Hadron Form Factors at Large Transfer Momentum

Saul D. Cohen

BOSTON UNIVERSITY

(with Huey-Wen Lin)

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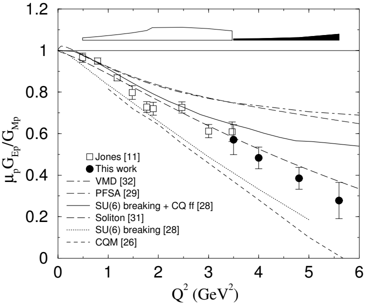
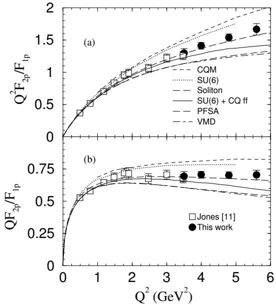
Outline

1. Experimental Motivation
2. Methodology
3. Nucleon Form Factors
4. Transverse Distributions

Form Factors at High Momentum Transfer

- Experimental results for neutrons available only up to ≈ 4 GeV^2
- JLab 12-GeV upgrade should shed some light on things
- Does G_E^p cross zero around 7 GeV^2?

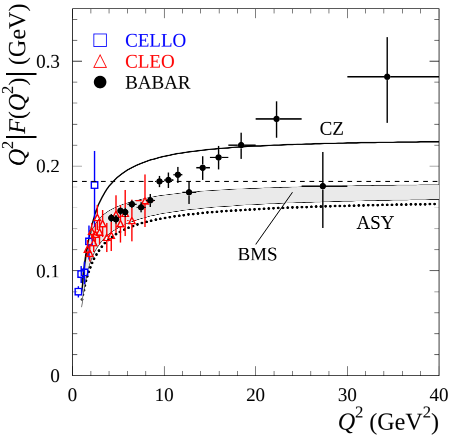




Isn't Perturbative QCD Good Enough?

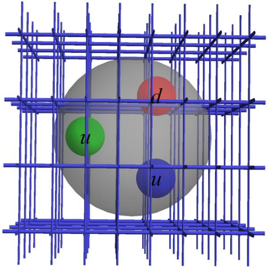
Recent BaBar results for $\gamma^* \gamma \to \pi^0$

- Disagreement with PQCD over a wide range: $4\ \mathrm{GeV}^2 < Q^2 < 40\ \mathrm{GeV}^2$
- An opportunity for lattice QCD

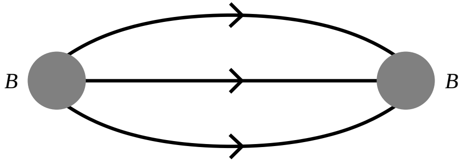


Anisotropic 2+1-Flavor Clover Ensembles

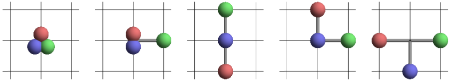
- Anisotropy
  improves resolution of excited states
- Clover fermion action
- Quenched Ensembles
  - $16^3 \times 64$, $\xi = 3$
  - $a_t^{-1} \approx 6.0$ GeV ($a_s \approx 0.10$ fm)
  - Wilson gauge action
  - Valence $M_\pi \in \{480, 720, 1080\}$ MeV
- 2+1-Flavor Ensembles
  - $16^3 \times 128$, $\xi = 3.5$
  - $a_t^{-1} \approx 5.6$ GeV ($a_s \approx 0.12$ fm)
  - RG-improved gauge action
  - $M_\pi \in \{450, 580, 875\}$ MeV



Baryon Correlators



- $B(x) = \epsilon^{abc} \left[q_1^{a\mathrm{T}}(x) C\gamma_5 q_2^b(x) \right] q_1^c(x)$
- Use various Gaussian smearings at source and sink
  - Quenched: $\sigma \in \{0.5, 2.5, 4.5\}$
  - 2+1-flavor: $\sigma \in \{0.5, 1.5, 2.5, 3.5, 4.5\}$
- More sophisticated operators may be used



