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Mesons and baryons masses with low mode averaging

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Introduction

- preliminary study for further works (possibly in the large N limit)
- light quark masses $\Rightarrow$ deflation technique with low lying eigenvectors
- eigenvectors $\Rightarrow$ low-mode averaging to improve the signals
  T. DeGrand, S. Schaefer, Nucl. Phys. B (2004)
- comparison of LMA using eigenvectors of the massive Dirac operator M and the Hermitian one $\gamma_5 M$.
- study of LMA for baryons

Low-mode averaging

The meson two point function can be written as:

$$C^{aa}(t) = \sum_{\overline{x}\,\overline{y}\,t_0} \mathrm{Tr}[\Gamma\, G(\overline{x}, t+t_0; \overline{y}, t_0)\Gamma\, G(\overline{y}, t_0; \overline{x}, t+t_0)]$$

We can split the Green functions as a sum of high and low modes:

$$G = G_H + G_L$$

And reorganize the all-to-all correlator as:

$$C^{aa} = C_{HH} + C_{HL} + C_{LH} + C_{LL} = C_{LL}^{aa} + \sum_p (C^{pa} - C_{LL}^{pa})$$

We define the low mode averaged two point function as:

$$C_{lma} = \frac{C^{aa}}{V} \approx \frac{C_{LL}^{aa}}{V} + C^{pa} - C_{LL}^{pa}$$

Low-mode averaging

The statistical mean of the two quantities is the same:

$$\langle C_{lma} \rangle = \langle C^{pa} \rangle$$

but error bars are smaller due to more sampling per lattice.

Using the eigenvectors of the Hermitian Dirac operator:

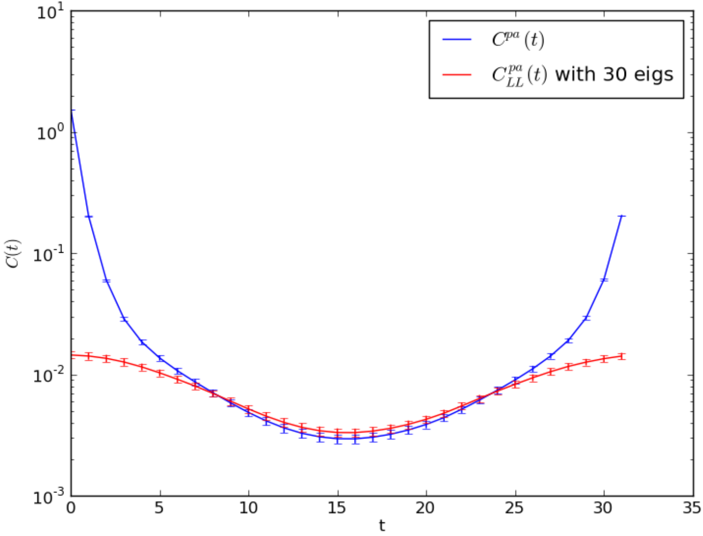
$$Q|v_i\rangle = \lambda_i |v_i\rangle, \qquad Q = \gamma_5 M$$

$$G = M^{-1} = Q^{-1}\gamma_5 = \sum_i \frac{1}{\lambda_i} |v_i\rangle\langle v_i|\gamma_5$$

we can write the meson two point function in terms of the eigenvectors:

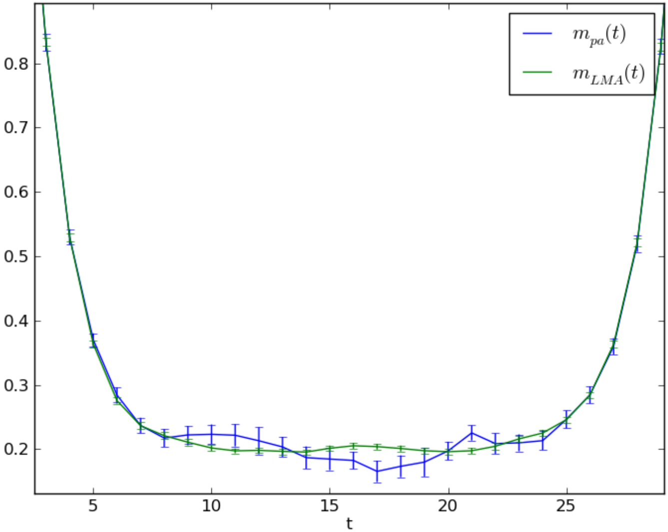
$$C_{LL}^{aa}(t) = \sum_{i\,j\,t_0} \frac{1}{\lambda_i \lambda_j} \left\{ \sum_{\overline{x}} \langle v_j(x)|\gamma_5\Gamma\,|v_i(x)\rangle \right\} \left\{ \sum_{\overline{y}} \langle v_i(y)|\gamma_5\Gamma\,|v_j(y)\rangle \right\}$$

