Non-perturbative computation of renormalization constants of bilinears operators with 4 dynamical flavours

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Outline

The problem

Computing (RI'-MOM) RCs of various operators (e.g. P, $O_j^{\Delta F=2}$) for the action used by ETMC in the 2+1+1 project

Mass independent scheme ⇔ Extrapolation to zero quark mass

- is crucial for operators with non-zero anomalous dimension
- requires working at renormalized quark masses $\lesssim 100\mathrm{MeV}$ &

Due to fixed m_s and m_c, 2+1+1 ensembles are not well suited

⇒ perform dedicated simulations with 4 degenerate light quarks

⇒ compute RI'-MOM RC's in the χ-limit

- yields pure numbers as functions of $(a \times \mathrm{momentum})^2$
- scale is set from physical world (2+1+1): eg from f_π
- $Z_\mu = Z_P^{-1}$ and Z_P/Z_S are relevant for quark masses

Action and quark mass parameters (I)

For this study we consider the action (see later too):

$$S_L = S_{Iwa}^{\rm YM} + a^4 \sum_{x,f} \bar{\chi}_f \left[\gamma \cdot \widetilde{\nabla} - \frac{a}{2} \nabla^* \nabla + m_0 + i r_f \mu_q \gamma_5 \right] \chi_f(x)$$

or, by passing from twisted to physical quark basis via

$$\chi_f \to q_f = \exp[\tfrac{i}{2}(\tfrac{\pi}{2} - \theta_{0f})\gamma_5]\chi_f \,, \quad \bar{\chi}_f \to \bar{q}_f = \bar{\chi}_f \exp[\tfrac{i}{2}(\tfrac{\pi}{2} - \theta_{0f})\gamma_5]$$

$$S_L = S_{Iwa}^{\rm YM} + a^4 \sum_{x,f} \bar{q}_f \left[\gamma \cdot \widetilde{\nabla} - i\gamma_5 e^{i\theta_{0f}\gamma_5} (-\frac{a}{2} \nabla^* \nabla + m_{\rm cr}) + M_0 \right] q_f(x) \,,$$

with

$$M_0 = \sqrt{(m_0 - m_{\rm cr})^2 + \mu_q^2} \,, \; \sin\theta_{0f} = \frac{m_0 - m_{\rm cr}}{M_0} \,, \; \cos\theta_{0f} = \frac{\mu_q r_f}{M_0}$$

Action and **quark mass parameters** (II)

- Renormalized parameters conveniently chosen as

$$M = Z_P \hat{M} = \sqrt{Z_A^2 m_{\rm PCAC}^2 + \mu_q^2}\,, \quad \tan\theta = \frac{Z_A m_{\rm PCAC}}{\mu_q}$$

- $d = 4$ term of Symanzik LEL involves only M, not θ.
- Partially quenched setup (convenient for RC studies)

$$(M,\theta) \implies (M_{\rm sea}, \theta_{\rm sea}; M_{\rm val}, \theta_{\rm val})$$

[θ's referred to $f = 1$]

Action and quark mass parameters (III) ⋆

The lattice action before allows to compute RCs relevant for...

- operators made out of quark fields with ETMC $2+1+1$ action, which in an unphysical basis reads

$$S_{Iwa}^{\rm YM} + a^4 \sum_x (\bar\chi_{\ell 1}, \bar\chi_{\ell 2}) \left[\gamma\cdot\tilde\nabla - \tfrac{a}{2}\nabla^*\nabla + m_0 + i\mu_\ell\gamma_5\tau^3\right] \begin{pmatrix}\chi_{\ell 1}\\ \chi_{\ell 2}\end{pmatrix}(x)$$
$$+ a^4 \sum_x (\bar\chi_{h1}, \bar\chi_{h2}) \left[\gamma\cdot\tilde\nabla - \tfrac{a}{2}\nabla^*\nabla + m_0 + i\mu_\sigma\gamma_5\tau^3 - \mu_\delta\tau^1\right] \begin{pmatrix}\chi_{h1}\\ \chi_{h2}\end{pmatrix}(x)$$

