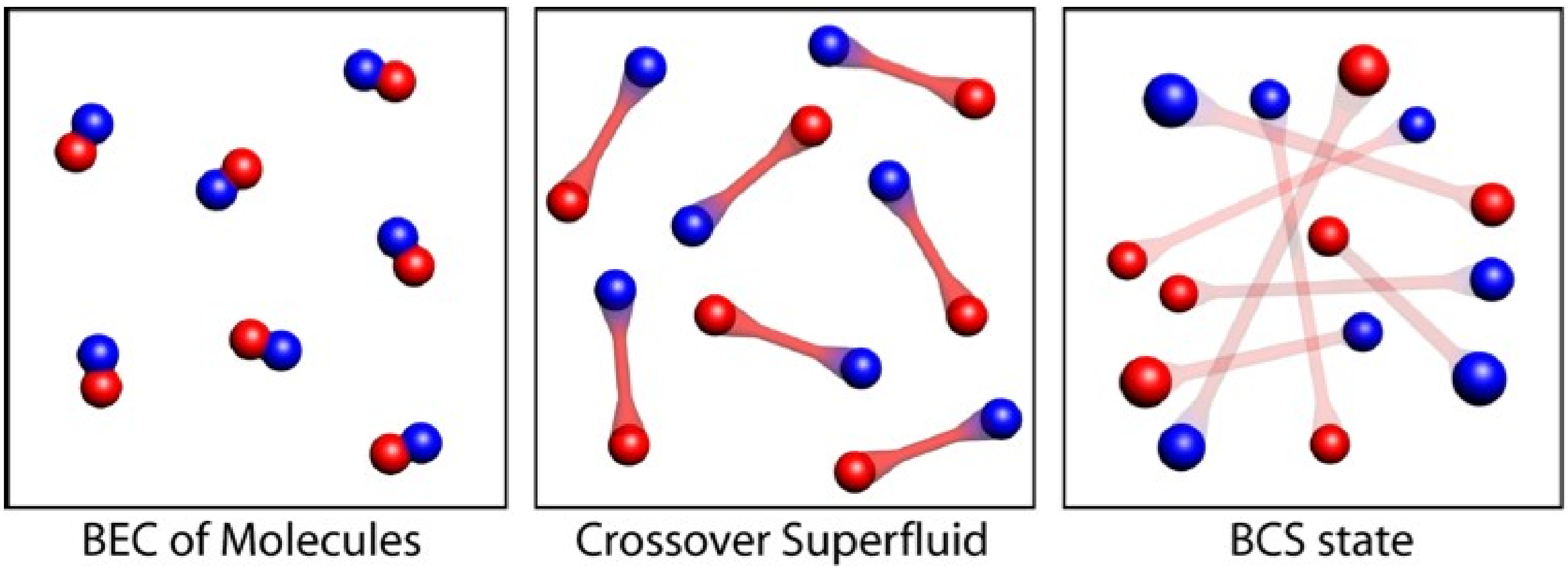
Our Goal

- High-precision studies of large, strongly interacting systems of fermions
  - → Unitary fermions - ideal laboratory
  - → Harmonic trap
    - Control systematics
    - Small S/N problem
    - Trap confinement enhances overlap with ground state
- Up to N=20 fermions, <1% errors



Unitary Fermions

- Vary strength of interaction between fermions → crossover from BEC state to BCS state
- Crossover point called unitarity



1/a>0 1/a=0 1/a<0

Figure: W. Ketterle and M. Zwierlein, arXiv:0801.2500

Unitary Fermions

→ p cot δ = 0 ➡ no intrinsic scale except the density

- → Displays universal features
- → Non-relativistic, conformal system
- → Strongly interacting – need non-perturbative techniques

• Other microscopic methods: Schrodinger Eq., GFMC, FN-DMC

• New lattice method – match Schrodinger Eq. to within 1%

Universal Quantities

Bertsch parameter

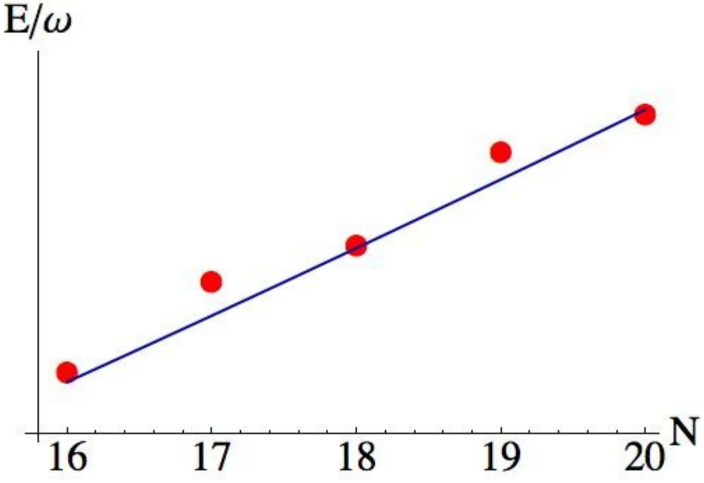
- $\mu_{int}(n) = \xi \mu_{free}(n)$
- Unitary fermions in an SHO:

$$E_{int}(N,\omega) = \sqrt{\xi} E_{free}(N,\omega)$$

Pairing gap

- Energy cost to break fermion pairs
- Even-odd splitting:

$$\Delta(N) = E(N) - \frac{1}{2}\left(E(N-1) - E(N+1)\right)$$



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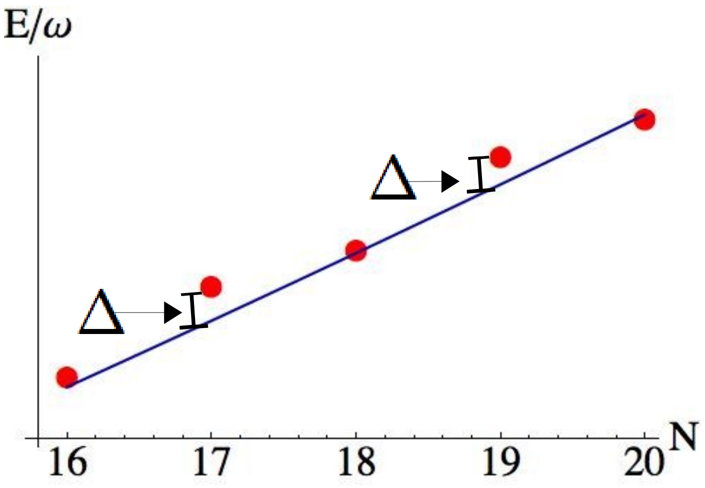


Lattice Theory

- Non-relativistic fermions interacting via point interactions
- No fermion determinant → quenched = exact
- Highly improved interactions give p cot δ = 0 to arbitrary order in p^2
- Harmonic trap easily implemented:

$$\mathcal{T} = \mathcal{D}^{-1/2}(1-\mathcal{V})\mathcal{D}^{-1/2}$$

$$\mathcal{V} = V_{int} + V_{SHO}$$

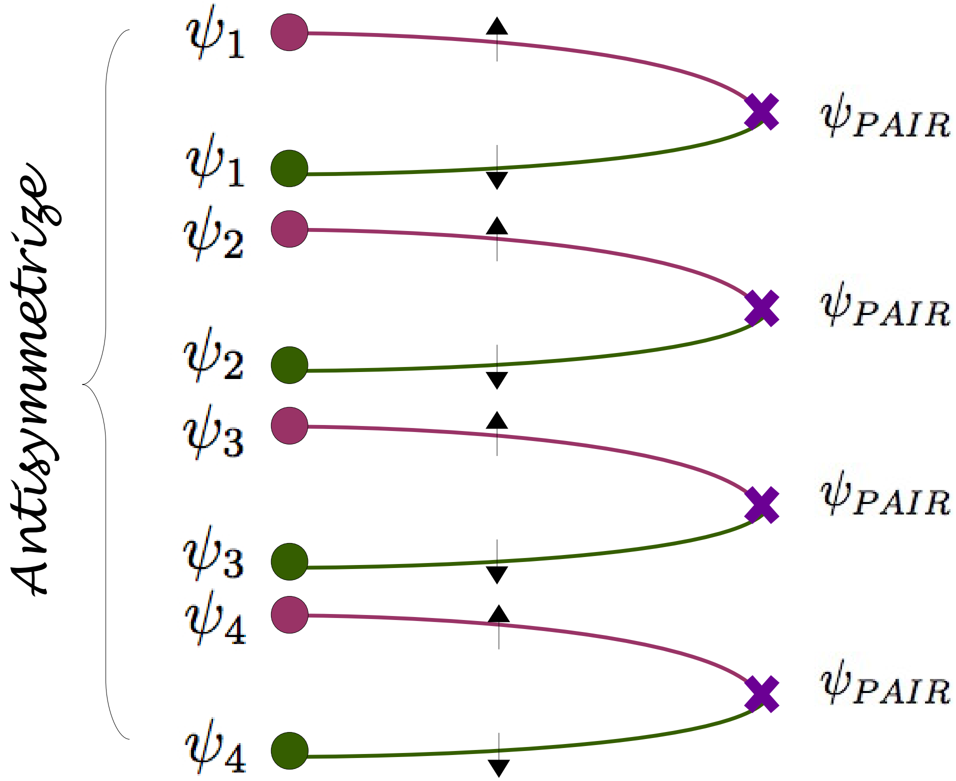
$$V_{SHO} = 1 - e^{-\frac{1}{2}m\omega^2\sum_{i=1}^{3}(L_i/2-x_i)^2}$$

* Details introduced in talk by M. Endres

N-body Correlators

- Slater determinant of single-particle SHO states

N-body Correlators

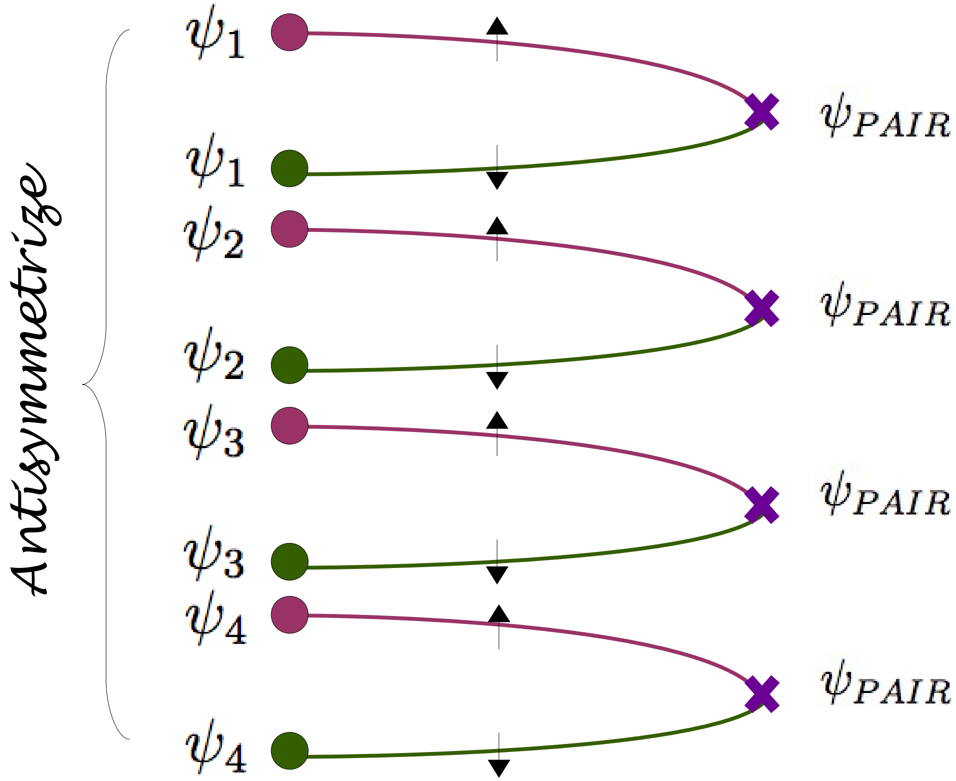


- Slater determinant of single-particle SHO states
- Include pair correlations*

$$\psi_{PAIR} \propto \frac{e^{-(x^2+y^2)/(2L_0^2)}}{|x-y|}$$

*J. Carlson, et al, Phys. Rev. Lett. **91** (2003)

