Lattice QCD Study for Gluon Propagator and Gluon Spectral Function

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Outline

We study Gluon Propagator in Landau gauge in SU(3) lattice QCD at β=5.7, 5.8 and 6.0 at quenched level.

(Landau gauge is one of the most popular gauges in QCD and keeps Lorentz covariance and global color symmetry.
It is often used in lattice QCD, Schwinger-Dyson formalism, and so on.)

We study Functional Form of Coordinate-Space Gluon Propagator.

(Our main interest is *Infrared and Intermediate region* of r = 0.1~1.0 fm, which is relevant for quark-hadron physics.)

Based on the obtained Gluon Propagator form, we derive Analytical expression of Gluon Spectral Function $\rho(\omega)$ for the first time.

Reference: T. Iritani, H. S, H. Iida, Phys. Rev. D80 (2009) 114505 (20 pages), "*Gluon-propagator functional form in the Landau gauge in SU(3) lattice QCD: Yukawa-type gluon propagator and anomalous gluon spectral function*".

Previous Lattice studies for Gluon Propagator in Landau gauge

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C. W. Bernard, C. Parrinello, A. Soni, Phys. Rev. D49,1585 (1994).
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P. Marenzoni et al., PLB318, 511 (1993); NPB455, 339 (1995).
"High statistics study of the gluon propagator in the Landau gauge at β = 6.0"
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F.D.R. Bonnet et al., Phys. Rev. D62, 051501 (2000); PRD64, 034501 (2001).
"Infrared behavior of the gluon propagator on a large volume lattice"
K. Langfeld, H. Reinhardt, J. Gattnar, NPB621, 131 (2002).
"Gluon propagators and quark confinement"
S. Furui, H. Nakajima, PRD69, 074505 (2004)."Infrared features of the Landau gauge QCD"
P.O. Bowman et al., PRD70, 034509 (2004)."Unquenched gluon propagator in Landau gauge"
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P. J. Silva and O. Oliveira, Phys. Rev. D 74, 034513 (2006).
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A. Cucchieri, T. Mendes, O. Oliveira, P.J. Silva, PRD76, 114507 (2007).
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A. Cucchieri and T. Mendes, Phys. Rev. Lett. 100, 241601 (2008).
"Constraints on the IR behavior of the gluon propagator in YM theories"
I. L. Bogolubsky, E.-M. Ilgenfritz, M. Mueller-Preussker, A. Sternbeck, PLB 676, 69 (2009).
"Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared"

There are so many studies.
This is only a part of the List…

Landau Gauge Fixing and Gluon Field in Lattice QCD

In Euclidean QCD, Landau gauge has a Global definition to minimize "Total amount of Gauge-field Fluctuation",

$$R \equiv \int d^4x \ \mathrm{Tr}\{A_\mu(x)A_\mu(x)\} = \frac{1}{2}\int d^4x A_\mu^a(x)A_\mu^a(x)$$

by the gauge transformation.

In the global definition, Landau gauge has a clear physical interpretation that it maximally suppresses Gauge-field Fluctuation.

In Lattice QCD, Landau gauge fixing is defined by maximization of

$$R_{\mathrm{latt}} = \sum_x \sum_\mu \mathrm{Re}\ \mathrm{Tr}\ U_\mu(x)$$

Gluon fields (hermite and traceless) are defined from link-variables as

$$A_\mu(x) \equiv \frac{1}{2ia}\left[U_\mu(x) - U_\mu^\dagger(x)\right] - \frac{1}{2iaN_c}\mathrm{Tr}\left[U_\mu(x) - U_\mu^\dagger(x)\right]$$

minimization of gluon-field fluctuation justifies expansion by lattice spacing a.

Gluon Propagator in Landau Gauge in Lattice QCD

Gluon Propagator is defined by Two-point function,

$$D_{\mu\nu}^{ab}(x,y) \equiv \langle A_\mu^a(x) A_\nu^b(y) \rangle = D_{\mu\nu}^{ab}(x-y).$$

In Laudau gauge, Lorentz and Color Structure of Gluon Propagator is simple.

→ We only have to consider

Scalar Combination of Gluon Propagator,

$$D(r) \equiv \frac{1}{3(N_c^2-1)} D_{\mu\mu}^{aa}(x) = \frac{1}{3(N_c^2-1)} \langle A_\mu^a(x) A_\mu^a(0) \rangle$$

as a function of *Four-dimensional Euclidean space-time distance*,

$$r \equiv |x| \equiv (x_\mu x_\mu)^{1/2}.$$

We mainly investigate Functional Form of Coordinate-Space Gluon Propagator in Landau Gauge in Lattice QCD, since coordinate-space variable is more directly obtained in Lattice QCD.

Calculation Condition of Lattice QCD for Gluon Propagator

Condition of our Lattice QCD calculation:

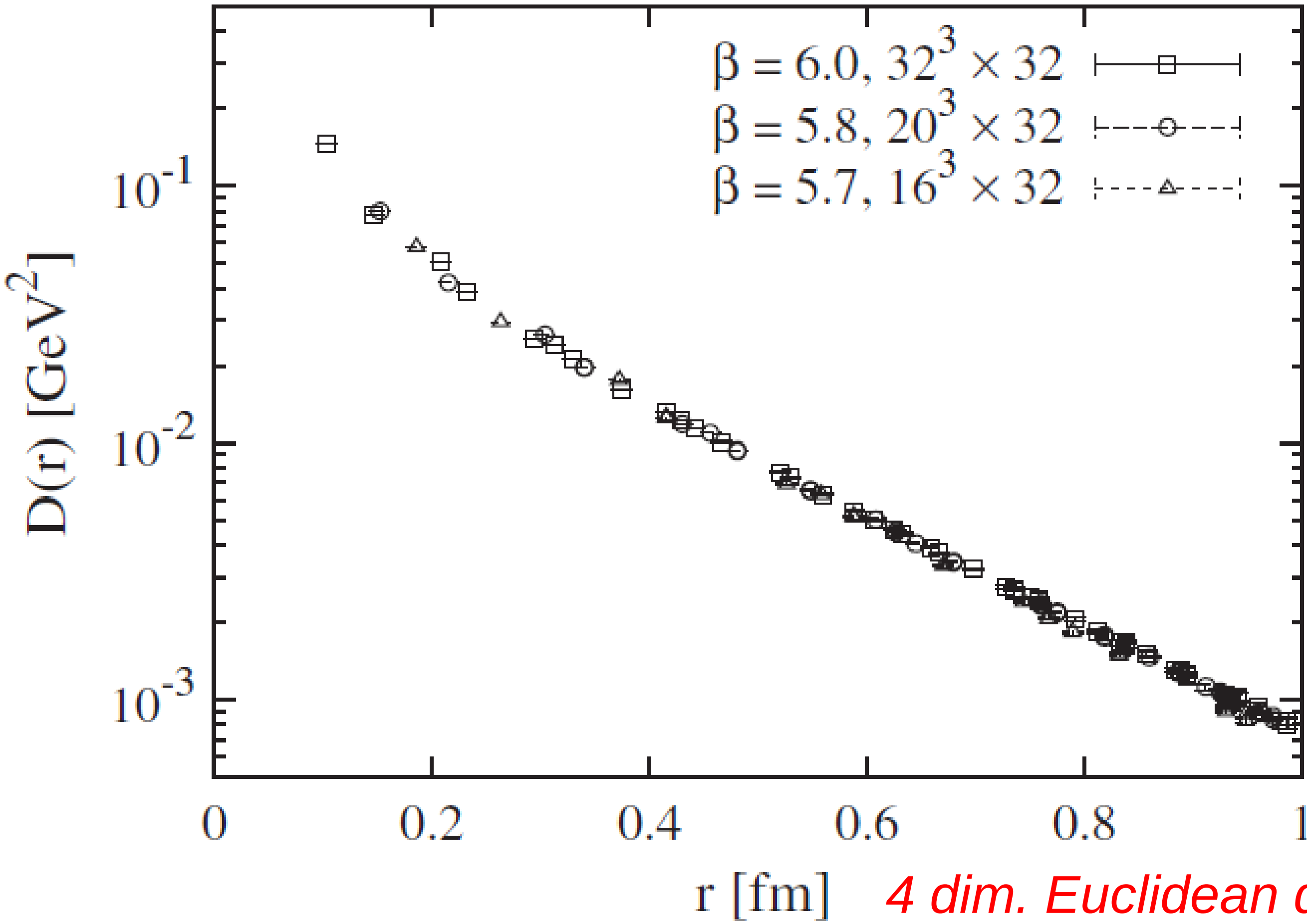
- quenched level
- standard plaquette action
- lattice spacing: ***a*** = 0.1~0.19fm (β = 5.7 ~ 6.0)
- various lattice volume

TABLE I. The lattice parameter β, lattice size, and the gauge-configuration number N_{conf}. The corresponding lattice spacing a and the lattice volume in the physical unit are added. The lattice spacing a is determined so as to reproduce the string tension $\sqrt{\sigma} = 427$ MeV.

β	Lattice size	a [fm]	Volume [fm^4]	N_{conf}
5.7	$16^3 \times 32$	0.186	$2.976^3 \times 5.952$	50
5.8	$20^3 \times 32$	0.152	$3.040^3 \times 4.864$	40
6.0	$32^3 \times 32$	0.104	$3.328^3 \times 3.328$	30

Coordinate Gluon Propagator in Landau Gauge in Lattice QCD

$$D(r) \equiv \frac{1}{3(N_c^2-1)} D^{aa}_{\mu\mu}(x) = \frac{1}{3(N_c^2-1)} \langle \mathcal{A}^a_\mu(x)\mathcal{A}^a_\mu(0)\rangle$$



$r \equiv |x| \equiv (x_\mu x_\mu)^{1/2}$

4 dim. Euclidean distance

FIG. 1. Lattice-QCD results of the scalar-type gluon propagator $D(r) \equiv \sum_{a=1}^{8}\sum_{\mu=1}^{4}\langle A^a_\mu(x)A^a_\mu(0)\rangle/24$ as the function of the four-dimensional Euclidean distance $r \equiv (x_\alpha x_\alpha)^{1/2}$ in the Landau gauge at $\beta = 5.7$, 5.8, and 6.0.

Comparison with Massive Propagator

To analyze gluon propagator, we first consider free massive-vector propagator form, using Stueckerberg form Lagrangian in Euclidean metric

$$\mathcal{L} = \frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)^2 + \frac{1}{2}m^2 A_\mu^a A_\mu^a - \frac{1}{2\alpha}\left(\partial_\mu A_\mu^a\right)^2$$

($\alpha = 0$ corresponds to Landau gauge.)