- operators involving Osterwalder-Seiler valence quarks: S_L above with $m_0 = m_{\rm cr}$, $\mu_q > 0$ (maximal twist)

[Frezzotti-Rossi'04][Herdoiza Lat10-Plenary]

We consider only bilinears: $\Gamma \Leftrightarrow S, P, V, A, T$

RI'MOM scheme

- $Z_q^{-1} \frac{i}{12} \mathrm{Tr} \left[\frac{\tilde{\not p} S_f(p)^{-1}}{\tilde{p}^2} \right]_{\tilde{p}^2=\mu^2} = 1$ any f [and r_f]
- $Z_q^{-1} Z_O^{(ff')} \mathrm{Tr} \left[\Lambda_\Gamma^{(ff')}(p,p) P_\Gamma \right]_{\tilde{p}^2=\mu^2} = 1$ $f \neq f'$ [$r_{f'} = -r_f$]

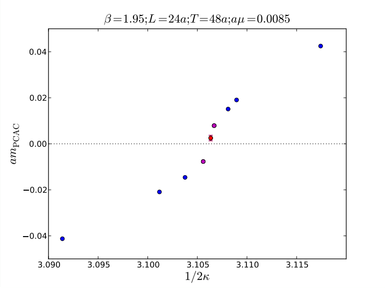
We need to compute:

- The quark propagator: $S_f(p) = a^4 \sum_x e^{-ipx} \langle \chi_f(x) \bar{\chi}_f(0) \rangle$
- the Green function:
  $G_\Gamma^{(ff')}(p,p) = a^8 \sum_{x,y} e^{-ip(x-y)} \langle \chi_f(x) (\bar{\chi}_f \Gamma \chi_{f'})(0) \bar{\chi}_{f'}(y) \rangle$
- and the amputated vertex:
  $\Lambda_\Gamma^{(ff')}(p,p) = S_f^{-1}(p) G_\Gamma^{(ff')}(p,p) S_{f'}^{-1}(p)$

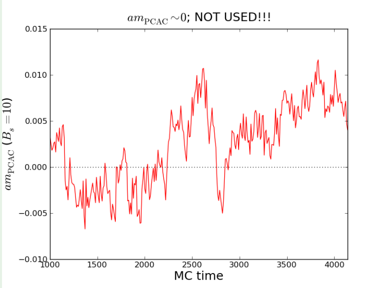
Numerical strategy at $N_f = 4$ “light” sea quarks

Simplicity & control of $O(a)$ VS no fine-tuning & reliable $m_{\rm PCAC}$

Some facts about $am_{\rm PCAC} = \dfrac{\sum_{\vec{x}} a\partial_0 \langle (\bar{\chi}_f \gamma_5 \gamma_0 \chi_{f'})(x)\ (\bar{\chi}_{f'} \gamma_5 \chi_f)(0)\rangle}{2\sum_{\vec{x}} \langle (\bar{\chi}_f \gamma_5 \chi_{f'})(x)\ (\bar{\chi}_{f'} \gamma_5 \chi_f)(0)\rangle}$



(a)



(b)

Numerical strategy at $N_f = 4$ "light" sea quarks

If $a\mu_q \lesssim 0.01$ at $0.08[0.09] fm \Leftrightarrow \beta = 1.95[1.90]$

- considerable fine tuning in $1/2\kappa$ is needed to work at maximal twist
- $\sigma_{\mathrm{stat}}[am_{\mathrm{PCAC}}]$ difficult to evaluate when $am_{\mathrm{PCAC}} \ll 0.01$

Various setups still possible:

A) maximal twist, at larger M's $\longrightarrow$ need some fine tuning work,

B) out of maximal twist, at larger M's $\longrightarrow$ remove $O(a)$ effects by $\theta_{\mathrm{val/sea}}$-average

