B meson spectrum and decay constants from $N_{\rm f} = 2$ simulations

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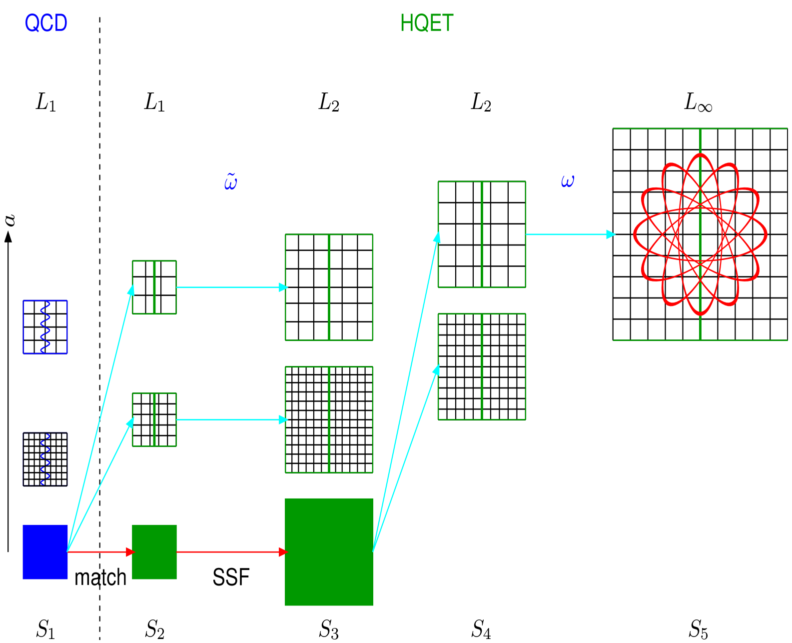
- Strategy and configurations set up
- B spectrum and b quark mass
- f_B decay constant

Strategy and simulations set up

Extraction of f_B, m_b and $m_{B^*} - m_B$ from lattice computation using Heavy Quark Effective Theory expanded up to $1/m$.

$$\mathcal{L}^{\mathrm{HQET},1/\mathrm{m}} = \mathcal{L}^{\mathrm{stat}} + m_{\mathrm{bare}}\mathcal{O}^{\mathrm{c.t.}} - \omega_{\mathrm{kin}}\mathcal{O}^{\mathrm{kin}} - \omega_{\mathrm{spin}}\mathcal{O}^{\mathrm{spin}}$$
$$A_0^{\mathrm{HQET},1/\mathrm{m}} = Z_A^{\mathrm{HQET}} A_0^{\mathrm{stat}} + c_A^{(1)} A_0^{(1)} + c_A^{(2)} A_0^{(2)}$$



Bare parameters of the HQET Lagrangian and currents need to be tuned.

It is done non perturbatively by imposing in a small volume $L_1 \sim$ 0.5 fm several matching conditions between correlators defined in QCD and their HQET counterpart:

$$\underbrace{\Phi_i^{\mathrm{QCD}}}_{\text{cont lim}} = \underbrace{f_{ij}(\omega_k)\Phi_j^{\mathrm{HQET}}}_{\text{finite } a}$$

Ultraviolet divergences of HQET are absorbed in the ω_k coefficients, determined in the $N_{\mathrm{f}} = 2$ Schrödinger Functional set up [talk by N. Garron]

$$m_B = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} E^{\mathrm{kin}} + \omega_{\mathrm{spin}} E_{\mathrm{spin}}$$
$$f_B\sqrt{m_B/2} = Z_A^{\mathrm{HQET}}(1 + b_A^{\mathrm{stat}} m_q) p^{\mathrm{stat}} \left(1 + \omega_{\mathrm{kin}} p^{\mathrm{kin}} + \omega_{\mathrm{spin}} p^{\mathrm{spin}} + c_A^{(1)} p^{\mathrm{A}^{(1)}}\right)$$

