Axial and pseudoscalar form-factors of the $\Delta^+(1232)$

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Outline

- Motivation
- Lattice calculation
- *preliminary* FF results
- Multipole decomposition
- Goldberger-Treiman Relation

Motivation

Form factors:

- describe the structure of hadrons
- provide input for phenomenological model builders & χPTs: e.g effective $\pi\Delta\Delta$ couplings
- test the Goldberger-Treiman relations

$\longrightarrow$ First lattice QCD calculation of axial form factors of the Δ^+ baryon.

Axial vertex decompositions

Isoscalar axial vertex:

$$A^{\mu}(x) = \overline{\psi}(x)\gamma^{\mu}\gamma_5 \frac{\tau^3}{2}\psi(x)$$

$$\langle\Delta(p_f, s_f)|A^{\mu}|\Delta(p_i, s_i)\rangle = \overline{u}_{\alpha}(p_f, s_f)\left[\mathcal{O}^{\mu \mathrm{A}}\right]^{\alpha\beta} u_{\beta}(p_i, s_i),$$

with

$$\mathcal{O}^{\mu \mathrm{A}} = -g^{\alpha\beta}\left(g_1(q^2)\gamma^{\mu}\gamma^5 + g_3(q^2)\frac{q^{\mu}}{2M_{\Delta}}\gamma^5\right) + \frac{q^{\alpha}q^{\beta}}{4M_{\Delta}^2}\left(h_1(q^2)\gamma^{\mu}\gamma^5 + h_3(q^2)\frac{q^{\mu}}{2M_{\Delta}}\gamma^5\right)$$

and Rarita Schwinger spinors $\overline{u}$, u.

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Pseudoscalar vertex decompositions

Similarly, for the pseudoscalar vertex:

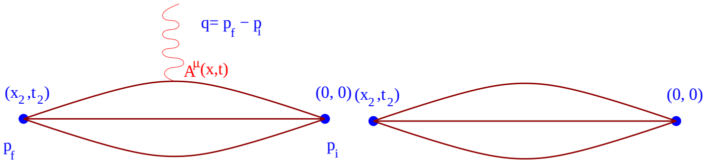
$$P(x) = \overline{\psi}(x)\gamma_5 \frac{\tau^3}{2}\psi(x)$$

$$\langle \Delta(p_f, s_f)|P|\Delta(p_i, s_i)\rangle = \overline{u}_\alpha(p_f, s_f) \left[\mathcal{O}^{\mathrm{PS}}\right]^{\alpha\beta} u_\beta(p_i, s_i),$$

with

$$\mathcal{O}^{\mathrm{PS}} = -g^{\alpha\beta}\left(\tilde{g}(q^2)\gamma^5\right) + \frac{q^\alpha q^\beta}{4M_\Delta^2}\left(\tilde{h}(q^2)\gamma^5\right)$$

3-point & 2-point functions



The two-point and three-point functions of interest are:

$$G^{\mathrm{A}}_{\sigma\mu\tau}(\Gamma^\nu, \vec{q}, t) = \sum_{\vec{x},\vec{x}_f} e^{+i\vec{x}\cdot\vec{q}}\Gamma^\nu_{\alpha'\alpha}\langle\chi_{\sigma\alpha}(\vec{x}_f, t_f)A_\mu(\vec{x}, t)\overline{\chi}_{\tau\alpha'}(0,\vec{0})\rangle$$

$$G^{\mathrm{PS}}_{\sigma\tau}(\Gamma^\nu, \vec{q}, t) = \sum_{\vec{x},\vec{x}_f} e^{+i\vec{x}\cdot\vec{q}}\Gamma^\nu_{\alpha'\alpha}\langle\chi_{\sigma\alpha}(\vec{x}_f, t_f)P(\vec{x}), t\overline{\chi}_{\tau\alpha'}(0,\vec{0})\rangle,$$

$$G_{\sigma\tau}(\Gamma^\nu, \vec{p}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{p}}\Gamma^\nu_{\alpha'\alpha}\langle\chi_{\sigma\alpha}(\vec{x}_f, t)\overline{\chi}_{\tau\alpha'}(0,\vec{0})\rangle$$

with

$$\Gamma^4 = \frac{1}{4}(\mathbf{1}+\gamma^4), \qquad \Gamma^k = \frac{i}{4}(\mathbf{1}+\gamma^4)\gamma_5\gamma_k, \qquad k = 1, 2, 3.$$

Some algebra...