We have chosen B) with $aM \in [0.014, 0.033]$

Runs

ensemble	$a\mu_{\mathrm{sea}}$	$am_{\mathrm{sea}}^{\mathrm{PCAC}}$	$a\mu_{\mathrm{val}}$	$am_{\mathrm{val}}^{\mathrm{PCAC}}$
1m	0.0085	-0.041	[0.0085, . . . , 0.0298]	-0.022
1p	0.0085	+0.042	[0.0085, . . . , 0.0298]	+0.019
3m	0.0180	-0.016	[0.0060, . . . , 0.0298]	-0.016
3p	0.0180	+0.015	[0.0060, . . . , 0.0298]	+0.016
2m	0.0085	-0.021	[0.0085, . . . , 0.0298]	-0.021
2p	0.0085	+0.019	[0.0085, . . . , 0.0298]	+0.019
4m	0.0085	-0.015	[0.0060, . . . , 0.0298]	-0.015
4p	0.0085	+0.015	[0.0060, . . . , 0.0298]	+0.015

$O(a)$ improvement via θ-average

Based on the symmetry of the lattice action S_L under

$$\mathcal{P} \times (\theta_0 \to -\theta_0) \times \mathcal{D}_d \times (M_0 \to -M_0)$$

one can prove that the $O(a^{2k+1})$ artifacts occurring in the vev of (multi)local operators O that are invariant under $\mathcal{P} \times (\theta_0 \to -\theta_0)$

- are quantities that change sign upon sign change of θ_0 (or θ)
- are absent in θ-averages: $\frac{1}{2}\left[\langle O \rangle|_{\hat{M},\theta} + \langle O \rangle|_{\hat{M},-\theta}\right]$

The same holds for form factors invariant under $\mathcal{P} \times (\theta_0 \to -\theta_0)$ . . . e.g. for the RC-estimators at all $\hat{M}$'s (and $\tilde{p}^2$'s in RI-MOM)

In PQ setup: $(M, \theta) \quad \Rightarrow \quad (M^{\rm sea}, \theta^{\rm sea}; M^{\rm val}, \theta^{\rm val})$ [θ's referred to $f = 1$]

Current analysis

On the ensembles Ep/m (E=1,2,...) with $(M_{\rm sea}^{\rm Ep/m}, \theta_{\rm sea}^{\rm Ep/m})$ we compute RC-estimators for several $(M_{\rm val}, \theta_{\rm val})$'s and $\tilde{p}^2$'s

so far $0.013 \lesssim aM^{\rm val} \lesssim 0.033$ & $0.4 \lesssim |\theta_{\rm val}| \lesssim 1.2$ $(\theta_{\rm val}/m_{\rm PCAC}^{\rm val} > 0)$

1. valence chiral limit: via linear fit to the RC-estimator dependence on $(M_{\rm val}^{PS})^2$, with term $\sim (M_{\rm val}^{PS})^{-2}$ for $\Gamma = P, S$ [ignoring $\theta^{\rm val}$-depend; good χ^2]

(M1) build (with PT-evolution in E_Γ) RC-estimators at ren. scale $1/a$, i.e.

$$Z_\Gamma((a\Lambda)^{-2}; (a\tilde{p})^2) = Z_\Gamma(\tilde{p}^2/\Lambda^2; (a\tilde{p})^2)\ E_\Gamma^{PT}(\tilde{p}^2 \to a^{-2})$$

and get $Z_\Gamma((a\Lambda)^{-2}; 0)$ by extrapolation in $(a\tilde{p})^2$

(M2) build RC-estimator at ren. scale $\tilde{p}_{\rm M2}^2 \sim 12.2\ {\rm GeV}^2$ i.e.

$$Z_\Gamma((a\Lambda)^{-2}; (a\tilde{p}_{\rm M2})^2) = Z_\Gamma(\tilde{p}_{\rm M2}^2/\Lambda^2; (a\tilde{p}_{\rm M2})^2)\ E_\Gamma^{PT}(\tilde{p}_{\rm M2}^2 \to a^{-2})$$

and get $Z_\Gamma((a\Lambda)^{-2}; 0)$ by extrapolation in $(a\tilde{p}_{\rm M2})^2)$ (by averaging around $\tilde{p}_{\rm M2}^2$)

2. remove residual $O(a)$ artifacts in RCs via θ-average (see below)
3. sea chiral limit: $(M_{\rm sea})^2 \to 0$ taking θ-dependence into account

Some analysis possible and improvements planned...