For free massive field, 4-dim. Euclidean Coordinate-space Propagator is described with modified Bessel function

$$D(r) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\tilde{D}(p^2) = \frac{1}{4\pi^2}\frac{m}{r}K_1(mr)\,,\quad \tilde{D}(p^2) = \frac{1}{24}\tilde{D}_{\mu\mu}^{aa}(p) = \frac{1}{p^2+m^2}$$

For large r, $D(r)$ behaves as $D(r) \sim r^{-3/2}e^{-mr}\,,\ K_1(mr) \simeq \sqrt{\frac{\pi}{2mr}}e^{-mr}$

To estimate effective gluon mass, we compare lattice QCD data with modified Bessel function, $D_{\rm mass}(r) = A\frac{m}{r}K_1(mr)$

Coordinate Gluon Propagator v.s. Massive Propagator

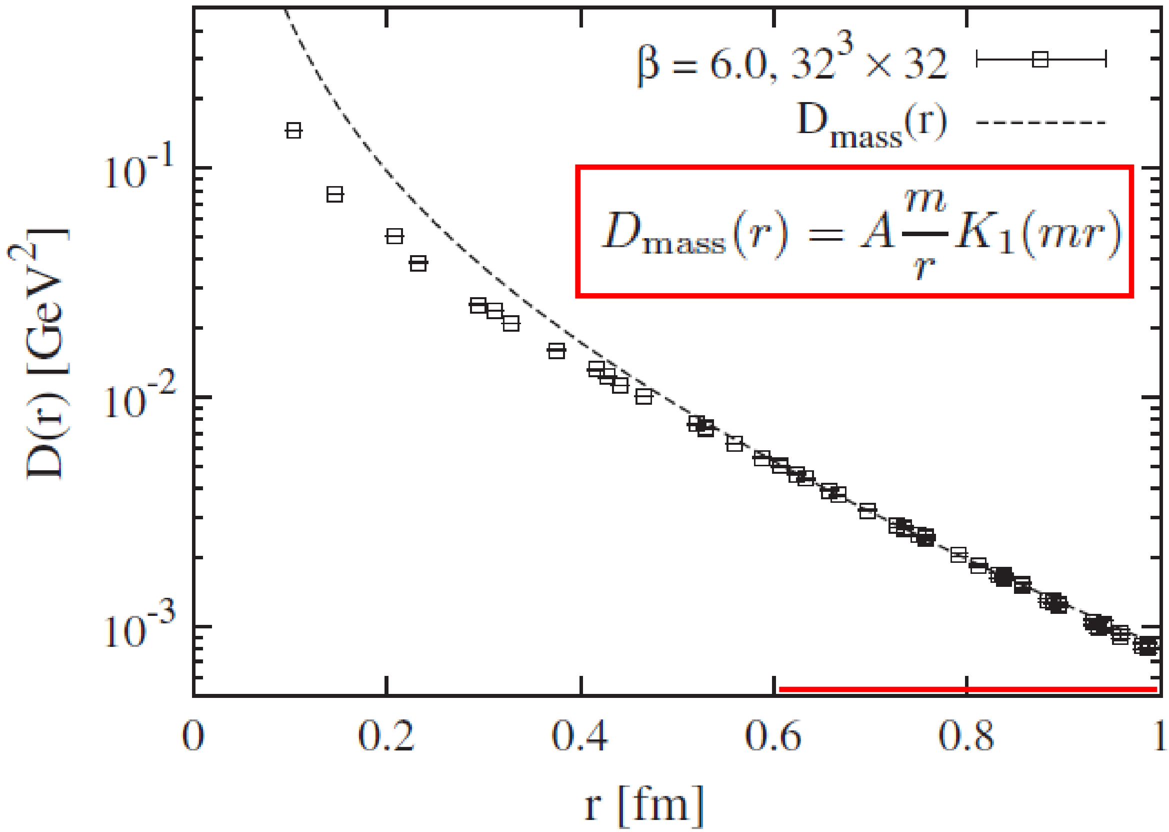


FIG. 3. A typical example of the fit analysis of the lattice gluon propagator $D(r)$ with the fit function $D_{\rm mass}(r)$ of the massive-vector propagator denoted by the dashed line. The fit is done for the lattice data at $\beta = 6.0$ in the fit range of $r = 0.6$–1.0 fm.

In infrared region of r = 0.6 ~1.0fm, lattice data seem to be reproduced by massive form with gluon mass m about 500MeV. But, massive form cannot describe Gluon Propagator in whole region of r = 0.1 ~ 1.0fm.

Effective Mass Plot of Gluons and Zero-spatial-momentum Propagator

We investigate the Effective Mass Plot of Gluons

$$M_{\rm eff}(t) = \ln\{D_0(t)/D_0(t+1)\}$$

defined from zero-spatial-momentum gluon propagator $D_0(t)$

$$D_0(t) \equiv \frac{1}{24}\sum_{\vec{x}} \langle A_\mu^a(\vec{x},t) A_\mu^a(\vec{0},0)\rangle = \sum_{\vec{x}} D(r)$$

In the actual lattice calculation, we use wall-to-wall correlator to improve statistics.
In numerical analysis, we take account of temporal periodicity used in lattice QCD.

Effective Mass Plot of Gluons in Landau Gauge

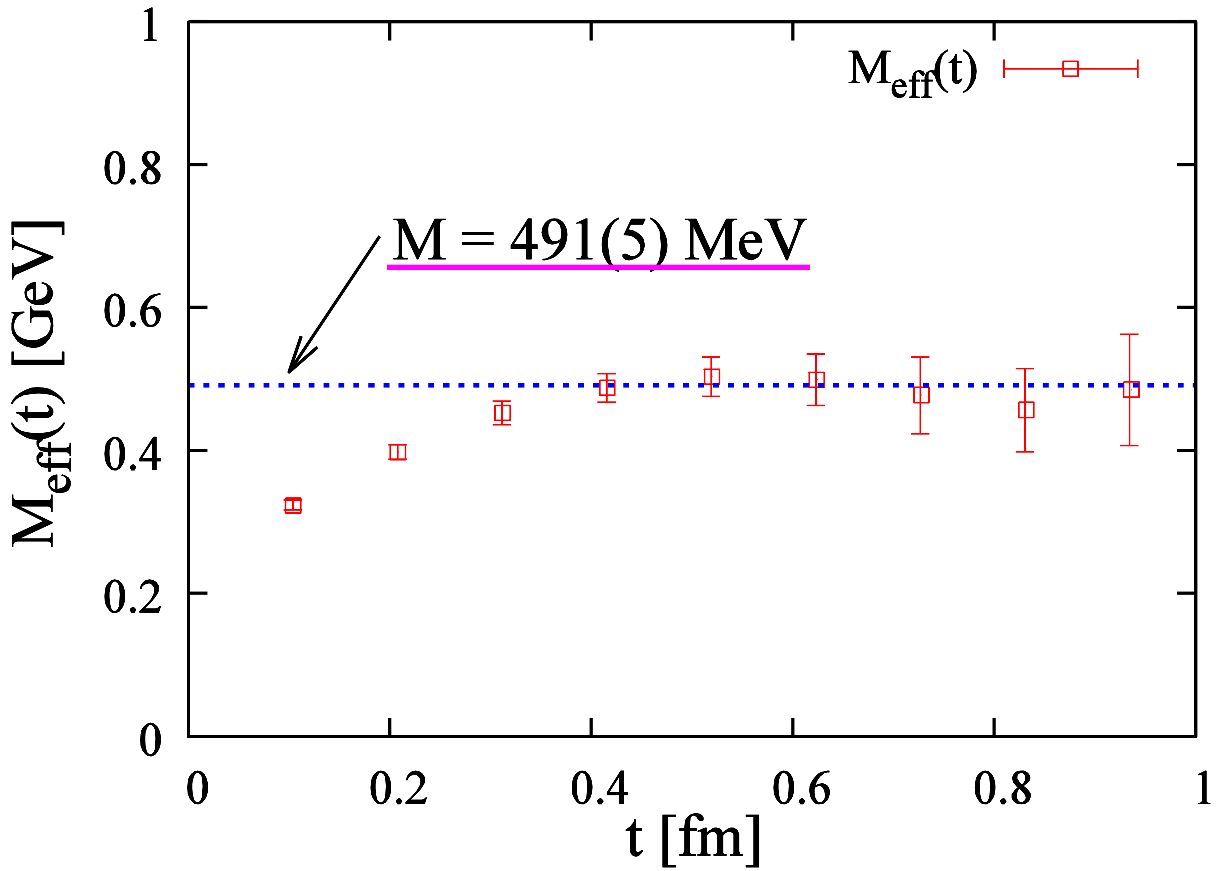


FIG. 4. The effective mass $M_{\text{eff}}(t)$ of gluons in the Landau gauge in lattice QCD at $\beta = 6.0$, i.e., $a = 0.104$ fm.

Gluon effective mass M_{eff} (t) is estimated about 500MeV.
Note that Gluon Effective Mass M_{eff} (t) is *Increasing Function* of time-variable t, unlike hadron or color-singlet case.

This tendency has been indicated by many previous lattice studies.

Function Form Analysis of Landau-Gauge Gluon Propagator

Coordinate-space Landau-gauge Gluon Propagator

is well described with

Four-dimensional Yukawa-type function

in the region of r = 0.1~1.0fm.

$$D(r) \equiv \frac{1}{24} D^{aa}_{\mu\mu}(r) = A \frac{m}{r} e^{-mr}$$

with $m \simeq 600\text{MeV}$ $A \simeq 0.16$

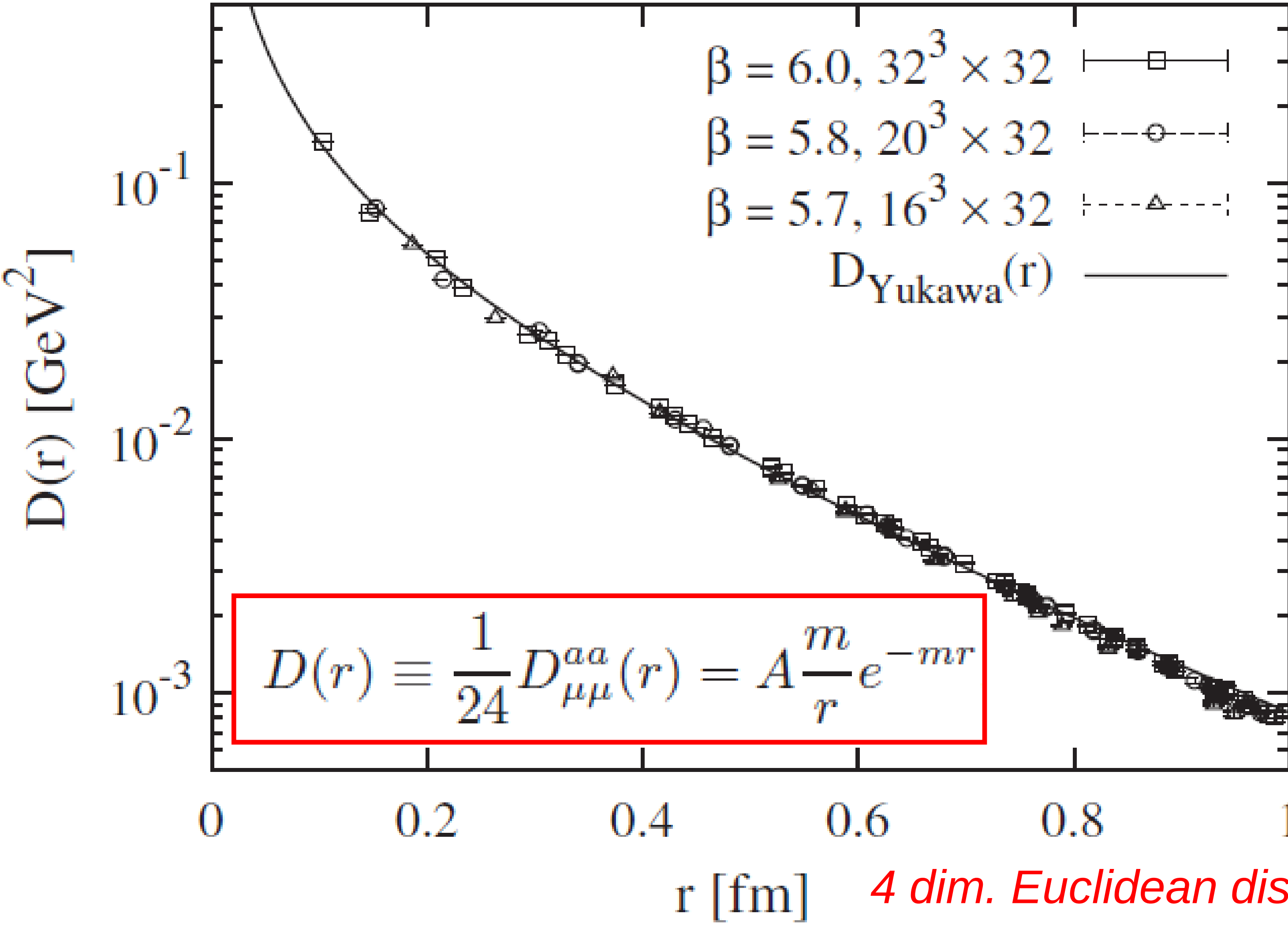
for $r \equiv (x_\alpha x_\alpha)^{1/2} = 0.1 \sim 1.0\text{fm}$

