The Aoki phase revisited

Alejandro Vaquero (Universidad de Zaragoza)
Vicente Azcoiti (Universidad de Zaragoza)
Giuseppe di Carlo (INFN, Gran Sasso)
Eduardo Follana (Universidad de Zaragoza)

Summary

- Motivation
- Simulations of the Aoki phase
  - First round: HMC algorithm
  - Second round: MFA algorithm
  - Third round: HMC + tiny external source
- Results and conclusions

(Notation) ⟶ $$\left\langle \left(i\bar{\psi}\gamma_5\psi\right)^q \right\rangle = \left\langle \left(\frac{1}{V}\sum_x i\bar{\psi}\gamma_5\psi\right)^q \right\rangle$$

Motivation

- Our work on the Aoki phase
  - V. Azcoiti, G. Di Carlo, E. Follana and A. Vaquero, Phys. Rev. D79, 014509 (2009).
  - S. Sharpe, Phys. Rev. D79, 054503 (2009).

Standard wisdom

$$\left\langle \left(i\bar{\psi}\gamma_5\psi \right)^{2n} \right\rangle = 0$$

$$\left\langle \left(i\bar{\psi}\gamma_5\tau_3\psi \right)^{2} \right\rangle \neq 0$$

Our claim

$$\left\langle \left(i\bar{\psi}\gamma_5\psi \right)^{2n} \right\rangle \neq 0$$

$$\left\langle \left(i\bar{\psi}\gamma_5\tau_3\psi \right)^{2} \right\rangle \neq 0$$

Motivation

- Two possible scenarios
  - New, unexpected Aoki-like phases appear
  - Infinite tower of sum-rules

$$\left\langle \left(i\overline{\psi}\gamma_5\psi \right)^{2n} \right\rangle = 0 \quad \forall n \in N \quad ???$$

- Simulation needed to choose an scenario

Motivation

- Rich numerical work inside the Aoki phase for QCD
  - S. Aoki
  - S. Aoki, A. Gocksch
  - S. Aoki, T. Kaneda, A. Ukawa, T. Umemura
  - E.-M. Ilgenfritz, W. Kerler, M. Müller-Preussker, A. Sternbeck, H. Stüben
- Most of them performed with an external source $i\bar{\psi}\gamma_5\tau_3\psi$

Motivation

- Why use the external source approach
  - Standard way of analyzing SSB
  - Helps convergence of solver (regularizes the small eigenvalues)
- Problems of the external source method
  - The term $i\overline{\psi}\gamma_5\psi$ leads to the ***sign problem***
  - We need another approach to verify our claims

Motivation

- Problems of the $i\bar{\psi}\gamma_5\tau_3\psi$ external source
  - Usually selects a vacuum PRD79, 014509 (2009)
  - First thermodynamic limit, then zero field limit
    - Too many simulations
    - Systematic errors in extrapolations
- Solution $\longrightarrow$ P.D.F. formalism
  - Reconstructs the complete P.D.F. for the observables. See Phys. Lett. B354, 111, (1995)

Motivation

- Quantities to be measured
  - $i\bar{\psi}_u\gamma_5\psi_u$ Parity o.p. 1 flavour $\left\langle \left(i\bar{\psi}_u\gamma_5\psi_u\right)^2 \right\rangle$
  - $i\bar{\psi}\gamma_5\psi$ Parity o.p. 2 flavour $\left\langle \left(i\bar{\psi}\gamma_5\psi\right)^2 \right\rangle$
  - $i\bar{\psi}\gamma_5\tau_3\psi$ Aoki o.p. $\left\langle \left(i\bar{\psi}\gamma_5\tau_3\psi\right)^2 \right\rangle$
- We always measure the second moment

First round: Hmc algorithm

- The Aoki phase features small eigenvalues
  - We need a good solver $\longrightarrow$ GCR + SAP
    - See M. Lüscher, Comput. Phys. Commun. 156 (2004) 209
  - We need a small stepsize $\longrightarrow$ Autocorrelation
  - Eigenvalue-crossing is not allowed
- HMC is not ergodic inside the Aoki phase (in the absence of an external source)