θ-dependence in chiral limit extrapolation of RC estimators (I)$\star$

$Z_{q,\Gamma}$-estimators (in general non θ-averaged) at $M > 0$ are

- $\mathcal{P} \times (\theta \to -\theta)-$even form factors [see their expression in physical quark basis]
- yield $Z_{q,\Gamma}$ that are independent of θ in the χ–limit: $M \to 0$

$\Rightarrow$ θ-dependence $=$ cutoff effect on the M-corrections wrt χ–limit,

hence described by the M-dependent terms in Symanzik's LEL

$$aL_5^{(M)} \Rightarrow aM^2\cos\omega[\bar\psi\psi]\,,\;\; aM^2\sin\omega[\bar\psi i\gamma_5\tau^3\psi]\,,\;\; aM\cos\omega L_4^{\rm YM}\,,\;\; aM\cos\omega[\bar\psi\gamma\cdot D\psi]$$

$$a^2L_6^{(M)} \Rightarrow a^2M[\bar\psi i\sigma\cdot F\psi]\,,\;\; a^2M[\bar\psi(-D\cdot D)\psi]\,,\;\; a^2M^2\cos(2\omega)L_4^{\rm YM}\,,\;\; a^2M^2\cos(2\omega)[\bar\psi\gamma\cdot D\psi]\,,\ldots$$

$\omega = \frac{\pi}{2} - \theta$, $\psi = (q_1, q_2)$ or (q_3, q_4): $\Leftrightarrow$ Sharpe-Wu '04 + off-shell terms kept & spurion symmetry used

θ-dependence in chiral limit extrapolation of RC estimators (II)$\star$

θ-averaged $Z_{q,\Gamma}$-estimators are O(a) improved $\Rightarrow$ admit a Symanzik description where only one insertion of $a^2 L_6$ & two of aL_5 appear

In particular: θ-dependence at O(a^2) gets considerably simplified after θ-average

From $\mathcal{P} \times (\theta \to -\theta)$ symmetry and form aL_5 and $a^2 L_6$ one finds

$$Z_\Gamma^{\mathrm{est}} - Z_\Gamma^{(\chi)} = R_1 M + R_2 M^2 + a^2 \rho_3 M \cos(2\theta) + a^2 \rho_4 M^2 \cos(2\theta)$$

up to corrections of order M^3 or higher – negligible in our data.

Terms linear in M reflect χ-SSB and are suppressed $\sim \Lambda^2_{QCD}/p^2$

In our PQ setup (with M_{val}, θ_{val} & M_{sea}, θ_{sea}) and current analysis

- valence χ–limit before θ-average: good fits in terms of $(M_{PS}^{\mathrm{val}})^2$
- sea χ–limit after θ-average: fit with $R_2 M_{\mathrm{sea}}^2 + a^2 \rho_4 M_{\mathrm{sea}}^2 \cos(2\theta_{\mathrm{sea}})$ is very good [tried ansatz with $M^2 \to M$, too: fit quality similar, results $\sim$ identical (up to few 0.001)]

The $(M_{\mathrm{PS}}^{\mathrm{val}})^2$-dependence of RC-estimators @ $(a\tilde{p})^2 = 1.5$

