On the universal O(N) scaling behavior of (2+1)-flavor QCD

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based on:
S. Ejiri *et al.*, PRD 80 (2009) 094505.

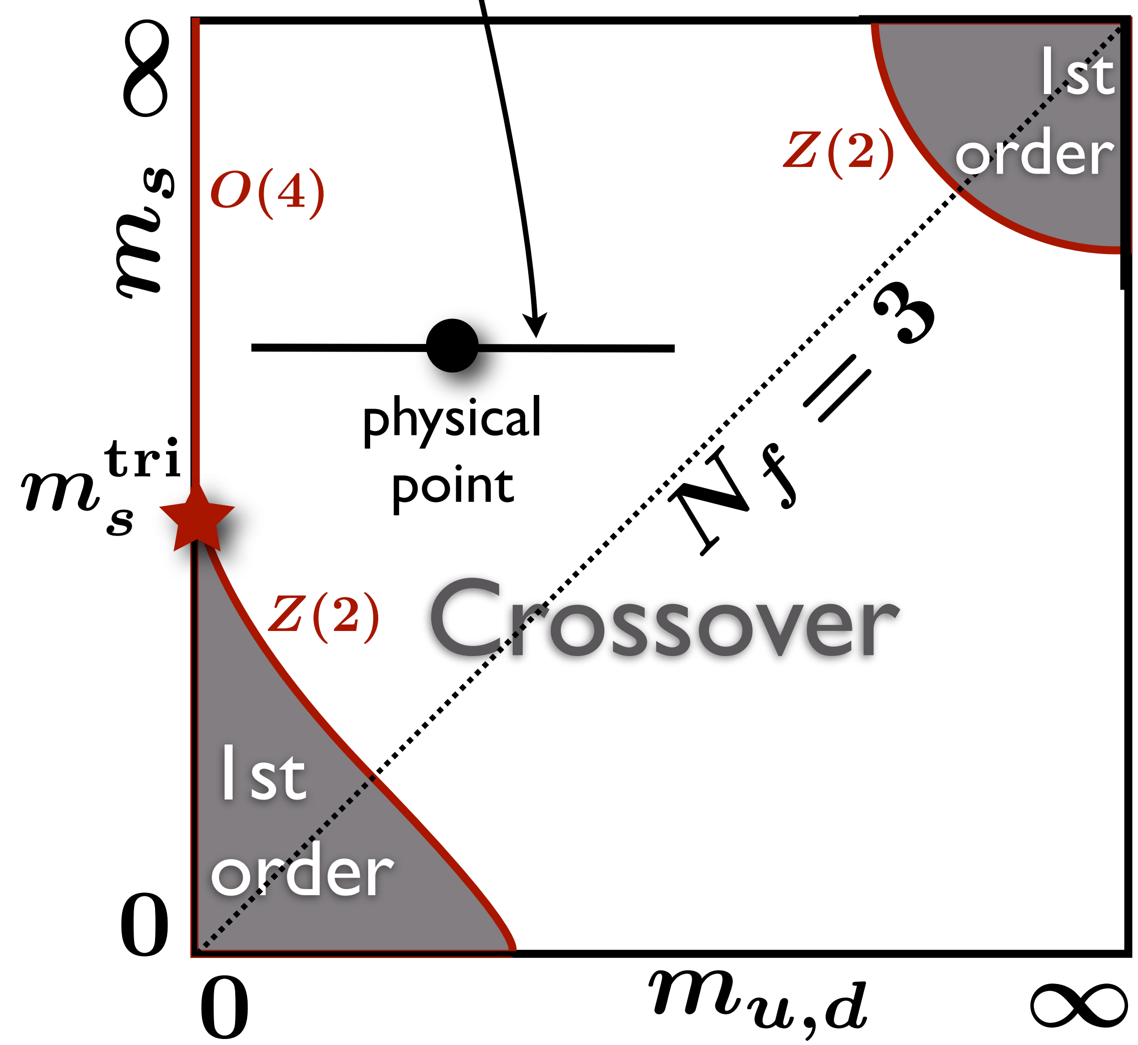
Critical Behavior of QCD (I)

- chiral symmetry of 2-flavor QCD
  $SU_L(2) \times SU_R(2) \simeq O(4)$
- hence, if m_s is large in (2+1)-flavor QCD:
  expect universal behavior as of 3d-$O(4)$ spins in the vicinity of T_c and the chiral limit
- so far no clear evidence from simulations
- staggered fermions preserve a flavor non-diagonal $U(1)$-part of chiral symmetry even at $a > 0$
  $\longrightarrow$ look for $O(2)$-critical behavior

Simulations with improved staggered fermions (p4fat3)

