The Deconfinement Phase Transition in QCD near the heavy quark limit

H. Saito for WHOT-QCD Collaboration

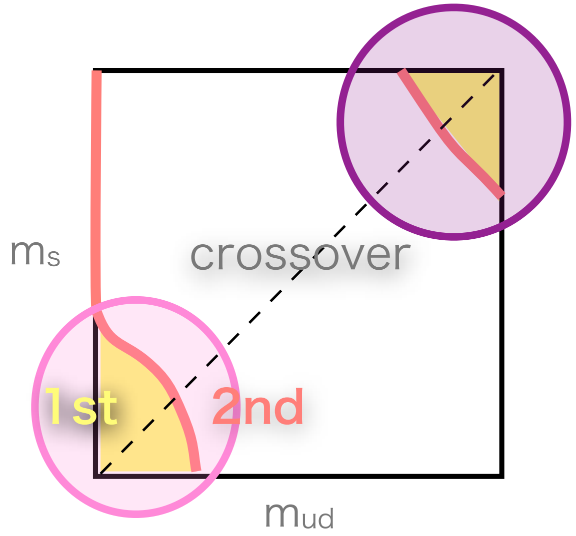
University of Tsukuba

Lattice 2010, Sardinia, Italy, 14-19 June 2010

WHOT-QCD Collaboration:
S. Aoki[1], S. Ejiri[2], T. Hatsuda[3], K. Kanaya[1], Y. Maezawa[4], H. Ohno[1], H. Saito[1], and T. Umeda[5]
[1]Univ. of Tsukuba, [2]Niigata Univ., [3]Univ. of Tokyo, [4]RIKEN, [5]Hiroshima Univ.

Introduction

- Low temperature ***Confinement*** ⟷ High temperature ***QGP***
- What is the order of QGP transition?
  - Quark mass dependence
  - Try a method using a histogram in QCD with heavy quarks.
  - Our goal: light quarks at μ≠ 0