N-body Correlators

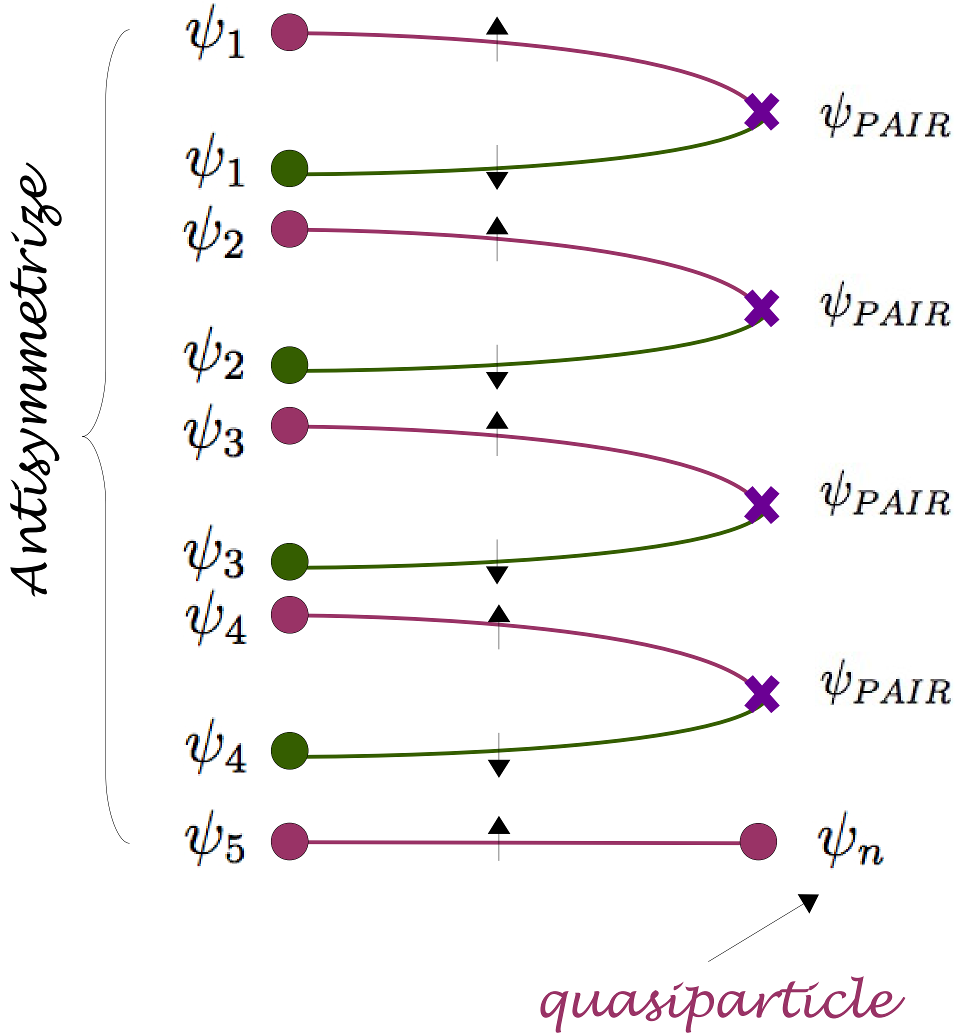


- Slater determinant of single-particle SHO states
- Include pair correlations

$$\psi_{PAIR} \propto \frac{e^{-(x^2+y^2)/(2L_0^2)}}{|x-y|}$$

$$C_N(\tau) = \begin{vmatrix} \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_{N/2}\rangle \\ \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_{N/2}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_{N/2}\rangle \end{vmatrix}$$

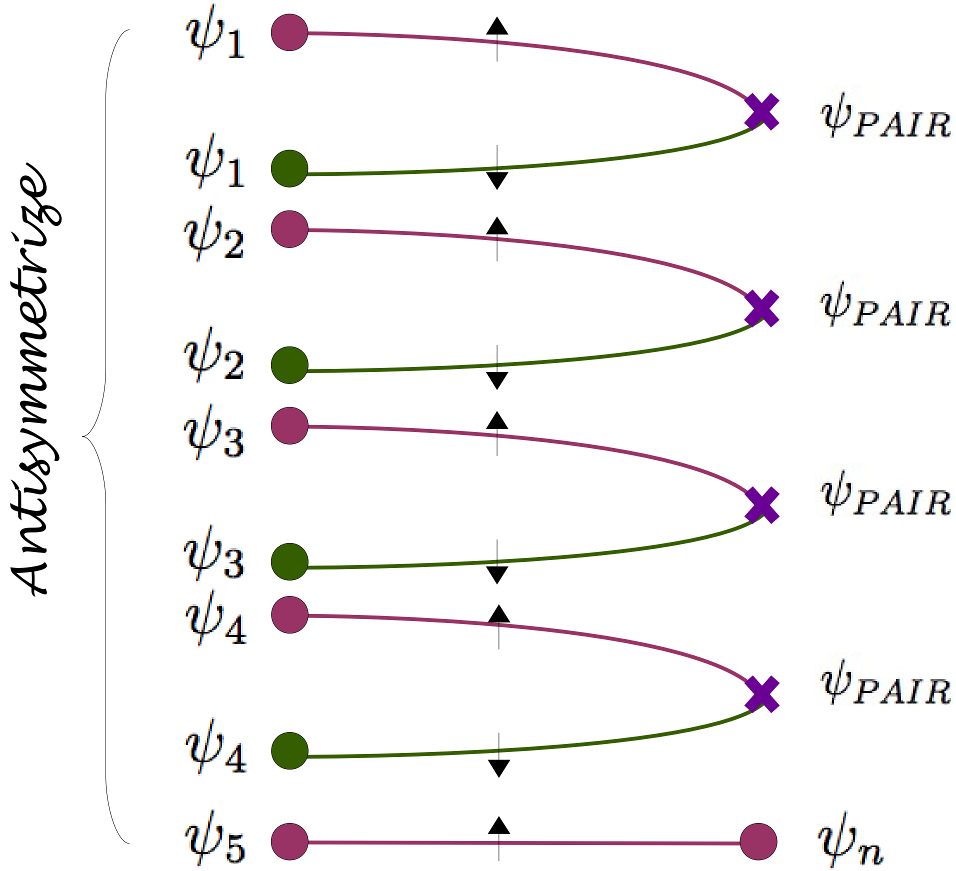
N-body Correlators



- Slater determinant of single-particle SHO states
- Include pair correlations
- For odd N, add single particle state at sink

→ Replace nth row in slater matrix

N-body Correlators



- Slater determinant of single-particle SHO states
- Include pair correlations
- For odd N, add single particle state at sink
- For SHO, either is sufficient

Systematic Errors

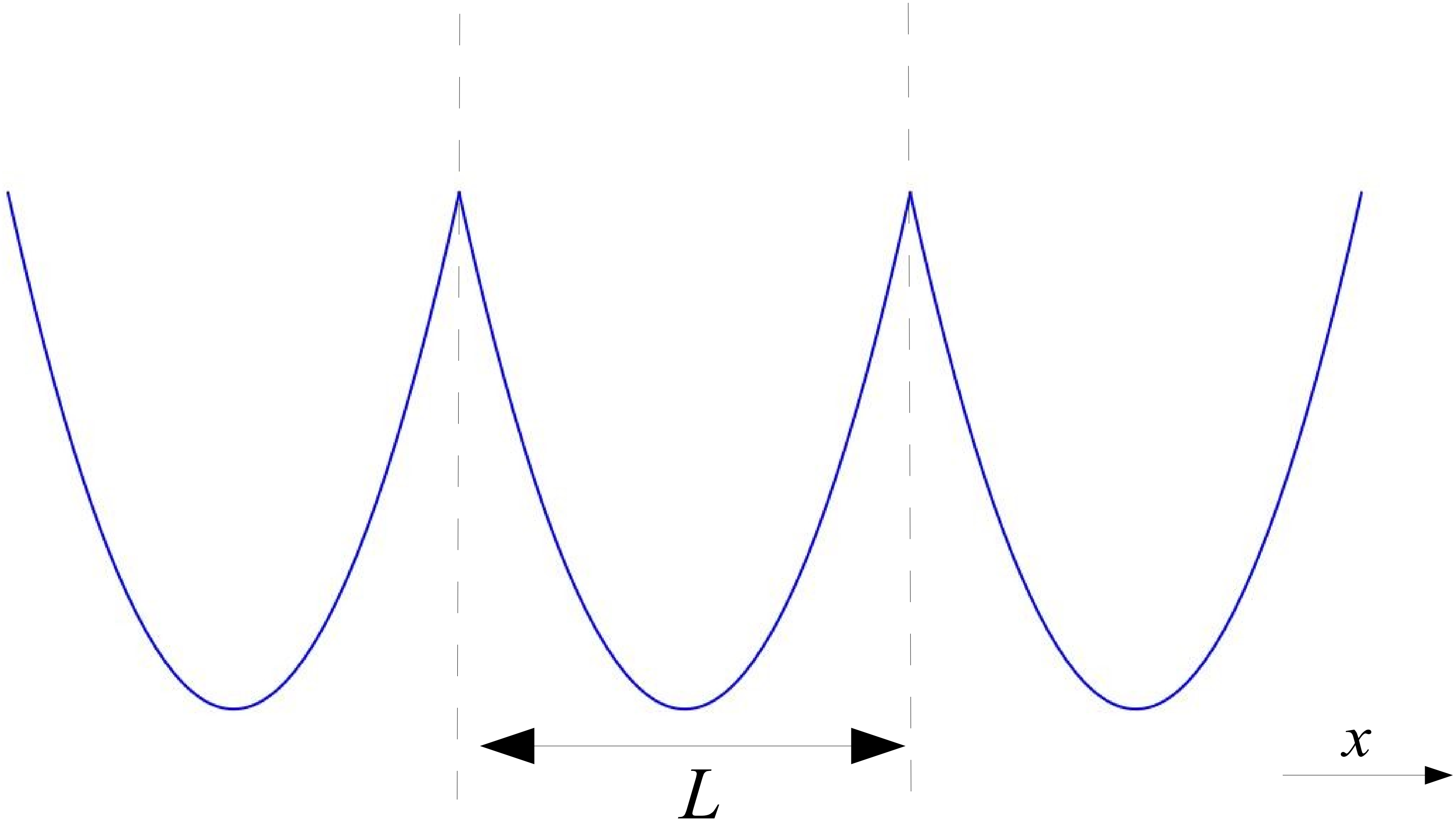
- Tunable scales set finite volume, finite lattice spacing (b_t, b_s) effects
  - $\omega b_t \longrightarrow$ temporal discretization error
  - $b_s/L_0 \longrightarrow$ spatial discretization error
  - $L_0/L \longrightarrow$ finite volume error

