Ghost-gluon coupling, power corrections and $\Lambda_{\overline{MS}}$ from twisted-mass QCD at $N_f = 2$

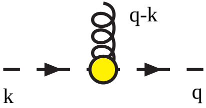
Mario Gravina
Laboratoire de Physique Théorique
CNRS and Universite Paris-Sud, Orsay

Villasimius, 15^{th} june 2010

In collaboration with: B. Blossier, Ph. Boucaud, O. Pène (LPT, Orsay)
F. De soto (Univ. Pablo de Olavide, Sevilla), V. Morenas (Univ. Blaise Pascal, Aubière)
J. Rodríguez-Quintero (Univ. de Huelva, Huelva)

Ghost-gluon vertex

$$\widetilde{\Gamma}^{abc}_{\nu}(-q,k;q-k) =$$



$$= i g_0 f^{abc}\left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k)\right)$$

$$\widetilde{\Gamma}_R = \widetilde{Z}_1 \Gamma$$

Strong coupling

$$g_R(\mu^2) = \lim_{\Lambda\to\infty} Z_g^{-1}(\mu^2,\Lambda^2) g_0(\Lambda^2) = \lim_{\Lambda\to\infty} \frac{Z_3^{1/2}(\mu^2,\Lambda^2)\widetilde{Z}_3(\mu^2,\Lambda^2)}{\widetilde{Z}_1(\mu^2,\Lambda^2)} g_0(\Lambda^2)$$

Taylor scheme: **zero incoming ghost momentum**[1]

$$\widetilde{Z}_1(\mu^2,\Lambda^2) \equiv 1$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2,\Lambda^2)\widetilde{Z}_3^2(\mu^2,\Lambda^2)$$

---

[1] J. C. Taylor, Nuclear Physics B33 (1971) 436

Lattice: $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\left(G^{(2)}\right)^{ab}_{\mu\nu}(p^2,\Lambda) = \frac{G(p^2,\Lambda)}{p^2}\,\delta_{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$
$$\left(F^{(2)}\right)^{a,b}(p^2,\Lambda) = -\delta_{ab}\,\frac{F(p^2,\Lambda)}{p^2}$$

$$G_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} Z_3^{-1}(\mu^2,\Lambda)\,G(p^2,\Lambda)$$
$$F_R(p^2,\mu^2) = \lim_{\Lambda\to\infty} \widetilde{Z}_3^{-1}(\mu^2,\Lambda)\,F(p^2,\Lambda)$$

Non-perturbative renormalization: **MOM scheme**

$$G_R(\mu^2,\mu^2) = F_R(\mu^2,\mu^2) = 1$$

Strategy

Matching lattice-perturbation theory

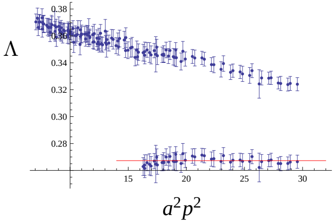
4-loops perturbation theory[3]: $p \gg \Lambda_{QCD}$

$$\alpha_T(\mu^2) = \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2 + \frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right)$$
$$+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right), \; t = \ln\frac{\mu^2}{\Lambda_T^2}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi\sum_{i=0}\widetilde{\beta}_i\left(\frac{\alpha_T}{4\pi}\right)^{i+2}, \quad \frac{\Lambda_{\overline{\rm MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}} = 0.541449$$

Quenched QCD: $N_f = 0$ [2]

Matching lattice-perturbation theory



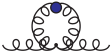
[2] Ph. Boucaud *et al.*, Phys. Rev. D **79** (2009) 014508

[3] K. G. Chetyrkin, Nucl. Phys. B **710** (2005) 499

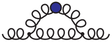
OPE power corrections

$$(F^{(2)})^{ab}(q^2) = (F^{(2)}_{\rm pert})^{ab}(q^2) + w^{ab}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \dots\,, \quad w^{ab} = 2\times$$

$$(G^{(2)})^{ab}_{\mu\nu}(q^2) = (G^{(2)}_{\rm pert})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu}\frac{\langle A^2\rangle}{4(N_C^2-1)} + \dots\,, \quad w^{ab}_{\mu\nu} =$$



$+\ 2\times$



$$F_R(q^2,\mu^2) = F_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)\,,\quad G_R(q^2,\mu^2) = G_{R,\rm pert}(q^2,\mu^2)\left(1+\frac{3}{q^2}\frac{g_R^2\langle A^2\rangle_{R,\mu^2}}{4(N_C^2-1)}\right)$$

