Extracting Scattering Parameters via Isospin Chemical Potential

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Lattice 2010
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Proposal

Explore two separate topics:

Scattering on the Lattice

Isospin Chemical Potential

- Tie these two concepts together to extract scattering information previously inaccessible from lattice calculations

Hadronic Scattering

- Understanding strongly mediated scattering is important for describing low-energy phenomena

Meson-Meson $\pi\pi$, $K\pi$, $\cdots$

Baryon-Meson πN , KN , $\pi\Sigma$, $\cdots$

Baryon-Baryon NN , $\Lambda\Xi$, $\cdots$

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Meson-Baryon

$$\pi^+\Sigma^+ \to \pi^+\Sigma^+$$

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Initial

Final

$$\pi^+ \left\{ \begin{array}{c} u \\ \bar{d} \end{array} \right. \qquad \left. \begin{array}{c} u \\ \bar{d} \end{array} \right\} \pi^+$$

$$\Sigma^+ \left\{ \begin{array}{c} u \\ u \\ s \end{array} \right. \qquad \left. \begin{array}{c} u \\ u \\ s \end{array} \right\} \Sigma^+$$

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Meson-Baryon

$$\pi^+\Sigma^+ \rightarrow \pi^+\Sigma^+$$

"Non-annihilation" Contribution

Initial

Final

Meson-Baryon (cont.)

$$\pi^+\Sigma^+ \to \pi^+\Sigma^+$$

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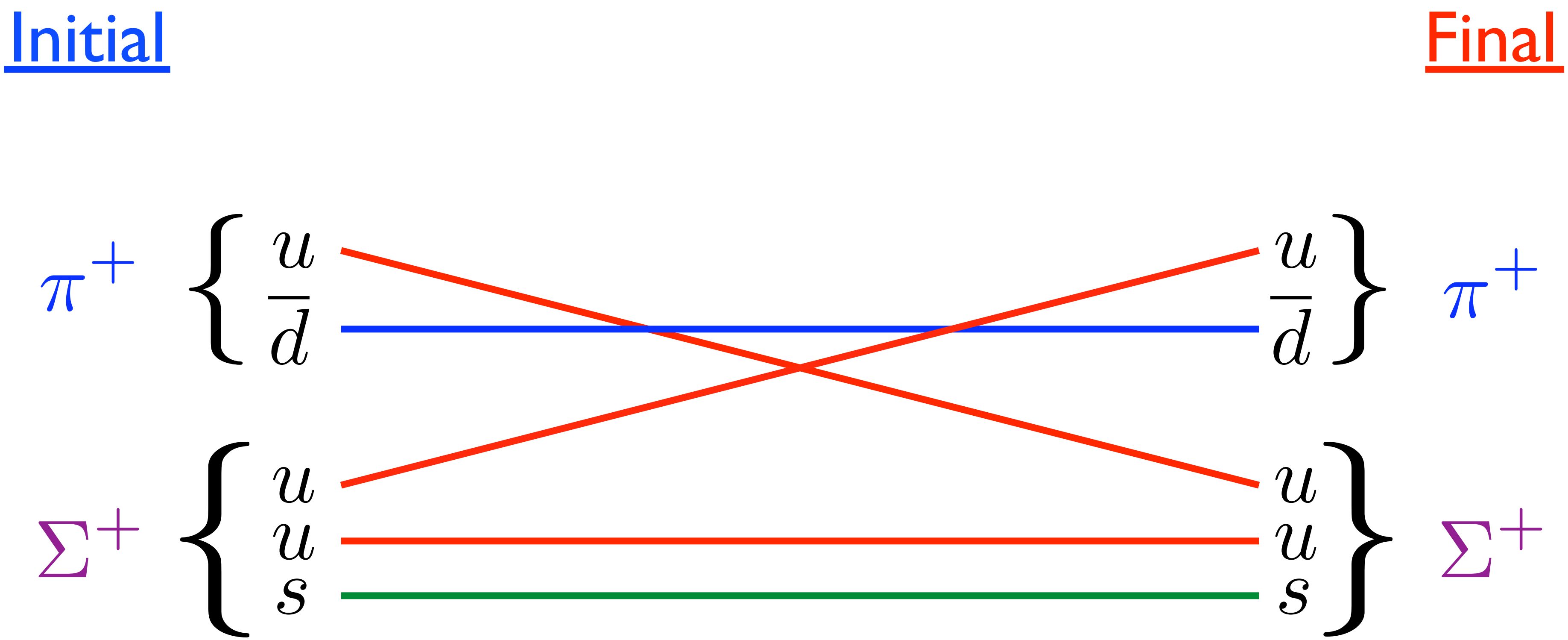