range of simulations
$(N_\tau = 4)$
$m_q = (2/5 - 1/80) m_s$

m_l/m_s	m_π
1/80	75 MeV
1/40	105 MeV
1/20	150 MeV

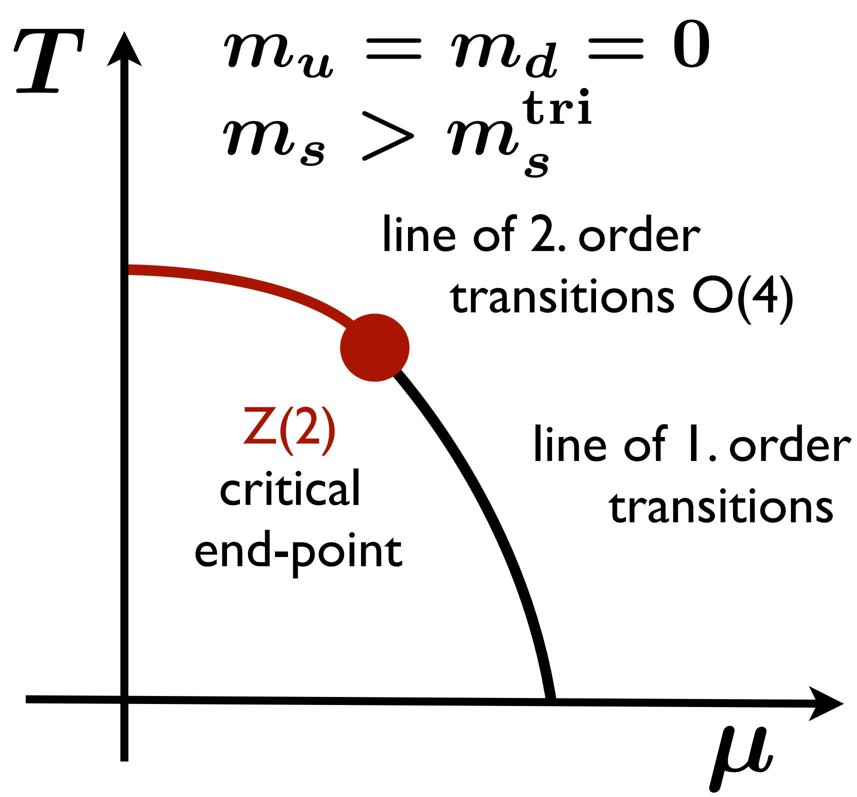


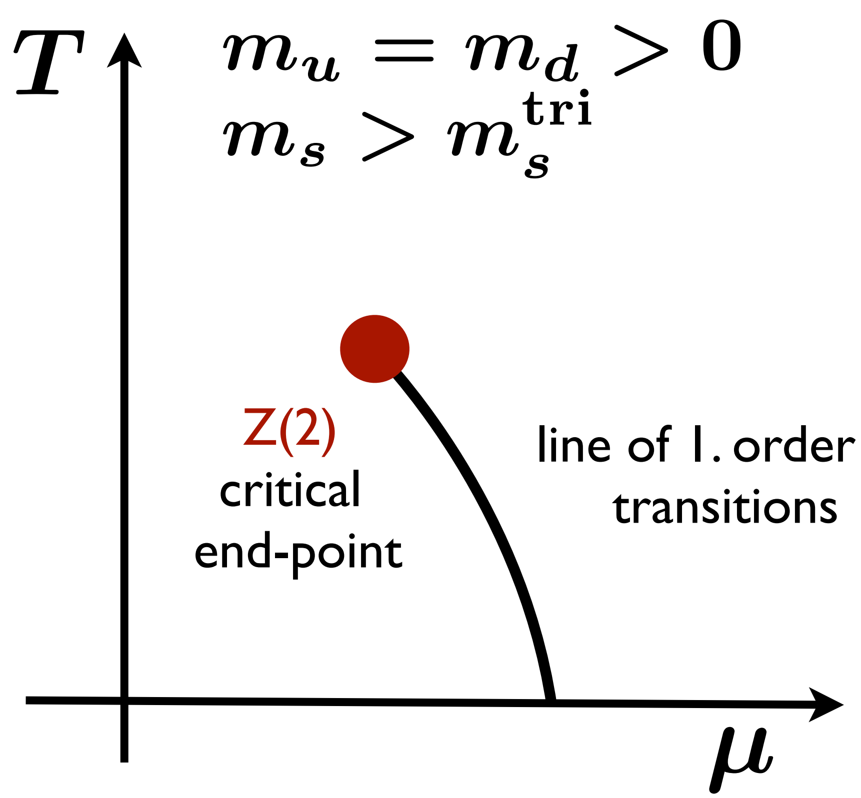
Critical Behavior of QCD (II)

- situation at nonzero chemical potential is very unclear
- direct simulations MC simulations are prohibited by the sign-problem

$\longrightarrow$ use Taylor expansion approach

expected (T, μ)-phase diagrams:





Outline

★ **Introduction**

★ **Scaling of chiral condensate (The magnetic EoS)**

- introduce some analogies between spin models and QCD
- fit p4fat3-data ($N_\tau = 4, 8$) to magnetic scaling function f_G

  $\longrightarrow$ determine important non universal constants of QCD

★ **Scaling of chiral and mixed susceptibilities**

- fit p4fa3-data to scaling functions f_χ and f'_G

  $\longrightarrow$ more sesitivity to the universality class

  $\longrightarrow$ predictions on the critical line

★ **Summary**

The scaling hypothesis

- Thermodynamics in the vicinity of a critical point:

free energy density:

$$-\frac{1}{V}\ln Z = f_s(t,h) + f_r(T,V,H)$$

(singular part) (regular part)

where

$$t = \frac{1}{t_0}\frac{T-T_c}{T_c}$$

(reduced temperature)

$$h = \frac{H}{h_0}$$

(external field)

QCD:

$H \sim m_q$

(quark mass)

our choice:
$H = m_l/m_s$

assume:

$$f_s(t,h) = b^{-d} f_s(b^{y_t}t, b^{y_h}h)$$

choose: $b = h^{-1/y_h}$

$$f_s(t,h) = h_0 h^{1+1/\delta} f_M(z) \quad \text{with} \quad z = t/h^{1/\beta\delta}$$

("magnetic version" of the fee energy density) (scaling variable)

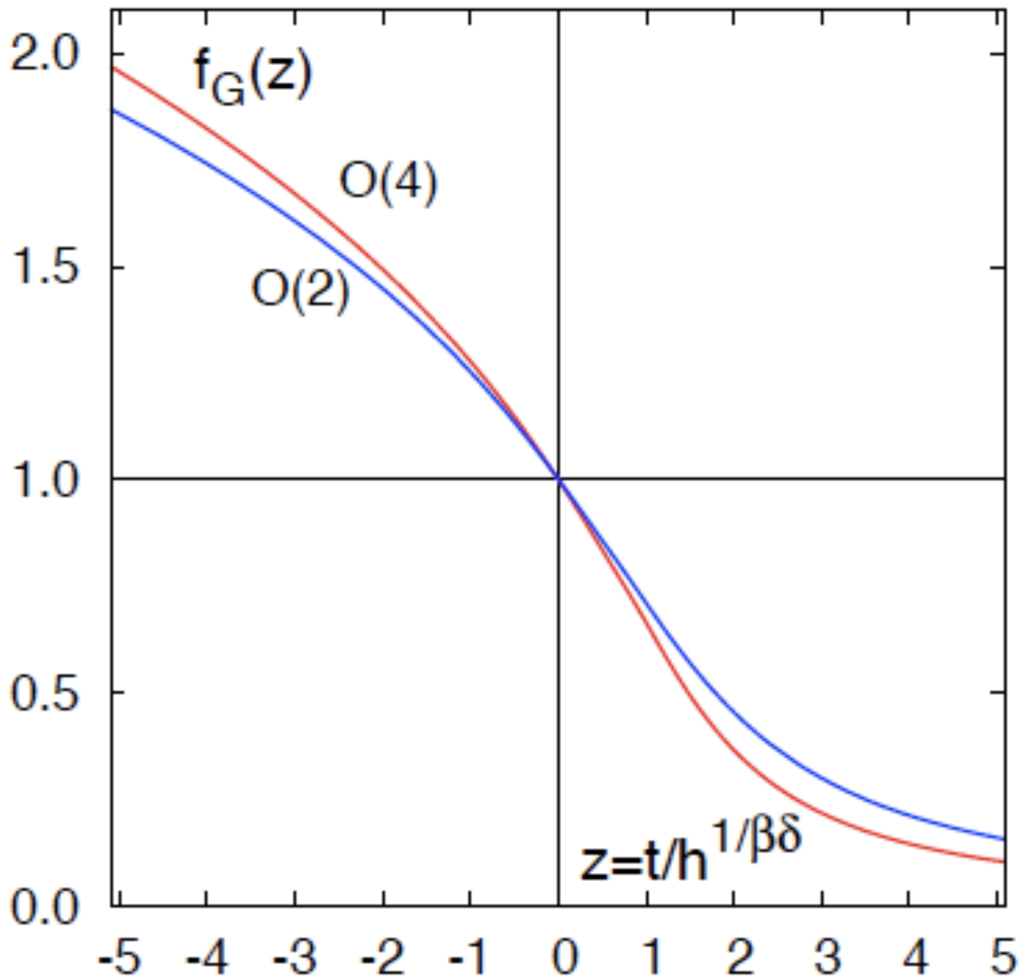
Magnetic EoS in $O(N)$-spin models

- order parameter (magnetization):

$$M = -\frac{\partial f_s(t,h)}{\partial H} = \frac{1}{h_0}\frac{\partial f_s(t,h)}{\partial h} \equiv h^{1/\delta} f_G(z)$$

universal scaling function

$$f_G(z) = -\left[\left(1+\frac{1}{\delta}\right) f_M(z) - \frac{z}{\beta\delta} f'_M(z)\right]$$



- scaling variable:

$$z = t/h^{1/\beta\delta}$$

- scaling function and critical exponents are known to high precision in condensed matter literature [e.g. Engels *et al.*]