Four-dimensional Euclidean space-time distance

m : Yukawa-damping mass parameter

A : a dimensionless parameter

Yukawa-type Function of Landau-Gauge Gluon Propagator



$$D(r) \equiv \frac{1}{24} D^{aa}_{\mu\mu}(r) = A \frac{m}{r} e^{-mr}$$

4 dim. Euclidean distance

Yukawa-damping mass parameter

$$m \simeq 600\text{MeV}$$

$$m = 0.624 \text{ GeV}$$

$$A = 0.162$$

For the whole region of *r* = 0.1~1.0fm, ***Coordinate-space Landau-gauge Gluon Propagator*** is fairly well described with ***<u>Yukawa-type function</u>*** in *Four-dimensional* Euclidean space-time.

Function Form of Landau-Gauge Gluon Propagator

Four-dimensional Euclidean Yukawa-function $D_{\rm Yukawa}(r)$ corresponds to a ***new-type propagator***,

$$\tilde{D}_{\rm Yukawa}(p^2) \equiv \int d^4x e^{ip\cdot x} D_{\rm Yukawa}(r) = \frac{4\pi^2 Am}{(p^2+m^2)^{3/2}}$$

Momentum-space Landau-gauge Gluon Propagator is well described with the new-type propagator corresponding to ***Four-dimensional Yukawa-type function*** in the region of p = 0.5 ~ 3GeV.

$$\tilde{D}(p^2) = \frac{1}{24}\tilde{D}^{aa}_{\mu\mu}(p^2) = \frac{4\pi^2 Am}{(p^2+m^2)^{3/2}}$$

with $m \simeq 600\mathrm{MeV}$ $A \simeq 0.16$ (the same values)

for $0.5\ \mathrm{GeV} \le p \le 3\ \mathrm{GeV}$

Yukawa-type Propagator for Landau-Gauge Gluon Field

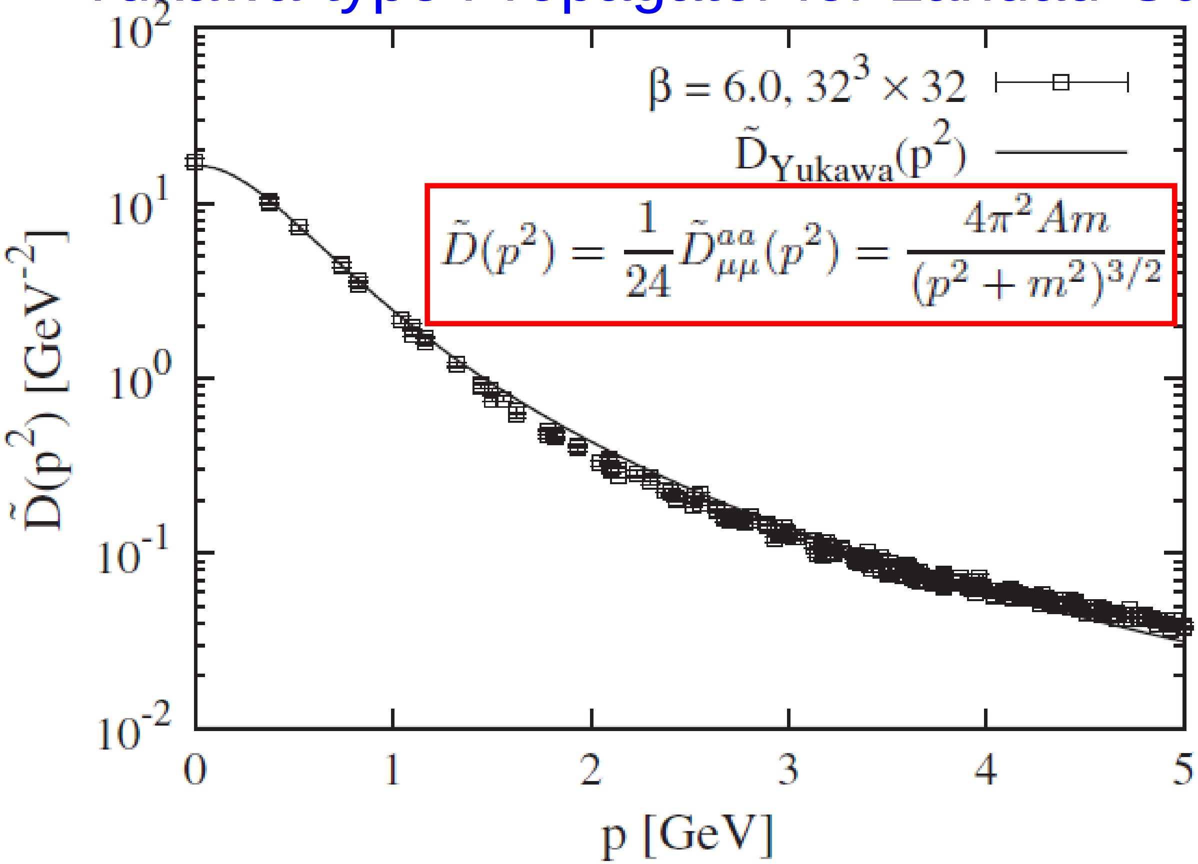


FIG. 8. The Yukawa-type propagator in the momentum space, i.e., $\tilde{D}_{\rm Yukawa}(p^2) = 4\pi^2 Am(p^2+m^2)^{-3/2}$ (solid line) with $m =$ 0.624 GeV and $A = 0.162$, the same values used in Fig. 7. The horizontal axis is $p \equiv (p_\alpha p_\alpha)^{1/2}$. The symbols denote the lattice-QCD data of the scalar-type gluon propagator $\tilde{D}(p^2)$ in the Landau gauge at $\beta = 6.0$, where the momentum is defined as $p_\mu = \frac{2}{a}\sin(\frac{\pi n_\mu}{L_\mu})$.

Yukawa-damping mass parameter

$$m \simeq 600\text{MeV}$$

$$m = 0.624 \text{ GeV}$$

$$A = 0.162$$

(same values as before)

This agreement is *not* so trivial because there are some deviations between the actual gluon propagator and Yukawa-type function in UV and Deep-IR regions.

Momentum-space Landau-gauge Gluon Propagator **is also well described with** ***4-dim Fourier transformation*** **of** ***Yukawa function*****.**

Yukawa-type Gluon Propagator for Landau-Gauge Gluons

Landau-gauge Gluon Propagator is well described with
Four-dimensional Yukawa-type function for r = 0.1~1.0fm.

coordinate space

$$D(r) \equiv \frac{1}{24} D^{aa}_{\mu\mu}(r) = A \frac{m}{r} e^{-mr}$$

for $r \equiv (x_\alpha x_\alpha)^{1/2} = 0.1 \sim 1.0\mathrm{fm}$

4-dim. Euclidean distance

momentum space

$$\tilde{D}(p^2) = \frac{1}{24} \tilde{D}^{aa}_{\mu\mu}(p^2) = \frac{4\pi^2 A m}{(p^2 + m^2)^{3/2}}$$

for $0.5\ \mathrm{GeV} \le p \le 3\ \mathrm{GeV}$

with $m \simeq 600\mathrm{MeV}$ $\quad A \simeq 0.16$

This Yukawa-type propagator is an approximate function for infrared/intermediate region relevant for quark-hadron physics. Such an Analytical form of Gluon Propagator would be useful for Nonperturbative Analysis of QCD phenomena.

Zero-spatial-momentum propagator for Yukawa-type gluon propagator

For Yukawa-type propagator, zero-momentum propagator $D_0(t)$ is expressed with modified Bessel function $K_1(mt)$

$$D_0(t) = 4\pi A t K_1(mt)$$

Derivation:

$$\begin{aligned} D_0(t) &\equiv \frac{1}{24}\sum \langle A_\mu^a(\vec{x},t) A_\mu^a(\vec{0},0)\rangle = \sum D(r) \\ &= 4\pi A m \int_0^\infty dx\, x^2 \frac{1}{\sqrt{x^2+t^2}} e^{-m\sqrt{x^2+t^2}} \\ &= 4\pi A m \int_t^\infty dr \sqrt{r^2-t^2}\, e^{-mr} \\ &= 4\pi A m t^2 \int_1^\infty d\bar{r} \sqrt{\bar{r}^2-1}\, e^{-\bar{r}mt} \\ &= 4\pi A m t^2 \frac{1}{mt} K_1(mt) = 4\pi A t K_1(mt), \end{aligned}$$