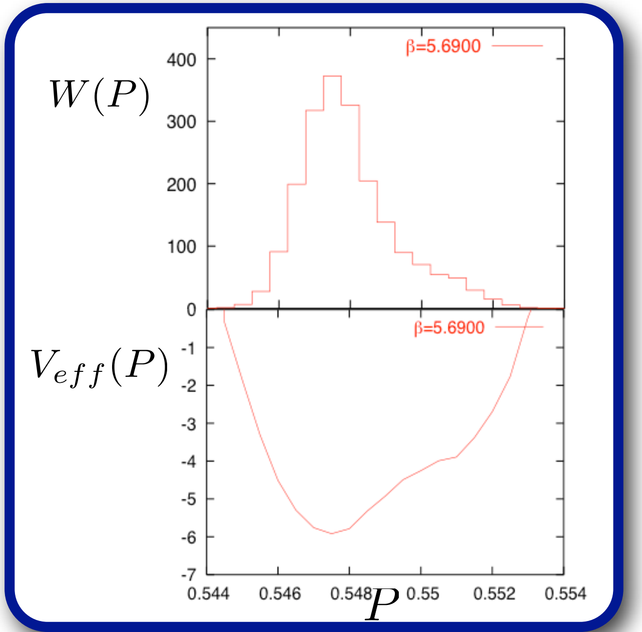


Effective Potential

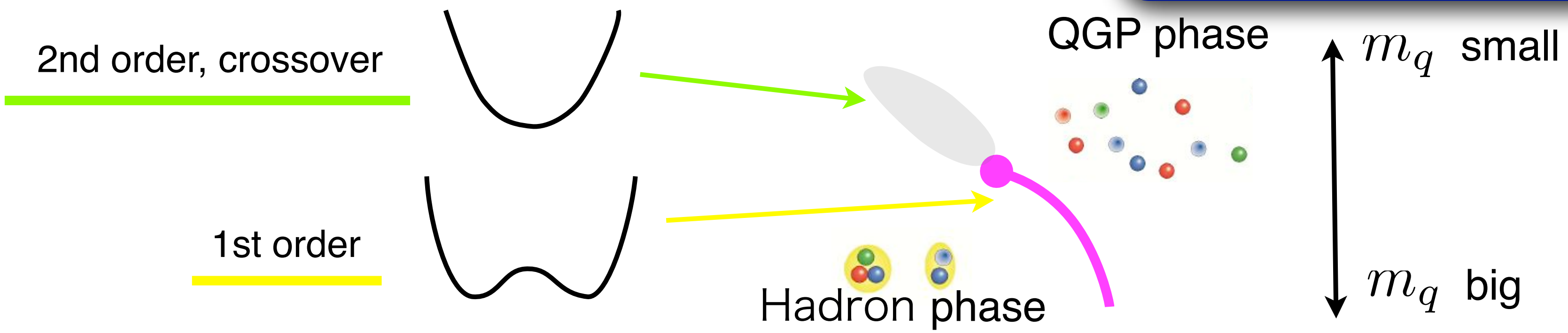
- Effective Potential defined by a histogram of plaquette value
  - Definition: $V_{eff}(P) = -\ln W(P)$

$$W(P') = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\delta(P(U) - P')e^{-S}$$

  - Ex. 1st order transition for pure gauge theory



- Quark mass dependence



Effective Potential (Quark mass dependence) 1

- Histogram

$$W(P') = R(P', \kappa) W_0(P', \beta)$$

$$W_0(P', \beta) \equiv \int \mathcal{D}U \delta(P(U) - P') e^{-S_g(\beta)}$$

$$R(P', \kappa) \equiv \frac{\int \mathcal{D}U \delta(P(U) - P')[\det M(U, \kappa)]^{N_f}}{\int \mathcal{D}U \delta(P(U) - P')}$$

- Effective potential

$$V(P, \kappa, \beta) = \boxed{- \ln R(P, \kappa)} + V_0(P, \beta)$$

Reweighting in κ

$$V_0(P, \beta) \equiv - \ln W_0(P, \beta)$$

- Action

Plaquette action:

$$S_g = -\frac{\beta}{3} \sum_n \sum_{\mu < \nu} Retr P_{\mu\nu}$$

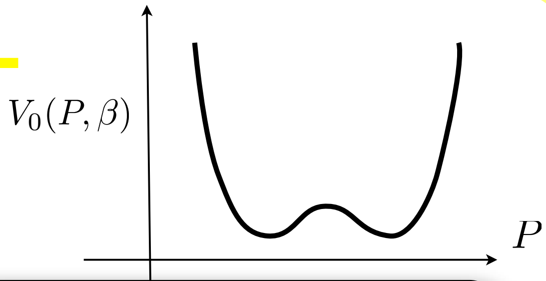
Wilson Fermion:

$$S_F = \sum_n \bar{\psi}_n \psi_n - \kappa \sum_{n,\mu} \bar{\psi}_n [(r - \gamma_\mu) U_{n,\mu} \psi_{n+\hat{\mu}} + (r + \gamma_\mu) U^\dagger_{n-\hat{\mu},\mu} \psi_{n-\hat{\mu}}] \equiv \sum \bar{\psi} M(U, \kappa) \psi$$

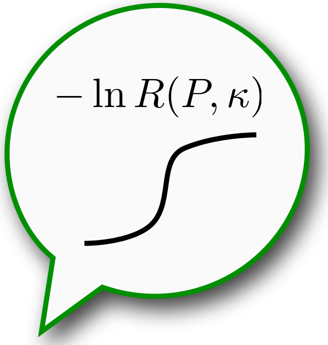
Effective Potential (Quark mass dependence) 2

- The shape of the V_{eff} discriminates the order of the transition.