Variational Method

Given a two-point correlator

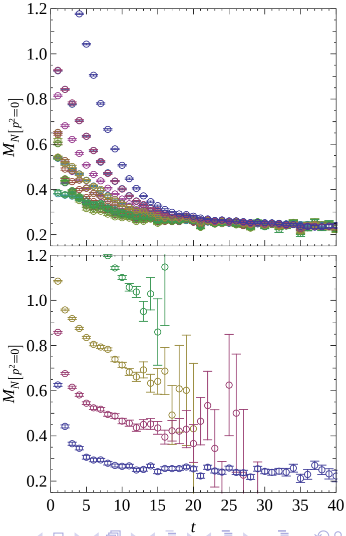
$$C_{ij}(t) = \langle O_j(t) O_i(0) \rangle = \sum_{n=0}^{\infty} Z_{in}^{\dagger} Z_{jn} e^{-E_n t},$$

take the generalized eigenvalues with respect to intermediate time t_0

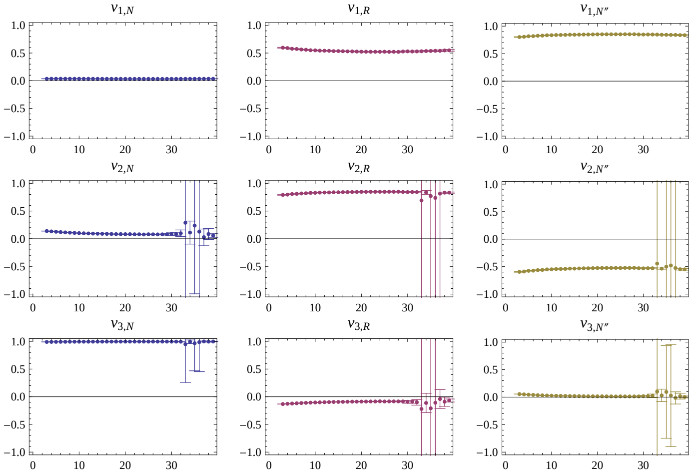
$$C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} \psi = \lambda(t, t_0) \psi.$$

For sufficiently large $t - t_0$,

$$\lambda_n(t, t_0) = e^{-(t-t_0)E_n}.$$



Variational Overlaps

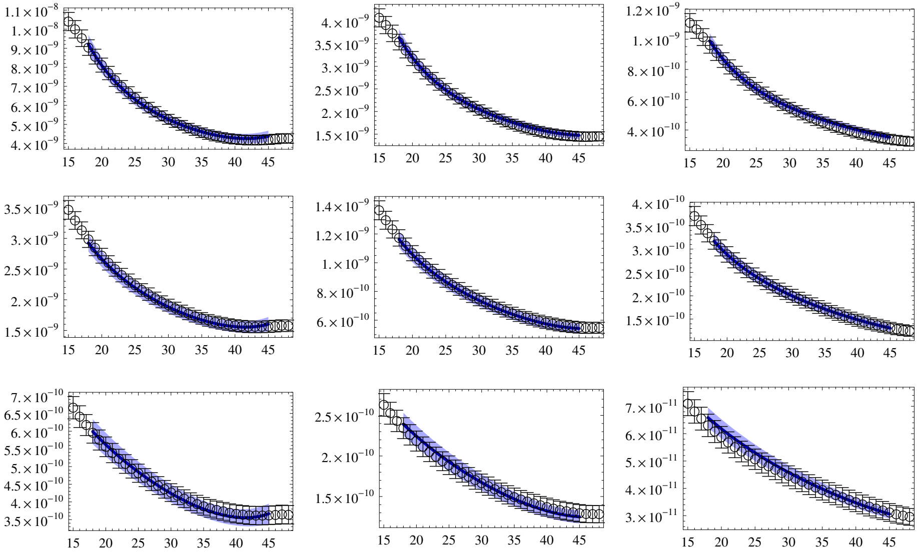


Extract from Three-Point Functions

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) = Z_V \sum_n \sum_{n'} f_{n,n'}(p_f,p_i;E_f,E_i;t_i,t,t_f) \times \sum_{s,s'} T_{\alpha\beta} u_{n'}(\vec{p}_f,s')_\beta \langle N_{n'}(\vec{p}_f,s')\,|V_\mu|\,N_n(\vec{p}_i,s)\rangle \overline{u}_n(\vec{p}_i,s)_\alpha$$

$$f = \frac{Z^*_{Bn'} Z_{An}}{4E_f E_i}\sqrt{\frac{2E_f}{E_f+M_i}}\, e^{-E_f(t_f-t)} e^{-E_i(t-t_i)}$$