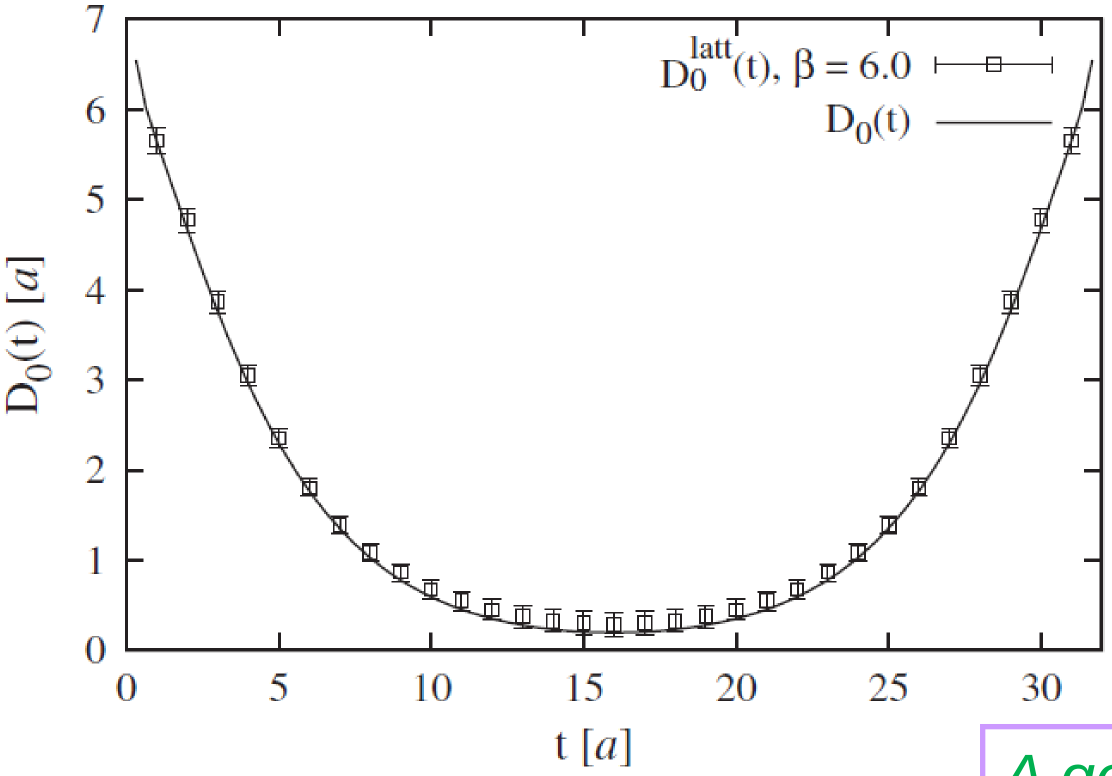
This is continuum formalism with infinite spatial volume.
For the actual comparison with lattice QCD data,
we take account of temporal periodicity, used in lattice calculations.

$$D_0(t) = 4\pi A [t K_1(mt) + (N_t - t) K_1(m(N_t - t))]$$

Zero-spatial-momentum Gluon Propagator

Lattice QCD data and Analytical result derived from Yukawa-type propagator

$$D_0(t) = 4\pi A t K_1(mt)$$ with temporal periodicity



$m = 0.624$ GeV

$A = 0.162$

(same values as before)

FIG. 9. The zero-spatial-momentum propagator $D_0(t)$ of gluons in the Landau gauge. The symbols are the lattice-QCD data at $\beta = 6.0$, and the solid line denotes the theoretical curve of Eq. (49), derived from the Yukawa-type propagator with $m = 0.624$ GeV and $A = 0.162$, the same values in Fig. 7.

A good agreement between lattice QCD data and theoretical curve derived from Yukawa-type propagator.

Effective mass plot for Yukawa-type Gluon Propagator

For Yukawa-type propagator, Effective Mass Plot of Gluons is also expressed with modified Bessel function $K_1(mt)$.

$$M_{\text{eff}}(t) = \ln \frac{D_0(t)}{D_0(t+1)} = \ln \frac{tK_1(mt)}{(t+1)K_1(m(t+1))}$$

This is continuum formalism with infinite spatial volume.
In actual comparison, we take account of temporal periodicity used in lattice QCD.

For large t, the effective mass is much simplified as

$$M_{\text{eff}}(t) \simeq m - \frac{1}{2}\ln\left(1+\frac{1}{t}\right) \simeq m - \frac{1}{2t}$$

Therefore, mass parameter $m \simeq$ 600MeV has a definite physical meaning of *Effective Gluon Mass in Infrared region*.

The value $m \simeq$ 600MeV for *infrared effective gluon mass* is almost the same as *phenomenologically conjectured value*.

[J.M. Cornwall, Phys. Rev. D26, 1453 (1982).]

Effective Mass Plot of Gluons obtained from Yukawa-type propagator

$$M_{\rm eff}(t) = \ln\frac{D_0(t)}{D_0(t+1)} = \ln\frac{tK_1(mt)}{(t+1)K_1(m(t+1))}$$

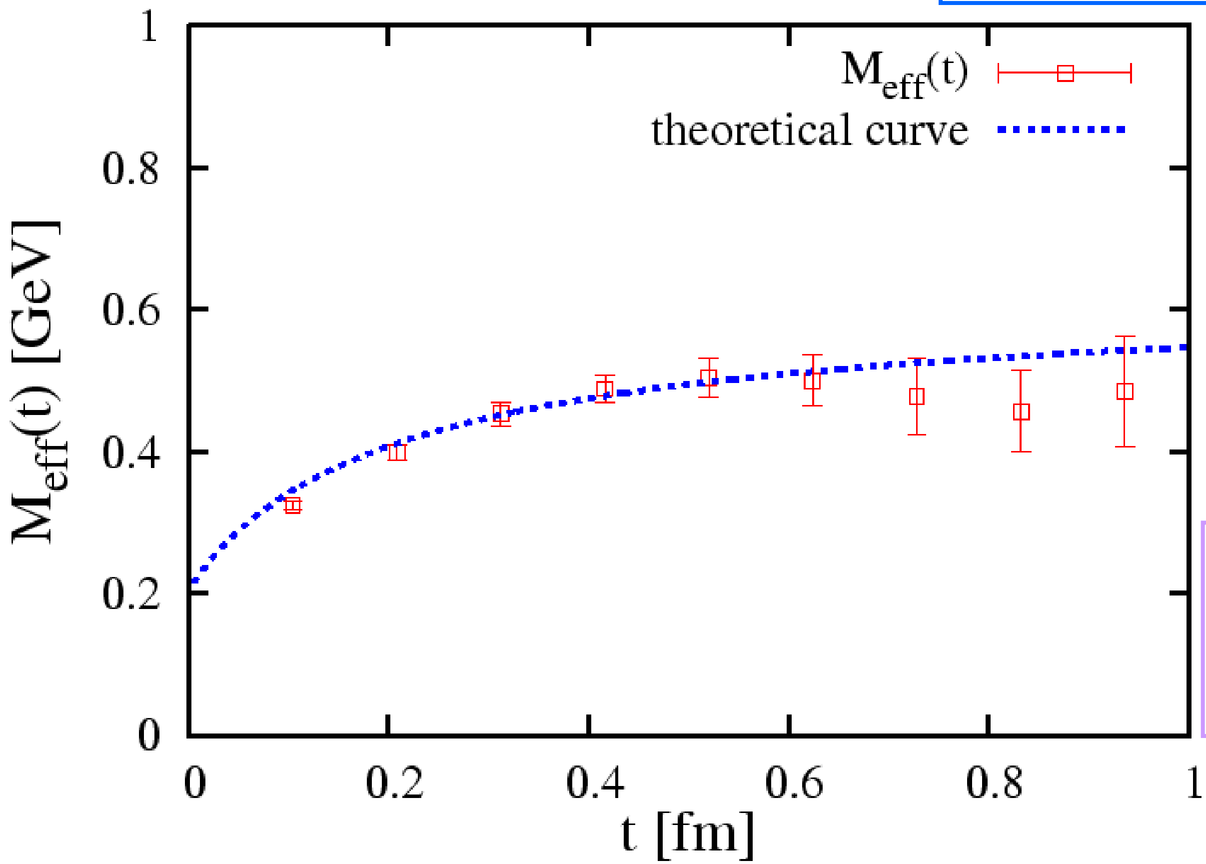


FIG. 10. The effective mass $M_{\rm eff}(t)$ of gluons in the Landau gauge. The symbols denote the lattice-QCD data at $\beta = 6.0$, and the solid line denotes the theoretical curve of Eq. (55) derived from the Yukawa-type propagator with $m = 0.624$ GeV, the same value used in Fig. 7.

$$m \simeq 600\text{MeV}$$

Infrared Effective Gluon Mass

Increasing behavior of Effective Mass is reproduced

A good agreement between lattice QCD data and the theoretical curve derived from Yukawa-type propagator

Spectral Function of the Gluon field derived from Yukawa-type propagator

From analytical expression of zero-momentum propagator $D_0(t)$, we can derive Spectral Function $\rho(\omega)$ of Gluon field, associated with Yukawa-type Gluon propagator.

(For simplicity, we take continuum formalism with infinite space-time.)

Relation between spectral function $\rho(\omega)$ and temporal propagator $D_0(t)$ is given by *Laplace transformation*

$$D_0(t) = \int_0^\infty d\omega \; \rho(\omega) \; e^{-\omega t}.$$

(When the spectral function is given by a δ-function such as $\rho(\omega) \sim \delta(\omega - \omega_0)$, which corresponds to a single mass spectrum, one finds a familiar exponential damping correlator $D_0(t) \sim \exp(-\omega_0 t)$.)

For the physical state like hadrons, spectral function $\rho(\omega)$ gives probability factor, and is non-negative definite in the whole region of ω.

Spectral Function of the Gluon field derived from Yukawa-type propagator

We can derive analytical expression of
Spectral Function ρ(ω) of Gluon field by
Inverse Laplace transformation of temporal propagator $D_0(t)$.

$$\rho(\omega) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\; e^{\omega t}\; D_0(t)$$

$$= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\; e^{\omega t}\; 4\pi A t K_1(mt)$$

$$= \frac{1}{2\pi i}\int_{c'-i\infty}^{c'+i\infty} dt'\; e^{\omega' t'}\; \frac{4\pi A}{m^2} t' K_1(t'),$$

$\omega' \equiv \omega/m, \quad t' \equiv mt, \quad c' \equiv mc$

Spectral Function of the Gluon field derived from Yukawa-type propagator

Using an integral expression of modified Bessel function,

$$K_1(t) = \int_1^\infty d\omega\ e^{-\omega t}\frac{\omega}{(\omega^2-1)^{1/2}}$$
$$= \int_0^\infty d\omega\ e^{-\omega t}\frac{\omega}{(\omega^2-1)^{1/2}}\theta(\omega-1)$$

we obtain formula of Inverse Laplace transformation of modified Bessel function:

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\ e^{\omega t}\ K_1(t) = \frac{\omega}{(\omega^2-1)^{1/2}}\theta(\omega-1)$$

By differentiating this by ω, we find the following formula

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\ e^{\omega t}\ tK_1(t)$$
$$= -\frac{1}{(\omega^2-1)^{3/2}}\theta(\omega-1) + \frac{\omega}{(\omega^2-1)^{1/2}}\delta(\omega-1)$$
$$= -\frac{1}{(\omega^2-1)^{3/2}}\theta(\omega-1) + \frac{1}{\{2(\omega-1)\}^{1/2}}\delta(\omega-1)$$

Spectral Function of the Gluon field derived from Yukawa-type propagator

Then, we obtain Spectral Function $\rho(\omega)$ as

$$\rho(\omega) = \frac{1}{2\pi i}\int_{c'-i\infty}^{c'+i\infty} dt'\ e^{\omega' t'}\ \frac{4\pi A}{m^2} t' K_1(t'),$$

$$= -\frac{4\pi A/m^2}{(\omega'^2-1)^{3/2}}\theta(\omega'-1) + \frac{4\pi A/m^2}{\{2(\omega'-1)\}^{1/2}}\delta(\omega'-1)$$

$$= -\frac{4\pi A m}{(\omega^2-m^2)^{3/2}}\theta(\omega-m) + \frac{4\pi A/\sqrt{2m}}{(\omega-m)^{1/2}}\delta(\omega-m).$$

Eventually, we derive Spectral Function $\rho(\omega)$ of Gluon field, associated with 4-dim Yukawa-type propagator:

$$\rho(\omega) = -\frac{4\pi A m}{(\omega^2-m^2)^{3/2}}\theta(\omega-m) + \frac{4\pi A/\sqrt{2m}}{(\omega-m)^{1/2}}\delta(\omega-m)$$

(For more rigorous derivation, we avoid the singularity at $\omega = m$ by a suitable regularization, T. Iritani, H.S., H. Iida, Phys. Rev. D80 (2009) 114505 (20 pages).)