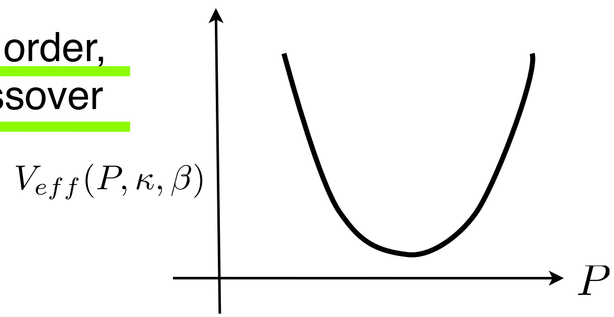
1st order



$$V_0(P,\beta) = -\ln W_0(P,\beta)$$



2nd order,
crossover



$$V(P,\kappa,\beta) = -\ln R(P,\kappa) + V_0(P,\beta)$$

Quark mass dependence

- Quark determinant by hopping parameter expansion

$$\det M(\kappa) = 1 + \kappa^4 N_f \left(2^4 \sum_n \sum_{\nu<\mu} \mathrm{Retr} P_{\mu\nu}(n) + 2^6 \sum_{\mathrm{n}} \mathrm{Retr}\Omega(\mathrm{n}) \right) + \mathcal{O}(\kappa^6)$$

$$P_{\mu\nu}(n) = U_{n,\mu} U_{n+\hat{\mu},\nu} U^\dagger_{n+\hat{\nu},\mu} U^\dagger_{n,\nu}$$: plaquette

$$\Omega(\mathbf{n}) = U_{(\mathbf{n},0),\hat{4}} U_{(\mathbf{n},1),\hat{4}} U_{(\mathbf{n},2),\hat{4}} U_{(\mathbf{n},3),\hat{4}}$$: polyakov loop

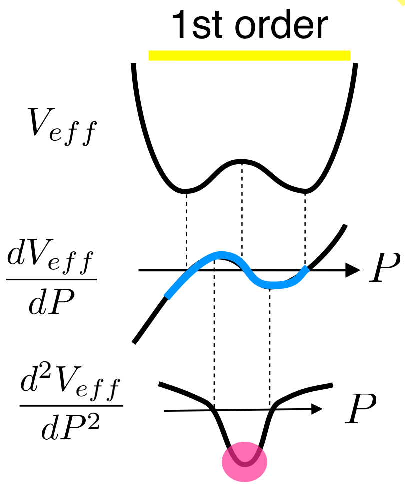
- We can choose the κ values freely.

Derivative

- Analysis by using the derivative/secondary derivative

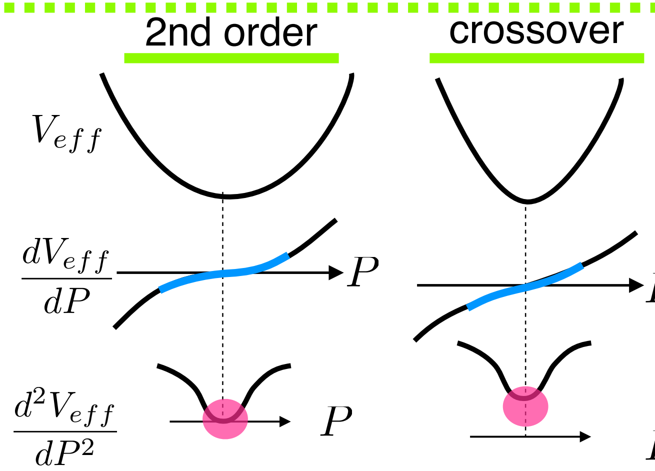
"S shape"

Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **Negative!**



Monotonically increasing shape

Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **0/Positive!**



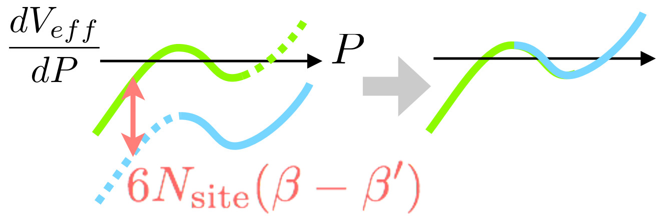
- To avoid a large number of confs to identify a 1st order transition

From a reweighting analysis,

$$\frac{dV_0(P,\beta)}{dP} = \frac{dV_0(P,\beta')}{dP} - 6N_{\rm site}(\beta-\beta')$$

const.