..leads to:

$$G_{\sigma\tau}(\Gamma^\nu, \vec{p}, t) = \frac{M_\Delta}{E_{\Delta(p)}} |Z|^2 \, e^{-E_{\Delta(p)}t} \mathrm{tr}\left[\Gamma^\nu \Lambda^E_{\sigma\tau}(p)\right] + \text{excited states}$$

$$G^{\mathrm{A}}_{\sigma\mu\tau}(\Gamma^\nu, \vec{q}, t) = \frac{M_\Delta}{E_{\Delta(p)}} |Z|^2 \, e^{-M_{\Delta(p)}(t_f-t)} e^{-E_{\Delta(p)}t} \mathrm{tr}\left[\Gamma^\nu \Lambda^E_{\sigma\sigma'}(p) \mathcal{O}^{E,A}_{\sigma'\mu\tau'} \Lambda^E_{\tau\tau'}(p)\right] + \text{e. s.}$$

$$G^{\mathrm{PS}}_{\sigma\tau}(\Gamma^\nu, \vec{q}, t) = \frac{M_\Delta}{E_{\Delta(p)}} |Z|^2 \, e^{-M_{\Delta(p)}(t_f-t)} e^{-E_{\Delta(p)}t} \mathrm{tr}\left[\Gamma^\nu \Lambda^E_{\sigma\sigma'}(p) \mathcal{O}^{E,PS}_{\sigma'\tau'} \Lambda^E_{\tau'\tau}(p)\right] + \text{e. s.}$$

(using Euclidean quantities)

Rid ourselves of unknowns

Eliminate unknown Z-factors and leading time-dependence by looking at ratios:

$$R^{A}_{\sigma\mu\tau}(\Gamma,\vec{q},t) = \frac{G^{A}_{\sigma\mu\tau}(\Gamma,\vec{q},t)}{G_{kk}(\Gamma^4,\vec{0},t_f)}\sqrt{\frac{G_{kk}(\Gamma^4,\vec{p}_i,t_f-t)G_{kk}(\Gamma^4,\vec{0},t)G_{kk}(\Gamma^4,\vec{0},t_f)}{G_{kk}(\Gamma^4,\vec{0},t_f-t)G_{kk}(\Gamma^4,\vec{p}_i,t)G_{kk}(\Gamma^4,\vec{p}_i,t_f)}},$$

$$R^{PS}_{\sigma\tau}(\Gamma,\vec{q},t) = \frac{G^{PS}_{\sigma\tau}(\Gamma,\vec{q},t)}{G_{kk}(\Gamma^4,\vec{0},t_f)}\sqrt{\frac{G_{kk}(\Gamma^4,\vec{p}_i,t_f-t)G_{kk}(\Gamma^4,\vec{0},t)G_{kk}(\Gamma^4,\vec{0},t_f)}{G_{kk}(\Gamma^4,\vec{0},t_f-t)G_{kk}(\Gamma^4,\vec{p}_i,t)G_{kk}(\Gamma^4,\vec{p}_i,t_f)}}$$

,

At large $t_f - t$ and t one finds this is constant!

$$R_{\sigma(\mu)\tau}(\Gamma,\vec{q},t)^X \longrightarrow C\Pi^X_{\sigma(\mu)\tau} = C\,\mathrm{tr}\left[\Gamma\Lambda_{\sigma\sigma'}\mathcal{O}^X_{\sigma(\mu)\tau}\Lambda_{\tau'\tau}\right],$$

with

$$C \equiv \sqrt{\frac{3}{2}}\left[\frac{2E_{\Delta(p_i)}}{M_\Delta} + \frac{2E^2_{\Delta(p_i)}}{M^2_\Delta} + \frac{E^3_{\Delta(p_i)}}{M^3_\Delta} + \frac{E^4_{\Delta(p_i)}}{M^4_\Delta}\right]^{-\frac{1}{2}},$$

$$\Pi^{X}_{\sigma(\mu)\tau} = \mathrm{tr}\left[\Gamma \Lambda_{\sigma\sigma'} \mathcal{O}^{X}_{\sigma(\mu)\tau} \Lambda_{\tau'\tau}\right],$$

- compute on the lattice specific summation combinations of σ and τ
- work out R.H.S as linear combinations of the form-factors (g_1, g_3, h_1, h_3),
- coefficients are functions of E_i, m_Δ, Q^2 calculated from the above
- solve linear system for each q^2

An example: "Axial Type II"

$$\Pi^{II\mathrm{A}}_{\mu}(q) = \sum_{\sigma,\tau=1}^{3} T_{\sigma\tau}\,\mathrm{tr}\left[\Gamma^4 \Lambda_{\sigma\sigma'}(p_f)\mathcal{O}_{\sigma'\mu\tau'}\Lambda_{\tau'\tau}(p_i)\right]$$

with $\Gamma^4 = \frac{1}{4}(\gamma_4 + \mathbf{1})$ $\quad T_{\sigma\tau} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$

For $\mu = 4$ we find

$$\Pi^{II\mathrm{A}}_{\mu=4}(\vec{p}, E) = \frac{i}{9m^2}\left[(E+4m)(g_1 - \tau g_3) + \frac{\tau}{2}(E+m)(h_1 - \tau h_3)\right](p_1 + p_2 + p_3)$$