Simultaneous Fitting



Dirac and Pauli Form Factors

- General Form:

$$\langle N_2 \left| V_\mu \right| N_1 \rangle(q) =$$

$$\overline{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x}$$

- Ground-State Only:

$$\langle N \left| V_\mu \right| N \rangle(q) = \overline{u}_N(p') \left[F_1(q^2) \gamma_\mu + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_N} \right] u_N(p) e^{-iq\cdot x}$$

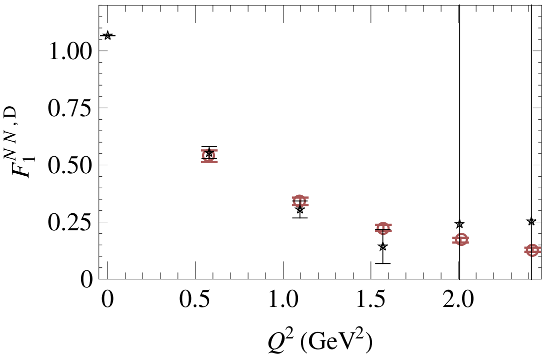
- Sachs Form Factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2) \quad (1)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \quad (2)$$

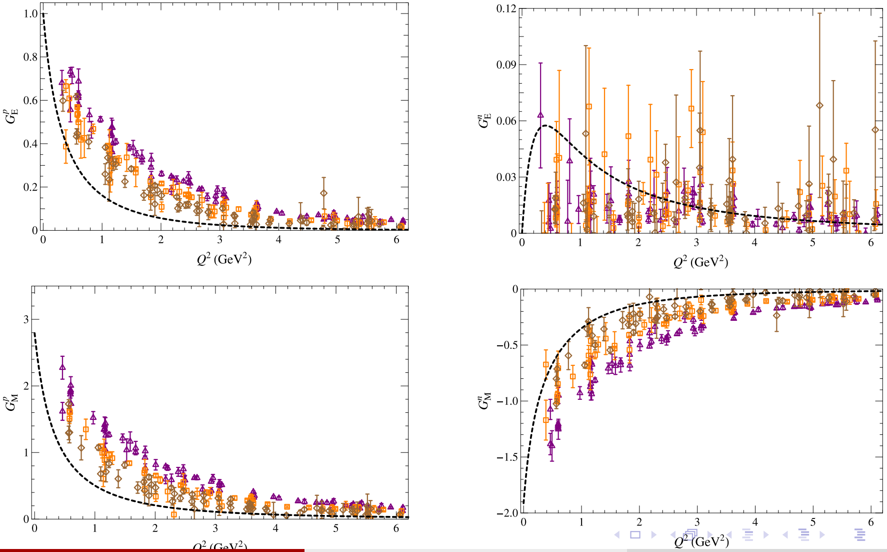
Comparison with Ratio Method

$$\frac{C^{(3)}_{ss}(t_i,t,t_f;p_i,p_f)}{C^{(2)}_{ss}(t_i,t_f;p_f)}\left(\frac{C^{(2)}_{ps}(t,t_f,p_i)C^{(2)}_{ss}(t_i,t,p_f)C^{(2)}_{ps}(t_i,t_f,p_f)}{C^{(2)}_{ps}(t,t_f,p_f)C^{(2)}_{ss}(t_i,t,p_i)C^{(2)}_{ps}(t_i,t_f,p_i)}\right)^{\frac{1}{2}}$$