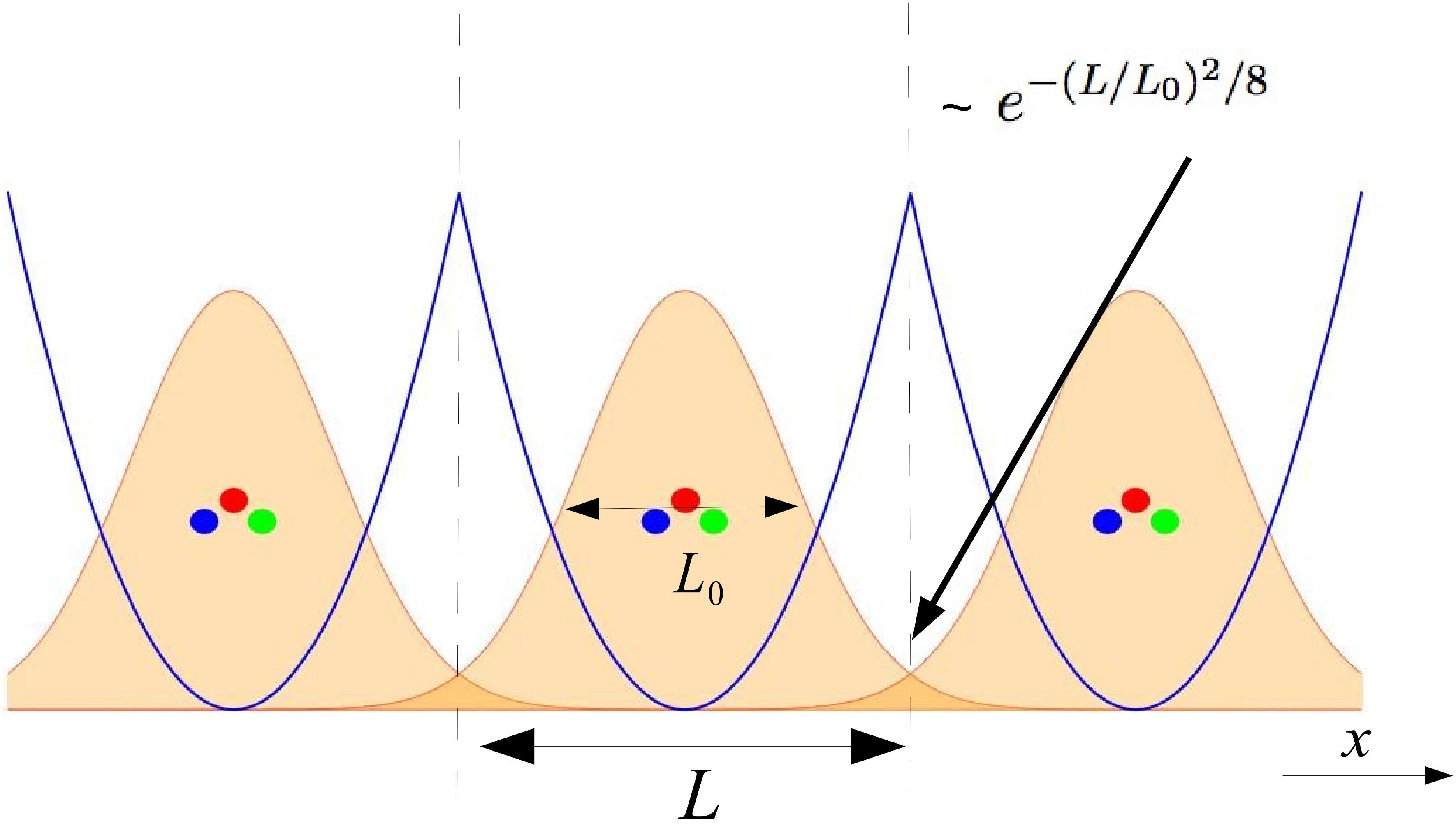
$$L_0 = (m\omega)^{-1/2}$$

- Temporal discretization error
  - Exponential form of SHO offers some improvement
  - Ensure small b_t errors by choosing small ω

Spatial Errors

Position space potential: PeriodicBC

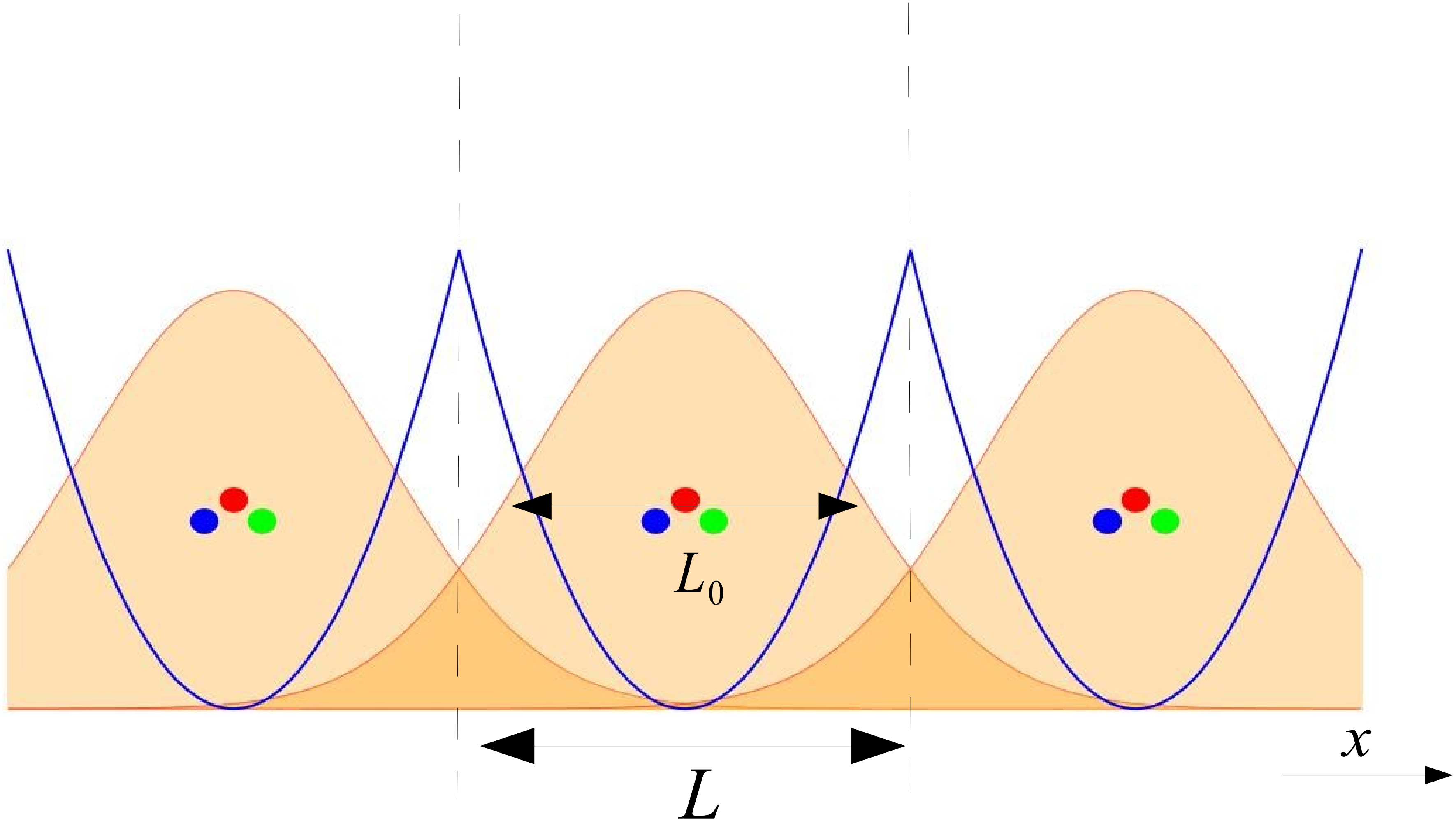




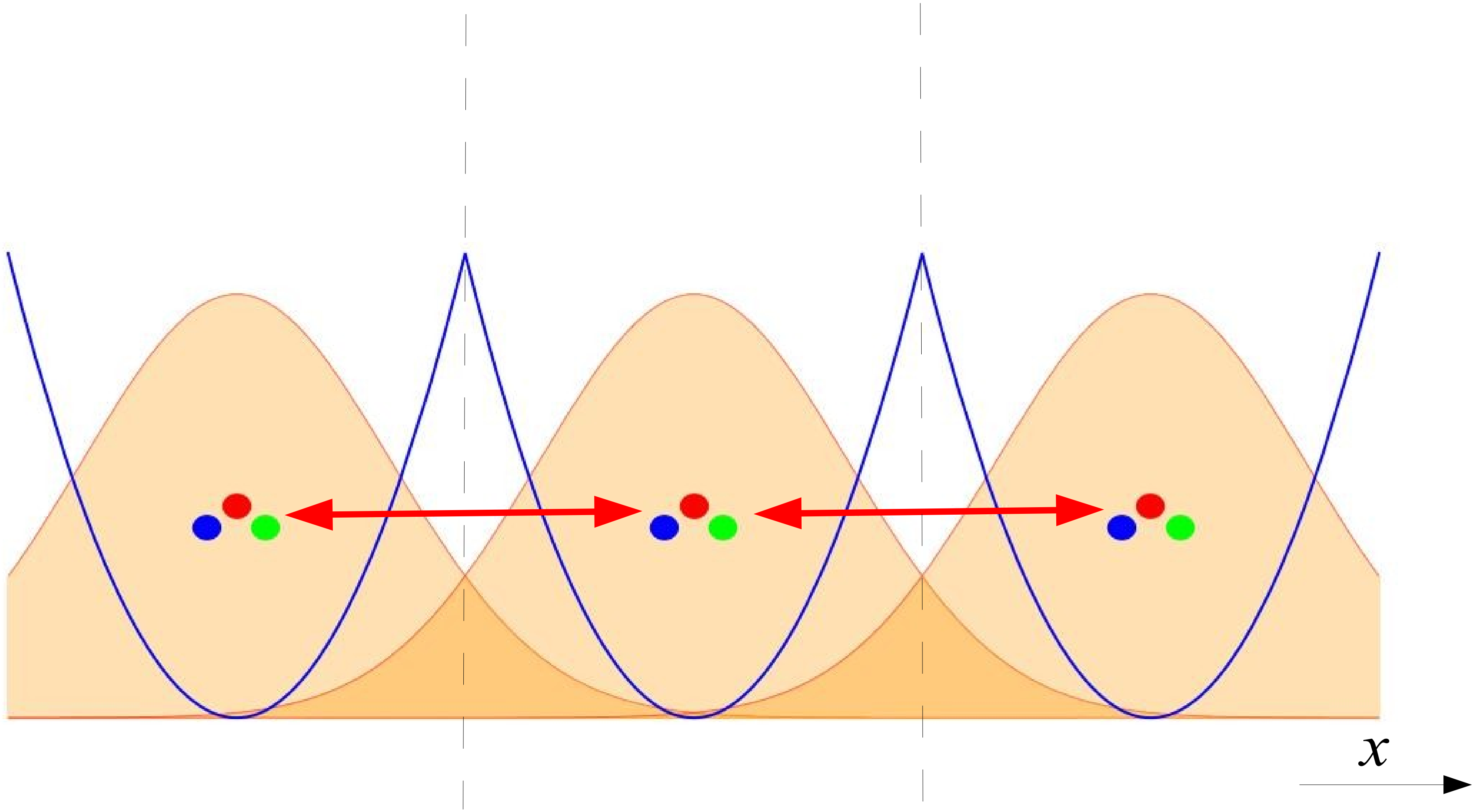
$\sim e^{-(L/L_0)^2/8}$
L_0
L
x

(ω fixed)

