Hyperon Form Factors in $N_f=2+1$ QCD

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Lattice 2010, Villasimius, Sardinia, Italy
June 14-19, 2010

Outline

- Motivation
- Simulation Parameters
- Extracting matrix elements from 3-point functions
- Preliminary results
  - Hyperon electromagnetic form factors
  - Hyperon semi-leptonic decays
- Summary and Outlook

Motivation

Hyperon electromagnetic form factors

- Charge and magnetic distribution of hyperons
- Examine into the role of SU(3) flavour symmetry breaking in these distributions
- Insights into the role of hidden flavour (e.g. strangeness in the proton)

Hyperon semi-leptonic decay form factors

$$\Sigma^- \to n\ell\nu_\ell \quad \text{and} \quad \Xi^0 \to \Sigma^+\ell\nu_\ell$$

- Provide an alternative method for determining the CKM matrix element $|V_{us}|$
- The axial semi-leptonic form factor at $q^2=0$ gives g_A/g_V
- $\Xi^0 \to \Sigma^+\ell\nu_\ell$ is analogous to usual β decay $n \to p\ell\nu_\ell$
  - expect $g_A/g_V \approx 1.26$

Simulation Details

- N_f=2+1 flavours dynamical O(a) improved clover fermions [Phys.Rev.D79 094507]
- Code & algorithm: BQCD Poster by Y. Nakamura & H. Stüben
- Tuned so singlet quark mass ($m_u + m_d + m_s$) kept fixed at physical value

Talks by R.Horsley & P. Rakow

- So far, three point functions only calculated on 24^3 enembles Also F.Winter

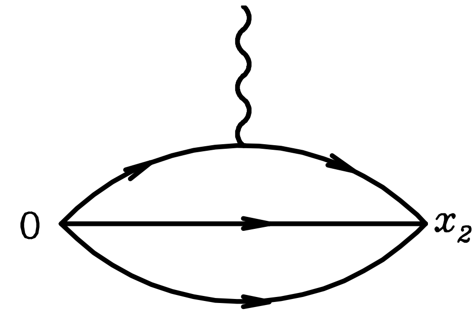
κ_l	κ_s	$N_S^3 \times N_T$	m_π [MeV]	m_K [MeV]
0.120830	0.121040	$24^3 \times 48$	462	402
0.120900	0.120900	$24^3 \times 48$	425	425
0.120950	0.120800	$24^3 \times 48$	394	437
0.121000	0.120700	$24^3 \times 48$	358	453
0.121040	0.120620	$24^3 \times 48$	337	460
0.121040	0.120620	$32^3 \times 64$	335	461
0.121095	0.120512	$32^3 \times 64$	281	479
0.121145	0.120413	$32^3 \times 64$	248	488

>2000 trajs

a=0.083 fm

Extraction of matrix elements

$$\langle\Omega|T\left(\chi(\vec{x}_2,t)\,\mathcal{O}(\vec{x}_1,\tau)\,\overline{\chi}(0)\right)|\Omega\rangle$$



Three-point function at the baryon level

$$G^{B\mathcal{O}B}(t,\tau)=\sum_{s,s'}e^{-E_{p'}(t-\tau)}e^{-E_p\tau}\langle\Omega|\chi|p',s'\rangle\langle p',s'|\mathcal{O}|p,s\rangle\langle p,s|\overline{\chi}|\Omega\rangle$$

E.g. the matrix element of the electromagnetic current can be extracted from a ratio of 3pt/2pt and has the general form