HYP1, $\theta_0 = 0.5$, $(\theta_1, \theta_2) = (0, 0.5)$

β	$L\,M_Q$	$a\,m_{\rm bare}$	$\ln(Z_A^{\rm HQET})$	$c_A^{(1)}/a$	$\omega_{\rm kin}/a$	$\omega_{\rm spin}/a$
5.2	13	1.112(23)	-0.169(32)	0.141(78)	0.465(06)	0.687(27)
	15	1.364(26)	-0.176(30)	0.172(74)	0.424(05)	0.614(24)
	18	1.714(29)	-0.187(21)	0.224(50)	0.371(03)	0.487(16)
5.3	13	0.881(20)	-0.142(33)	0.156(84)	0.518(06)	0.776(32)
	15	1.107(22)	-0.148(31)	0.187(79)	0.473(06)	0.693(29)
	18	1.420(25)	-0.159(21)	0.243(54)	0.414(04)	0.549(20)
5.5	13	0.470(16)	-0.105(35)	0.116(0.100)	0.652(08)	1.011(43)
	15	0.654(17)	-0.113(33)	0.160(94)	0.595(08)	0.903(39)
	18	0.907(19)	-0.122(23)	0.237(64)	0.521(05)	0.715(27)

HYP2, $\theta_0 = 0.5$, $(\theta_1, \theta_2) = (0, 0.5)$

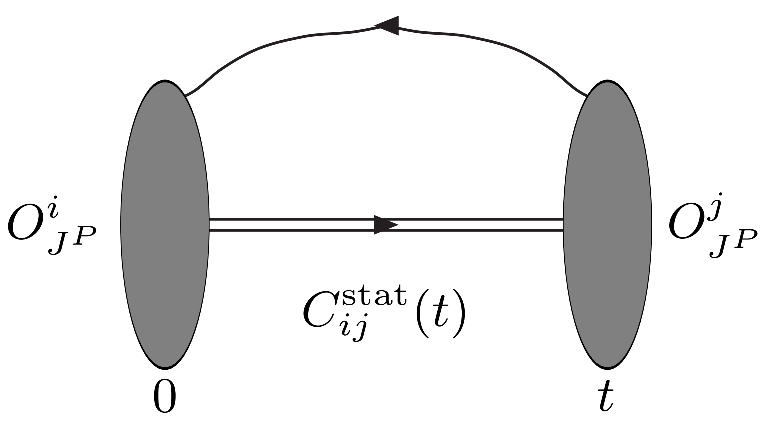
β	$L\,M_Q$	$a\,m_{\rm bare}$	$\ln(Z_A^{\rm HQET})$	$c_A^{(1)}/a$	$\omega_{\rm kin}/a$	$\omega_{\rm spin}/a$
5.2	13	1.132(23)	-0.126(29)	-0.518(73)	0.470(06)	0.854(34)
	15	1.385(26)	-0.121(27)	-0.431(70)	0.429(05)	0.763(30)
	18	1.737(29)	-0.115(19)	-0.305(47)	0.376(04)	0.605(20)
5.3	13	0.898(20)	-0.126(30)	-0.536(79)	0.522(07)	0.937(39)
	15	1.125(22)	-0.120(28)	-0.446(75)	0.476(06)	0.837(35)
	18	1.440(25)	-0.111(20)	-0.314(51)	0.417(04)	0.663(24)
5.5	13	0.479(16)	-0.136(32)	-0.638(96)	0.656(08)	1.160(50)
	15	0.664(17)	-0.128(30)	-0.530(90)	0.599(08)	1.036(44)
	18	0.920(19)	-0.115(21)	-0.370(62)	0.525(05)	0.821(31)

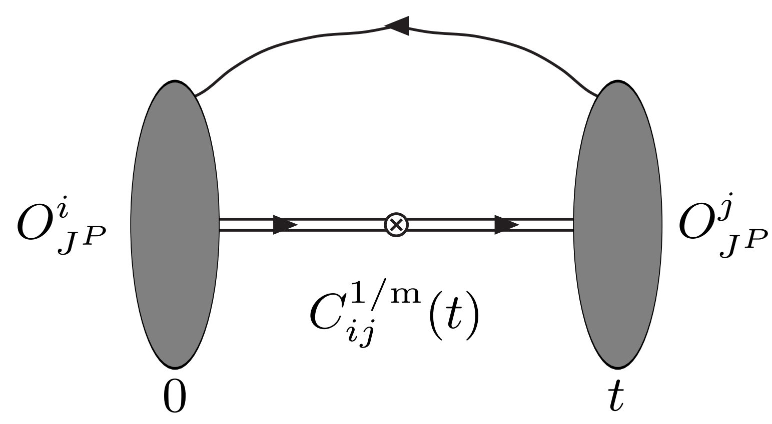
Measurement of HQET matrix elements using configurations ensembles with $N_\mathrm{f} = 2$ $\mathcal{O}(a)$ improved Wilson-Clover fermions produced by CLS (Berlin, Cern, Madrid, Mainz, Rome, Valencia).