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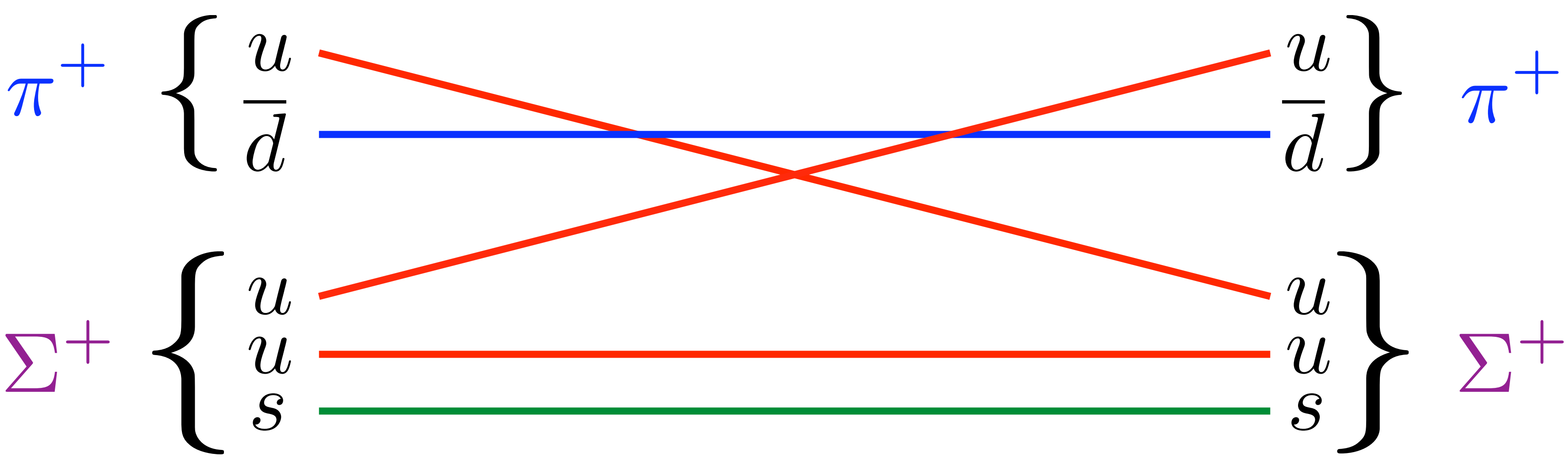