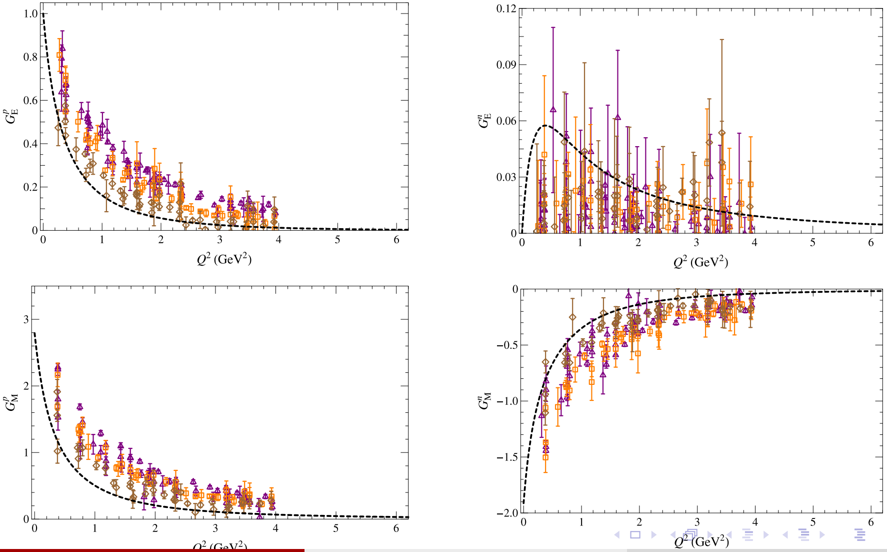
Sachs Form Factors

Quenched Results

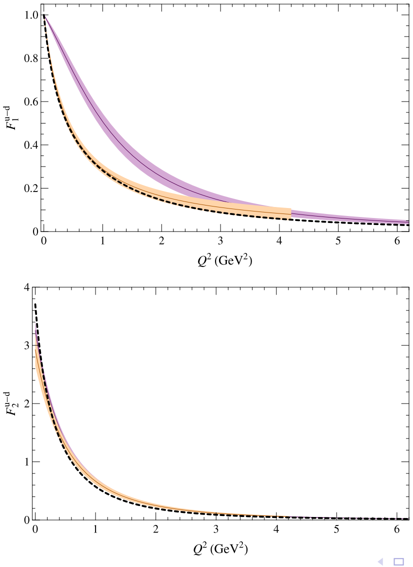


Sachs Form Factors

Dynamical Results



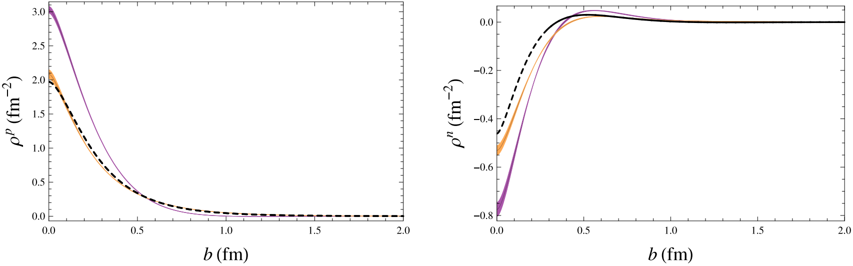
Isovector Form Factors



Transverse Charge Distribution

To avoid relativistic distortions, we calculate densities in a plane transverse to a direction of infinite boost:

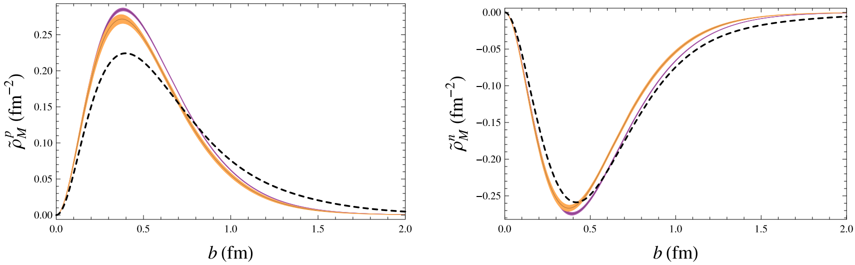
$$\rho(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$