- scaling function includes Goldstone effect in the limit of $z \longrightarrow -\infty$

$$z \rightarrow -\infty : \quad h \rightarrow 0, t < 0 \qquad M \sim (-t)^\beta + c(t)\sqrt{h}$$

Lattice setup

- lattice action: improved staggered fermions (p4fat3), (2+1)-flavor
- algorithm: exact RHMC
- strange quark mass: fixed to physical values, $N_\tau = 4: \; am_s = 0.065$
  $N_\tau = 8: \; am_s = 0.024$
  $\rightarrow \; m_{\bar{s}s} \simeq 669$ MeV
- light quark mass: $m_q/m_s = 1/80: \; \rightarrow \; m_\pi = 75$ MeV
  $m_q/m_s = 1/20: \; \rightarrow \; m_\pi = 150$ MeV
- statistics (measurements separated by 10 trajectories):

lattice dim.	m_q/m_s	statistics	lattice dim.	m_q/m_s	statistics
$32^3 \times 4$ ▲	1/80	$\mathcal{O}(20000)$			
$32^3 \times 4$ ▲	1/40	$\mathcal{O}(20000)$			
$16^3 \times 4$ ▲	1/40	$\mathcal{O}(30000)$	$32^3 \times 8$	1/40	just started
$16^3 \times 4$ ▲	1/20	$\mathcal{O}(40000)$	$32^3 \times 8$ ✦	1/20	$\mathcal{O}(20000)$
$16^3 \times 4$ ▲	1/10	$\mathcal{O}(40000)$	$32^3 \times 8$ ★	1/10	$\mathcal{O}(30000)$
$16^3 \times 4$ ▲	1/5	$\mathcal{O}(40000)$	$32^3 \times 8$ ★	1/5	$\mathcal{O}(30000)$
$16^3 \times 4$ ▲	2/5	$\mathcal{O}(40000)$			
$\beta = 3.2800$,	...	3.3300	$\beta = 3.4800$,	...	3.5400

▲ S. Ejiri *et al.* [**RBC-Bielefeld-GSI**], PRD 80 (2009) 094505.

✦ M. Cheng *et al.* [**RBC-Bielefeld-GSI**], PRD 81 (2010) 054504.

★ A. Bazavov *et al.* [**HotQCD**], PRD 80 (2009) 014504.

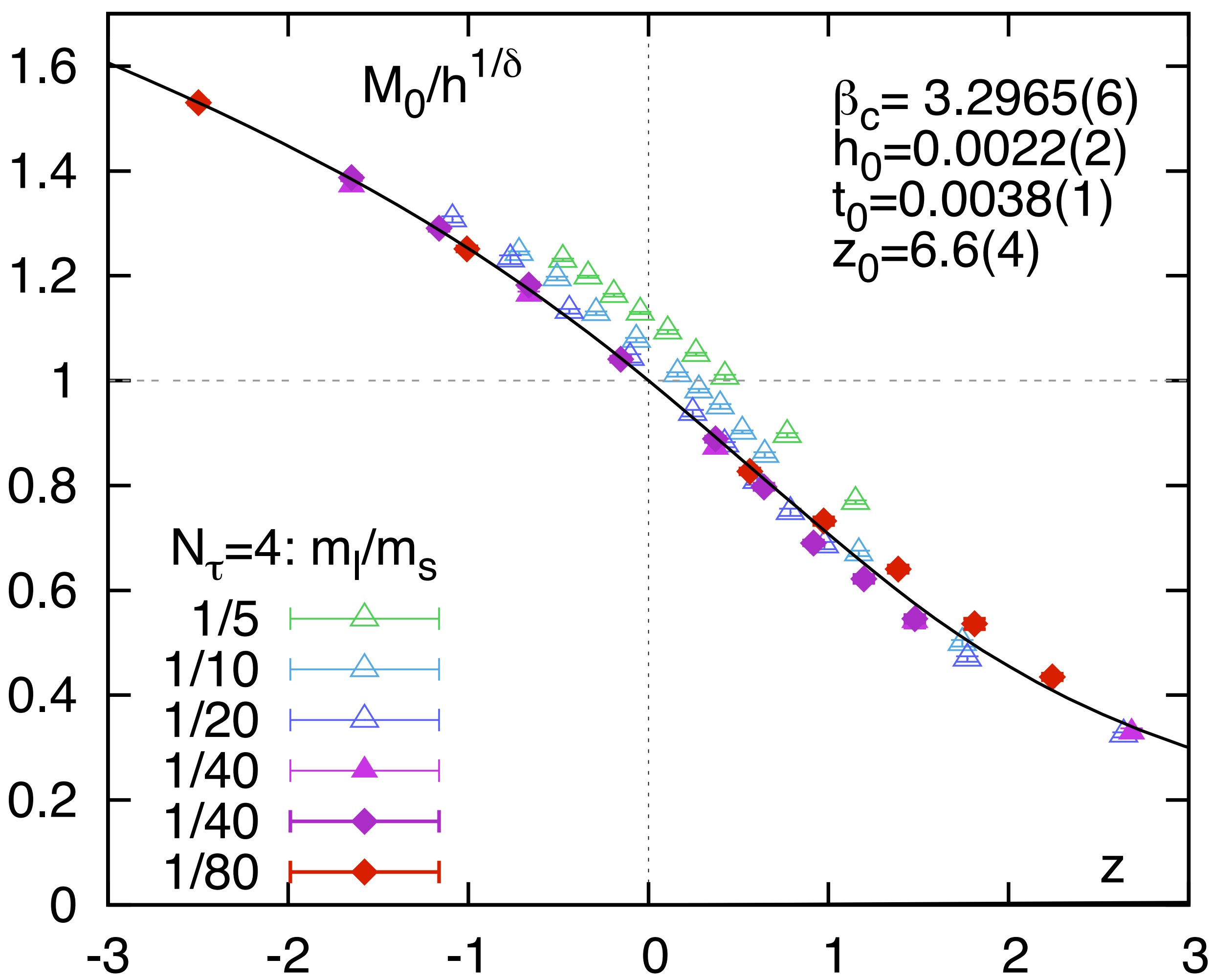
Magnetic EoS in QCD (Nt=4)

