Glueball masses with exponentially improved statistical precision

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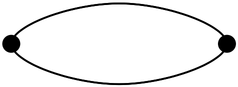
Outline

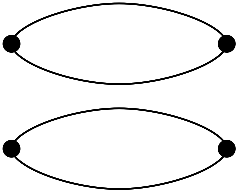
- Exponential growth of the noise to signal ratio in lattice QCD and YM theories
- Basic ideas for the Symmetry-Constrained Monte Carlo
  - The example of Parity (including results)
- Extension to other symmetries
- The strategy for the 0^{++} glueball (including results)
- Conclusions and outlook

Exponential growth of the signal to noise ratio (Parisi '84, Lepage '89)

Consider a point to point correlation function interpolating (eg) a meson. The signal is given by the expectation value of



while the a priori variance is given by the expectation value of

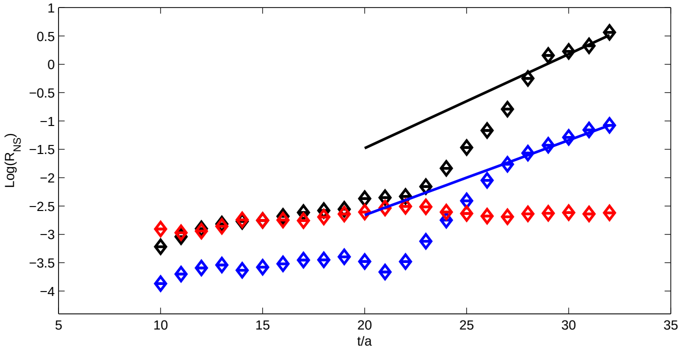


Luckily Wick-contractions are done *before squaring*, for the variance. Then a multi-pion state dominates, otherwise it would be the vacuum (as for YM).

pion $R_{NS} \propto const$

ρ $R_{NS} \propto \exp((m_\rho - m_\pi)t)$

N $R_{NS} \propto \exp((m_N - \frac{3}{2}m_\pi)t)$



O(2000) quenched confs ($\beta = 6.2,\ \kappa = 0.1526$) in APE, hep-lat/9611021

- For an operator interpolating a parity odd glueball

$$C_{O_G}(t) = \langle O_G(t) O_G(0) \rangle \to |\langle 0|O_G(0)|G^-\rangle|^2 \, e^{-M_{G^-} t} + \dots$$

the variance can be estimated as

$$\sigma^2 = \langle O_G^2(t) O_G^2(0) \rangle - \langle O_G(t) O_G(0) \rangle^2 \to \langle 0|O_G^2(0)|0\rangle^2 + \dots$$

- The noise to signal ratio at large time separations is given by

$$R_{NS}(t) \to \frac{\langle 0|O_G^2(0)|0\rangle}{|\langle 0|O_G(0)|G^-\rangle|^2} \, e^{M_{G^-} t} + \dots$$

$\Leftarrow$ On a given gauge configuration symmetries as parity are not preserved. All states are allowed to propagate despite the quantum numbers of the source.

$\Rightarrow$ For every gauge-field configuration the vacuum dominates. The signal emerges due to large cancellations in the gauge average.

$\Rightarrow$ In the standard approach glueball masses are extracted at rather short separations.

Decomposition of the path integral and boundary conditions

with periodic boundary conditions $Z = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V\rangle$

$$Z = Z^+ + Z^- \; , \quad Z^\pm = e^{-E_0\, T}\left[\frac{1\pm 1}{2} + \sum_{n=1} w_n^\pm\, e^{-E_n^\pm T}\right]$$

We introduce a parity transformation

$$\hat{\wp}\,|V\rangle = |V^\wp\rangle \; , \qquad V_k^\wp(\mathbf{x}) = V_k^\dagger(-\mathbf{x} - \hat{k}) \; ,$$

with $\hat{\mathrm{V}}_k(\mathbf{x})|V\rangle = V_k(\mathbf{x})|V\rangle$ and

$$Z^{tw} = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V^\wp\rangle =$$

$$\sum_G \int D_3[V]\langle V|G\rangle\langle G|e^{-T\hat{H}}\hat{\wp}|G\rangle\langle G|V\rangle = Z^+ - Z^-$$

- We want to compute $\frac{Z^-}{Z}(T) = \frac{1}{2}\left(1 - \frac{Z^{tw}}{Z}\right)(T)$ where, compared to Z, the boundary conditions in Z^{tw} are parity twisted. At large T we should be able to extract the lightest parity odd glueball.
- We aim at a hierarchical integration scheme [Lüscher and Weisz, '01] and divide the system in thick time-slices of size d with boundaries updated at different rates wrt the internal dof.
- We start from the factorized expression for $Z(T)$

$$Z(T) = \int \prod_{i=0}^{T/d-1} \mathbf{D}_3[V_{id}]\, T^d[V_{(i+1)d}, V_{id}] \,, \quad \text{with}$$

$$T^d[V_{x_0+d}, V_{x_0}] = \langle V_{x_0+d} | \hat{T}^d | V_{x_0} \rangle$$

and by introducing

$$(T^-)^d[V_{x_0+d}, V_{x_0}] = \frac{1}{2}\left\{ T^d[V_{x_0+d}, V_{x_0}] - T^d[V_{x_0+d}, V_{x_0}^{\wp}] \right\}$$

we generalize it to Z^-/Z.