CLS based

β	a (fm)	$L^3 \times T$	m_π (MeV)	#	traj. sep.
5.2	0.08	$32^3 \times 64$	700	110	16
		$32^3 \times 64$	370	160	16
5.3	0.07	$32^3 \times 64$	550	152	32
		$32^3 \times 64$	400	600	32
		$48^3 \times 96$	300	192	16
		$48^3 \times 96$	250	350	16
5.5	0.05	$32^3 \times 64$	430	250	20
		$48^3 \times 96$	430	30	16

Computation of matrices $N \times N$ of static-light correlators $C_{ij}(t)$ (Gaussian smearing, APE-blocking); statistics increased by using stochastic all to all light quark propagators.





Contribution of excited states to correlators efficiently suppressed by solving a Generalised Eigenvalue Problem [C. Michael, '85; M. Lüscher and U. Wolff, '90] [ALPHA, B. B. *et al*, '09]

$$
\begin{aligned}
C^{ij}(t)\,v_n^j(t,t_0) &= \lambda_n(t,t_0)\,C^{ij}(t_0)\,v_n^j(t,t_0) \\
aE_n^{\rm eff}(t,t_0) &= -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right) \\
Q_n^{\rm eff}(t,t_0) &= \frac{O^i(t)v_n^i(t,t_0)}{\sqrt{v_n^i(t,t_0)C^{ij}(t)v_n^j(t,t_0)}}\left(\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right)^{t/2a}
\end{aligned}
$$

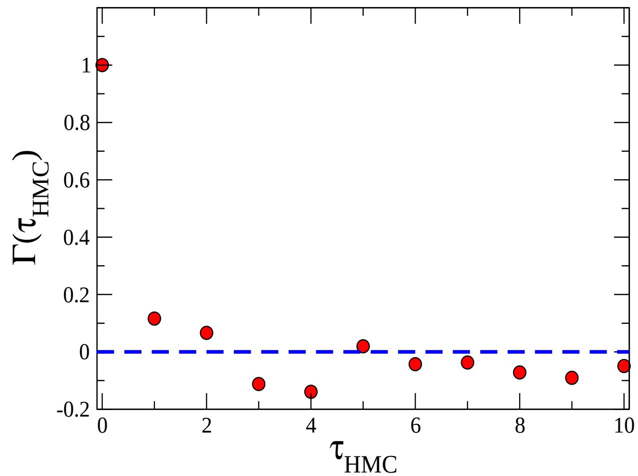
Estimate the $1/m$ corrections in HQET to static energies and matrix elements using GEVP is not an issue; it is enough to determine $\lambda_n^{\rm stat}$ and $v_n^{\rm stat}$:

$$
\begin{aligned}
E_n^{\rm eff}(t,t_0) &= E_n^{\rm eff,stat}(t,t_0)+\omega E_n^{\rm eff,1/m}(t,t_0)+O(\omega^2) \\
aE_n^{\rm eff,stat}(t,t_0) &= -\ln\left(\frac{\lambda_n^{\rm stat}(t+a,t_0)}{\lambda_n^{\rm stat}(t,t_0)}\right) \\
E_n^{\rm eff,1/m}(t,t_0) &= \frac{\lambda_n^{\rm 1/m}(t,t_0)}{\lambda_n^{\rm stat}(t,t_0)}-\frac{\lambda_n^{\rm 1/m}(t+a,t_0)}{\lambda_n^{\rm stat}(t+a,t_0)} \\
\frac{\lambda_n^{\rm 1/m}(t,t_0)}{\lambda_n^{\rm stat}(t,t_0)} &= v_{n\,i}^{\rm stat}(t,t_0)\left[\frac{C_{ij}^{\rm 1/m}(t)}{\lambda_n^{\rm stat}(t,t_0)}-C_{ij}^{\rm 1/m}(t_0)\right]v_{n\,j}^{\rm stat}(t,t_0)
\end{aligned}
$$