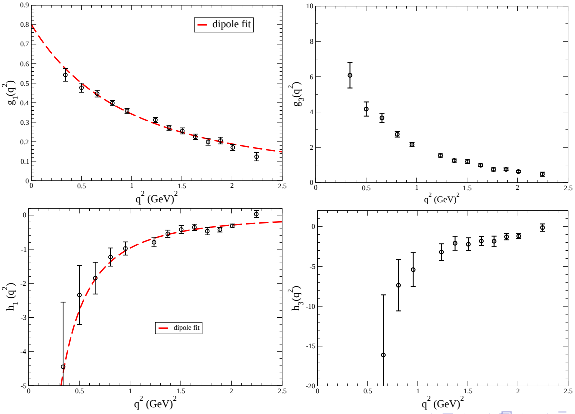
with $\tau = \frac{Q^2}{(2m_\Delta)^2}$

Lattice simulation

- Preliminary results from an un-improved quenched Wilson ensemble:
  $N_{\text{cfg}} = 200$, $32^3 \times 64$, $\beta = 6.0$, $a^{-1} = 2.14(6)$ GeV,
  $\kappa = 0.1554$
- Use sequential source method to get 3-pt functions

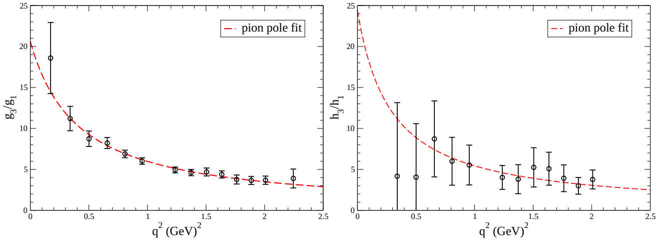
Axial form factor results

g_1, g_3, h_1 and h_3



Axial form factor ratios

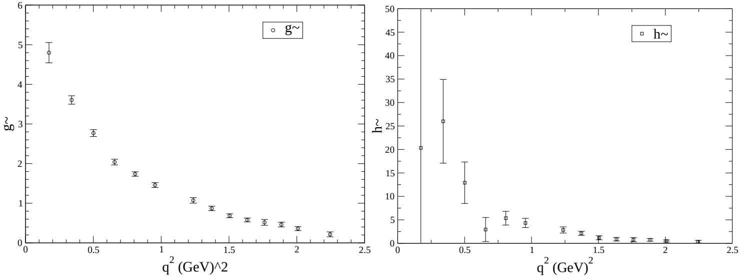
$g_3/g_1, h_3/h_1$



$m_\pi = 0.563(4)\text{MeV}$

Pseudoscalar form factor results

$\tilde{g}$ and $\tilde{h}$



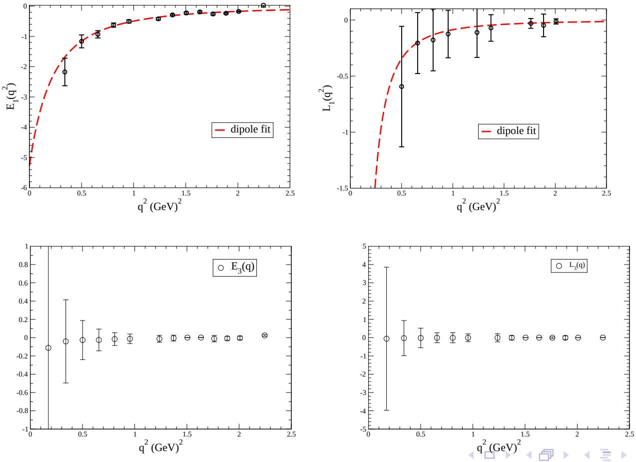
Multipole decomposition

Multipole form factors E_1, E_3, L_1, L_3 are related to physical quantities in the multipole expansion.

$$
\begin{aligned}
g_1 &= \frac{3}{\sqrt{2}} E_1 + \sqrt{3} E_3 \\
(g_1 - \tau g_3) &= \sqrt{1+\tau}(3L_1 - L_3) \\
\tau(1+\tau) h_1 &= -3\sqrt{2}\tau E_1 - (2+\tau)\sqrt{3} E_3 \\
\tau(1+\tau)(h_1 - \tau h_3) &= -\sqrt{1+\tau}(-6\tau L_1 + (5+2\tau) L_3
\end{aligned}
$$