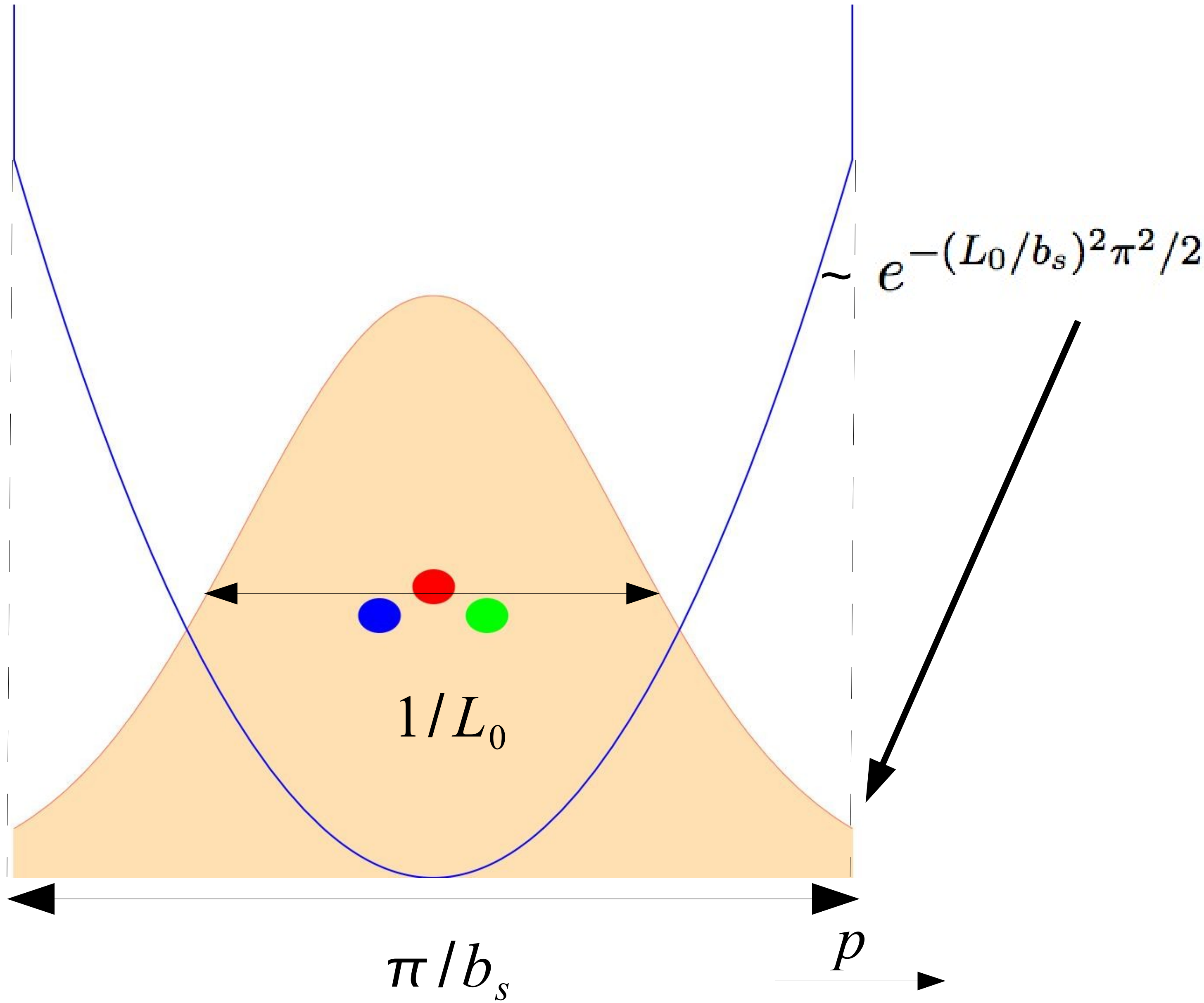
Increase L_0, keep L fixed



Interactions with image charges lower energy

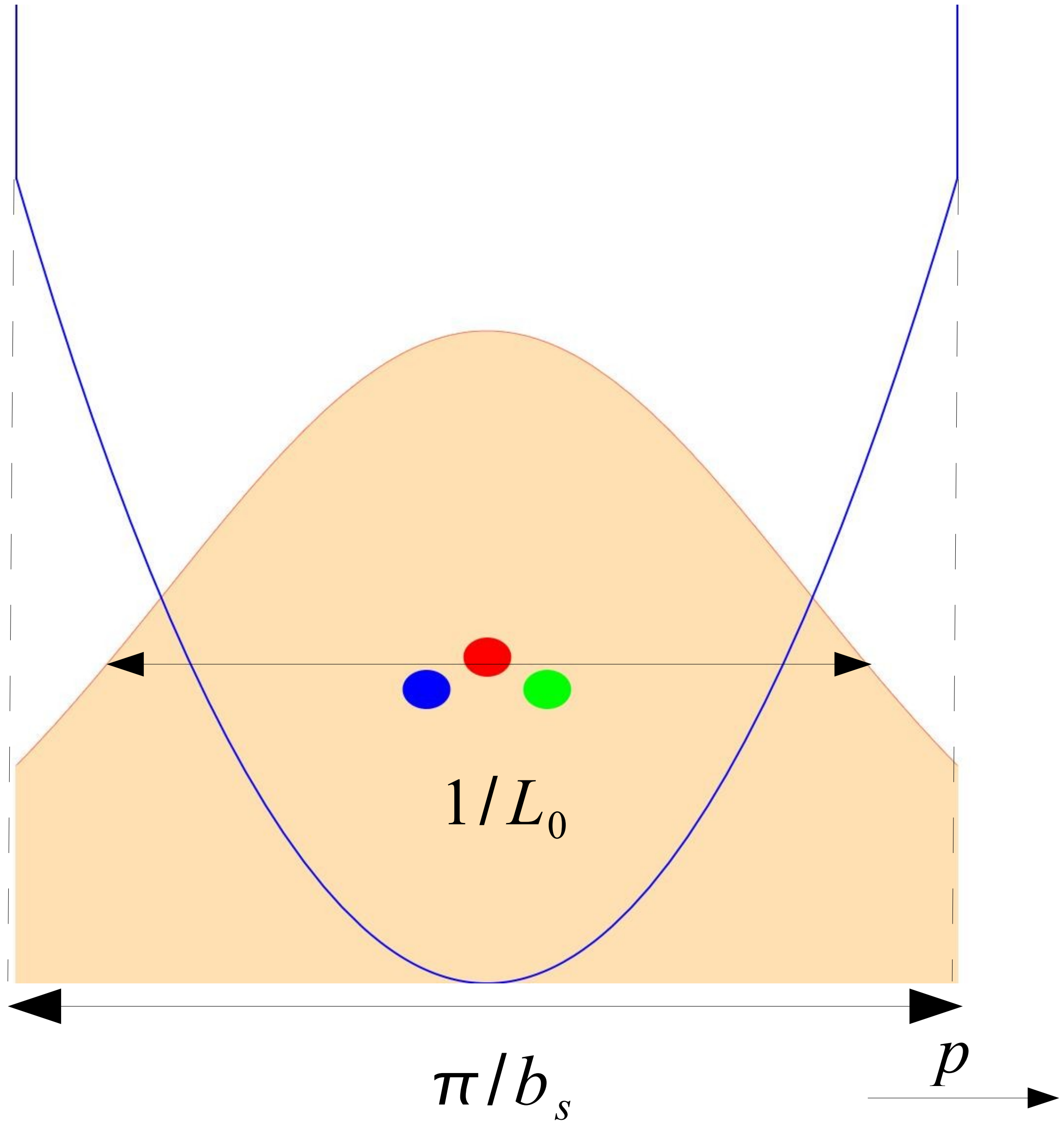


Momentum Space: Hard Cutoff

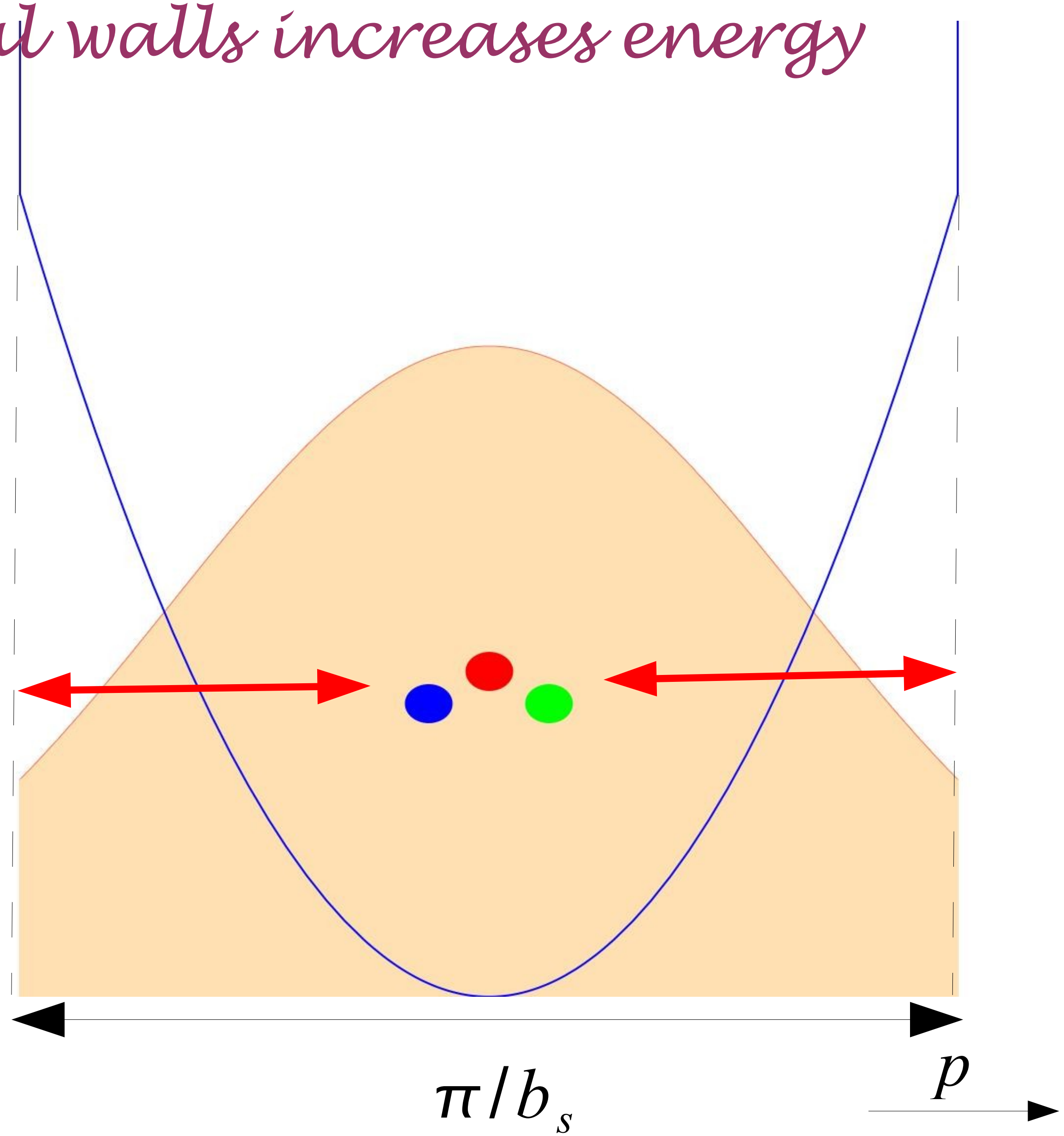


Reduce L_0

(ω fixed)