B spectrum and b quark mass

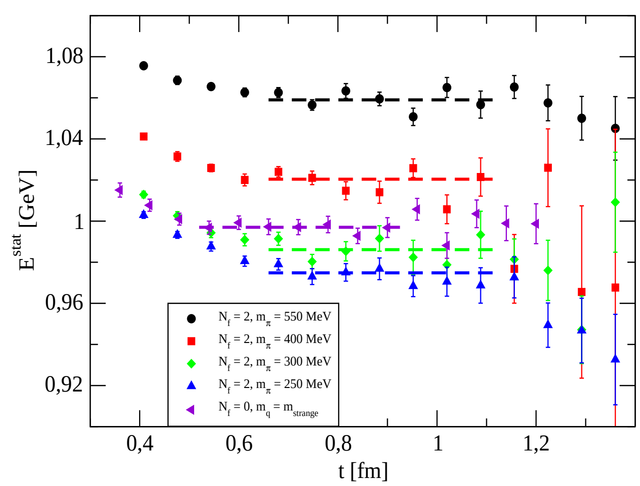
With our choice of separation in trajectories between 2 measurements, autocorrelation time of static-light correlators seem pretty small.

$a \sim 0.07$ fm $m_\pi = 400$ MeV $C_{\rm smear}(t \sim 1$ fm)



A comparison with quenched data indicates that the method to extract static energies is still working as far as the statistical uncertainty is concerned.

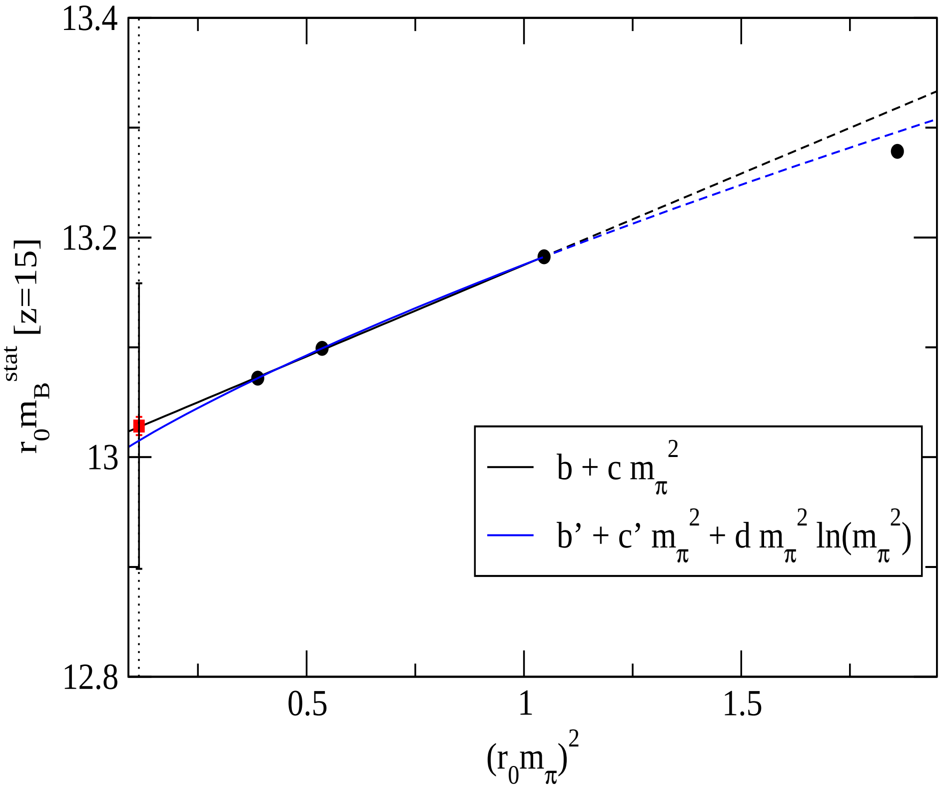
$a(N_{\rm f} = 2) \sim 0.07$ fm $a(N_{\rm f} = 0) \sim 0.06$ fm $N = 5$ $t_0 = 5a$



Correction terms in GEVP: $E_n^{\rm eff,stat}(t,t_0) = E_n^{\rm stat} + \beta_n^{\rm stat} e^{-\Delta E_{N+1,n} t}$

The time-range we have chosen to fit the plateaux is safe because the correction terms are found small at $t_0 > 0.3$ fm.