"no-deformation limit" $\longrightarrow E_3 = L_3 = 0,\ E_1 = \sqrt{2} L_1$

Multipole form-factors



Goldberger-Trieman relation

Recall

$$\langle \Delta(p_f, s_f)|P|\Delta(p_i, s_i)\rangle = \overline{u}_\alpha(p_f, s_f)\left[-g^{\alpha\beta}\left(\tilde{g}(q^2)\gamma^5\right) + \frac{q^\alpha q^\beta}{4M_\Delta^2}\left(\tilde{h}(q^2)\gamma^5\right)\right]^{\alpha\beta} u_\beta(p_i, s_i),$$

Define

$$2m_q\langle \Delta_{p_f}|P|\Delta_{p_i}\rangle \equiv \left(\frac{m_\Delta^2}{E_\Delta(\vec{p}_f)E_\Delta(\vec{p}_i)}\right)\frac{f_\pi m_\pi^2\left[g^{\alpha\beta}G_{\pi\Delta\Delta}(q^2) + \frac{q^\alpha q^\beta}{4m_\Delta^2}H_{\pi\Delta\Delta}(q^2)\right]}{(m_\pi^2 - q^2)}\overline{u}^\alpha\gamma^5\frac{\tau^3}{2}u^\beta$$

Now we identify

$$m_q\tilde{g} \equiv \frac{f_\pi m_\pi^2 G_{\pi\Delta\Delta}(q^2)}{(m_\pi^2 - q^2)} \qquad m_q\tilde{h} \equiv \frac{f_\pi m_\pi^2 H_{\pi\Delta\Delta}(q^2)}{(m_\pi^2 - q^2)}$$

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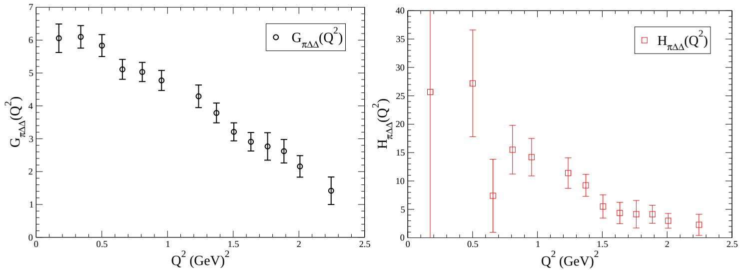
$\pi\Delta\Delta$ couplings



Goldberger Treiman relations

From the Ward identity, we can equate

$$\partial_\mu \langle \Delta | A^\mu | \Delta \rangle = 2m_q \langle \Delta | P | \Delta \rangle$$

LHS gives

$$\partial_\mu \langle \Delta | A^\mu | \Delta \rangle = 2m_\Delta \left[(g_1 + \tau g_3) g^{\alpha\beta} + (h_1 + \tau h_3) \frac{q^\alpha q^\beta}{4m_\Delta^2} \right] \overline{u}^\alpha \gamma^5 \frac{\tau^3}{2} u^\beta$$

$$2m_\Delta \left[(g_1 - \tau g_3) \right] = \frac{f_\pi m_\pi^2 G_{\pi\Delta\Delta}(q^2)}{(m_\pi^2 - q^2)}$$

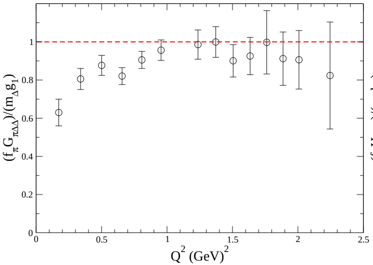
and

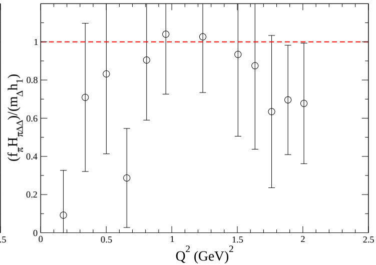
$$2m_\Delta \left[(h_1 - \tau h_3) \right] = \frac{f_\pi m_\pi^2 H_{\pi\Delta\Delta}(q^2)}{(m_\pi^2 - q^2)}$$

demanding that the g_3 and h_3 terms cancel the pole at $q^2 = m_\pi^2$ give us the Goldberger-Treiman relations:

$$f_\pi G_{\pi\Delta\Delta}(q^2) = m_\Delta g_1(q^2) \qquad f_\pi H_{\pi\Delta\Delta}(q^2) = m_\Delta h_1(q^2)$$

Goldberger-Treiman relations





Conclusions

- possible to extract Δ^+ axial form factors g_1, g_3, h_1, h_3 with lattice QCD
- possible to extract Δ^+ pseudoscalar form factors $\tilde{g}$, $\tilde{h}$
- $\pi\Delta\Delta$ couplings satisfy Goldberger-Treiman relation
- results are preliminary; future work will include mixed action (DWF on Asqtad sea)
- stay tuned

Goldberger-Treiman relations for $N \to \Delta$

$$G_{\pi N\Delta}(q^2)f_\pi = 2m_N C_5^A(q^2)$$