- Only “Non-annihilation” Contributions for this process

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$$\pi^- n \to \pi^- n$$

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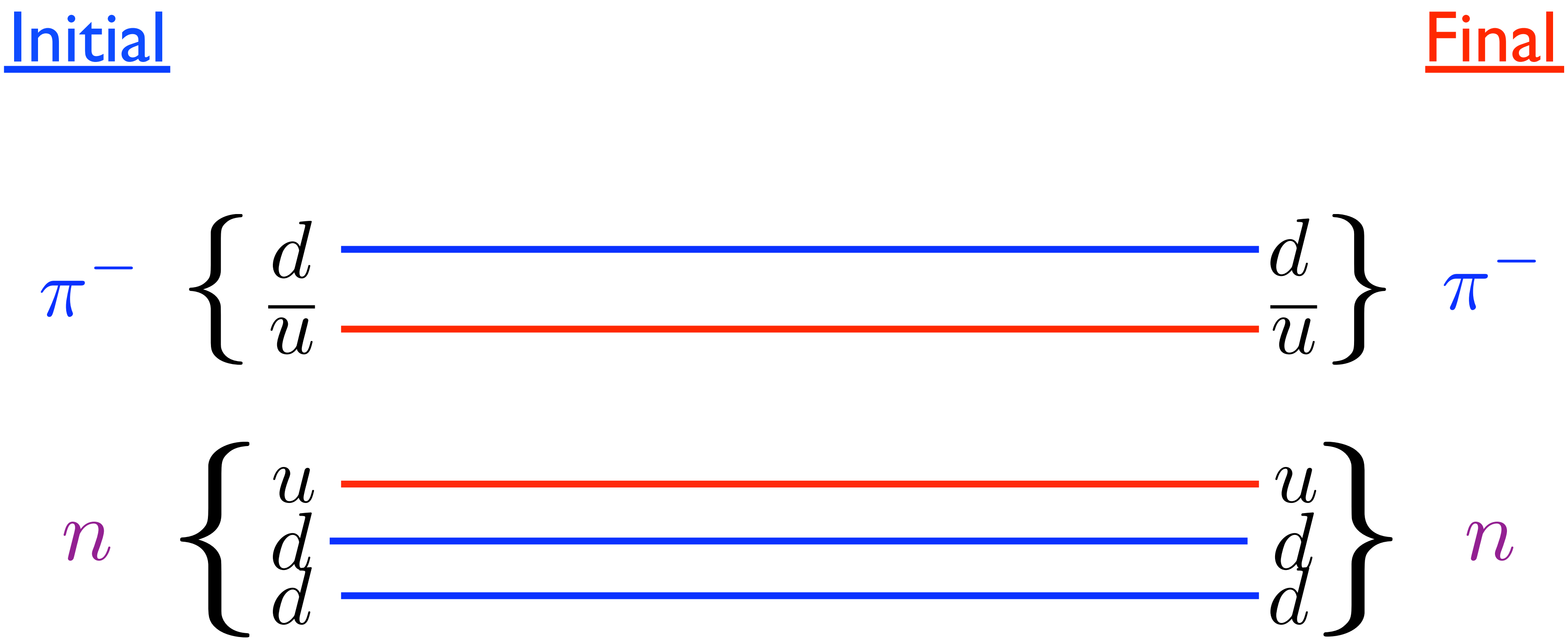

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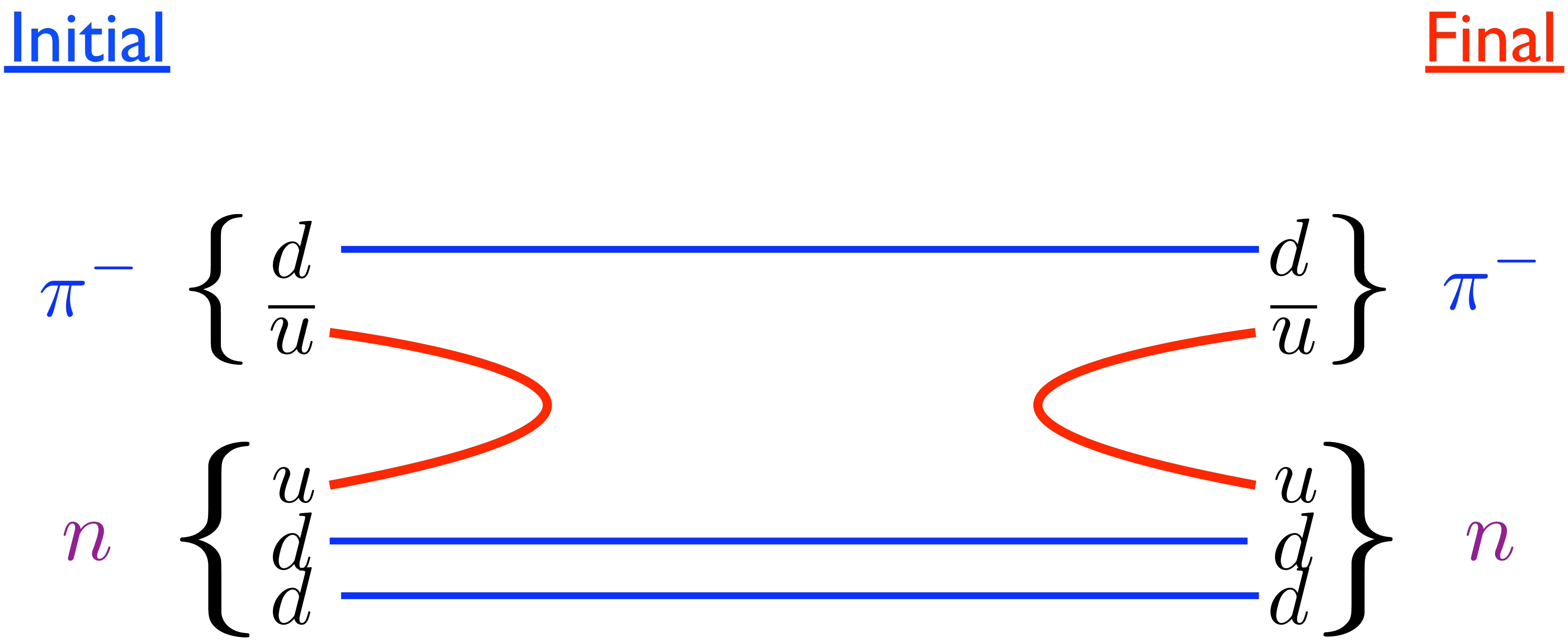