$a \sim 0.07$ fm $N = 5$ $t_0 = 5a$



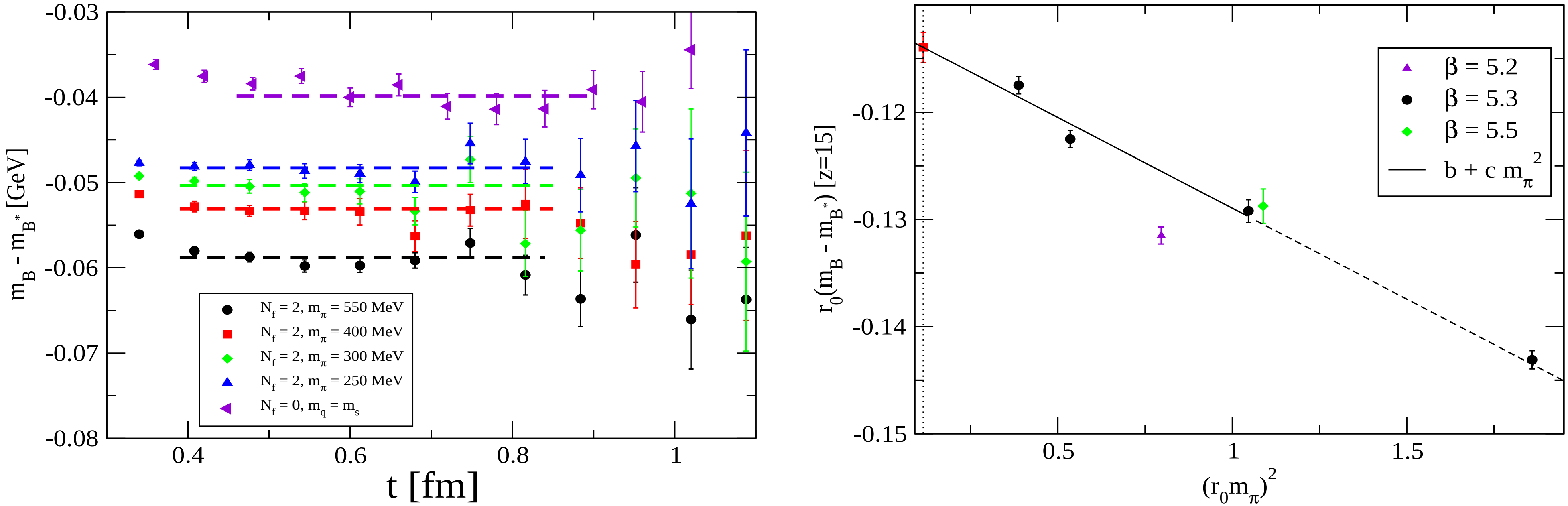
A systematic uncertainty is introduced by extrapolating to the chiral limit. However a larger source of uncertainty comes from the error on r_0/a, needed in the continuum extrapolation. Uncertainty from the scale: $r_0 = 0.475 \pm 0.025$ fm.

From $aE^{\rm stat}$, $am_{\rm bare}^{{\rm stat},\,I}$ and m_B, one can deduce m_b:

$$\overline{m_b}(m_b)^{\rm stat} = \underbrace{4.255(25)_{r_0}(50)_{\text{stat+renorm}}(?)_a \text{ GeV}}_{\text{Preliminary}}$$

The situation of $O(1/m)$ terms for energies is encouraging.

$N = 5 \quad t_0 = 4a$



Continuum limit of $r_0(m_{B^*} - m_B)$ to be checked.