⇨ combine data at several β



We obtain the derivative of $V_{\rm eff}$ in a wide range.

Simulation setup

- HeatBath

  $\det M(\kappa, P)$ is evaluated by using a hopping parameter expansion.

- β & Number of Configurations

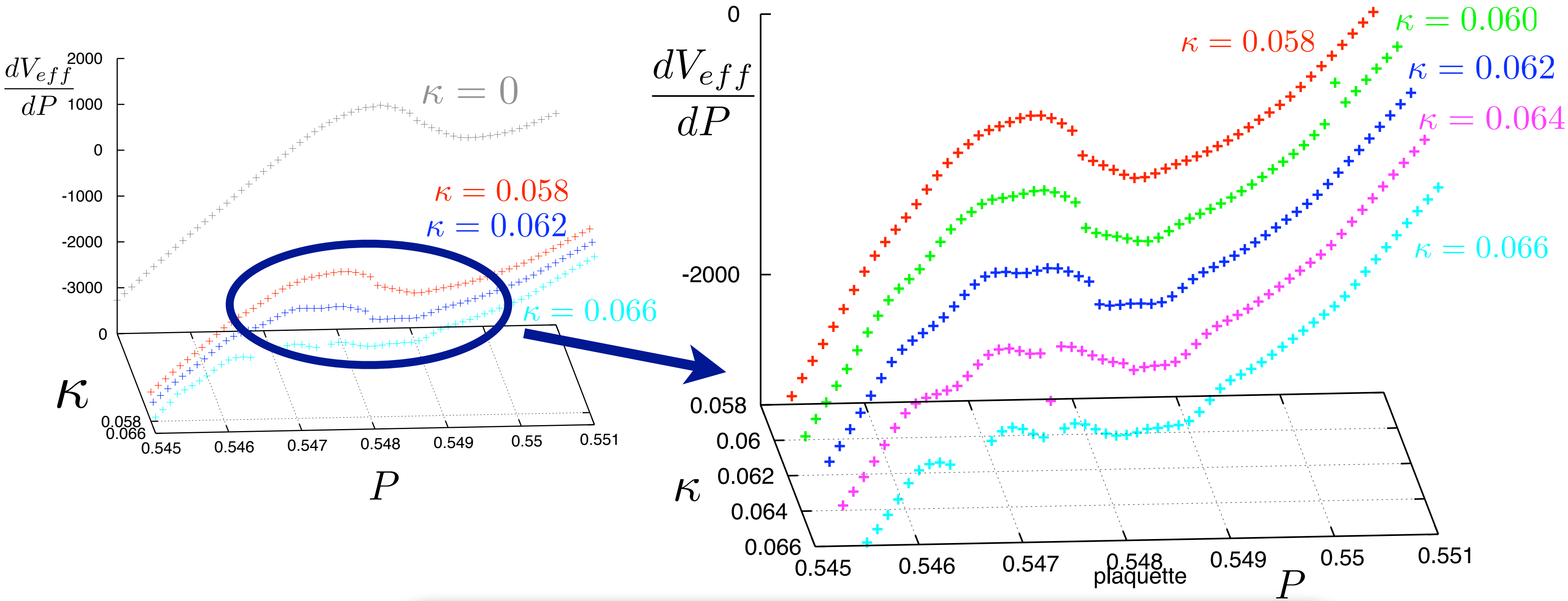
β	Conf. num.
5.68	100,000
5.685	430,000
5.69	500,000
5.6925	670,000
5.7	100,000

⇐ transition point

- Lattice size: 24^3 x 4

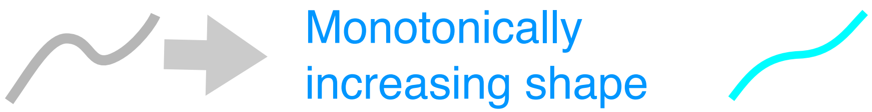
Result - 1st order transition disappears