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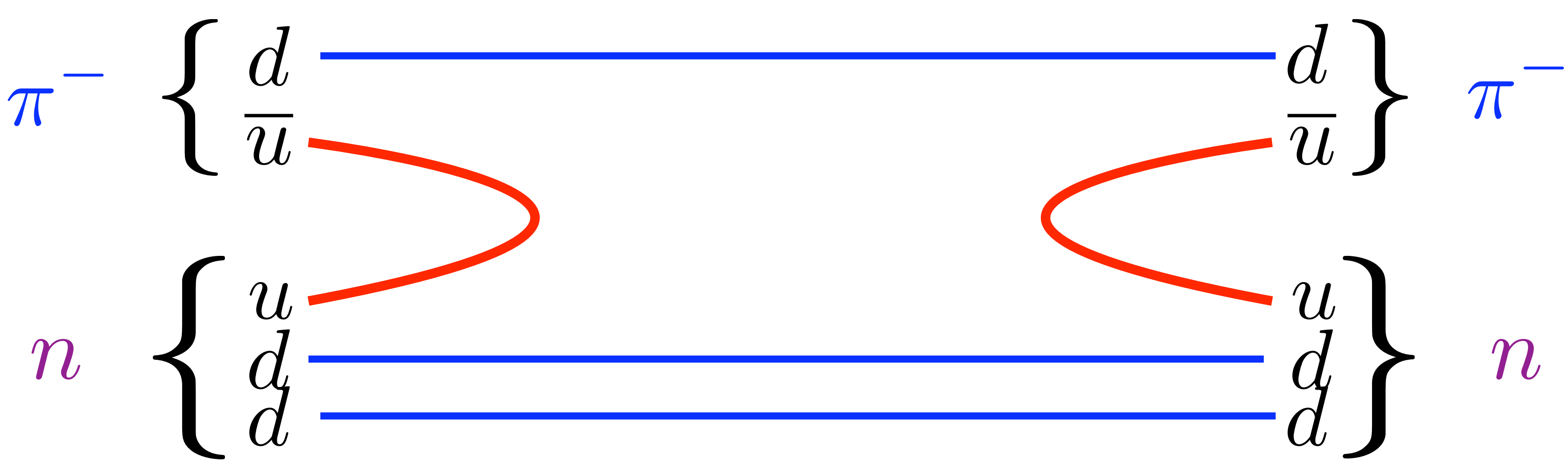