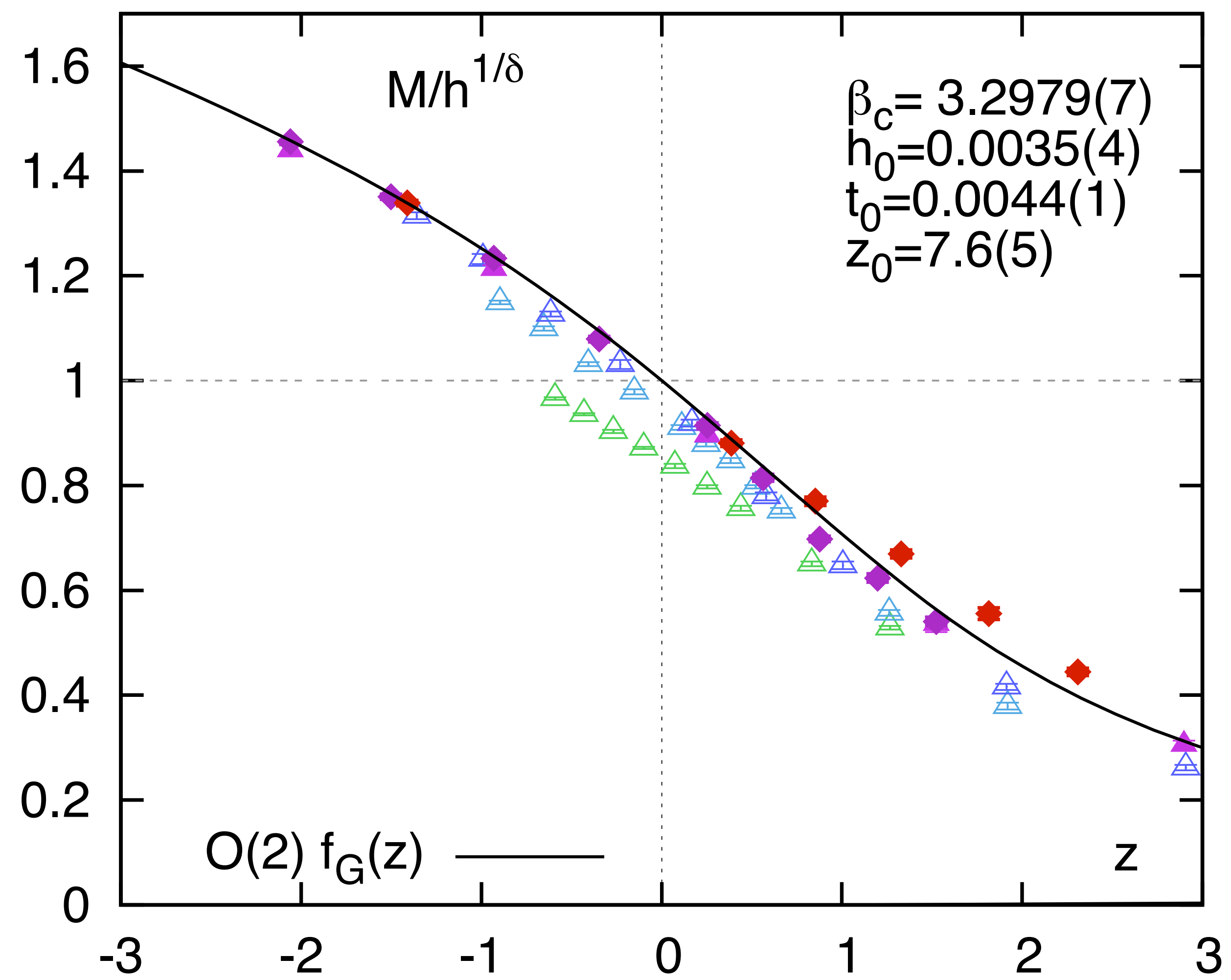
- two order parameter:

$$\left.\begin{aligned} M_0 &= m_s \langle\bar\psi\psi\rangle_l / T^4 \\ M &= m_s \left(\langle\bar\psi\psi\rangle_l - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_s\right)/T^4 \end{aligned}\right\} = h^{1/\delta} f_G(z)$$

(subtracted condensate to remove UV-div. $\sim m_l/a^2$)

- three fit parameter: critical temperature T_c (critical coupling β_c), normalization constants $t_0,\ h_0$