- The Derivative of V_{eff}



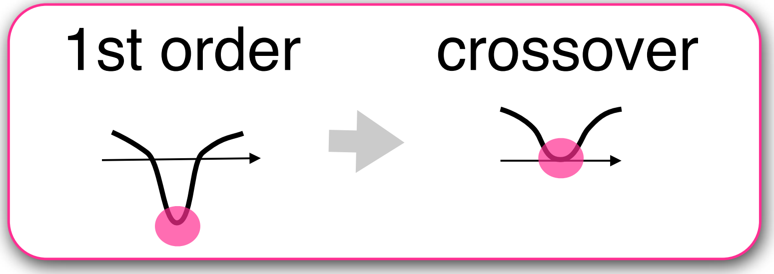
As κ increases,

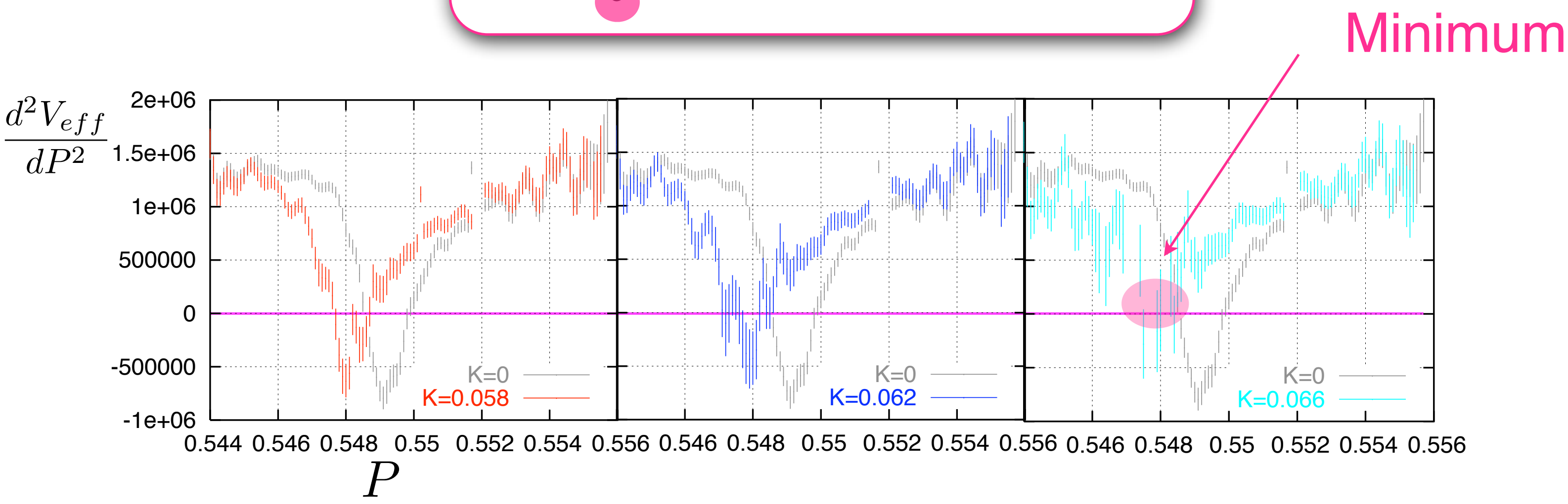
"S shape" → Monotonically increasing shape