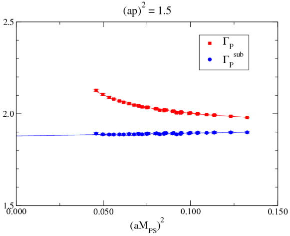
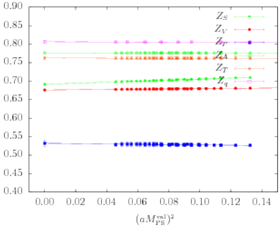
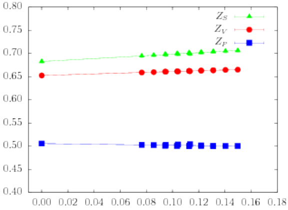


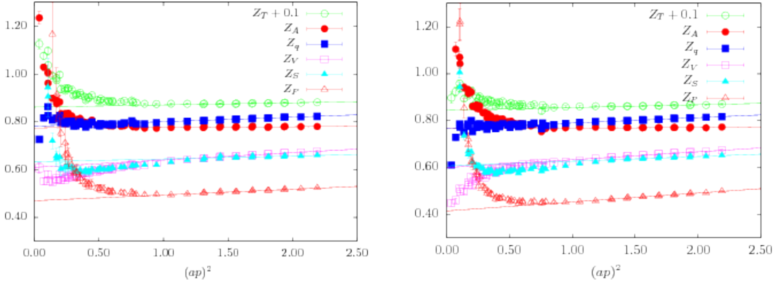
Figure: 4m, 4m and 2p





The $\tilde{p}^2$-dependence of RC-estimators @ $M^{\rm val} = 0$

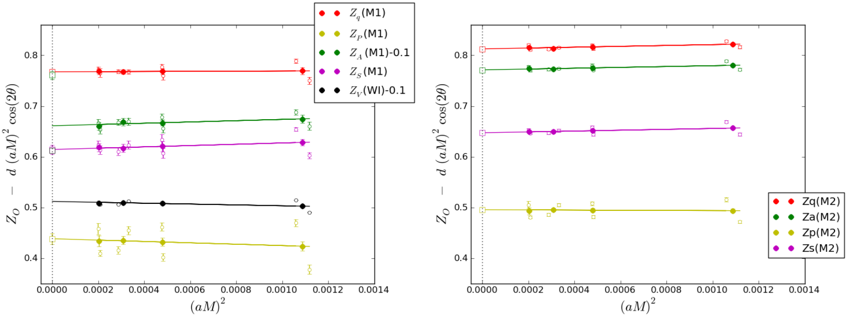
Figure: ensembles 1m and 2p



M1: intercept at $\tilde{p}^2 = 0$ of the shown best fit lines

M2: values at $\tilde{p}^2 = 12.2\ \mathrm{GeV}^2$, here corresponding to $a^2\tilde{p}^2 = 1.9$

The M^{sea}-dependence after θ-average [M1/M2 @ $M^{\text{val}} = 0$]



left: M1; right: M2

First analysis results for RCs [M1/M2 @ $M^{\rm sea} = M^{\rm val} = 0$]

VERY PRELIMINARY!

RC(RI')	M1	M2	Alternative
Z_A	0.761(08)	0.771(03)	OS-tm ...
Z_V	0.630(05)	0.674(03)	WI: 0.6120(05)
$Z_P(1/a)$	0.438(08)	0.496(04)	—
$Z_S(1/a)$	0.614(09)	0.647(03)	—
Z_P/Z_S	0.716(21)	0.767(08)	OS-tm ...
$Z_T(1/a)$	0.753(07)	0.768(03)	—
$Z_q(1/a)$	0.767(06)	0.813(02)	—

Conclusions & outlook

- Very encouraging results at $\beta = 1.95$ $\longrightarrow$ workable approach
- Usual & mild valence quark mass dependence observed
- Mild sea quark mass dependence, in particular after θ-average
- One (two) more $M_{\rm sea}$ point(s) at $\beta = 1.95$ to reduce errors
- Removal of $O(a^2)$ effects at 1-loop PT plus some analysis improvements are planned
- Extend work to other β's: 2.1 (in progress), 2.0 and 1.9