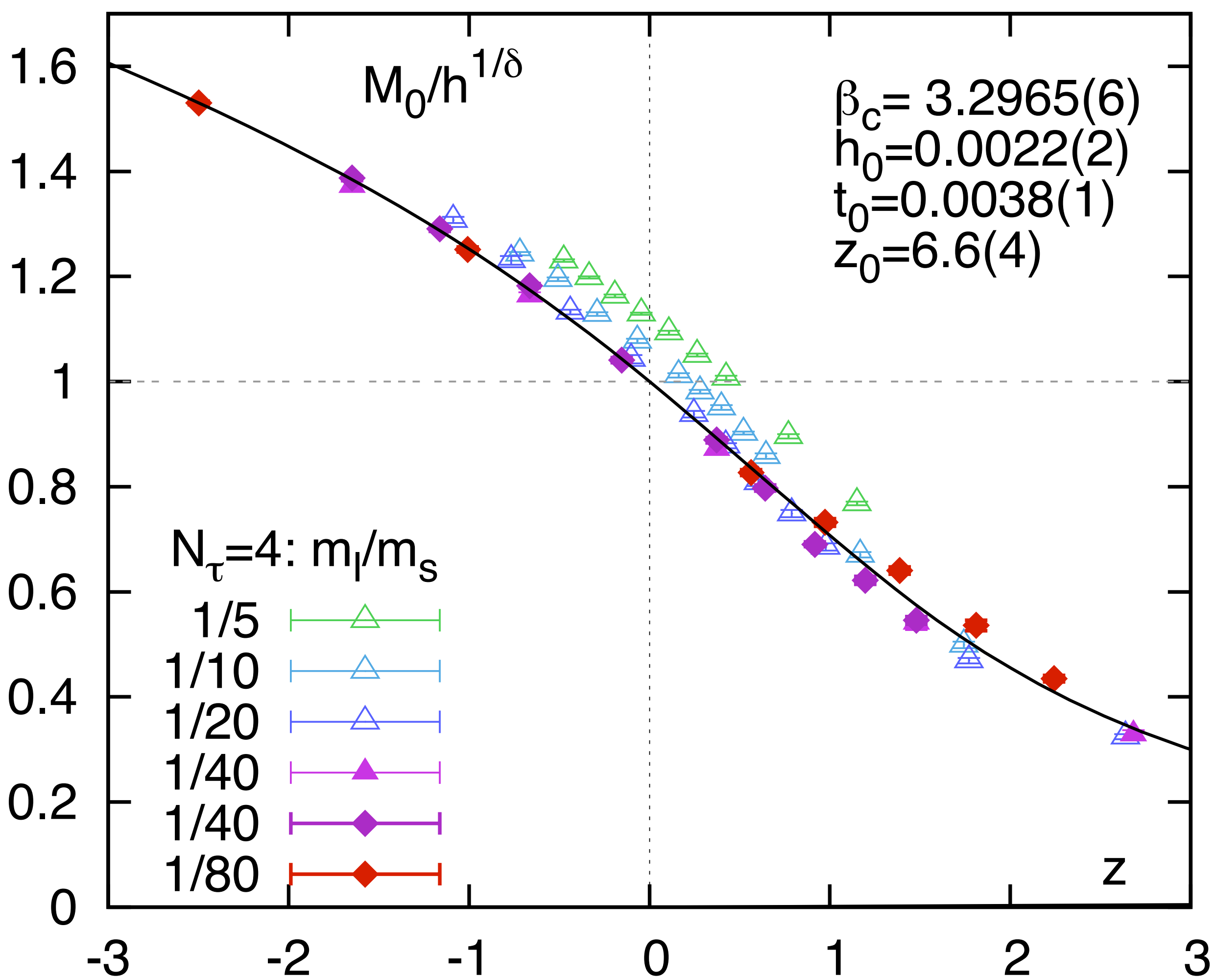


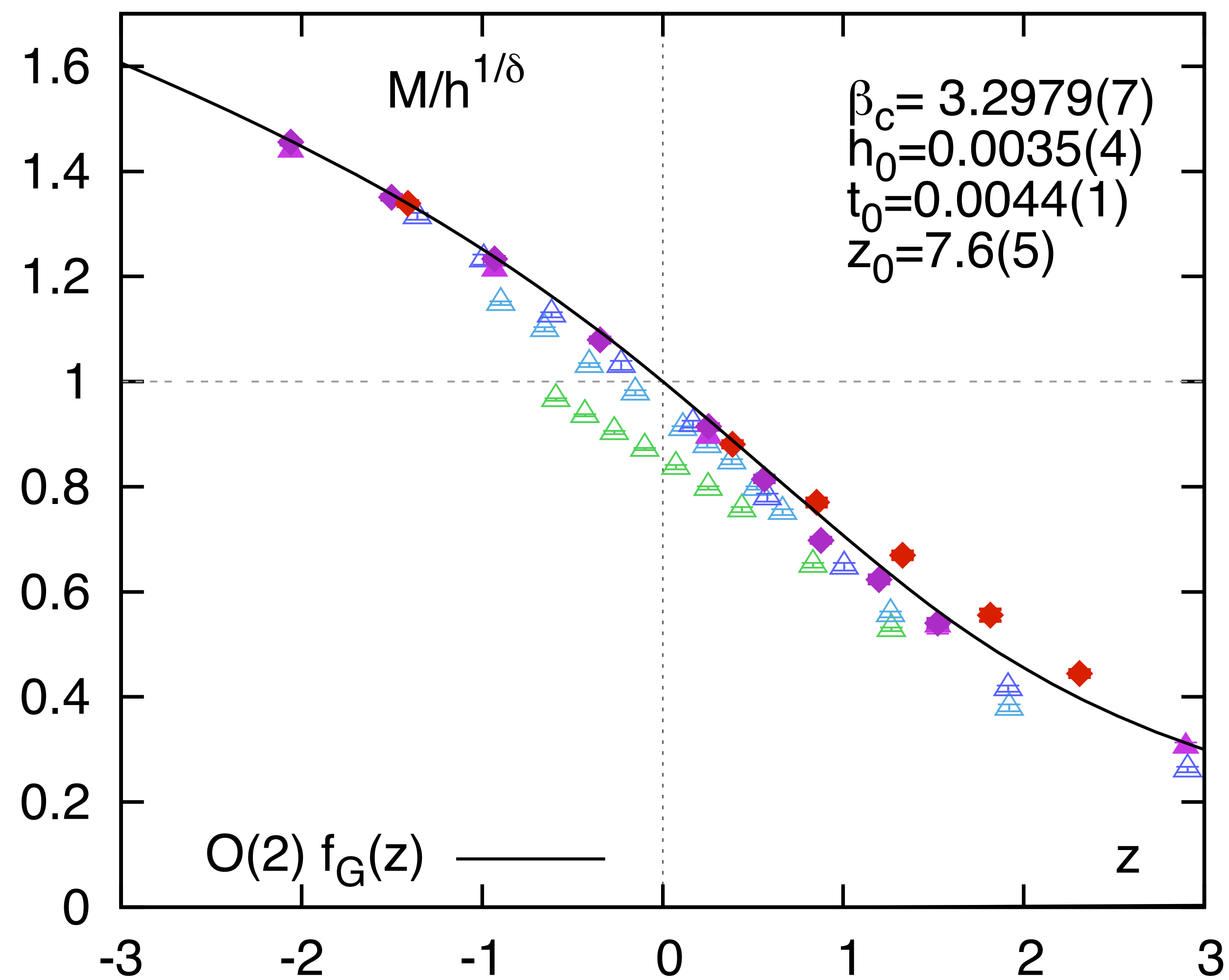


Magnetic EoS in QCD (Nt=4)

- O(2) slightly preferred, however, re-parametrization $z \rightarrow 1.2z$ moves O(2) onto O(4)

  $\longrightarrow$ scaling functions almost indistinguishable, we can not discriminate between O(2) and O(4)
- $z_0 = t_0/h_0^{1/\beta\delta}$ is independent under re-scaling ($t_0,\ h_0$ not)
- $z_0(m_s, a^2)$ might be a QCD invariant, which only depend on strange quark mass and lattice artifacts



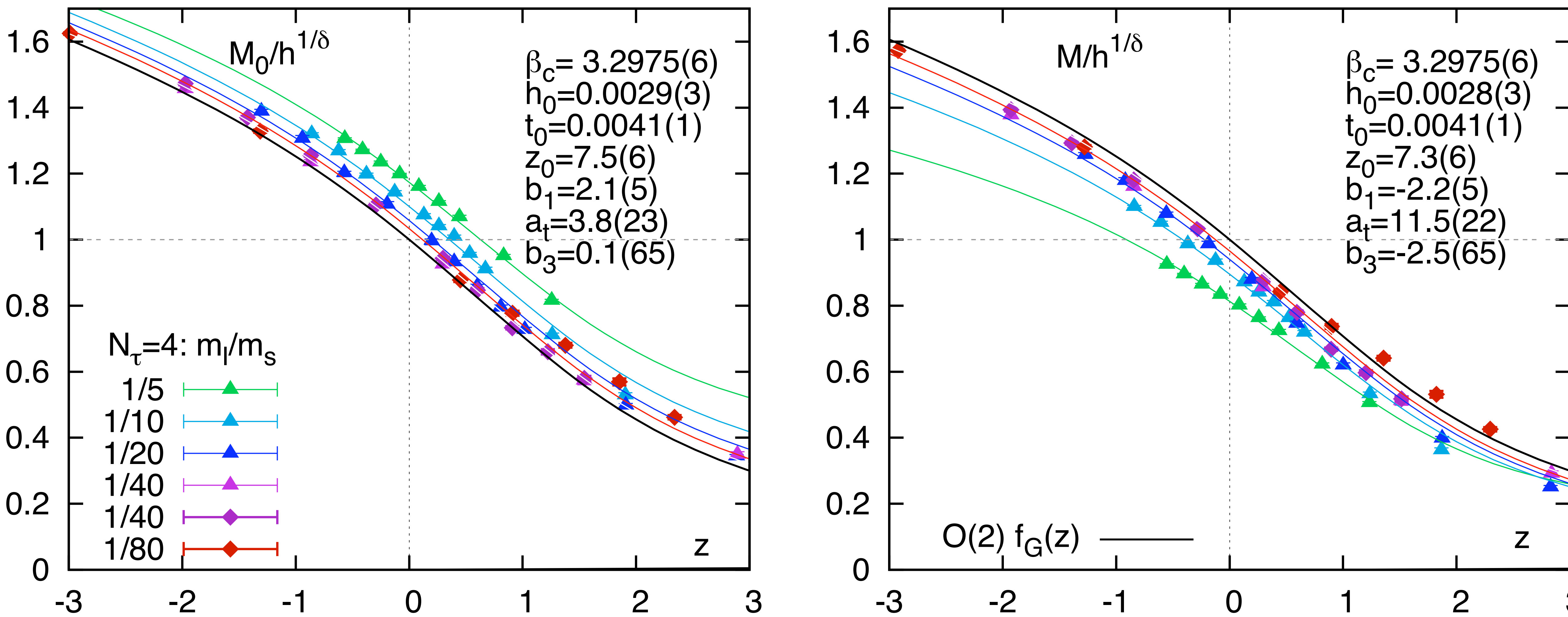


Deviations from scaling (Nt=4)