Leading logarithm [4]: $\alpha_T(\mu^2) = \alpha_T^{\rm pert}(\mu^2)\left(1+\frac{9}{\mu^2}\left(\frac{\alpha_T^{\rm pert}(\mu^2)}{\alpha_T^{\rm pert}(q_0^2)}\right)^{1-\gamma_0^{A^2}/\beta_0}\frac{g_T^2(q_0^2)\langle A^2\rangle_{R,q_0^2}}{4(N_C^2-1)}\right)$

$$1-\gamma_0^{A^2}/\beta_0 = 1-\frac{105-8N_f}{132-8N_f} = \frac{9}{44-\frac{8}{3}N_f}$$

Wilson coefficient is known at four-loop [5]

[4] R. Wilson, Phys. Rev. **179** (1969) 1499

[5] K. G. Chetyrkin and A. Maier, arXiv:0911.0594 [hep-ph]

4-loops perturbation theory[7]: $p \gg \Lambda_{QCD}$

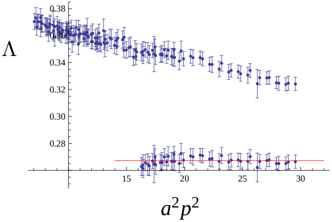
$$
\begin{aligned}
\alpha_T(\mu^2) &= \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4}\frac{1}{t^2}\left(\left(\log(t) - \frac{1}{2}\right)^2 + \frac{\widetilde{\beta}_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right) \\
&+ \frac{1}{(\beta_0 t)^4}\left(\frac{\widetilde{\beta}_3}{2\beta_0} + \frac{1}{2}\left(\frac{\beta_1}{\beta_0}\right)^3\left(-2\log^3(t) + 5\log^2(t) + \left(4 - 6\frac{\widetilde{\beta}_2\beta_0}{\beta_1^2}\right)\log(t) - 1\right)\right) \quad , t = \ln\frac{\mu^2}{\Lambda_T^2}
\end{aligned}
$$

$$
\beta_T(\alpha_T) = \frac{d\alpha_T}{d\ln\mu^2} = -4\pi\sum_{i=0}\widetilde{\beta}_i\left(\frac{\alpha_T}{4\pi}\right)^{i+2}, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}} = 0.541449
$$

Quenched QCD: $N_f = 0$ [6]

Matching lattice-perturbation theory

$$
\Lambda_{\overline{MS}}^{N_f=0} = 224^{+8}_{-5}\ \text{Mev},\ g_T^2\langle A^2\rangle = 5.1^{+0.7}_{-1.1}\ \text{Gev}^2
$$



[6] Ph. Boucaud *et al.*, Phys. Rev. D **79** (2009) 014508

[7] K. G. Chetyrkin, Nucl. Phys. B **710** (2005) 499

European Twisted Mass Collaboration

Fermions: twisted-mass action

$$S^{\mathrm{F}}_{\mathrm{tm}} = a^4 \sum_x \left\{ \bar{\chi}_x \left[D_{\mathrm{W}} + m_0 + i\gamma_5 \tau_3 \mu_q \right] \chi_x \right\}$$

Gauge fields: tlSym action

$$S_g = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\substack{\mu,\nu=1 \\ 1\leq\mu<\nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U^{1\times1}_{x,\mu,\nu})\} + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \{1 - \mathbb{R}\mathrm{eTr}(U^{1\times2}_{x,\mu,\nu})\} \right) , \quad \beta \equiv 6/g_0^2$$

$(b_0 = 1 - 8b_1, b_1 = -1/12)$ + **Maximal twist :** $\mathcal{O}(a^2)$

$$\begin{aligned} V = 24^3 \times 48 \quad \beta &= 3.9 \quad \mu = 0.004,\ 0.0064,\ 0.010 \\ V = 32^3 \times 64 \quad \beta &= 4.05 \quad \mu = 0.003,\ 0.006,\ 0.008,\ 0.012 \\ \beta &= 4.2 \quad \mu = 0.0065 \end{aligned}$$

Artefacts: $\mathcal{O}(a^2 \Lambda^2_{QCD})$, $\mathcal{O}(a^2 p^2)$, $\mathcal{O}(a^2 \mu^2)$

Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \mathrm{Re}\left[\sum_x \sum_\mu \mathrm{Tr}\left(1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x+\mu)\right)\right]$$

Gluon:

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\left(\frac{U_\mu(x) - U_\mu^\dagger(x)}{2iag_0}\right)$$

$$\left(G^{(2)}\right)^{a_1 a_2}_{\mu_1 \mu_2}(p) = \langle A^{a_1}_{\mu_1}(p) A^{a_2}_{\mu_2}(-p) \rangle$$

Ghost:

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \left\langle \left(M^{-1}\right)^{ab}_{xy} \right\rangle, \quad M(U) = -\frac{1}{N} \nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U_\mu^\dagger - U_\mu^\dagger(x)\eta(x)\right)$$

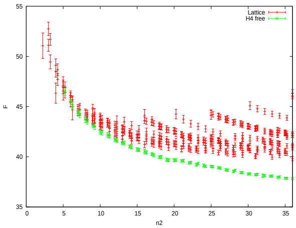
$O(4)$ breaking: $H(4)$ discretization artefacts[8]

Orbit labeled by $H(4)$-invariants: $p^{[2n]} = \sum_{\mu=1}^{4} p_\mu^{2n}$, $n = 1, 2, 3$

Momentum on the lattice: $\tilde{p}_\mu = \frac{1}{a} \sin a p_\mu \;,\;\; p_\mu = \frac{2\pi n}{Na} \qquad n = 0, 1, \cdots, N$

$$a^2\tilde{p}^2 \equiv \sum_{\mu=1}^{4} a^2\tilde{p}_\mu^2 = a^2p^2 + c_1 a^4 p^{[4]} + \cdots = a^2p^2 \left(1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \cdots\right)$$

If $\epsilon = a^2p^{[4]}/p^2 << 1$ ...

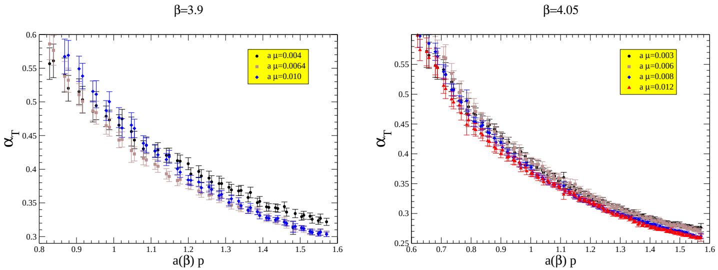


$$Q(a^2\tilde{p}_\mu^2, a^2\Lambda^2) \equiv Q\left(a^2p^2\left(1 + c_1 a^2\frac{p^{[4]}}{p^2} + \cdots\right), a^2\Lambda^2\right)$$
$$= Q(a^2p^2, a^2\Lambda^2) + \left.\frac{dQ}{d\epsilon}\right|_{\epsilon=0} a^2\frac{p^{[4]}}{p^2} + \cdots$$

$$R(a^2p^2, a^2\Lambda^2) = \left.\frac{dQ\left(a^2p^2\left(1 + c_1\epsilon + \cdots\right), a^2\Lambda^2\right)}{d\epsilon}\right|_{\epsilon=0}, \quad R = R_0 + R_1 a^2p^2$$

[8] F. de Soto and C. Roiesnel, JHEP **0709** (2007) 007

Quark mass artefacts



Triyng $\mathcal{O}(a^2\mu_q^2)$ dependence

$$\widehat{\alpha}_T(a^2p^2, a^2\mu_q^2) = \frac{g_0^2(a^2)}{4\pi}\widehat{G}(a^2p^2, a^2\mu_q^2)\widehat{F}^2(a^2p^2, a^2\mu_q^2)$$

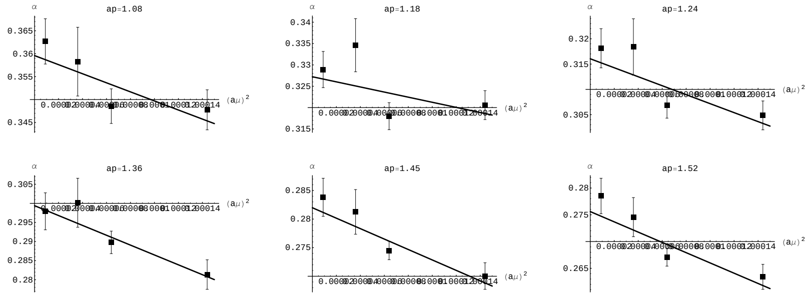
$$= \widehat{\alpha}_T(a^2p^2, 0) + \frac{\partial\widehat{\alpha}_T}{\partial(a^2\mu_q^2)}\left(a^2p^2\right)\, a^2\mu_q^2 + \cdots$$