Result - Identification of κ_{ep} value 1

- The Secondary Derivative of V_{eff}

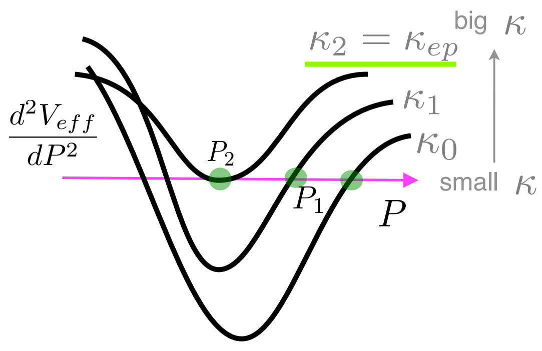




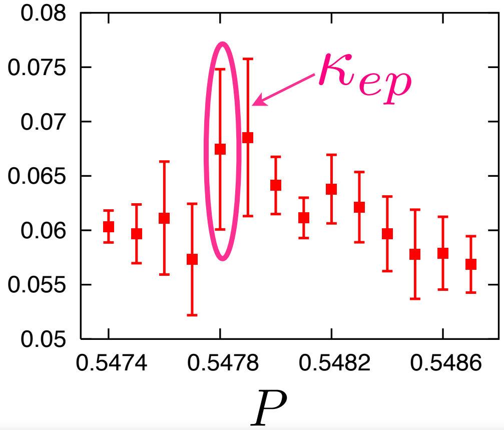
Result - Identification of κ_{ep} value 2

- Detailed analysis for the (pink oval) region

1. Assume that the secondary derivative of V_{eff} changes like this as κ increases.
2. Near the end point, search κ values where $\frac{d^2V_{eff}}{dP^2} = 0$ for each P → (green dot)
3. Maximum of these κ values is identified by κ_{ep}



κ value when $\frac{d^2V_{eff}}{dP^2} = 0$



$$N_f = 2 \qquad \kappa_{ep} = 0.068 \pm 0.007$$

Summary

Summary

- In heavy quark region

$N_f = 2 \qquad \kappa_{ep} = 0.068 \pm 0.007$

- As N_f increases, κ_{ep} decreases.

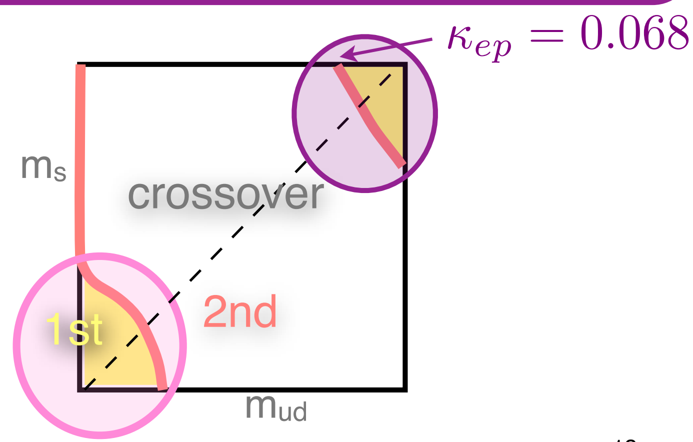
  It is easy to indicate this nature, since a κ dependence is trivial in hopping parameter expansion. We could get the κ_{ep} for 1 flavor which is consist with Alexandrou et al. (1990).

N_f	κ_{ep}	Alexandrou et al (1990,Z(3))
1	0.081(0.008)	~0.08
2	0.068(0.007)	
3	0.061(0.006)	

- Reweighting in κ works well to determine the κ_{ep}.