Spectral Function of Gluon Field obtained from Yukawa-type Propagator

$$\rho(\omega) = -\frac{4\pi A m}{(\omega^2 - m^2)^{3/2}}\theta(\omega - m) + \frac{4\pi A/\sqrt{2m}}{(\omega - m)^{1/2}}\delta(\omega - m)$$

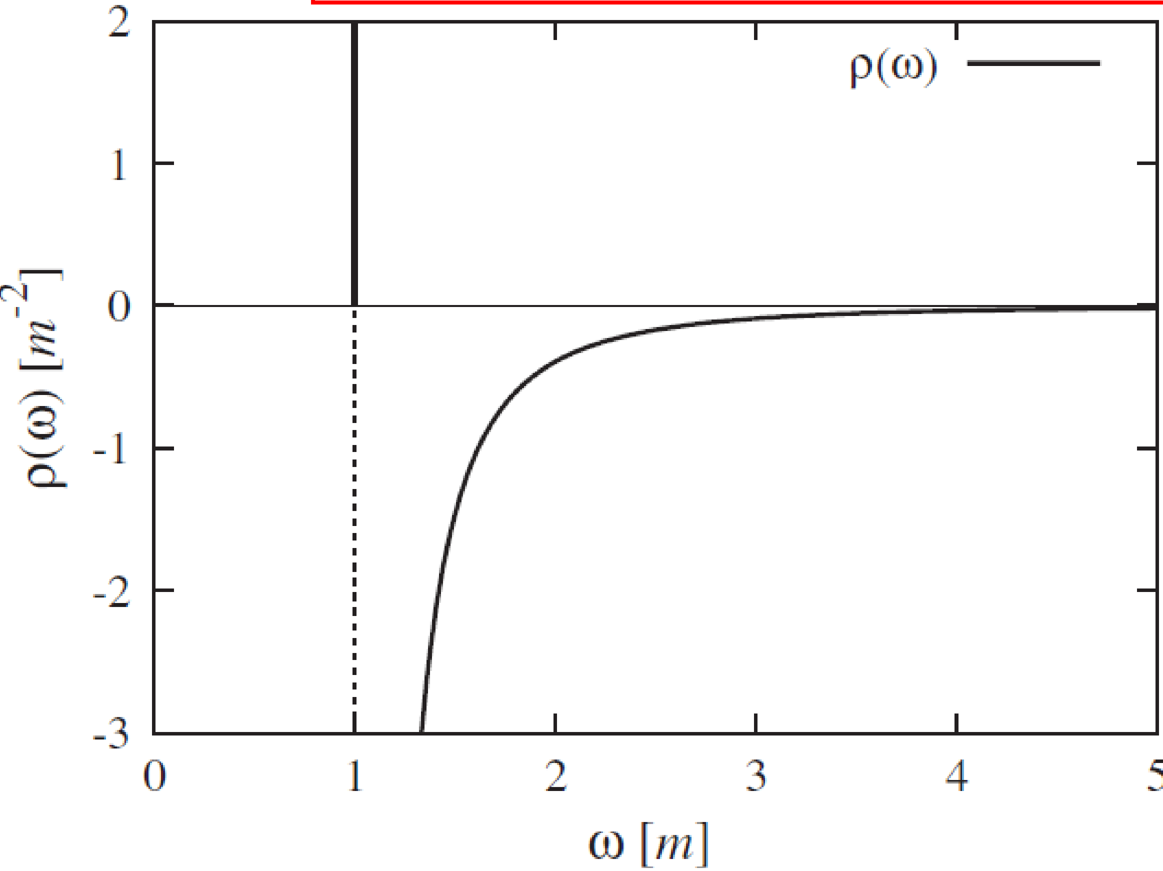


FIG. 11. The spectral function $\rho(\omega)$ of the gluon field, associated with the Yukawa-type propagator. The unit is normalized by the mass parameter $m \simeq 600$ MeV. As Eq. (67) indicates, $\rho(\omega)$ shows anomalous behaviors: it has a positive δ-functional peak with the residue of $+\infty$ at $\omega = m(+\varepsilon)$, and takes negative values for all of the region of $\omega > m$.

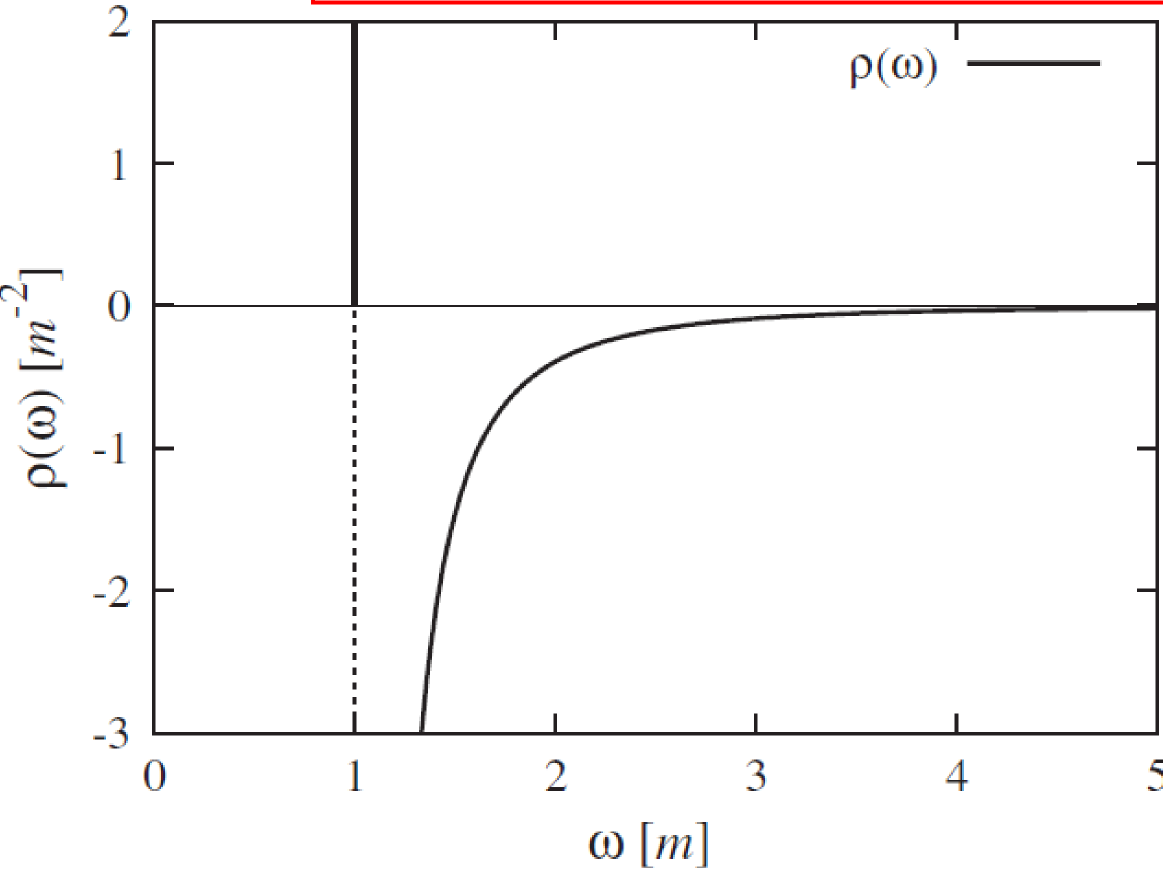
Remarkably, the obtained Gluon Spectral Function ρ(ω) is negative-definite for all the region of $\omega > m$, except for the positive δ-functional peak at $\omega = m$.

On finite-volume lattice, all the singularities are to be smeared, and ρ(ω) becomes finite everywhere. On the lattice, we conjecture that the spectral function ρ(ω) includes a narrow positive peak stemming from the δ-function at $\omega = m$ and a wider negative peak near $\omega \simeq m$ in the region of $\omega > m$.

T. Iritani, H.S., H. Iida, *Phys. Rev. D*80 (2009).

Spectral Function of Gluon Field obtained from Yukawa-type Propagator

$$\rho(\omega) = -\frac{4\pi A m}{(\omega^2 - m^2)^{3/2}}\theta(\omega - m) + \frac{4\pi A/\sqrt{2m}}{(\omega - m)^{1/2}}\delta(\omega - m)$$



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Gluon Spectral Function ρ(ω) includes both positive δ-functional peak with infinite residue and negative continuous part.

Coexistence of positive and negative parts in Gluon Spectral Function ρ(ω) is necessary to describe the increasing behavior of effective mass $M_{\mathrm{eff}}(t)$ of gluons.