Pion - Two point function

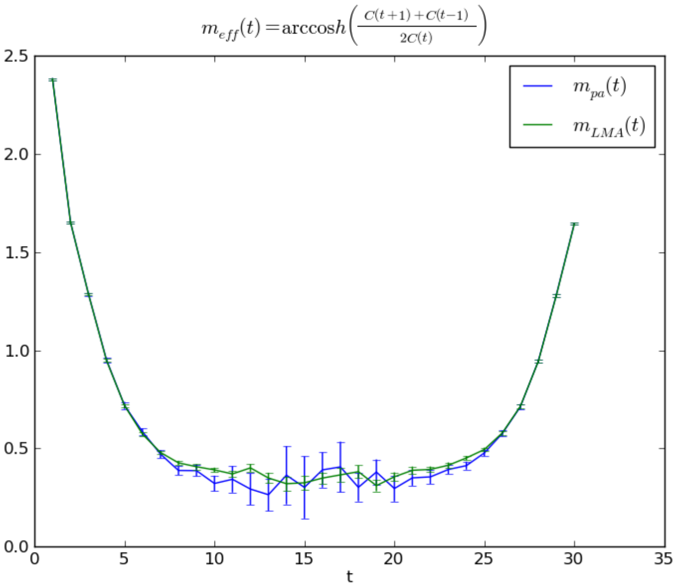


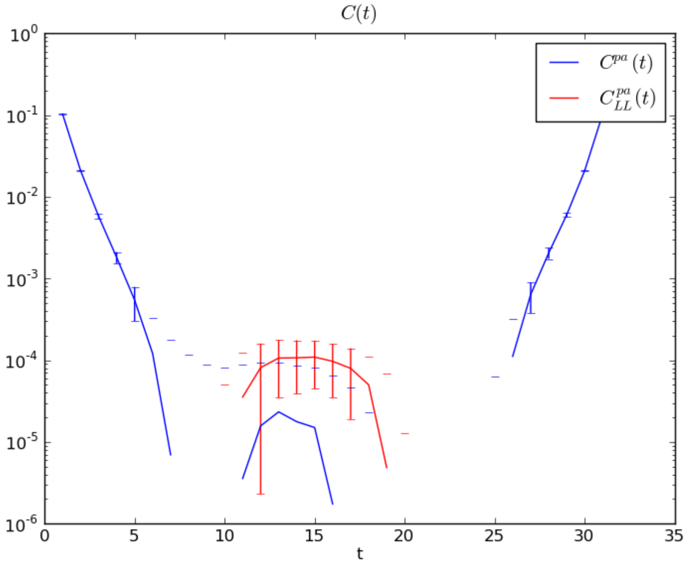
Pion - Effective mass

$$m_{eff}(t) = \text{arccosh}\left(\frac{C(t+1)+C(t-1)}{2C(t)}\right)$$



ρ - Effective mass





C(t)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
5
10
15
20
25
30
35
t
$C^{pa}(t)$
$C^{pa}_{LL}(t)$

Some considerations

Mesons with negative parity seem to look better than the positive parity ones, in particular:

- the pion π with the two point function:

$$C_\pi(t) = \sum_{\overline{x}\,\overline{y}\,i\,j\,t_0} \frac{1}{\lambda_i\,\lambda_j} \langle v_j(x)|(\gamma_5 \cdot \gamma_5 = 1)\,|v_i(x)\rangle\langle v_i(y)|(\gamma_5 \cdot \gamma_5 = 1)\,|v_j(y)\rangle$$

  seems to be the best one.

- the scalar a_0 with the two point function:

$$C_{a_0}(t) = \sum_{\overline{x}\,\overline{y}\,i\,j\,t_0} \frac{1}{\lambda_i\,\lambda_j} \langle v_j(x)|(\gamma_5 \cdot 1 = \gamma_5)\,|v_i(x)\rangle\langle v_i(y)|(\gamma_5 \cdot 1 = \gamma_5)\,|v_j(y)\rangle$$

  seems to be the worst one.

Can we find a way to have 1 also for the scalar?