- mass range $m_l/m_s < 1/20$ is well described by scaling function
- deviations from scaling substantial for $m_l/m_s > 1/20$
- include regular part into the fit:

$$M = h^{1/\delta} f_G(z) + a_t(T - T_c)H + b_1 H + b_3 H^3$$

$\longrightarrow$ results for β_c, t_0, h_0 are recovered within errors

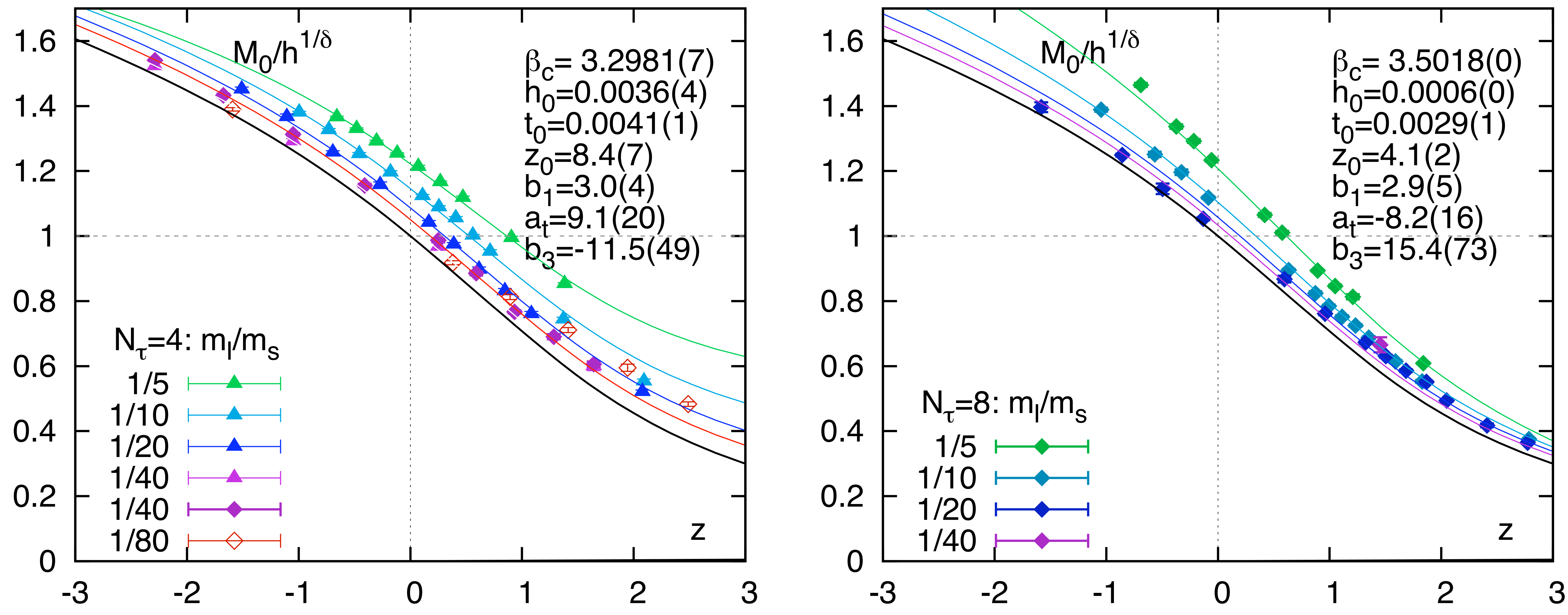


Magnetic EoS in QCD (Nt=8)

- $N_\tau = 8:$ fit w/o scaling violations not possible yet
- fit for $\beta_c, t_h, h_0, a_t, h_1, h_3$ (range $m_l/m_s \geq 1/40$) works reasonably well $\longrightarrow$ assume z_0 to be stable/reliable
- cutoff dependence:

N_τ	4	8
z_0	$7.5(9)$	$4.3(5)$

$\longrightarrow$ further studies are needed to control continuum limit



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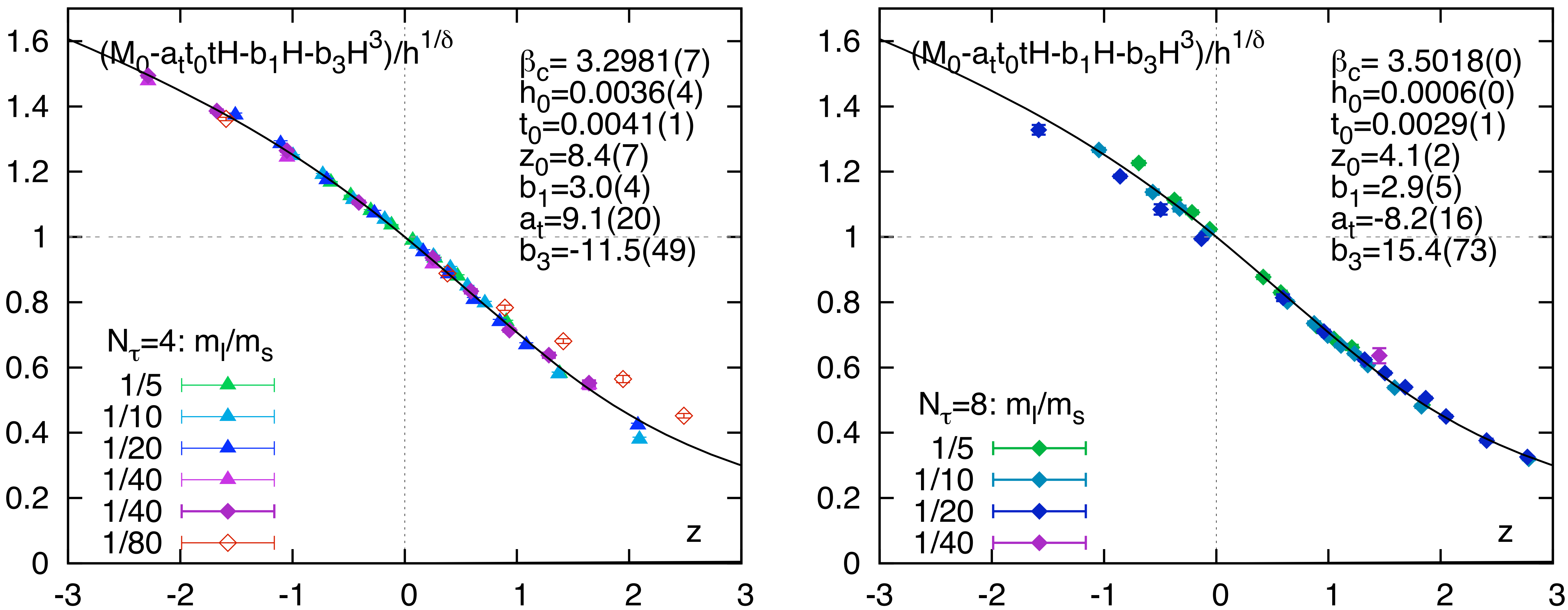
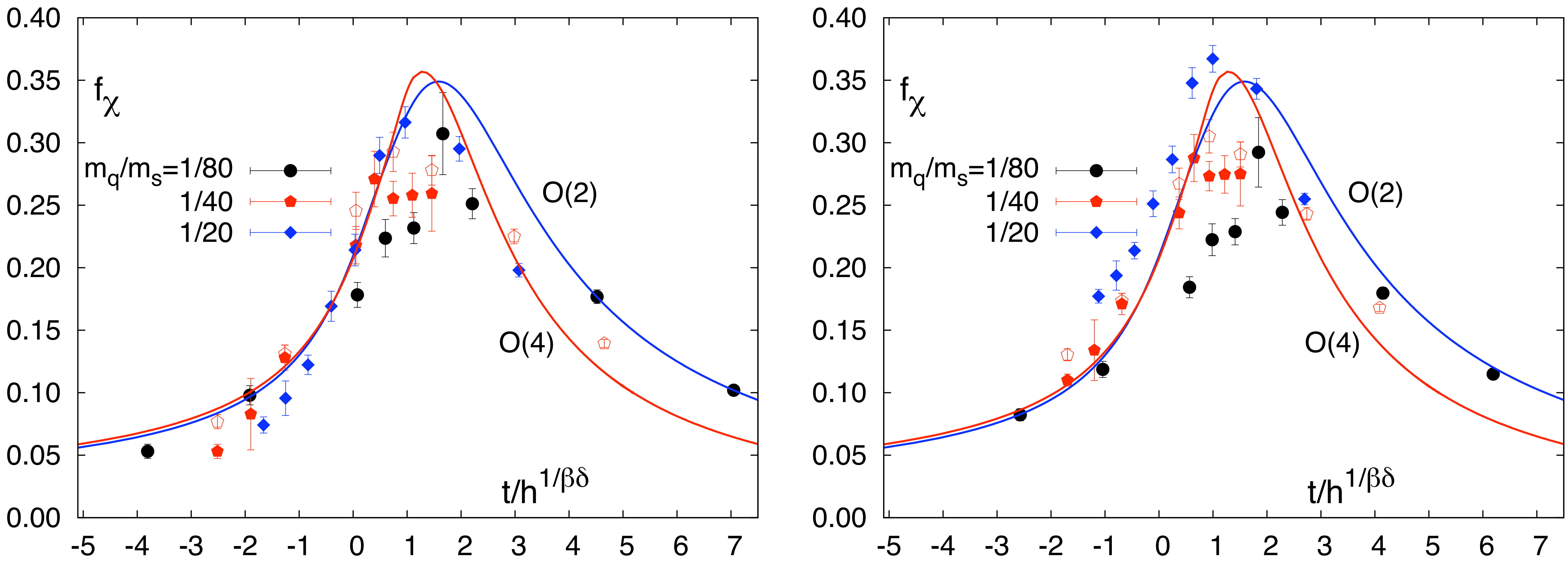