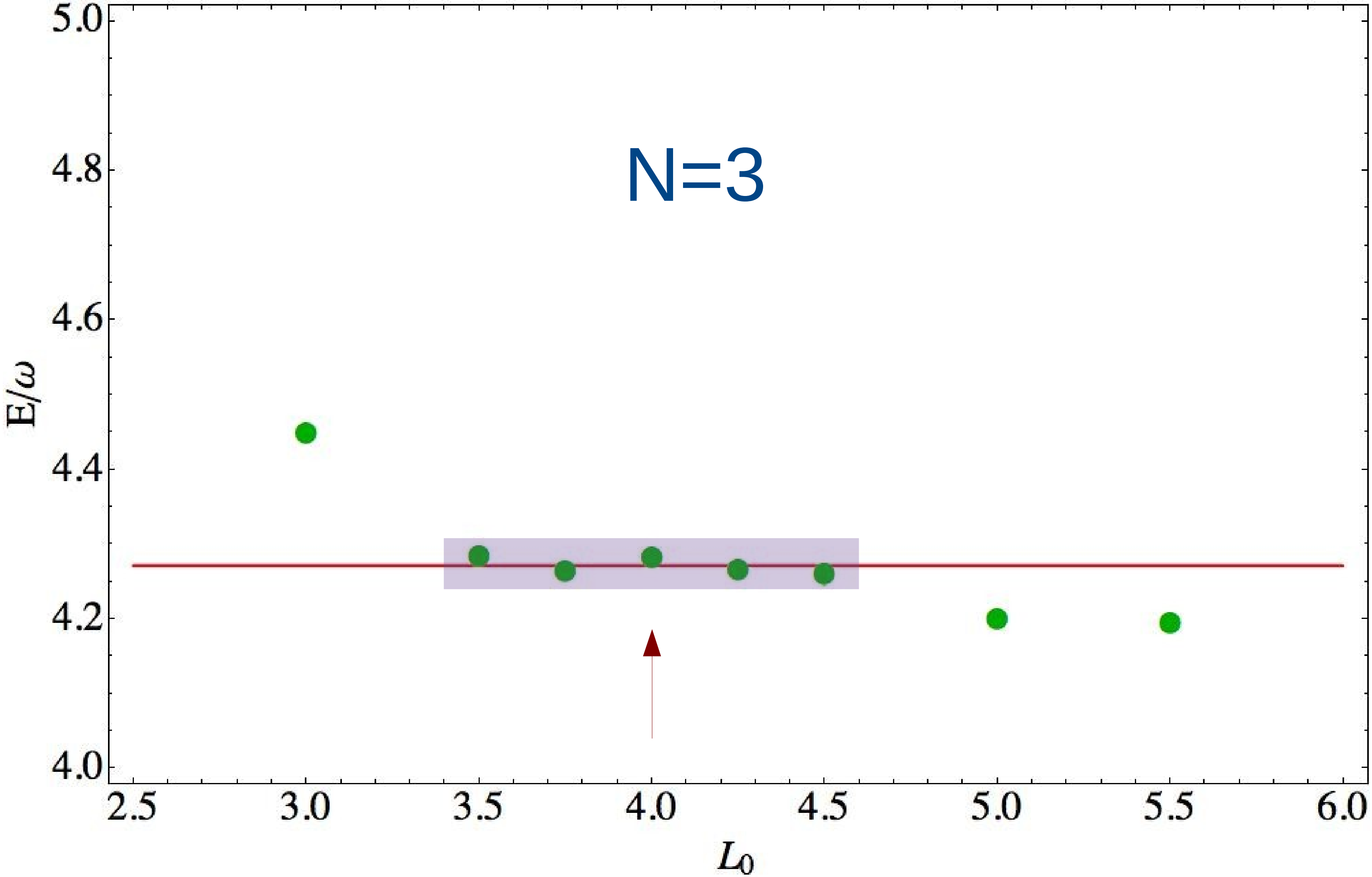
Reduce L_0 - more sensitivity to infinite potential walls increases energy



Spatial Errors

- Both finite volume and spatial discretization errors affected by changing L_0
  - → Finite volume errors push energy down for large L_0
  - → Discretization errors push energy up for small L_0
- Performed tests at various values of L_0 to choose ideal value

L = 32
ω = 0.013



F. Werner and Y. Castin, Phys. Rev. Lett. **97**. 150401 (2006)

L = 32
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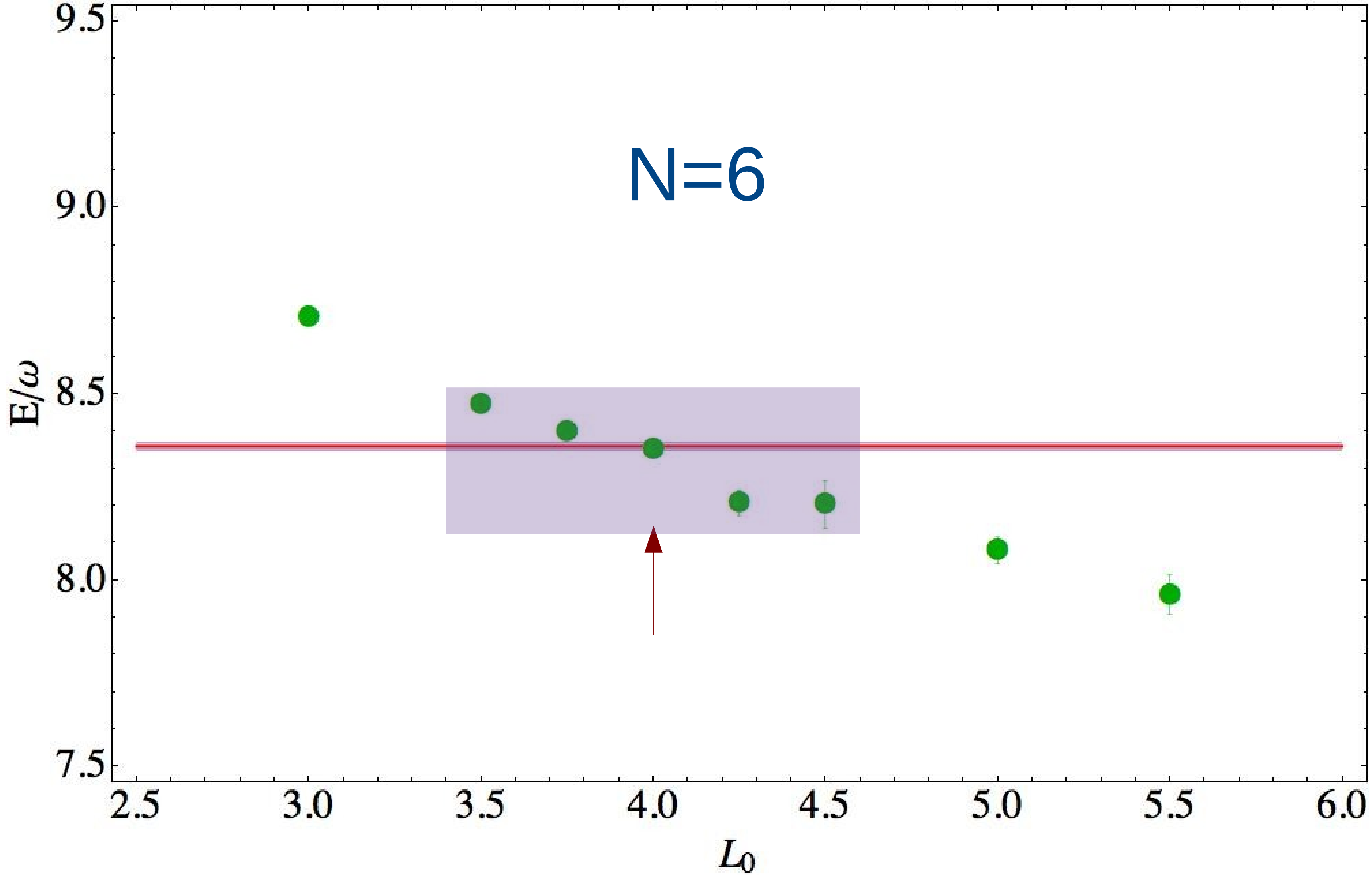
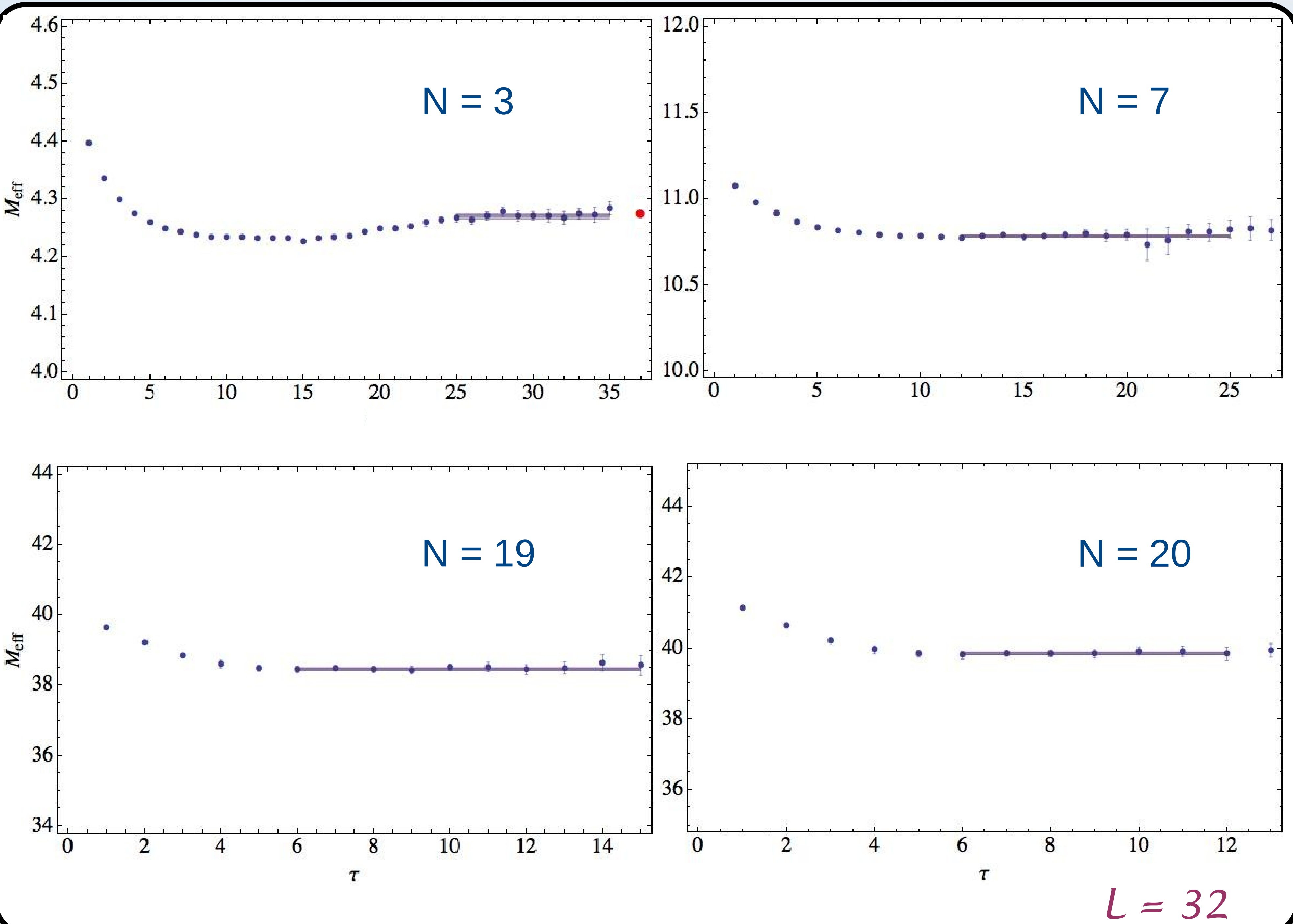
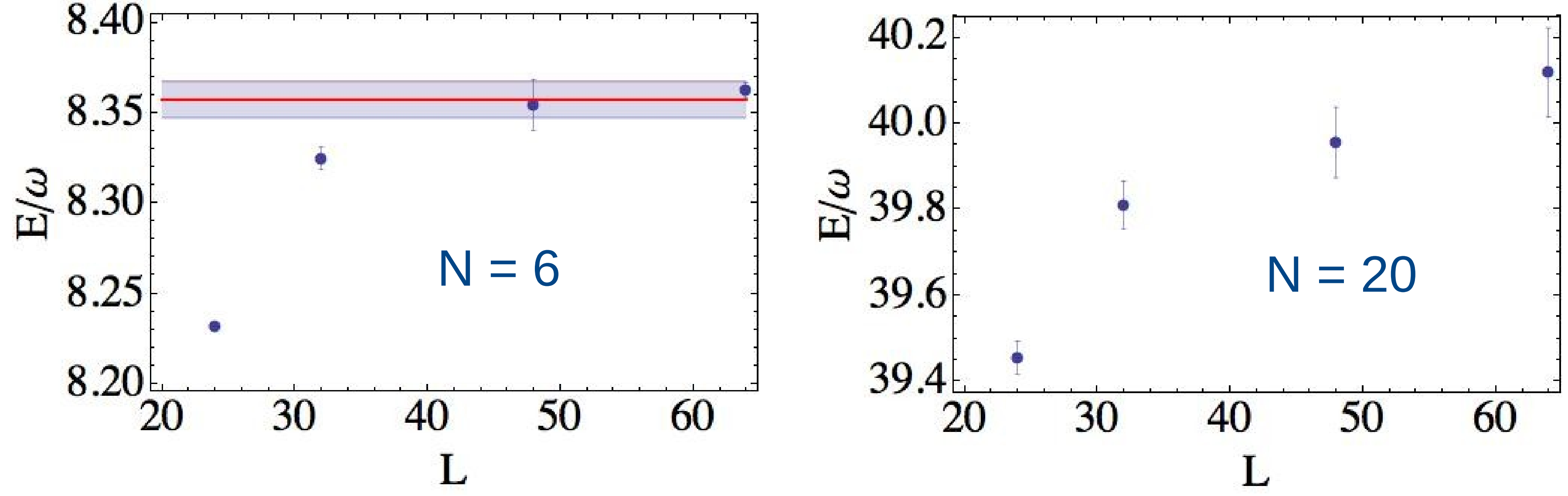