$r_0(m_{B_s^*} - m_{B_s})^{\mathrm{HQET},1/m}_{N_\mathrm{f}=0} = 0.075(8)$ [ALPHA, B. B. *et al*, '10]

Finally we obtain $\overline{m_b}(m_b)^{\mathrm{HQET},1/m}_{N_\mathrm{f}=2} = \underbrace{4.276(25)_{r_0}(50)_{\text{stat+renorm}}(?)_a \text{ GeV}}_{}$

Preliminary

$\overline{m_b}(m_b)^{\mathrm{HQET},1/m}_{N_\mathrm{f}=0} = 4.320(40)_{r_0}(48)$ GeV [M. Della Morte *et al*, '06]

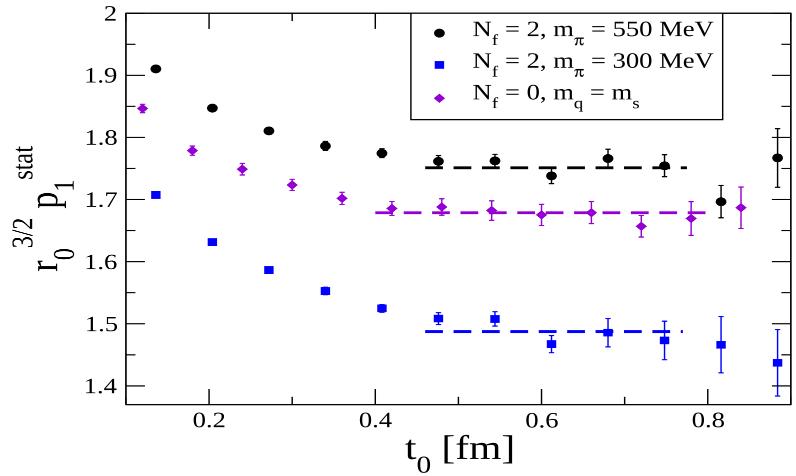
$\overline{m_b}(m_b)^{\text{sum rules}} = 4.163(16)$ GeV [K. Chetyrkin *et al*, '09]

Interesting to compare $\overline{m_b}(m_b)$ measured with the B spectrum with its counterpart estimated from the B_s spectrum in a partially quenched set up (need to know κ_s).

f_B decay constant

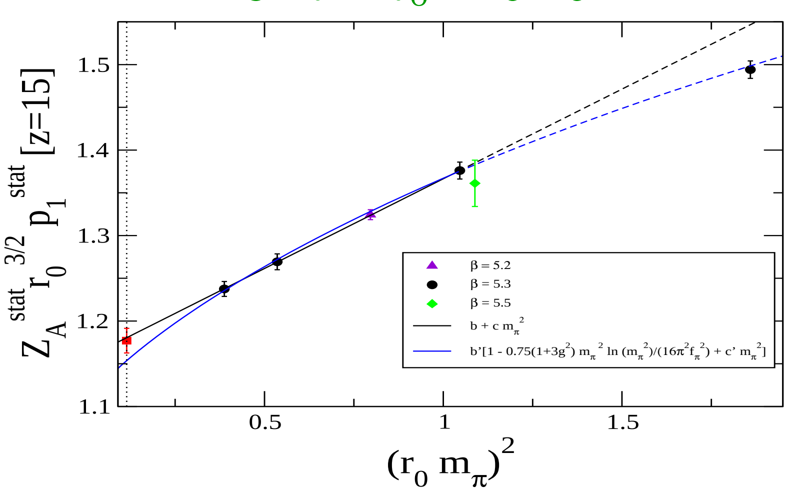
The GEVP method is working as in the quenched case for the static matrix element.

$N = 3 \quad t - t_0 \sim 0.25$ fm



Chiral extrapolation reliable; with $g = 0.497(3)$ [talk by M. Donnellan] HMχPT describes rather well our data. To be confirmed by one pion mass more, for instance lighter than 250 MeV.