T. Iritani, H.S., H. Iida, *Phys. Rev. D*80 (2009).

Spectral function and Effective Mass

Proof Effective mass is defined by temporal propagator $D_0(t)$

$$M_{\rm eff}(t) = \ln\{D_0(t)/D_0(t+1)\}$$

In spectral representation, temporal propagator is expressed as

$$D_0(t) = \sum_i c_i e^{-m_i t}$$

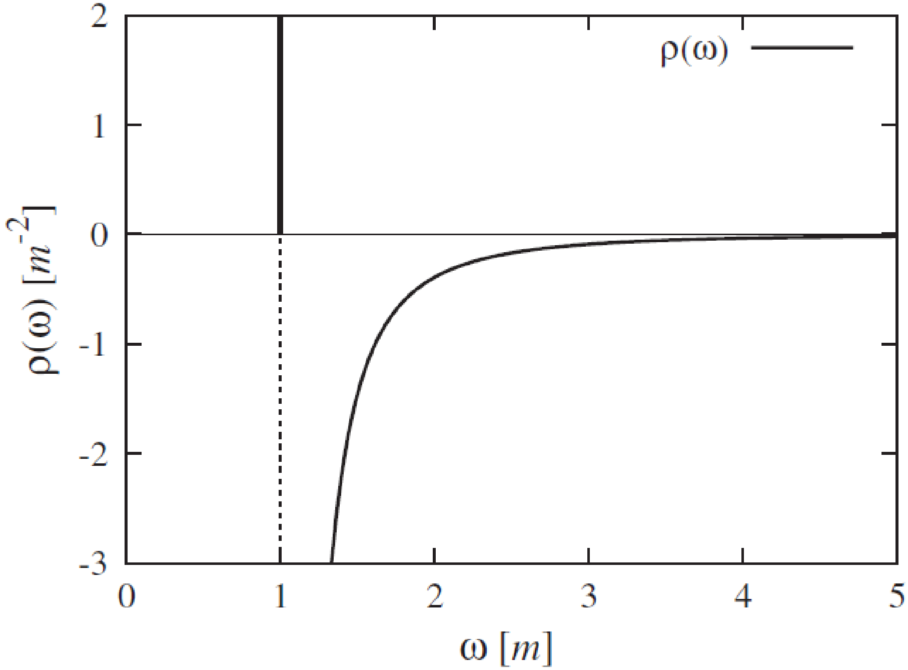
$$\frac{d}{dt}M(t) = -\frac{d^2}{dt^2}\ln\left(\sum_i c_i e^{-m_i t}\right) = -\frac{(\sum_i c_i e^{-m_i t})(\sum_i c_i m_i^2 e^{-m_i t}) - (\sum_i c_i m_i e^{-m_i t})^2}{(\sum_i c_i e^{-m_i t})^2}$$

If all the spectral weights are non-negative (or non-positive), time-derivative of effective mass $M_{\rm eff}(t)$ is always non-positive, due to Cauchy-Schwartz inequality, and effective mass $M_{\rm eff}(t)$ must be a decreasing function of time-variable t. This holds for all the hadronic correlators.

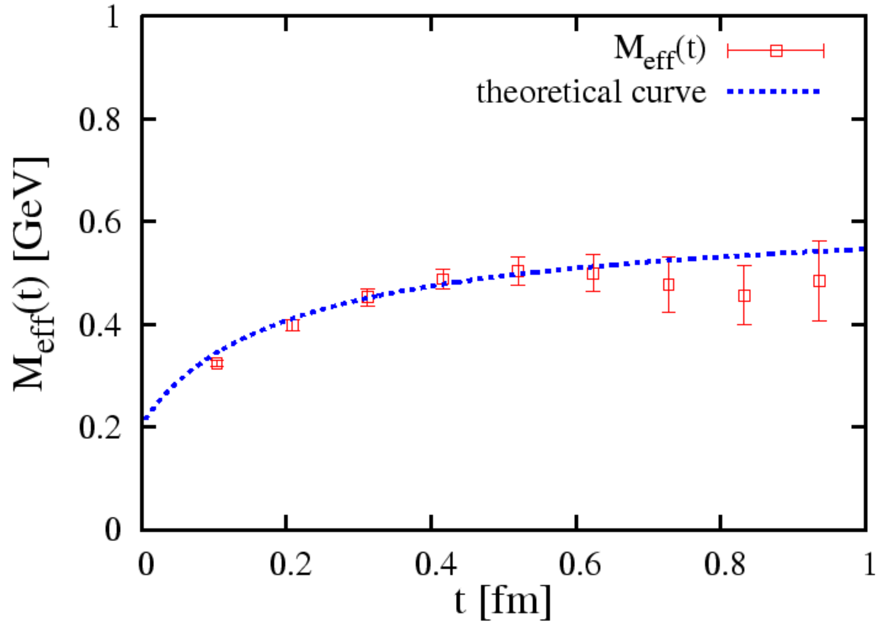
Hence, to describe the increasing behavior of effective mass $M_{\rm eff}(t)$ of gluons, the Spectral Function of Gluon must include Both Positive and Negative parts.

Spectral function and Effective Mass

$$\rho(\omega) = -\frac{4\pi A m}{(\omega^2 - m^2)^{3/2}}\theta(\omega - m) + \frac{4\pi A/\sqrt{2m}}{(\omega - m)^{1/2}}\delta(\omega - m)$$



⇒



Gluon Spectral Function ρ(ω) including positive and negative parts can realize increasing effective mass $M_{eff}(t)$ of gluons.

T. Iritani, H.S., H. Iida,
*Phys. Rev. D*80 (2009).

Possible Effective Dimensional Reduction in QCD

Next, we consider a possible physical meaning of 4-dimensional Yukawa-type propagation of gluons.

Landau-gauge Gluon Propagator is well described with *Four-dimensional Yukawa-type function* for r = 0.1~1.0fm.

$$D(r) \equiv \frac{1}{24} D^{aa}_{\mu\mu}(r) = A \frac{m}{r} e^{-mr} \qquad r \equiv |x| \equiv (x_\mu x_\mu)^{1/2}$$

Here, Yukawa function e^{-mr}/r is a natural form in 3-dim space, since it is obtained by 3-dim Fourier transformation of ordinary massive propagator $(p^2 + m^2)^{-1}$.

In fact, Yukawa-type propagator has "3-dimensional" property.

In this sense, as an interesting possibility, we propose to interpret this Yukawa-type behavior of Gluon Propagation as an "effective reduction of space-time dimension".

Effective Dimensional Reduction in Stochastic System
~ Parisi-Sourlas mechanism

Such a "dimensional reduction" sometimes occurs in stochastic systems, as Parisi and Sourlas pointed out for spin system in a random magnetic field.

[G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744-745 (1979).]

On Infrared Dominant diagrams, D-dimensional system coupled to Gaussian-Random external field is equivalent to (D − 2)-dimensional system without the external field.

For system coupled to Gaussian-random external source, space-time dimension of the theory seems to be reduced by two, owing to hidden supersymmetry.

Outline of Parisi-Sourlas mechanism

$$L(\varphi) = \frac{1}{2}(\partial\varphi)^2 + V(\varphi) = -\frac{1}{2}\varphi\Delta\varphi + V(\varphi)$$

In the presence of Gaussian random external field

$$L_{SS}[\Phi] \equiv -\frac{1}{2}\Phi\Delta_{SS}\Phi + V(\Phi)$$

SUSY structure
SUSY invariant

$$\Phi(x,\theta) = \varphi(x) + \bar{\theta}\psi(x) + \bar{\psi}(x)\theta + \theta\bar{\theta}\omega(x)$$

Superfield formalism

$L_{SS}[\Phi]$: θ–dependent part is a function of $x^2 + \bar{\theta}\theta$

$$\int dx^D d\theta\ f(x^2 + \bar{\theta}\theta) = \int d^{D-2}x\ f(x^2)$$

Dimensional Reduction

$(d\theta \equiv d\bar{\theta}\, d\theta)$

$$\int d^D x\, d\theta\, L_{SS}[\Phi(x,\theta)] = \int d^{D-2}x\, L_{SS}[\varphi(x)]$$

$$= \int d^{D-2}x\left(-\frac{1}{2}\varphi\Delta\varphi + V(\varphi)\right) = \underline{\int d^{D-2}x\ L(\varphi)}$$

Original theory in
2D-Reduced space-time
without external field

Possible Dimensional Reduction in QCD ~ Parisi-Sourlas mechanism

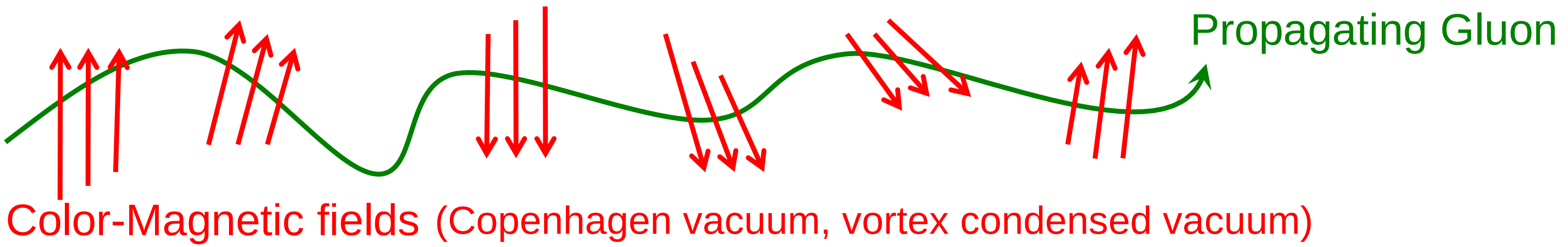
We note that Gluon propagation in QCD vacuum resembles the situation of system coupled to stochastic external field.

In fact, as is indicated by Large Positive Gluon Condensate in Minkowski space,

$$\frac{\alpha_s}{\pi}\left\langle G^a_{\mu\nu} G_a^{\mu\nu} \right\rangle = \frac{2\alpha_s}{\pi}\left\langle \mathrm{H}_a^2 - \mathrm{E}_a^2 \right\rangle = (200-300\,\mathrm{MeV})^4 > 0$$

⇒ QCD vacuum is filled with Color-Magnetic field, which is considered to be highly random at infrared scale.

Since gluons interact each other, propagating gluon is violently scattered by other gluon fields randomly condensed in QCD vacuum at infrared scale.



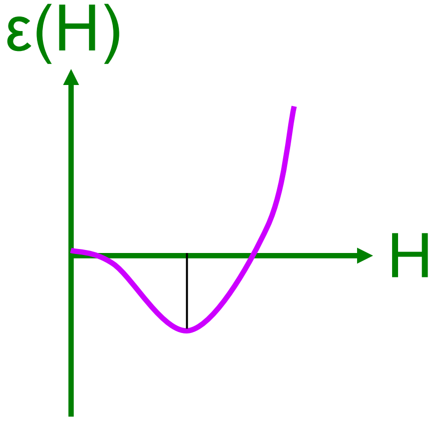
Schematic figure of Gluon Propagation in Quasi-Random Color-Magnetic field

Color Magnetic Instability of QCD~ Savvidy vacuum

G.K.Savvidy (1977):

Energy density ε(H) of SU(2) Yang-Mills theory in the presence of constant color-magnetic field H at 1 loop-level:

$$\varepsilon(H)-\varepsilon(0)=\frac{1}{2}H^2+\frac{11(gH)^2}{48\pi^2}\ln\frac{gH}{\mu^2}-i\frac{(gH)^2}{8\pi}$$



Minimum condition

$$\frac{\partial}{\partial H}\mathrm{Re}\{\varepsilon(H)\}=H+\frac{11g^2H}{24\pi^2}\left(\ln\frac{gH}{\mu^2}+\frac{1}{2}\right)=0$$