Future

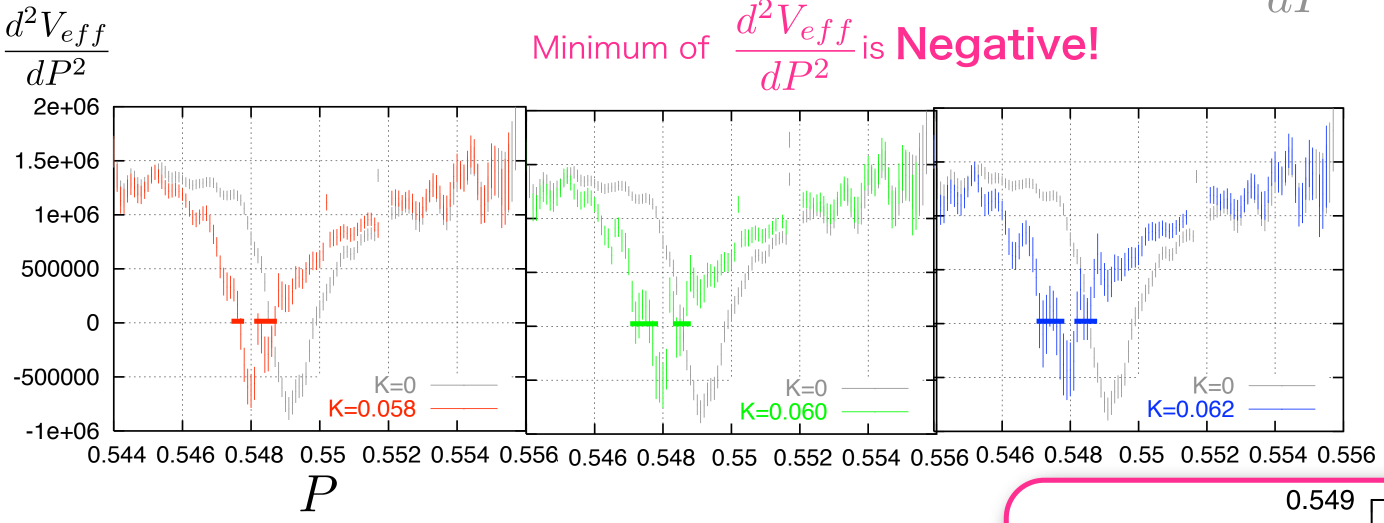
- Analyze the order of the transition of light quark



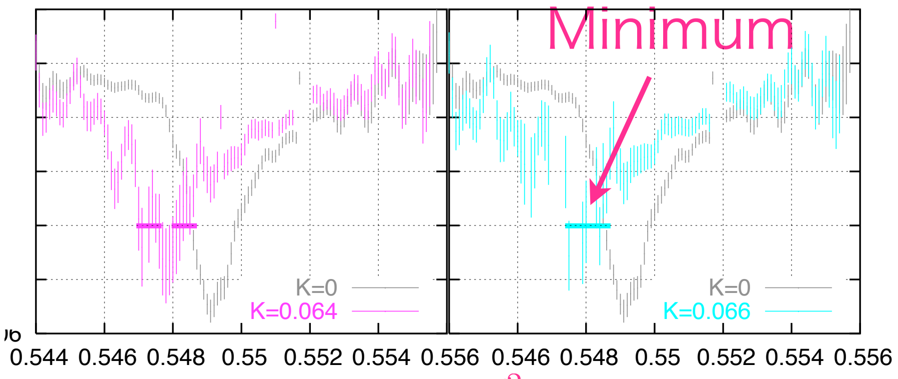
Result - Identification of κ_{ep} value (Detail 1)

- How to decide the candidates for minimum of $\frac{d^2V_{eff}}{dP^2}$.