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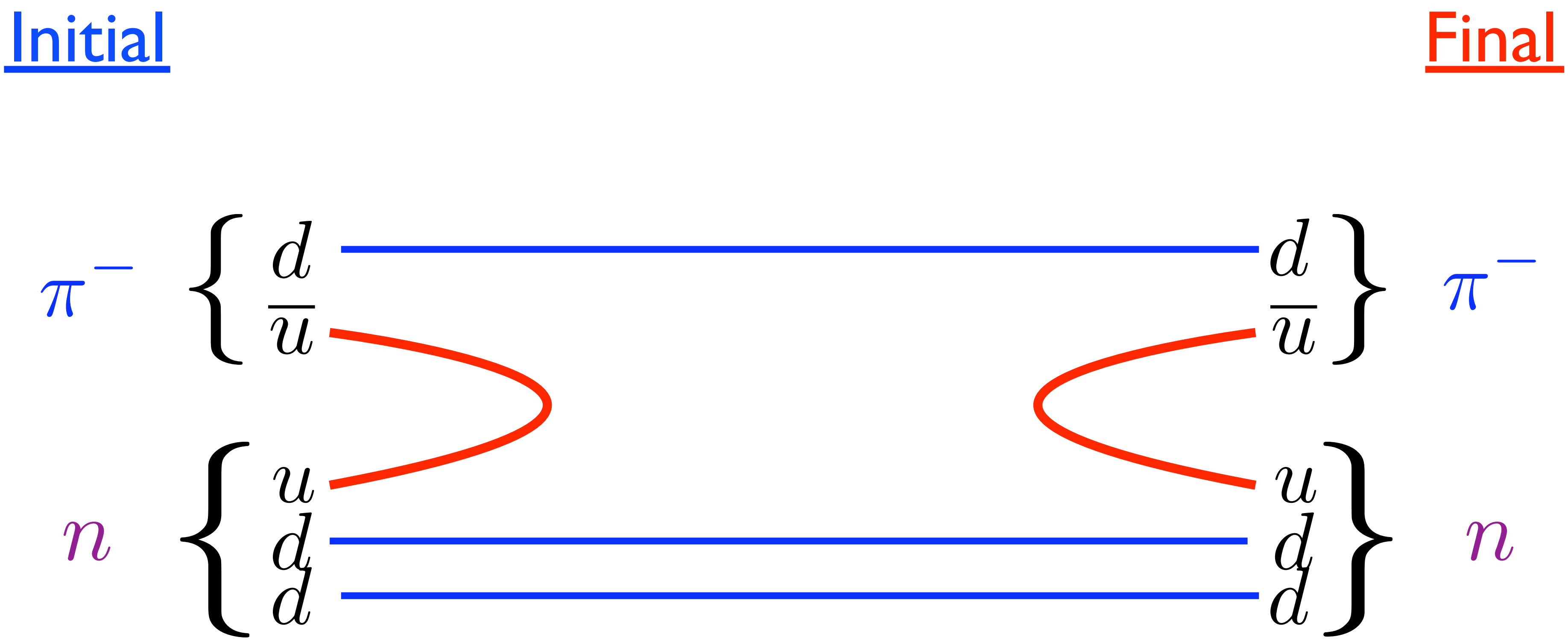

- Annihilation diagrams often prohibitively expensive for lattice

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  - VERY Expensive!
  (Whole matrix >> One Column)

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Our Solution: Measure something else!

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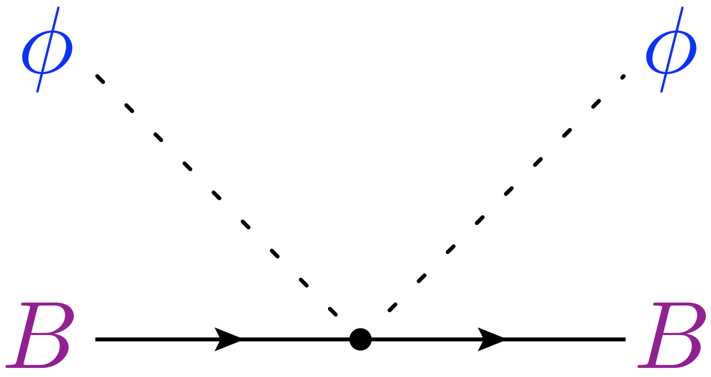