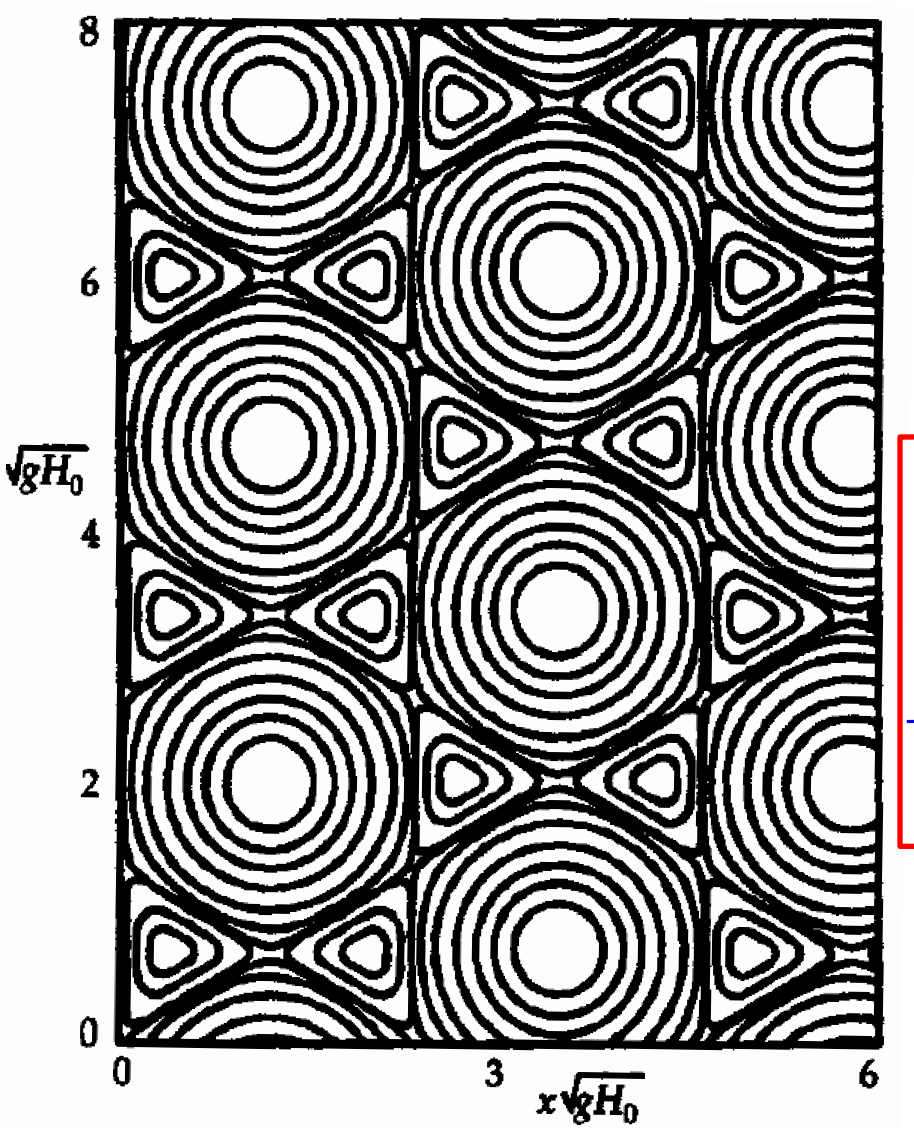
$$gH=\mu^2\exp\left[-\left(\frac{24\pi^2}{11g^2}+\frac{1}{2}\right)\right]$$

QCD has Color Magnetic Instability, and there occurs Spontaneous Generation of Color Magnetic field.

H $\neq 0$ i.e. $\langle G_{\mu\nu}G^{\mu\nu}\rangle > 0$

Color Magnetic Instability of QCD ~ Copenhagen vacuum

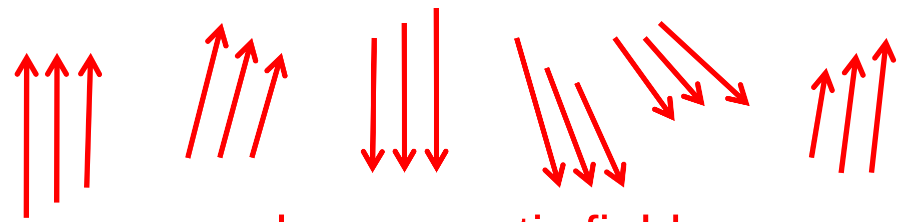
Ambjorn-Olesen NPB170 (1980)



Ambjorn-Olesen solution :
solution of 1 loop-level
effective action of QCD

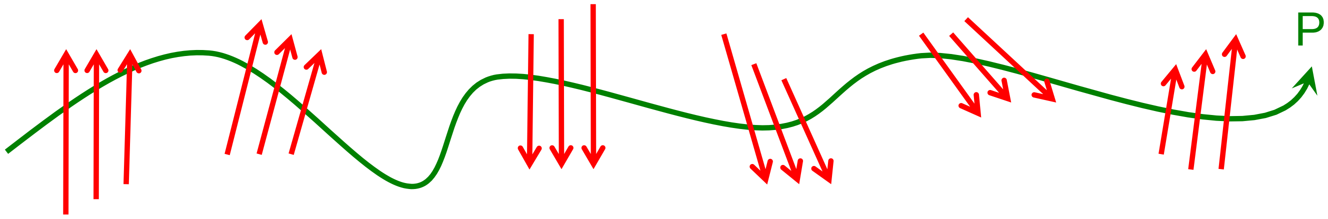
Color Magnetic Instability of QCD
→ Inhomogeneous Complicated
system of Color Magnetic field
~Copenhagen vacuum

To restore Lorentz symmetry,
Domain Structure appears
in QCD vacuum at macro scale
→Color Magnetic field is Randomly
oriented at infrared scale



color-magnetic fields

Possible Dimensional Reduction in QCD ~ Parisi-Sourlas mechanism



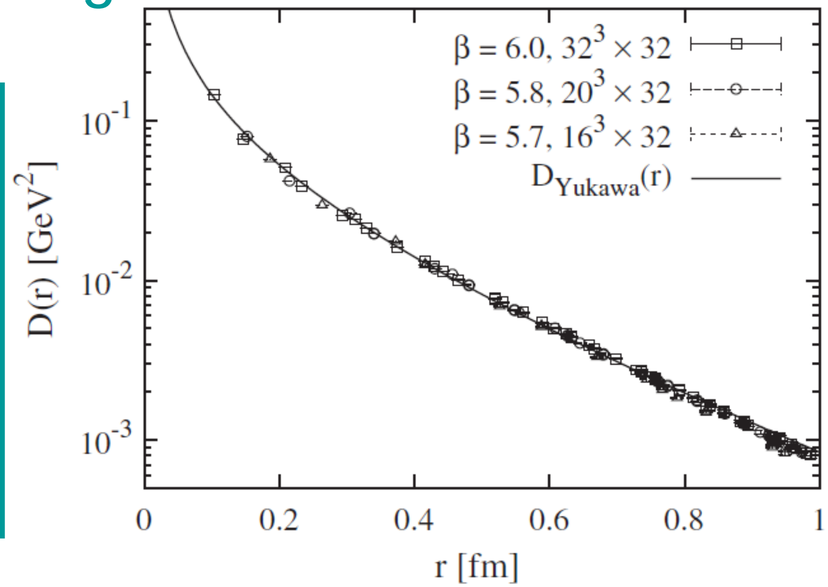
Schematic figure of Gluon Propagation in Quasi-Random Color-Magnetic field

As a generalization of the Parisi-Sourlas mechanism, we conjecture that Infrared Structure of a theory in the presence of Quasi-Random external field in Higher-dimensional space-time has a similarity to the theory without the external field in Lower-dimensional space-time.

From this viewpoint, Yukawa behavior of Gluon Propagation may indicate an "Effective Reduction of space-time Dimension" by one, due to Stochastic interaction between Propagating Gluon and other Infrared-Random Gluon fields condensed in the QCD vacuum.

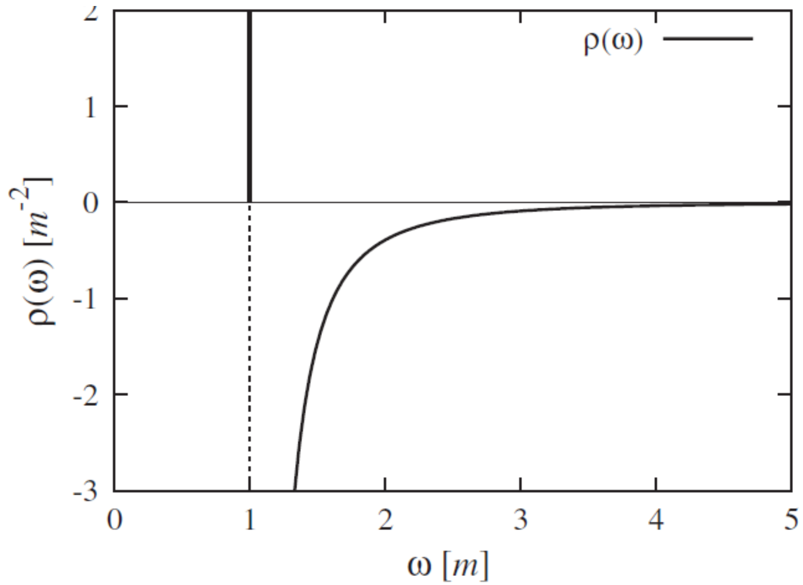
Summary and Concluding Remarks

Landau-gauge Gluon Propagator
$D_{\mu\mu}$ (x) is well described by
Yukawa-type function e^{-mr}/r with
m = 600MeV for r = 0.1~1.0 fm
in 4-dim. Euclidean space-time.



From Yukawa-type propagator,
we analytically derive

Gluon Spectral Function $\rho(\omega)$:
positive δ-functional peak and
negative continuous part



Reference: T. Iritani, H. S, H. Iida, Phys. Rev. D80 (2009) 114505 (20 pages),
"*Gluon-propagator functional form in the Landau gauge in SU(3) lattice QCD: Yukawa-type gluon propagator and anomalous gluon spectral function*".

Correction from Deep IR region

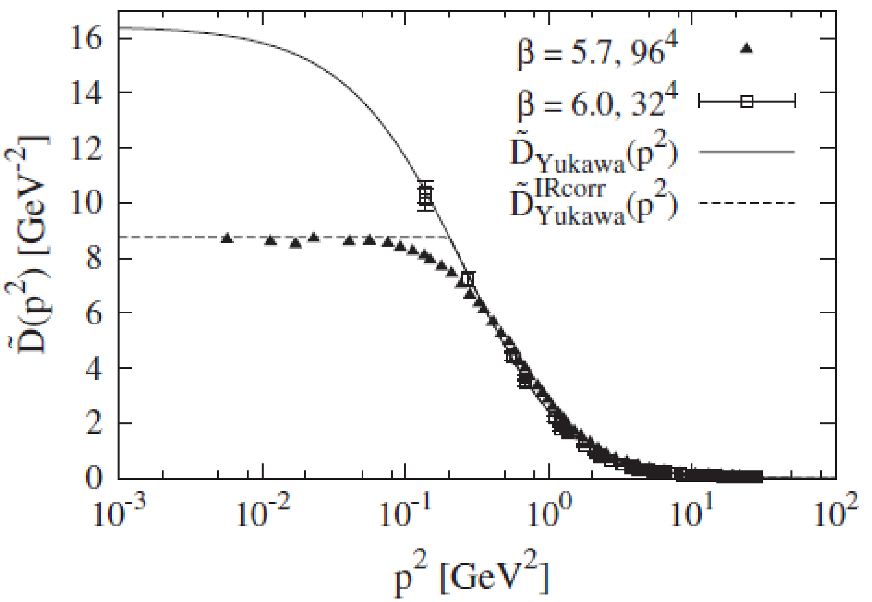


FIG. 13. The infrared behavior of the gluon propagator $\tilde{D}(p^2)$. The triangle symbols denote recent huge-volume lattice data taken from Ref. [32]. The solid line denotes the Yukawa-type propagator $\tilde{D}_{\rm Yukawa}(p^2)$, and the dashed line the deep-IR-corrected propagator $\tilde{D}^{\rm IR\,corr}_{\rm Yukawa}(p^2)$ with $p_{\rm IR} = 0.45$ GeV.