D. Blume, private communication

Results


N = 3
N = 7
N = 19
N = 20
M_{eff}
τ
L = 32



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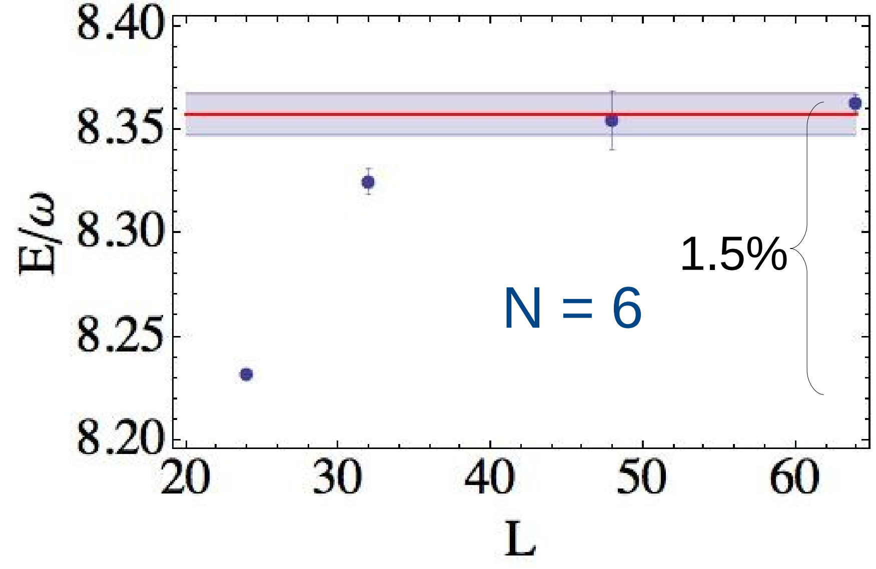

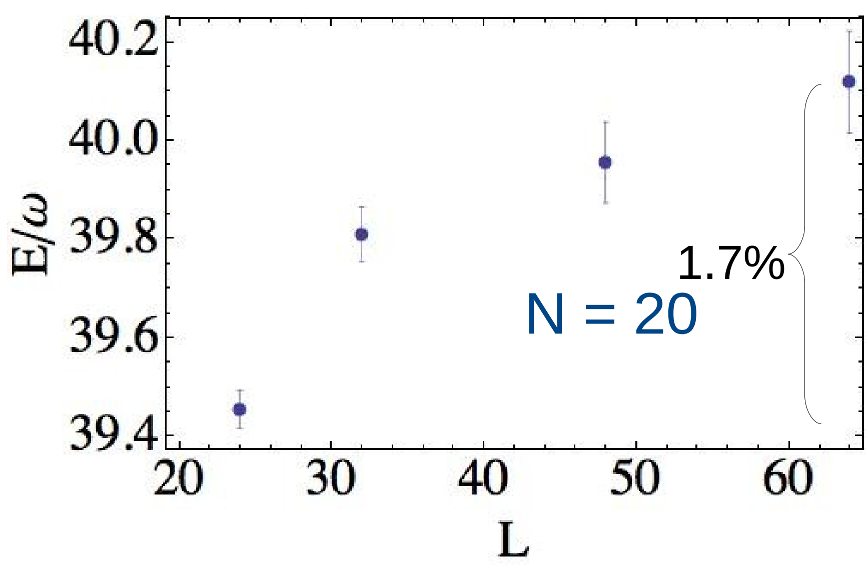

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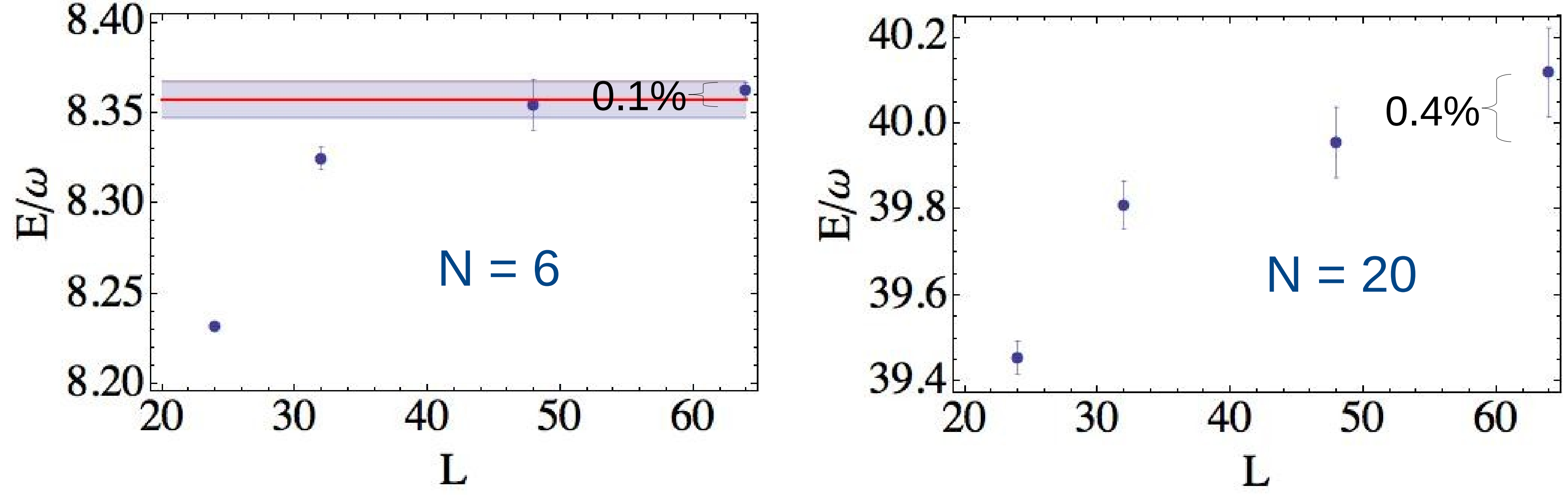