Scaling of chiral susceptibility (Nt=4)

- scaling function: $$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}f'_G(z)\right)$$

- chiral susceptibilities scale reasonably well

- f_χ more sensitive to universality class, however, statistics still not sufficient



Thermal fluctuations of the order parameter

- mixed susceptibility:

$$\chi_t \equiv \frac{\partial M}{\partial T} = \frac{1}{t_0 T_c}\frac{\partial M}{\partial t} = \frac{1}{t_0 T_c} h^{(\beta-1)/\beta\delta} f'_G(z)$$

where

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c}$$
(reduced temperature)

$$h = \frac{H}{h_0}$$
(external field)

- introducing chemical potential:

$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa_\mu \left(\frac{\mu_l}{T}\right)^2\right)$$

in the chiral limit: μ_l does not break chiral symmetry

$\longrightarrow$ couples only to reduced temperature

- (other) mixed susceptibility:

$$c_2^{\bar{\psi}\psi} \equiv \left.\frac{\partial^2 M}{\partial(\mu_l/T)^2}\right|_{\mu_l=0} = \frac{2\kappa_\mu}{t_0 T_c}\frac{\partial M}{\partial t} = \frac{2\kappa_\mu}{t_0 T_c} h^{(\beta-1)/\beta\delta} f'_G(z)$$

$$\propto \chi_t$$

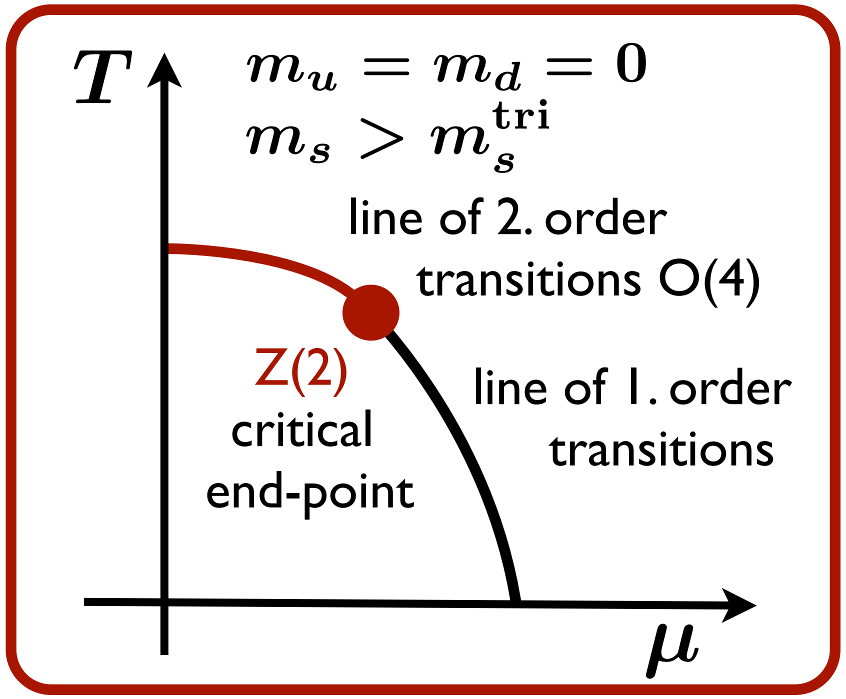
The critical line

- curvature of critical line in the ciral limit:

$$t = 0 \quad \longleftrightarrow \quad \frac{T}{T_c} = 1 - \kappa_\mu \left(\frac{\mu_l}{T}\right)^2$$

t_0, h_0, T_c known form scaling analysis of magnetic EoS

expected phase diagram



$\longrightarrow$ fit $2\kappa_\mu f_G'(z)$ to χ_t-data (one fit parameter)