Transverse Magnetization Distribution

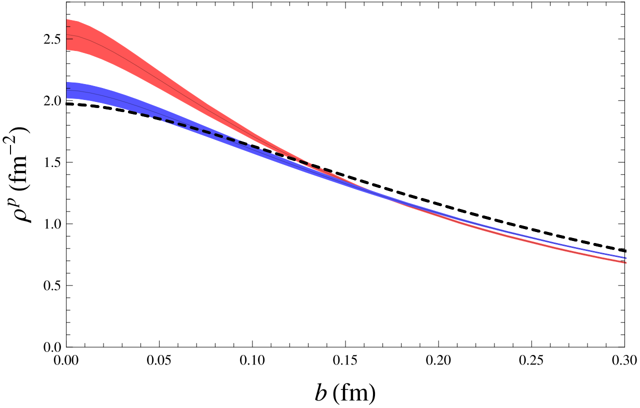
The transverse magnetization is defined with respect to a B-field in the $\hat{x}$ direction:

$$\tilde{\rho}_M(b) = b \sin^2 \phi \int_0^\infty \frac{Q^2\, dQ}{2\pi} J_1(bQ) F_2(Q^2)$$



How High Q^2 Is High Enough?

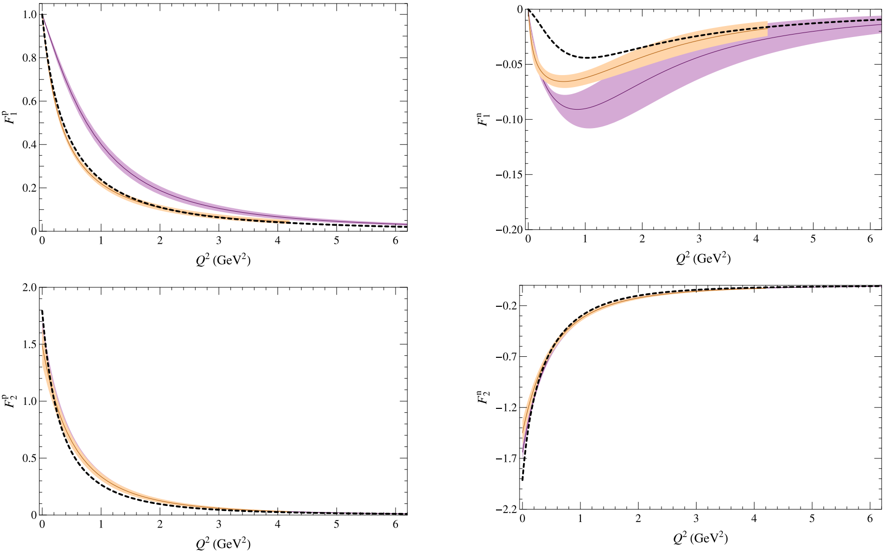
Consider what we would get for the density in the nucleon core if we only had data up to a typical Q^2 of 2 GeV^2



Summary

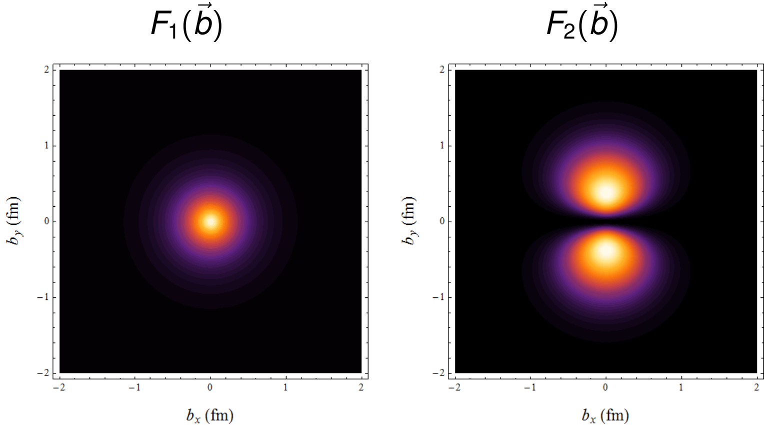
- Conclusions
  - Method allows extension to higher momentum transfer
  - Will extend reach to match future experiments
  - Essential for accurately determining densities in the hadronic core
- Future Work
  - Introduce more sophisticated operators
  - Nucleon axial and pion form factors (see H-W Lin's poster)

Quenched-Dynamical Fit Comparisons



Two-Dimensional Densities

Proton



Two-Dimensional Densities

Neutron