First round: Hmc algorithm

- Results in a 4^4 lattice (2 Flavours)
  - Aoki point β=2.0 κ=0.25
  - Physical point β=3.0 κ=0.22

Simulation	$\langle (i\bar{\psi}_u\gamma_5\psi_u)^2 \rangle$	$\langle (i\bar{\psi}\gamma_5\psi)^2 \rangle$	$\langle (i\bar{\psi}\gamma_5\tau_3\psi)^2 \rangle$
Aoki Cold [0]	(1.92 ± 0.03)x1e-02	(2.68 ± 0.12)x1e-02	(5.00 ± 0.09)x1e-02
Aoki Hot [0]	(1.87 ± 0.03)x1e-02	(2.70 ± 0.05)x1e-02	(4.79 ± 0.08)x1e-02
Aoki Hot [1]	(6.52 ± 0.72)x1e-03	(-4.49 ± 0.50)x1e-02	(7.10 ± 0.30)x1e-02
Physical [0]	(2.100±0.002)x1e-03	(4.151±0.003)x1e-03	(4.246±0.005)x1e-03

- Statistics ~ 10000 trajectories per point

Second round: MFA

- MFA relies on quenched generation weighted by the determinant
  - Phys. Rev. Lett. 65, 2239, (1990)
  - Eigenvalue crossing allowed
  - Determinant fluctuations suppressed
  - Poor sampling
- We need huge configuration ensembles to obtain meaningful measurements

Second round: MFA

- Results in a 4^4 and a 6^4 lattice

Simulation	$\langle (i\bar{\psi}_u \gamma_5 \psi_u)^2 \rangle$	$\langle (i\bar{\psi}\gamma_5\psi)^2 \rangle$	$\langle (i\bar{\psi}\gamma_5\tau_3\psi)^2 \rangle$
MFA 4^4 [All]	(2.25 ± 0.16)x1e-02	(-1.9 ± 2.8)x1e-02	(1.1 ± 0.4)x1e-01
MFA 4^4 [0]	(2.57 ± 0.46)x1e-02	(-0.5 ± 3.4)x1e-02	(1.0 ± 0.5)x1e-01
Hmc+Weights	(1.75 ± 0.17)x1e-02	(1.73 ± 0.15)x1e-02	(5.3 ± 0.6)x1e-02
MFA 6^4 [All]	(4.6 ± 1.0)x1e-02	(-1.3 ± 4.0)x1e-01	(3.1 ± 4.0)x1e-01
MFA 6^4 [0]	(4.6 ± 0.7)x1e-02	(6.4 ± 0.5)x1e-02	(1.2 ± 0.3)x1e-01

- Weights 4^4 [0] ~ 86.8% [1] ~ 13.2% Error 7.1%
- Statistics 4^4 ~ 2140000 configurations
  6^4 ~ 50000 configurations

Third round: Hmc + tiny external source

- If the source is small enough…
  - …regularizes the inverse of the dirac operator, so it can not diverge
  - …this enhances eigenvalue crossing
  - …if the external field is small enough (of order 1/V), it does not select a vacuum
- HMC becomes ergodic again

Results and Conclusions

- First time the Aoki phase without external source is simulated with dynamical fermions
  - Errors too large at this moment, but…
  - …data consistent among different methods
  - Need to perform simulations at larger volumes

Simulation	$\left\langle \left(i\bar{\psi}_u \gamma_5 \psi_u \right)^2 \right\rangle$	$\left\langle \left(i\bar{\psi} \gamma_5 \psi \right)^2 \right\rangle$	$\left\langle \left(i\bar{\psi} \gamma_5 \tau_3 \psi \right)^2 \right\rangle$
Aoki Cold [0]	(1.92 ± 0.03)x1e-02	(2.68 ± 0.12)x1e-02	(5.00 ± 0.09)x1e-02
MFA 4^4 [0]	(2.6 ± 0.5)x1e-02	(-0.5 ± 3.4)x1e-02	(1.0 ± 0.5)x1e-01
MFA 6^4 [0]	(4.6 ± 0.7)x1e-02	(6.4 ± 0.5)x1e-02	(1.2 ± 0.3)x1e-01