Minimum of $\frac{d^2V_{eff}}{dP^2}$ is **Negative!**

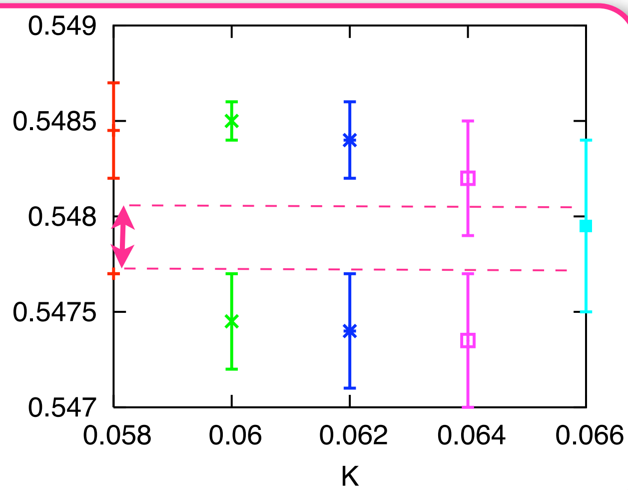


Using the plaquette value when $\frac{d^2V_{eff}}{dP^2} = 0$



Minimum of $\frac{d^2V_{eff}}{dP^2}$ is consistent with **0!**

Plaquette value when $\frac{d^2V_{eff}}{dP^2} = 0$

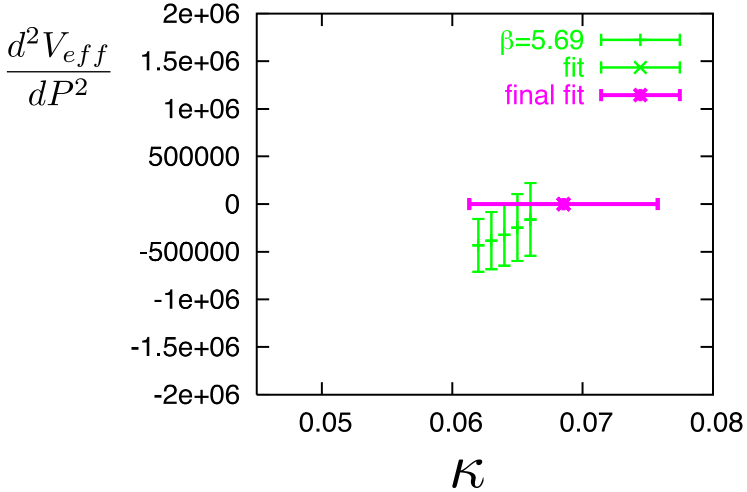


Result - Identification of κ_{ep} value (Detail 2)

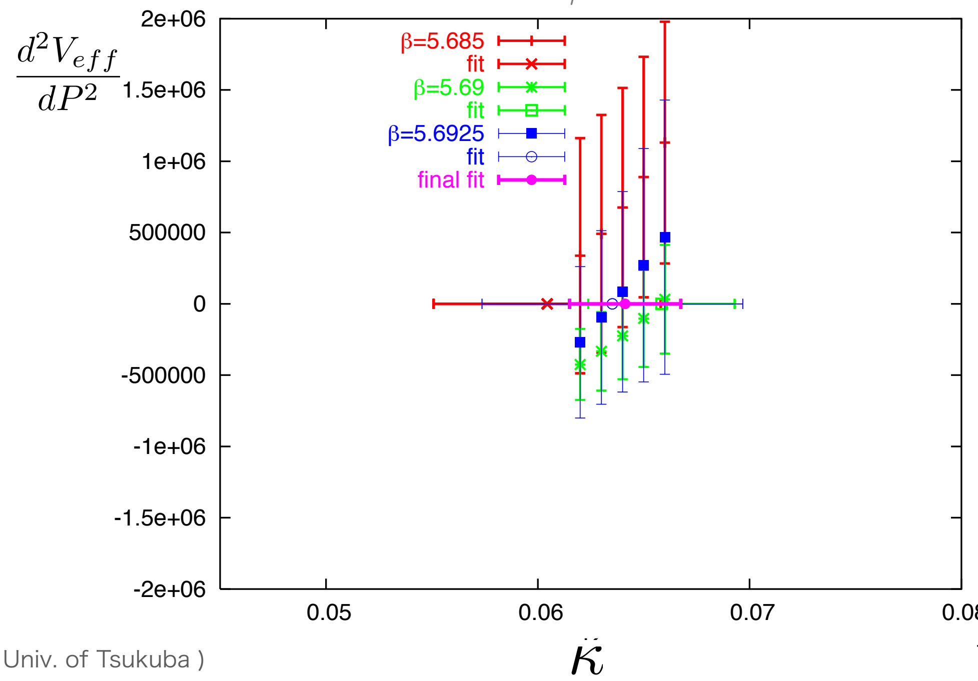
- How to decide the error of Kc

1. jacknife
2. χ^2 fit (for several β data)

single β data



several β data



Result - 1st order transition disappears

- The derivative of V_{eff} with error