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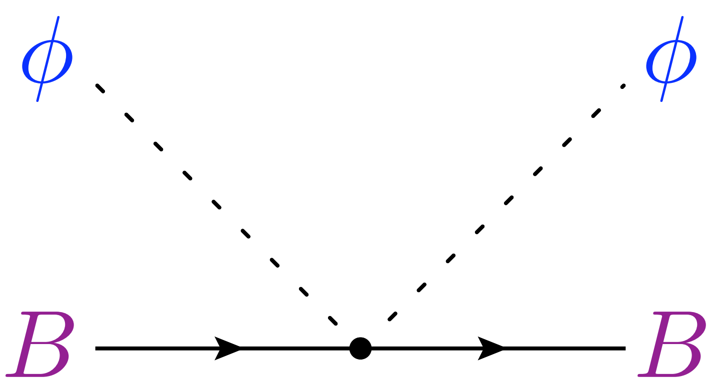

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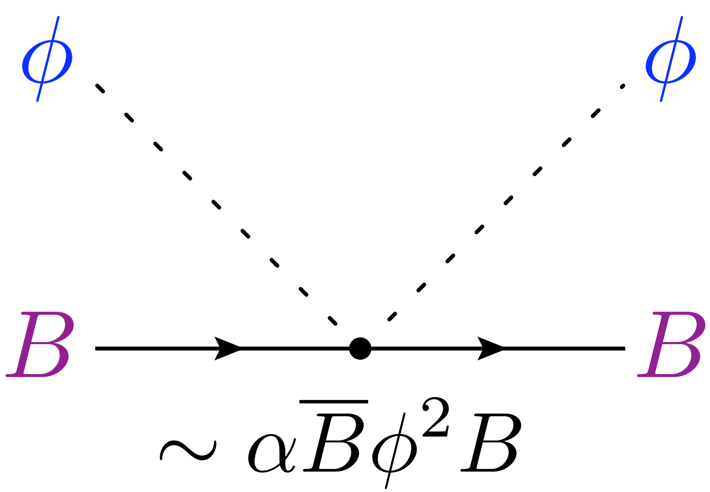

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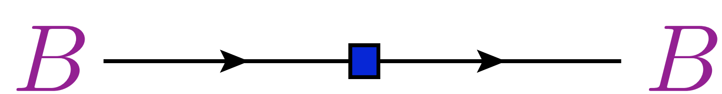

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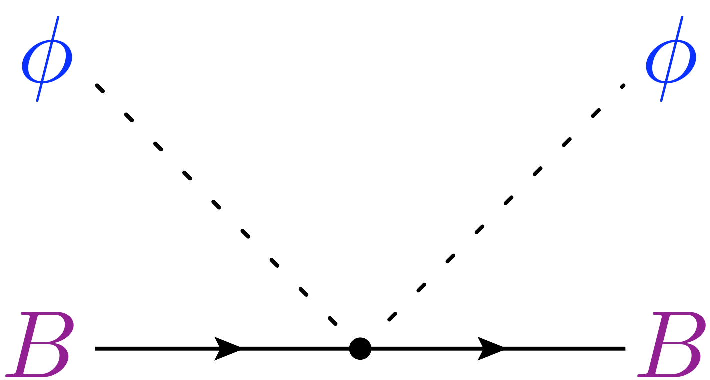

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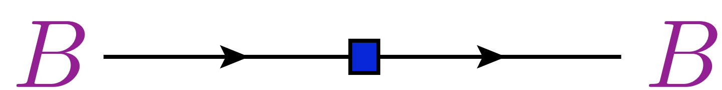