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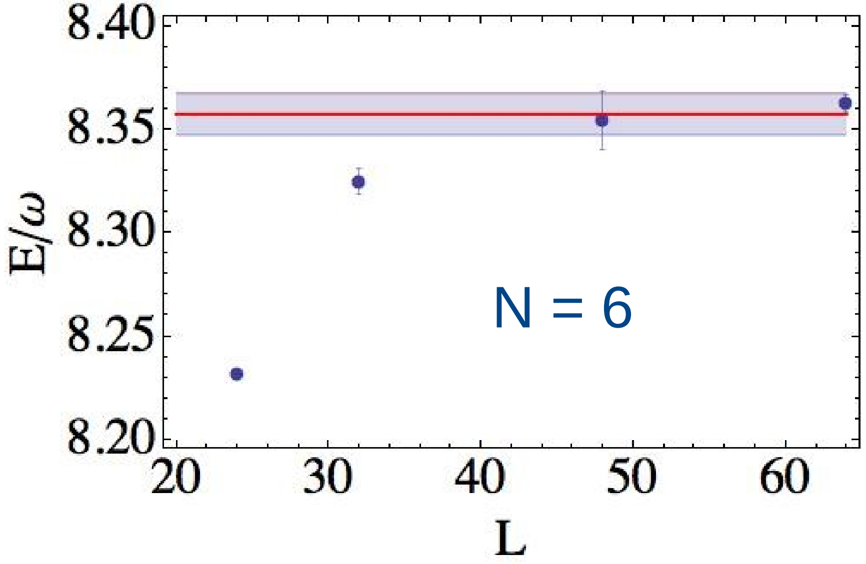
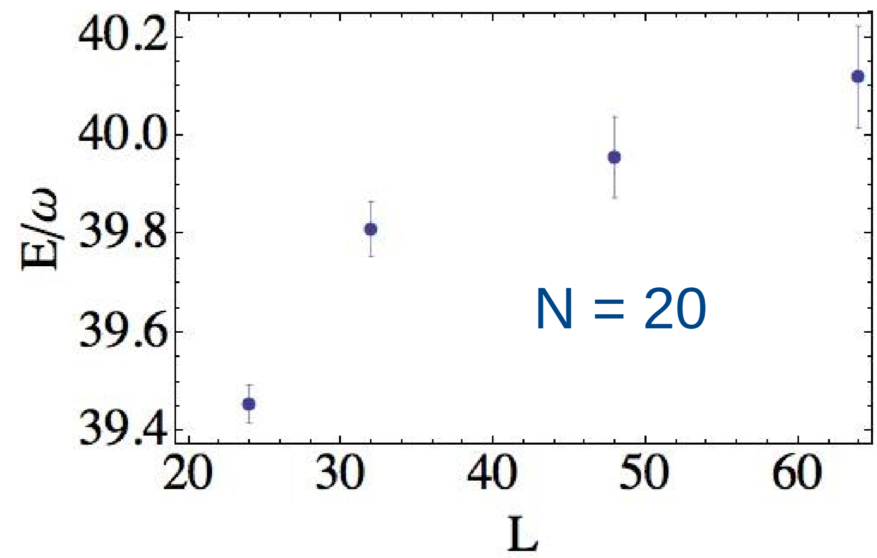
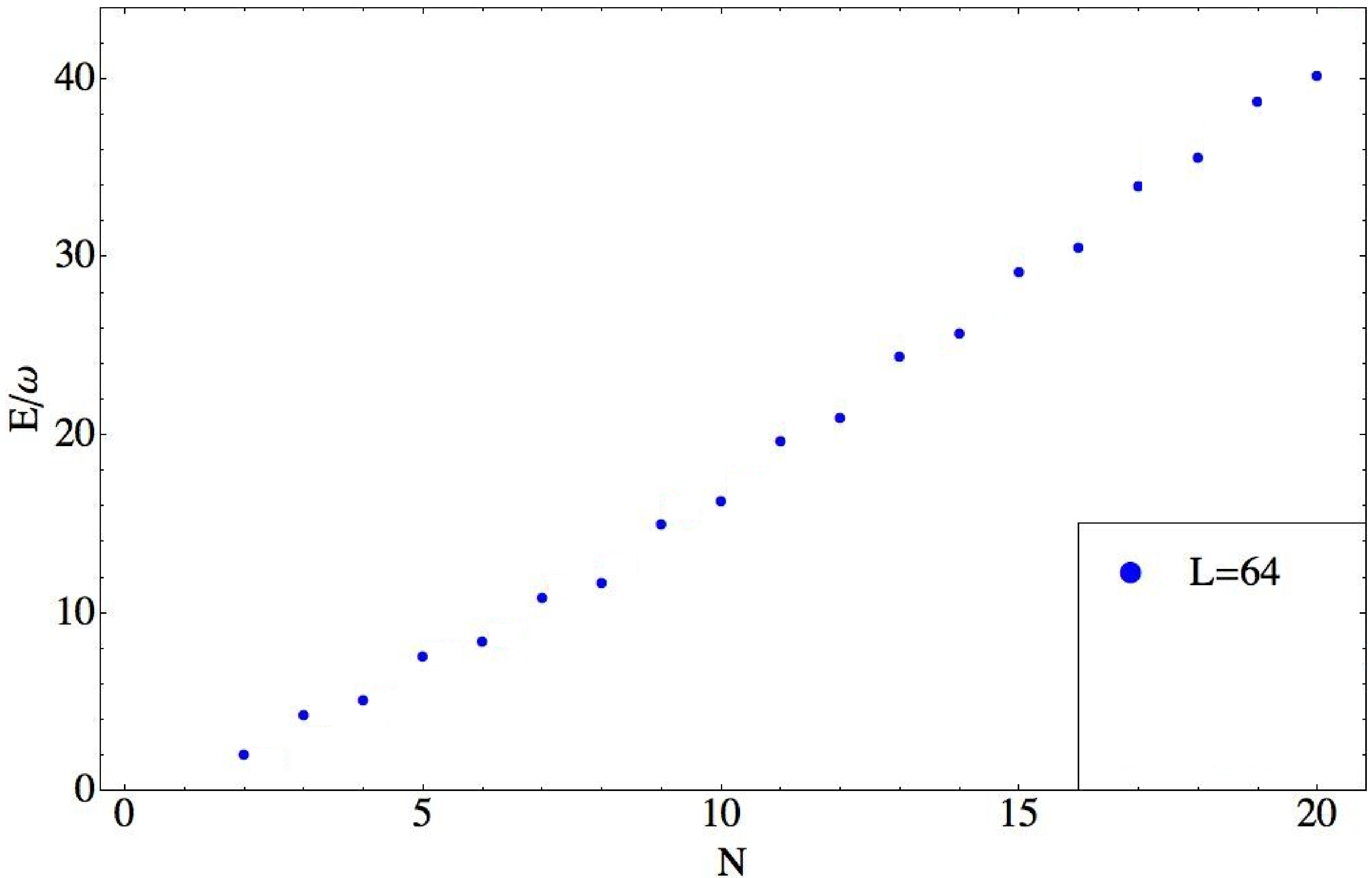




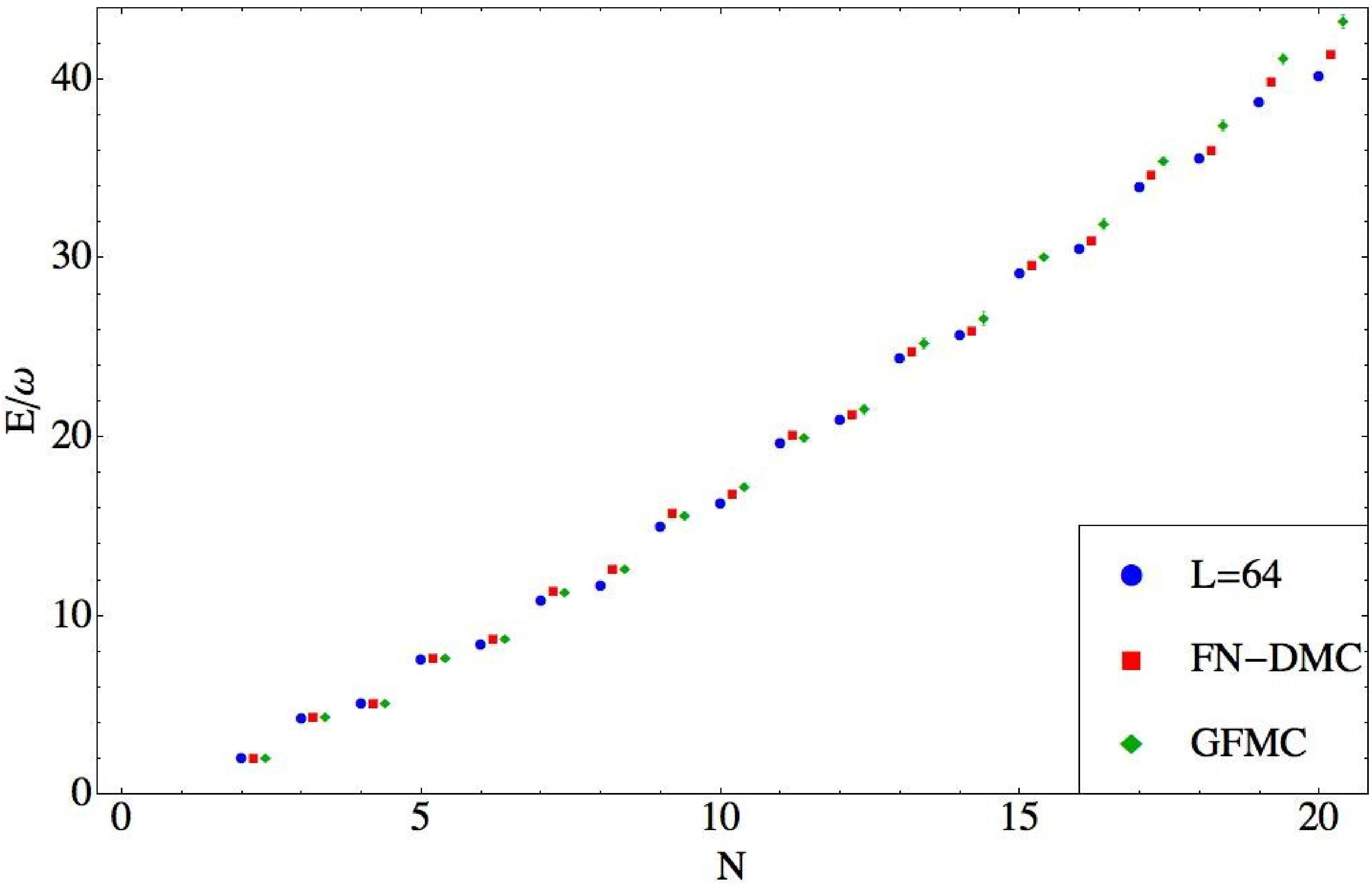
N	This Work	Comparison	% Deviation
3	4.253(2)(4)	4.2727*	0.5
4	5.058(1)(1)	5.028(20)†	0.6
5	7.513(3)(2)	7.457(10)‡	0.8
6	8.338(4)(5)	8.357(10)‡	0.2

PRELIMINARY

*F. Werner and Y. Castin, Phys. Rev. Lett. **97**. 150401 (2006)
†D. Blume, J. von Stecher, and C. Greene, Phys. Rev. Lett. **99**. 233201 (2007)
‡D. Blume, private communication