- The basic quantity to be computed for each *sub*-lattice of time extent d with Dirichlet boundary conditions is the ratio of partition functions

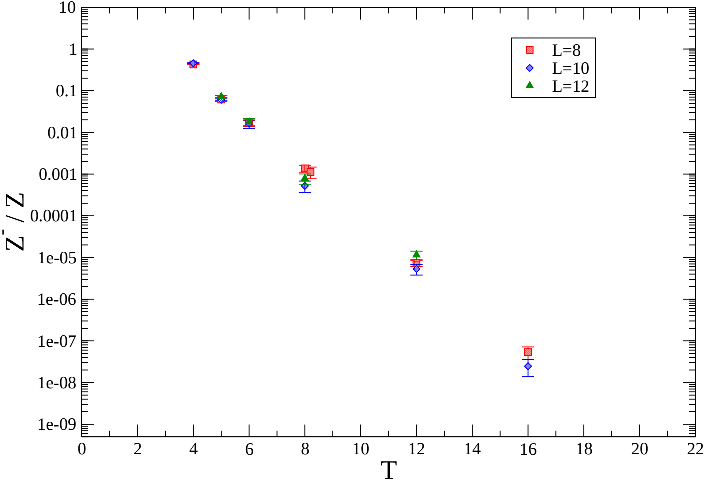
$$\frac{T^d\left[V^{\wp}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}$$

The product (over the thick-slices) of a simple linear function of that is then integrated numerically on the boundary configurations $V_{x_0=id}$.

- We need $\mathrm{O}((L/a)^3)$ MC simulations to estimate the ratio above. We have a $V^2 = (L/a)^6$ algorithm but we get rid of the exponential (in time) degradation of the signal, if we choose $d \geq 1/T_c$, such that the ratio above is of the right size $\mathrm{O}(e^{-M_G - d})$ and its fluctuations are reduced to the same level.

Results (Parity only)

Wilson action $\beta = 5.7$ ($a \simeq 0.17$ fm) and O(50) meas at each T/a.



- The algorithm works as expected. We see a clear signal up to a separation of about 3 fm.
- There is no strong dependence of the results from L for $1.4 \text{ fm} < L < 2 \text{ fm}$ ($\Rightarrow$ negligible “torelon” contribution)
- However, by using parity only it is difficult to correctly identify the dominating state. For example, a rather light parity odd state (maybe lighter than the lightest 0^{+-} glueball) could be

$$\frac{1}{\sqrt{2}}\left(|0^{++},\vec{p}\rangle - |0^{++},-\vec{p}\rangle\right) \ , \quad |\vec{p}| = 2\pi/L$$

- We want to consider the lattice YM symmetry groups
  - C and P, $g = 2$
  - spatial translations, $g = L^3$
  - central charge conjugations, Z_3^3, $g = 27$
  - spatial rotations, octahedral group, $g = 24$

Fixing an irreducible representation (quantum numbers) [DM and Giusti, to appear]

- The phase space of the theory can be factorized into regular representations of the group. In the partition function

$$Z(T) = \mathrm{Tr}\left[\hat{T}^{T}\right]$$

one inserts the identity I written as

$$I = \frac{1}{g}\sum_{i=1}^{g}\int \mathbf{D}_3[V]|V^{\Gamma^i}\rangle\langle V^{\Gamma^i}|$$

eg on the boundaries of our thick-slices.

- Then group theory tells us how to project on an irreducible representation μ

$$\hat{P}^{(\mu)} = \frac{n_\mu}{g}\sum_{i=1}^{g}\chi_i^{(\mu)^*}\hat{\Gamma}^i$$

- So, one has to compute

$$\frac{T^d\left[V^{\Gamma^i}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}, \quad i = 1\dots g$$

and then form linear combinations of them.

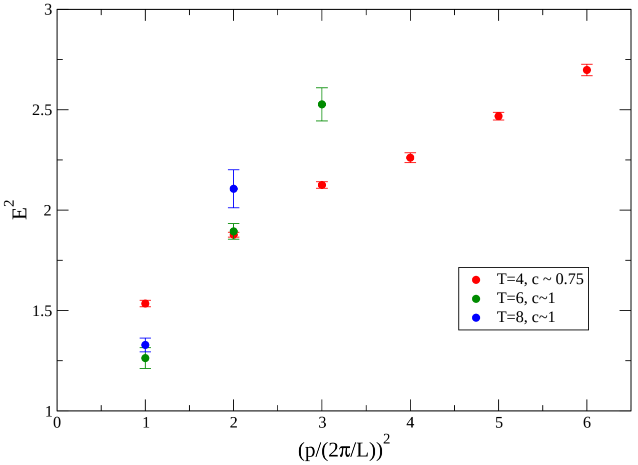
For example: The relative contribution of states with momentum $\vec{p}$ in the system with Dirichlet bc is ($\hat{P}(\vec{x})$ representing translations by $\vec{x}$)

$$\frac{(T^{\vec{p}})^d\left[V_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]} = \frac{1}{\sqrt{L^3}}\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\frac{T^d\left[V^{P(\vec{x})}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}$$

We will use this setup to extract the mass of the lightest 0^{++} glueball through the dispersive relation. By selecting non-zero momentum we get rid of the vacuum.

Results for the dispersion relation (fixing in addition C parity to be even)

$\beta = 5.7,\ L/a = 8$



Conclusions and outlook

- In the YM theory the noise to signal problem can be solved by enforcing the propagation in time of states with the desired quantum numbers only.
- We have shown that all quantum numbers can be fixed in this approach.
- We are now exploring the stochastic projection on the singlet component (eg zero momentum in order to avoid another L^3 factor in the scaling of the algorithm).
- In the near future we will also concentrate on the 0^{++} and 2^{++} glueball masses.