Non-Hermitian low modes

We use the left and the right eigenvectors of the massive Dirac operator M we construct the propagator:

$$M|r_i\rangle = \lambda_i |r_i\rangle, \qquad \langle l_i|M = \lambda_i \langle l_i| \qquad \Rightarrow \qquad M^{-1} = \sum_i \frac{1}{\lambda_i} |r_i\rangle\langle l_i|$$

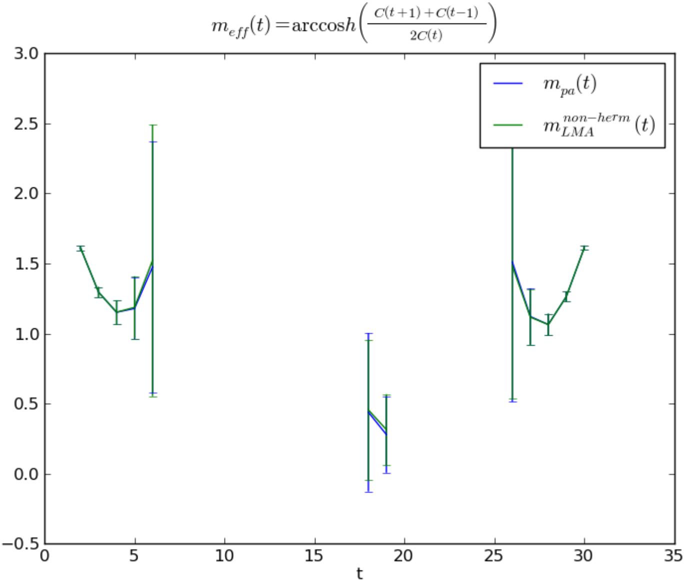
In this way the two point function for a_0 become:

$$C_{a_0}(t) = \sum_{\overline{x}\,\overline{y}\,i\,j\,t_0} \frac{1}{\lambda_i \lambda_j} \langle l_j(x)|1|r_i(x)\rangle\langle l_i(y)|1|r_j(y)\rangle$$

- One more advantage:
  If we change values of k, the eigenvectors remain the same, while the eigenvalues transform as:

$$\lambda_{i,\,new} = 1 + \frac{k_{new}}{k_{old}}(\lambda_{i,\,old} - 1)$$

a_0 - Non Hermitian case - Effective mass



Nucleon

Using the interpolator for the nucleon:

$$N(n) = \varepsilon_{abc} P_{\pm} u(n)_a \left(u(n)_b^T C\gamma_5 d(n)_c \right)$$

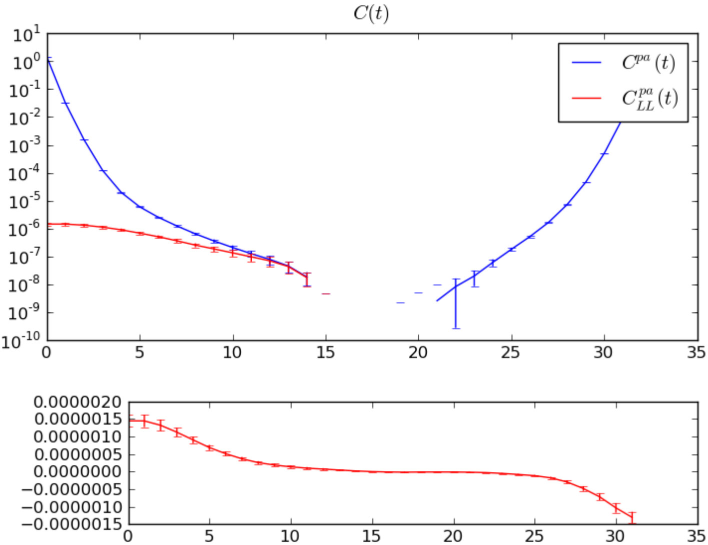
we compute the two point function:

$$\begin{aligned} \langle N\overline{N} \rangle = \quad & \langle \varepsilon_{abc}\varepsilon_{a'b'c'} (C\gamma_5)_{\alpha\beta} (C\gamma_5)_{\alpha'\beta'} (P_{\pm})_{\gamma\gamma'} d(n)_{b'\beta'} \overline{d}(m)_{b\beta} \\ & \times u(n)_{a'\alpha'} \overline{u}(m)_{a\alpha} u(n)_{c'\gamma'} \overline{u}(m)_{c\gamma} \rangle \end{aligned}$$

Which can be written in terms of the eigenvector components as:

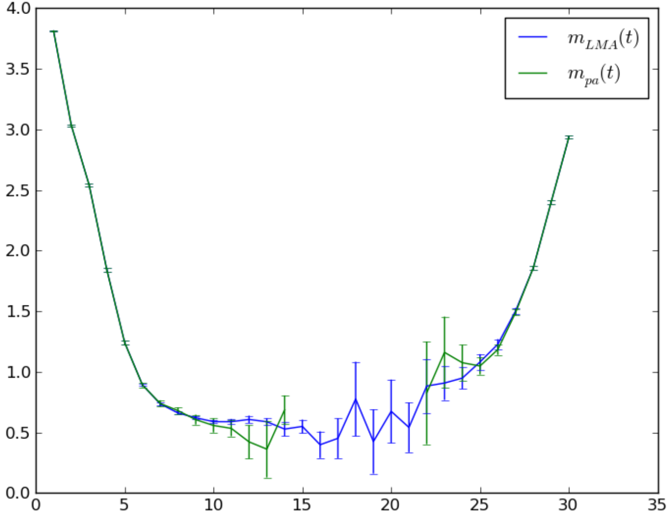
$$\begin{aligned} \langle N\overline{N} \rangle = \quad & \sum_{ijk} \frac{1}{\lambda_i \lambda_j \lambda_k} \left\{ \varepsilon_{a'b'c'} (C\gamma_5)_{\alpha'\beta'} \left[r_{i,b',\beta'}(n) r_{j,a',\alpha'}(n) r_{k,c',\gamma'}(n) \right] \right\} (P_{\pm})_{\gamma\gamma'} \\ & \times \left\{ \varepsilon_{abc} (C\gamma_5)_{\alpha\beta} l^*_{i,b,\beta}(m) \left[l^*_{j,a,\alpha}(m) l^*_{k,c,\gamma}(m) - l^*_{j,c,\gamma}(m) l^*_{k,a,\alpha}(m) \right] \right\} \end{aligned}$$

Nucleon - Two point function

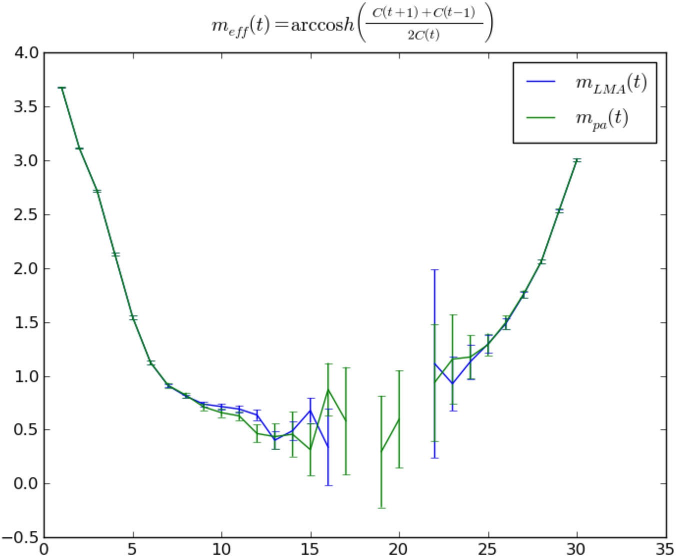


Nucleon - Effective mass

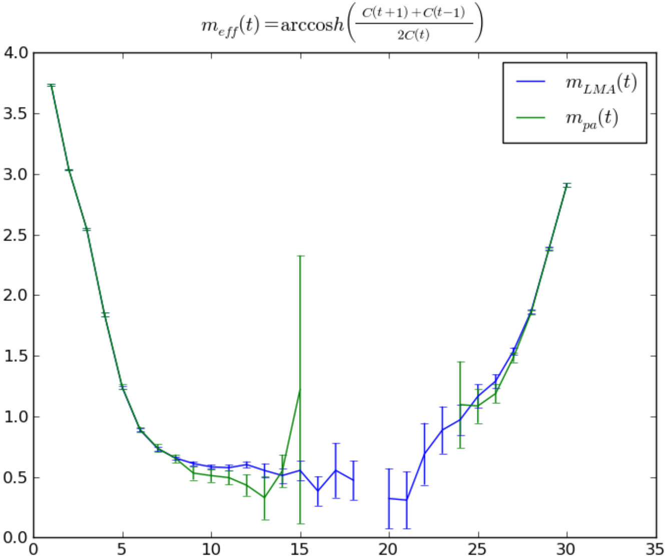
$$m_{eff}(t) = \text{arccosh}\left(\frac{C(t+1)+C(t-1)}{2C(t)}\right)$$



Δ - Effective mass



Λ - Effective mass



Conclusions

- The low-mode averaging using the Hermitian Dirac operator $\gamma_5 M$ works well for negative parity mesons π,ρ and positive parity baryons p,Δ,Λ.

	m_{pa}	m_{LMA}
π	0.192(4)	0.198(2)
ρ	0.35(2)	0.381(9)
p	0.51(14)	0.584(16)
Δ	0.872(28)	0.809(8)
Λ	0.720(36)	0.709(14)

- We do not see any of important improvement using the LMA with the non-Hermitian eigenmodes.

- the cost of the LMA for the proton goes as N^3_{eigs} and it can be very expensive.