$N = 3 \quad t - t_0 \sim 0.25$ fm

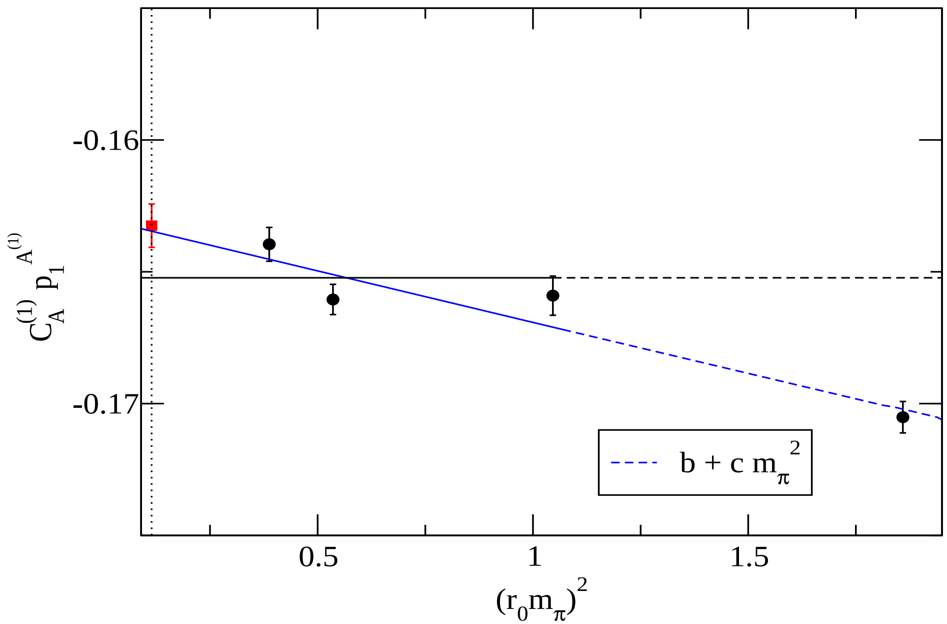


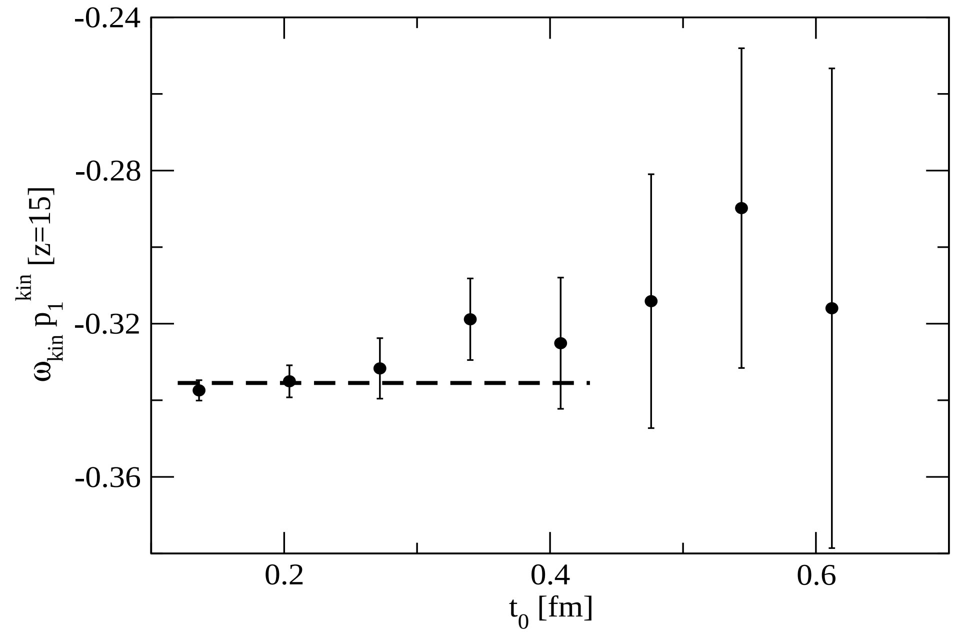
$$f_B\sqrt{m_B/2} = Z_A^{\rm HQET}(1+b_A^{\rm stat}m_q)p^{\rm stat}\left(1+\omega_{\rm kin}p^{\rm kin}+\omega_{\rm spin}p^{\rm spin}+c_A^{(1)}p^{{\rm A}^{(1)}}\right)$$

The situation of $O(1/m)$ terms correction to $f_B^{\rm stat}$ might be more problematic.

Uncertainties come from the chiral extrapolation and in the extraction of the signal itself.

$a \sim 0.07$ fm $\quad N=4 \quad t-t_0 \sim 4a$





However their contribution to uncertainties on $f_B^{\rm HQET,\,1/m}$ drops to ~ 2 %.

We are at the beginning of this analysis. In particular the extrapolation to the continuum limit has to be performed.

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