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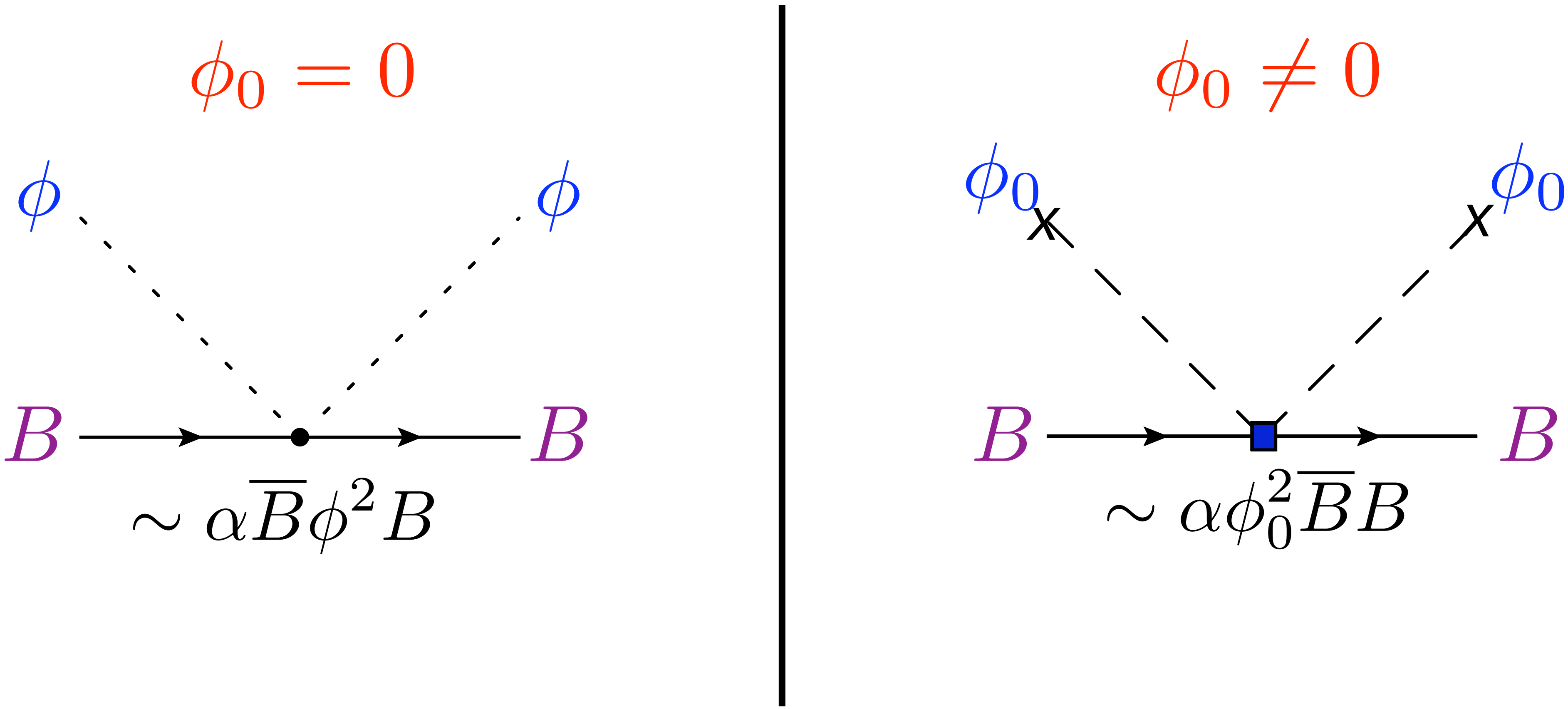

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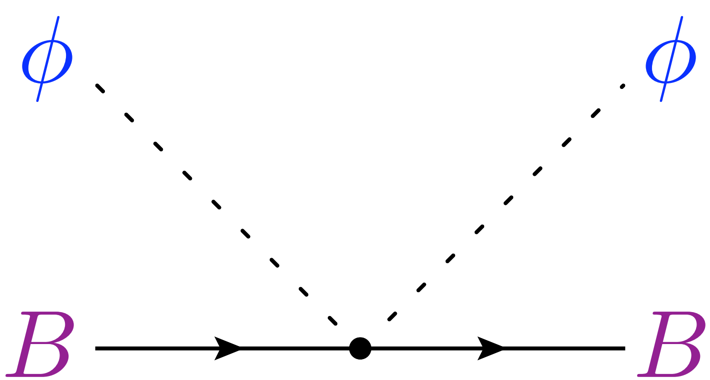