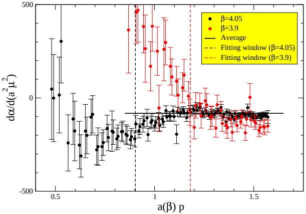
Quark mass artefacts

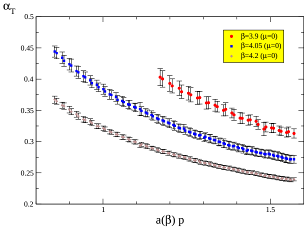
$$\widehat{\alpha}_T(a^2p^2, a^2\mu_q^2) = \alpha_T(p^2) + R_0(a^2p^2)\; a^2\mu_q^2$$

$$R_0(a^2p^2) \equiv \frac{\partial\widehat{\alpha}_T}{\partial(a^2\mu_q^2)}$$

Example: $\beta = 4.05$

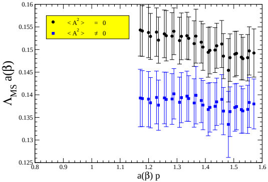




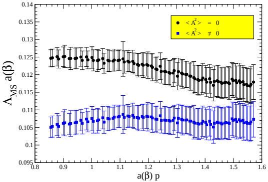


$R_0 = -92(11)$, $\quad p \geq p_{\min} \simeq 2.8$ GeV $\quad$ (a(3.9)=0.0801(14) fm[9])

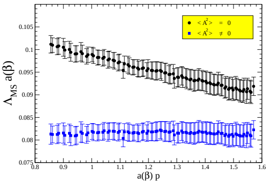
Requiring a plateau for $\Lambda_{\overline{MS}}$ in the window $ap \geq p_{\min}$



$\beta = 3.9$



$\beta = 4.05$



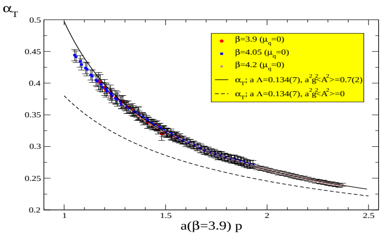
$\beta = 4.2$

[9] R. Baron *et al.* [ETM Collaboration], arXiv:0911.5061

Global fit and calibration of lattice spacing

$$\chi^2\left(a(\beta_0)\Lambda_{\overline{\rm MS}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)}\right) = \sum_{j=0}^{2}\sum_{i} \frac{\left(\Lambda_i(\beta_j) - \frac{a(\beta_j)}{a(\beta_0)} a(\beta_0)\Lambda_{\overline{\rm MS}}\right)^2}{\delta^2(\Lambda_i)}$$

Variables: $a(\beta_0)\Lambda_{\overline{\rm MS}}$, c, $\frac{a(\beta_1)}{a(\beta_0)}$, $\frac{a(\beta_2)}{a(\beta_0)}$



	This paper	String tension[10]
$a(3.9)/a(4.05)$	1.224(23)	1.255(42)
$a(3.9)/a(4.2)$	1.510(32)	1.558(52)
$a(4.05)/a(4.2)$	1.233(25)	1.241(39)
$\Lambda_{\overline{\rm MS}}a(3.9)$	0.134(7)	
$g^2\langle A^2\rangle a^2(3.9)$	0.70(23)	

[10] R. Baron *et al.* [ETM Col laboration], arXiv:0911.5061

Systematics

Higher orders for Wilson coefficient...

	One loop	Two loops	Three loops	Four loops
$\Lambda_{\overline{\rm MS}} a(3.9)$	0.134(7)	0.136(7)	0.137(7)	0.138(7)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.52(18)	0.44(14)	0.39(14)

Three-loop versus four-loop perturbative coupling constant...

	Four loops	Three loops
$a(3.9)/a(4.05)$	1.224(23)	1.229(23)
$a(3.9)/a(4.2)$	1.510(32)	1.510(29)
$a(4.05)/a(4.2)$	1.233(26)	1.234(25)
$\Lambda_{\overline{\rm MS}} a(3.9)$	0.134(7)	0.125(6)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.80(20)

Higer orders in OPE...unstable!

$$\alpha_{T,P4}(\mu^2) = \alpha_T(\mu^2) + \frac{c_4}{\mu^4}$$

Conclusions

$N_f = 2$:

$$\Lambda_{\overline{\rm MS}} = (330 \pm 23 \pm 22_{-33})\,\text{MeV}$$

$$g^2(q_0^2)\langle A^2\rangle_{q_0} = \left(4.2 \pm 1.5 \pm 0.7^{+?}\right)\text{GeV}^2\,, \quad q_0 \sim 10\,\text{GeV}$$

$N_f = 0$:

$$\Lambda_{\overline{\rm MS}} = 224^{+8}_{-5}\text{Mev}$$

$$g_T^2\langle A^2\rangle = 5.1^{+0.7}_{-1.1}\text{Gev}^2$$

Outlooks

ETMC new configurations:

$$N_f = 4$$

$$N_f = 2+1+1$$

THANK YOU!