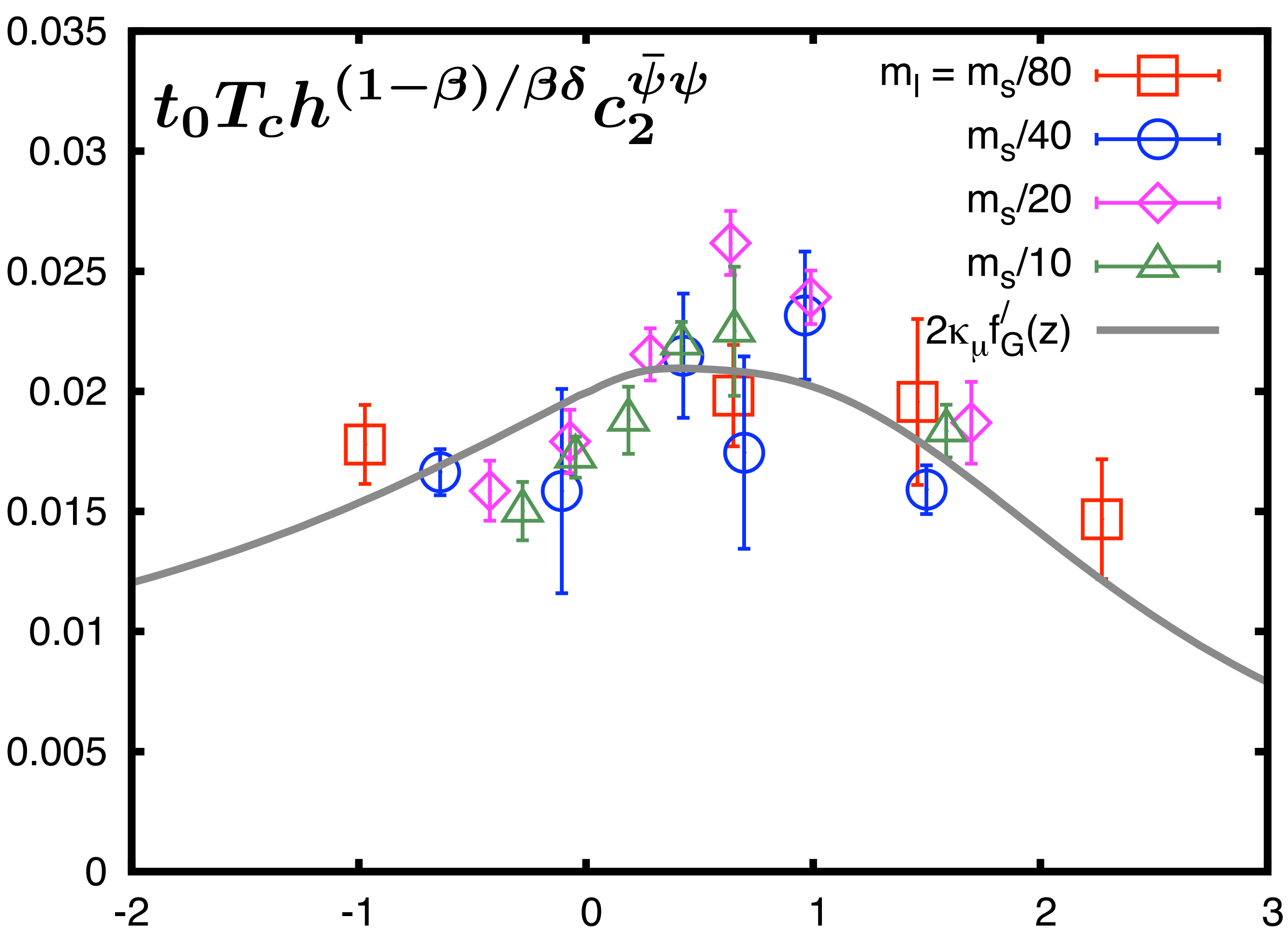
$\longrightarrow$ preliminary result from fit to O(2) scaling curve:

$$\kappa_\mu = 0.035(1)$$

$\longrightarrow$ for orientation: reweighting std. action, $m_l/m_s = 1/27$

$$\kappa_\mu = 0.0288(9)$$

Z. Fodor and S.D. Katz, JHEP 0404 (2004)



Summary

★ **The magnetic EoS**

- EoS consistent with 3d-O(N) scaling already at physical masses
- we find no evidence for nearby first order phase transitions

★ **Scaling of chiral susceptibilities**

- statistics not (yet) sufficient to discriminate between universality classes

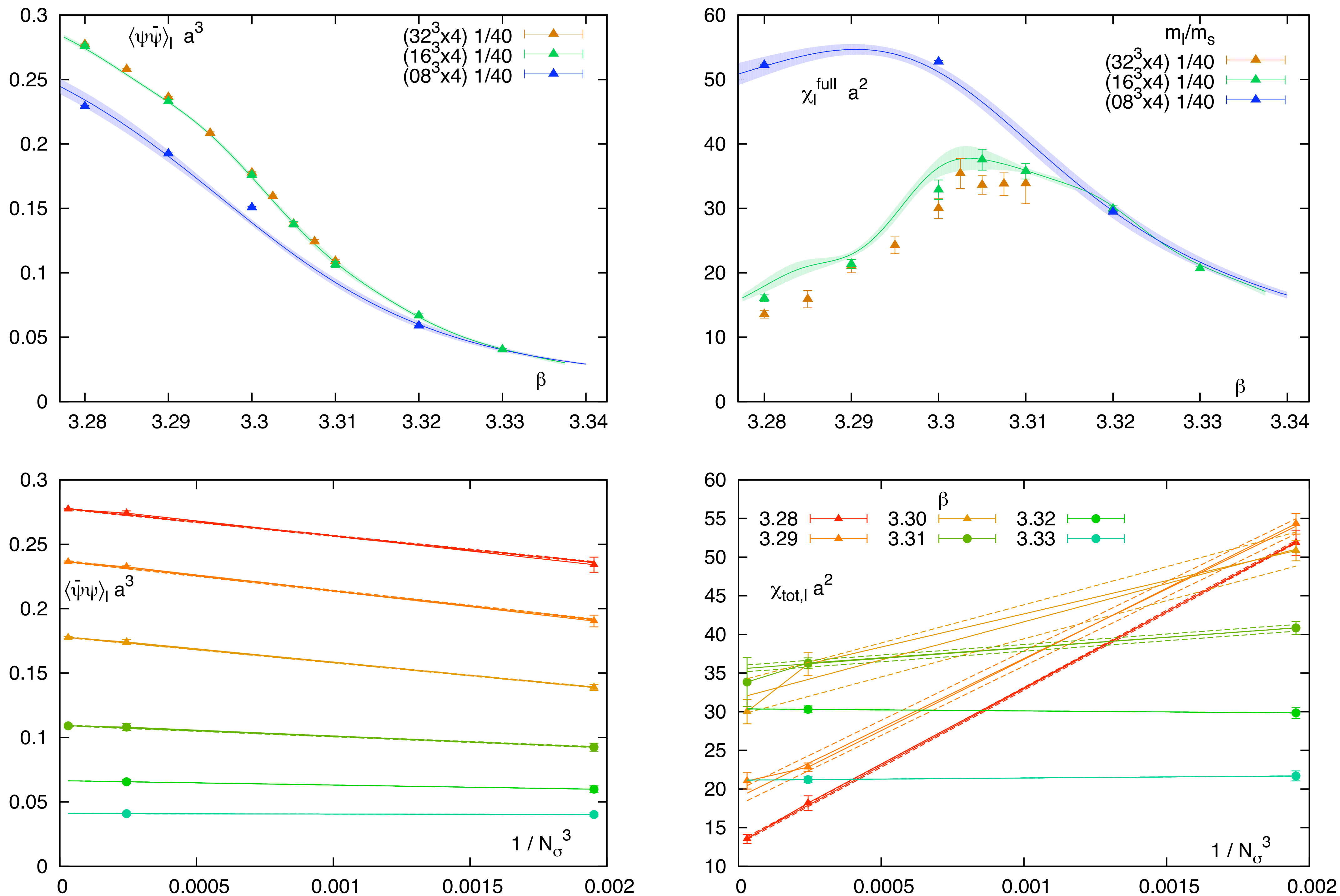
★ **Non-zero chemical potentails**

- the curvature of the critical line in the chiral limit can be extracted from scaling poperties of mixed susceptibilities

→ all this needs to be confirmed in the continuum limit

Check on finite size effects

- no evidence for finite size scaling $\longrightarrow$ crossover
- no strong temperature dependence of 1/V corrections
- thermodynamic limit well under control (for smalles mass: $m_\pi L \simeq 3$)



Scaling of chiral susceptibility (Nt=8)

- scaling function:

$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$

- full susceptibilities scale reasonably well (after subtraction of regular part)
- f_χ more sensitive to universality class, however, statistics still not sufficient