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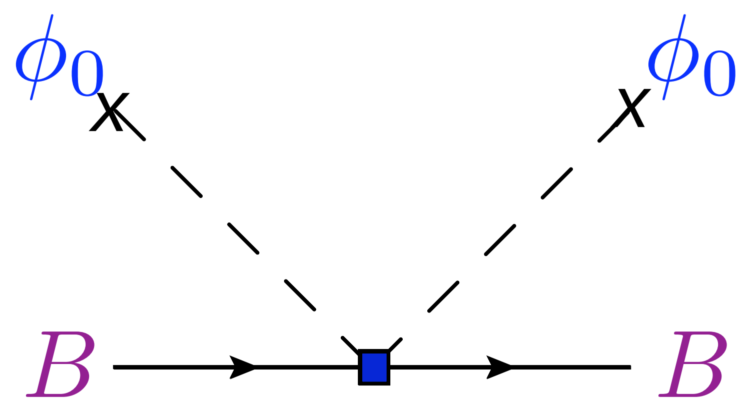

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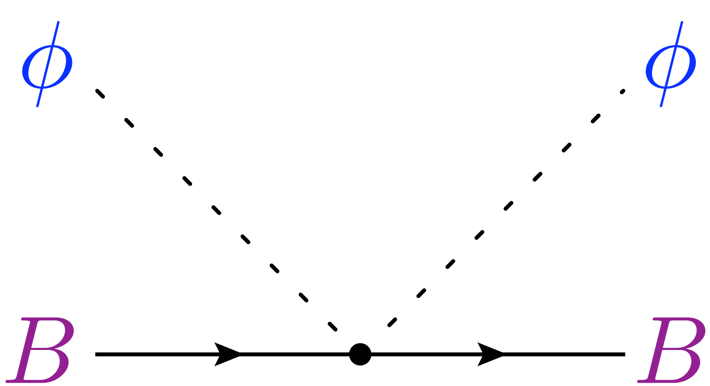

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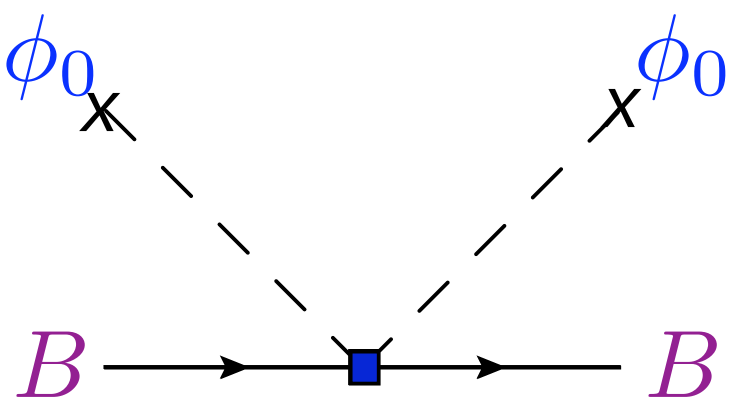

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Isospin Chemical Potential

$$\mathcal{L}_F = \overline{\psi}\Big[\Big(i(\not{D} + i\mu\gamma_0\frac{\tau^3}{2}\Big) - m_q\Big]\psi$$

$$m_q = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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2) Can form pion condensate

- Occurs when chemical potential exceeds critical value

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In the condensed phase:

$$\langle\overline{\psi}\psi\rangle = f^2\lambda\cos\alpha$$

$$i\langle\overline{\psi}\tau^2\gamma_5\psi\rangle = f^2\lambda\sin\alpha$$

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LO Sigma Term

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- Cannot disentangle c_2 from c_3

Can we extract more parameters?

Twisted boundary conditions with condensate:

- Can disentangle pair of low-energy constants

Two-flavor example: (c_2, c_3)

- Can extract single axial couplings

Two-flavor example: g_A

Follow up calculations

- Include strange quark (no strange chem. pot.)
  - $S = 2$ coupling to pions
  - $S = 1$ coupling to pions
- Match those couplings to three-flavor EFT
  - Give insight to kaon-nucleon
- EFT shift of meson masses in pion condensate
  - Extract parameters for meson-pion

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?

Final Word: Future Applications

- Spectroscopy with condensate (new "knob" to vary)

Mass shift ⟷ Condensate / Chemical Potential / $\boxed{\text{Scattering Parameters}}$

- Other quantities in presence of pion condensate
  - Heavy Quark Potential
  - Two meson phase shifts
  - ...
- Virial coefficients from thermodynamic lattices
  - Information on three or four meson interactions