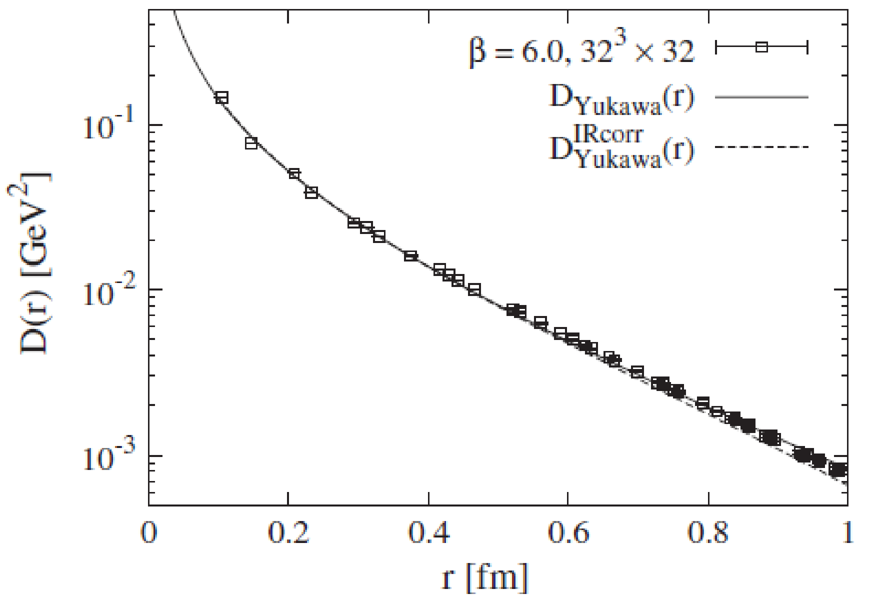


FIG. 14. The Yukawa-type propagator $D_{\rm Yukawa}(r)$ (solid line), and deep-IR-corrected propagator $D^{\rm IR\,corr}_{\rm Yukawa}(r)$ (dashed line), together with the lattice data. The difference between them is fairly small in the IR/IM region of $r = 0.1$–1.0 fm.

In Deep IR region, there is some deviation between huge-volume lattice data and Yukawa function. But, correction from Deep IR region is found to be very small in the region of $0.1\text{fm} < r < 1\text{fm}$.

Momentum Gluon Propagator in Landau Gauge in Lattice QCD

$$\tilde{D}(p^2) = \frac{1}{3(N_c^2-1)} \tilde{D}^{aa}_{\mu\mu}(p) = \sum_x e^{i\hat{p}\cdot x} D(r)$$

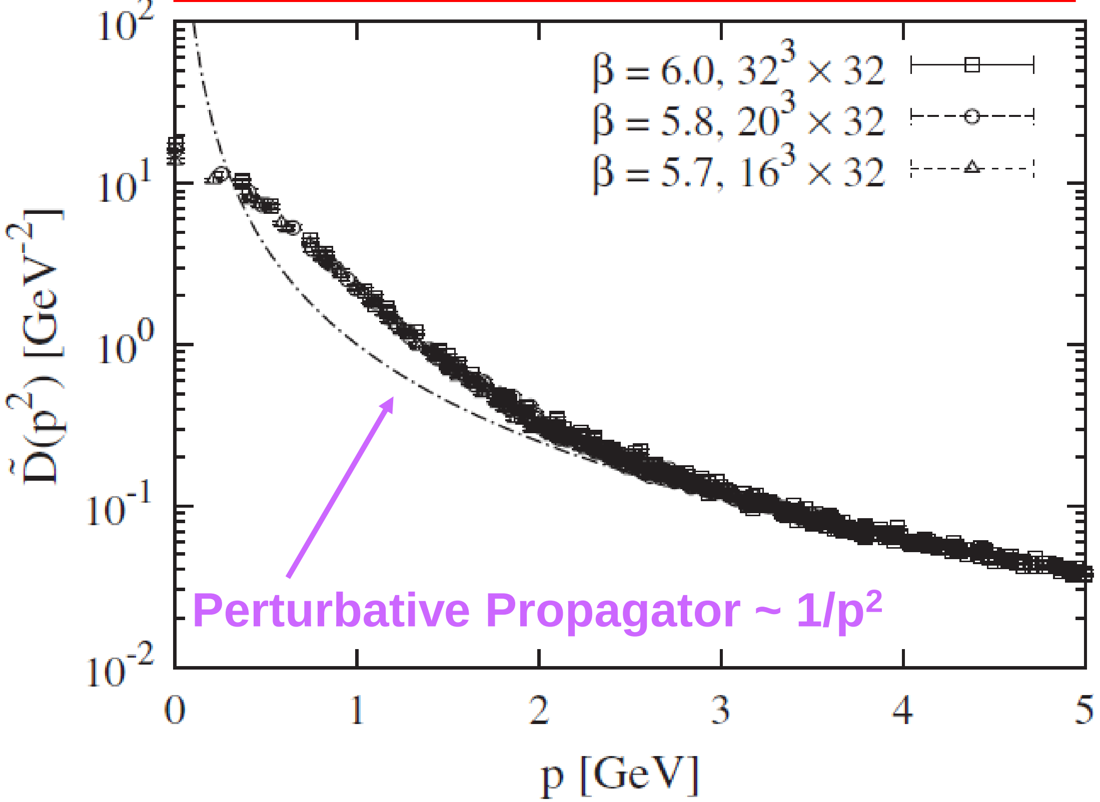


FIG. 2. Lattice-QCD results of the scalar-type gluon propagator $\tilde{D}(p^2) = \sum_x e^{i\hat{p}\cdot x} D(r)$ plotted against $p \equiv (p_\mu p_\mu)^{1/2}$ with the momentum $p_\mu = \frac{2}{a}\sin(\frac{\pi n_\mu}{L_\mu})$, in the Landau gauge at $\beta =$ 5.7, 5.8, and 6.0. We renormalize the propagator to satisfy the renormalize condition $D(p^2)|_{p^2=\mu^2} = 1/\mu^2$ at the scale $\mu =$ 4 GeV. The dash-dotted line denotes the tree-level massless propagator, $1/p^2$.

This lattice QCD result is almost the same as those of previous lattice studies.

The Gluon Propagator almost coincides with Perturbative Propagator above 3GeV.

Function Form Analysis of Landau-Gauge Gluon Propagator

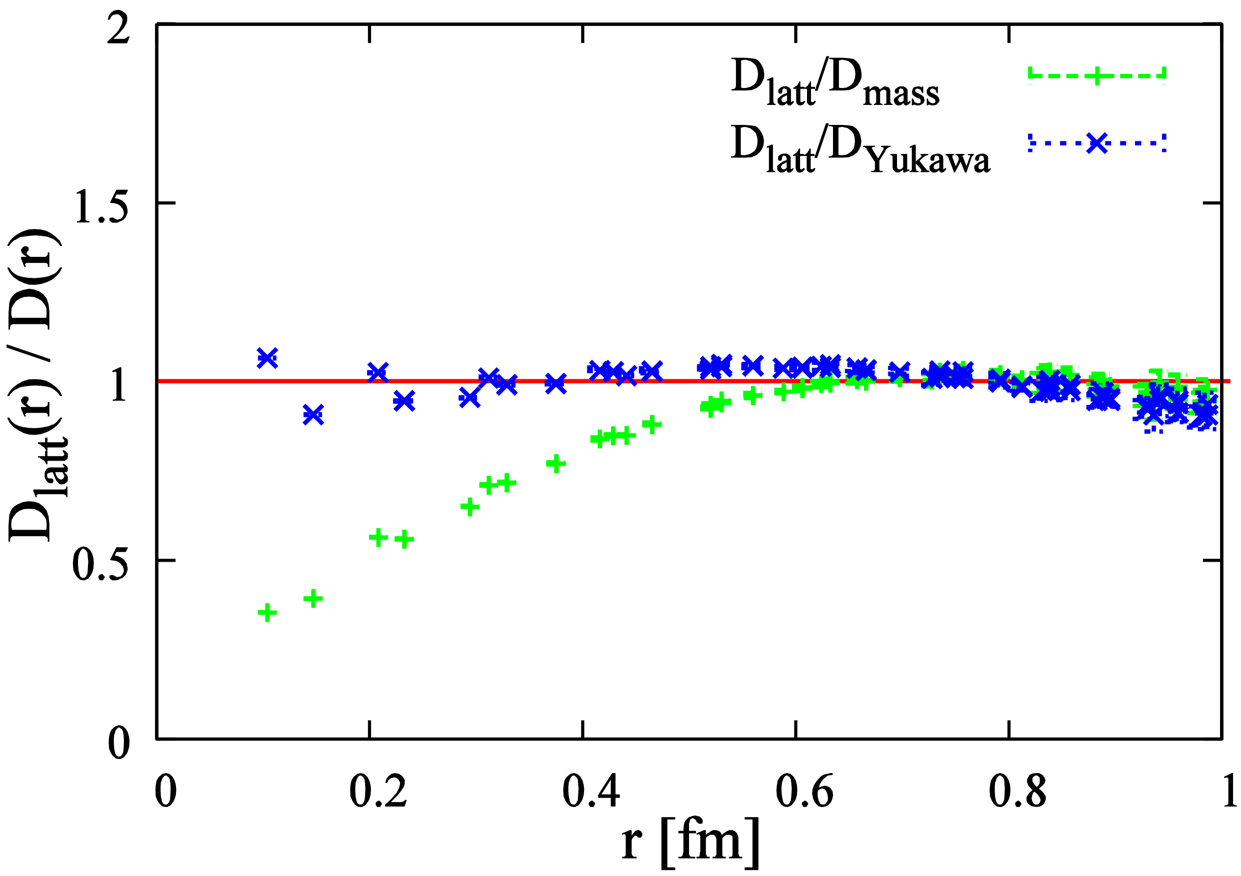


FIG. 6. The ratio of the lattice-QCD data $D_{\rm latt}(r)$ at $\beta = 6.0$ to the fit functions $D_{\rm mass}(r)$, $D_{\rm Yukawa}(r)$, and $D_{\rm dipole}(r)$ on the scalar-type gluon propagator, i.e., $D_{\rm latt}/D_{\rm mass}$, $D_{\rm latt}/D_{\rm Yukawa}$, and $D_{\rm latt}/D_{\rm dipole}$.