E/ω
N
L=64
0
10
20
30
40
0
5
10
15
20

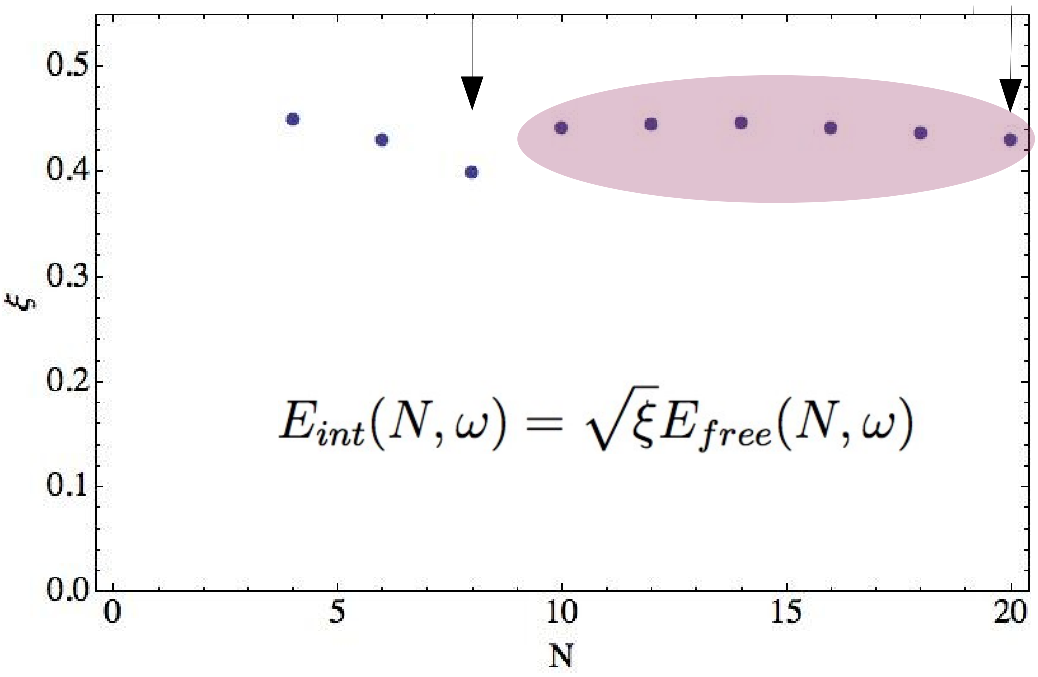


FN-DMC: D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

GFMC: S. Y. Chang and G. F. Bertsch, arXiv:physics/0703190

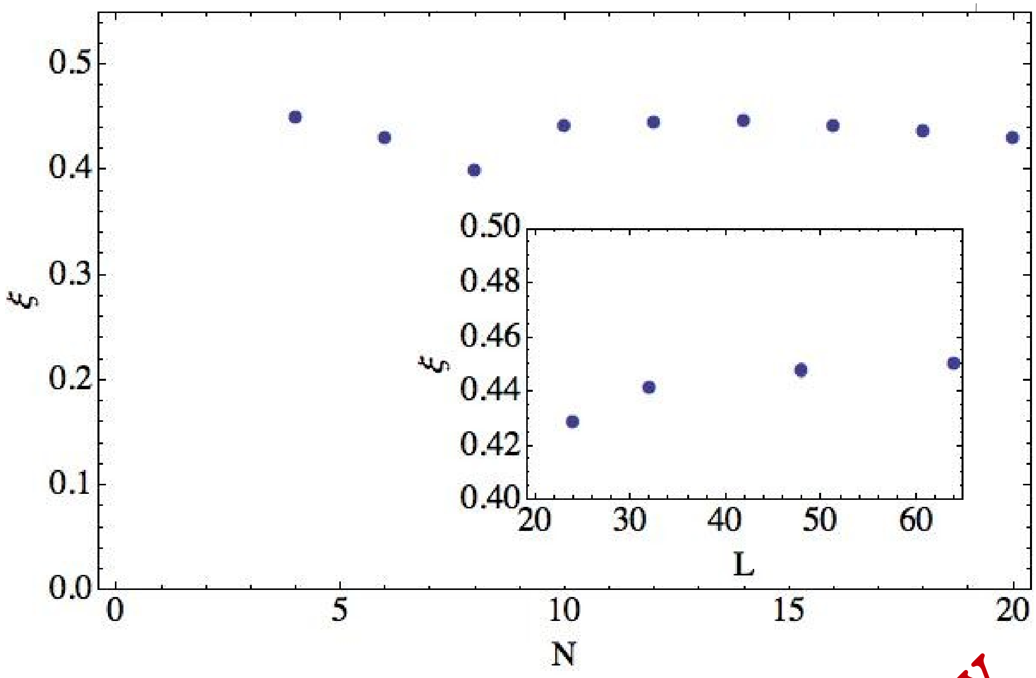
Bertsch Parameter

- Performed correlated fits to second shell
- Clear shell structure ➡ haven't reached thermodynamic limit



Bertsch Parameter

- Performed correlated fits to second shell
- Clear shell structure → haven't reached thermodynamic limit
- Finite volume effects smaller than 1%
- Results from SHO higher than from box



This Work: 0.450(1)
FN-DMC ~ 0.465
GFMC ~ 0.500

PRELIMINARY

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GFMC: J. Carlson, et al (2008)
Lattice EFT: D. Lee (2008)

Box:

PRELIMINARY

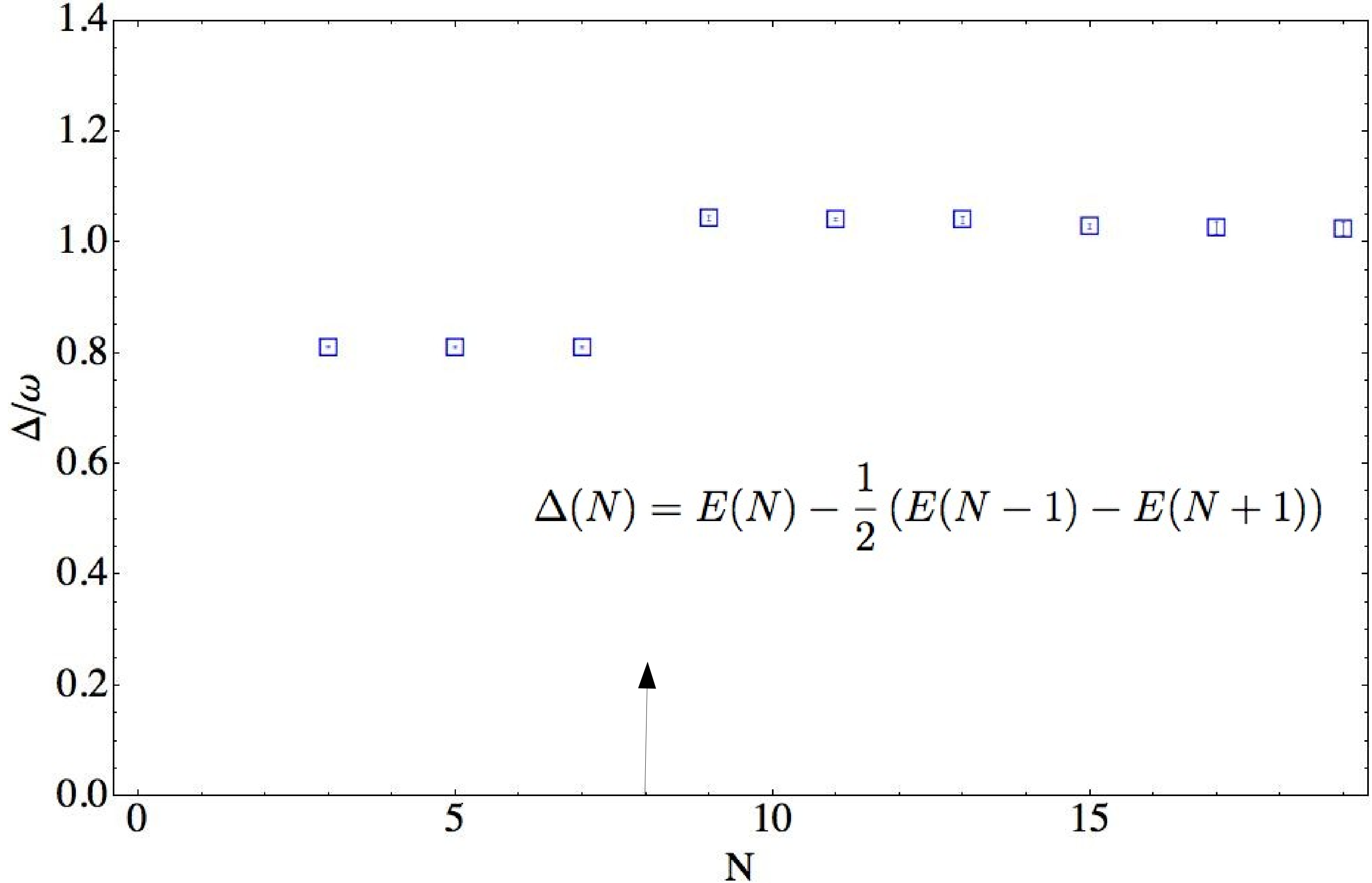
This Work: 0.412(4)
GFMC: 0.40(1)
Lattice EFT: 0.329(5)

SHO:

PRELIMINARY

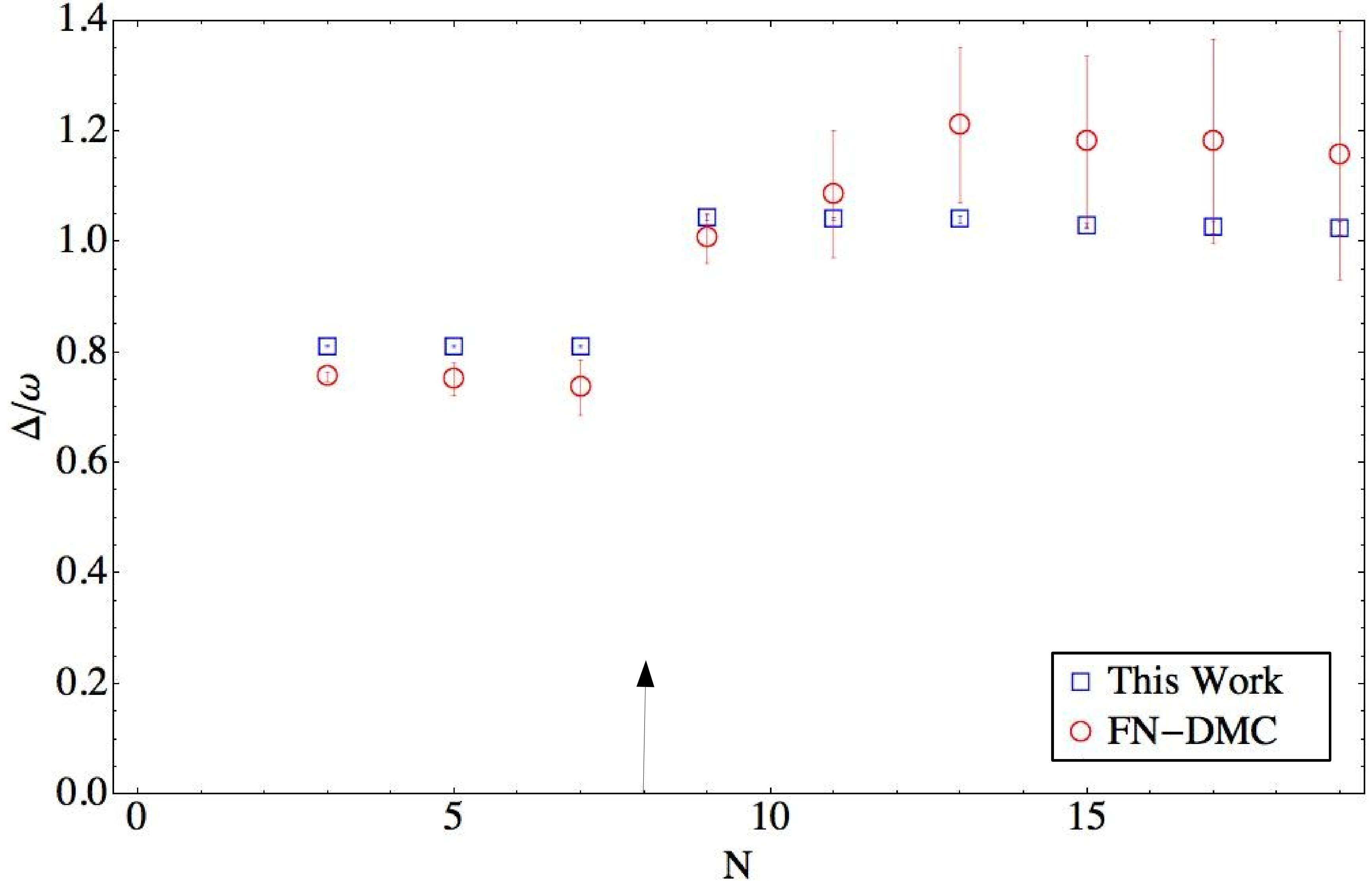
This Work: 0.450(1)
FN-DMC ~ 0.465
GFMC ~ 0.500

Gap



$$\Delta(N) = E(N) - \frac{1}{2}\left(E(N-1) - E(N+1)\right)$$

Gap



SHO Conclusions

- Possible to study large N to high precision
- Results consistent with high precision Schrodinger Eq. solutions for $N \leq 6$ to within 1%
- Better systematics and smaller errors than previous methods

Future directions

- Working to reduce lattice spacing errors further
- Ability to precisely tune p cot δ suggests applicability to nuclear systems
- Trap confinement may be useful for studying bound states of hadrons

Acknowledgements

Parts of these calculations were performed on New York Blue (BG/L) at Brookhaven National Laboratory