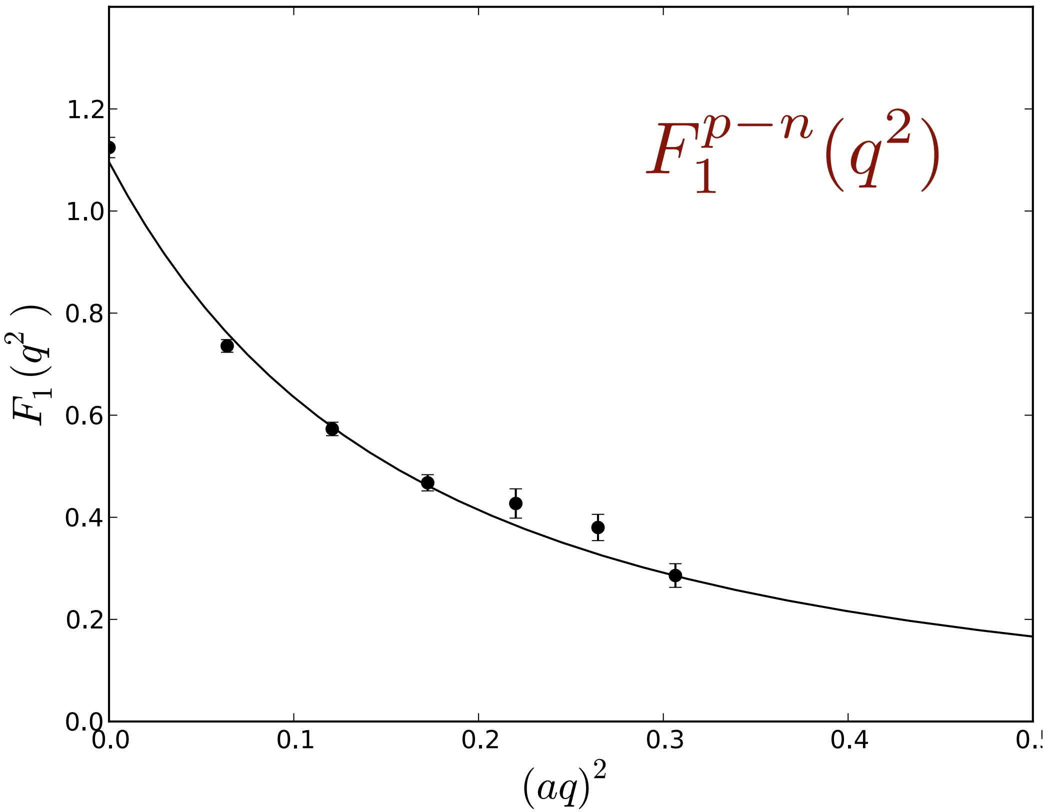
$$\langle p',s'|j_\mu|p,s\rangle=\overline{u}(p',s')\left(F_1(q^2)\gamma_\mu+F_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{2M}\right)u(p,s)$$

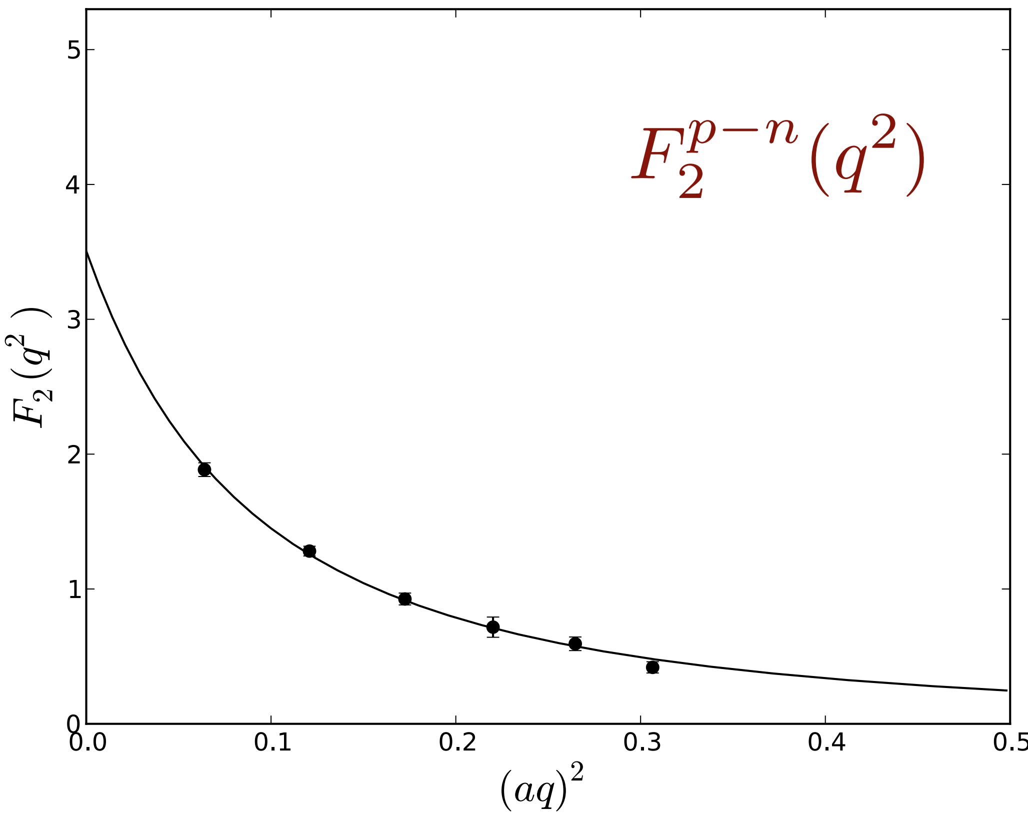
While for the vector semileptonic form factors between B' and B

$$\langle p',s'|V_\mu|p,s\rangle=\overline{u}(p',s')\left(F_1(q^2)\gamma_\mu+F_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_{B'}+M_B}+F_3(q^2)i\frac{q_\mu}{M_{B'}+M_B}\right)u(p,s)$$

Results

Hyperon electromagnetic form factors





$$m_\pi = 394 \text{ MeV}$$

Form factor radii & magnetic moments

Compare different quark sectors for

$$N, \ \Sigma, \ \Xi$$

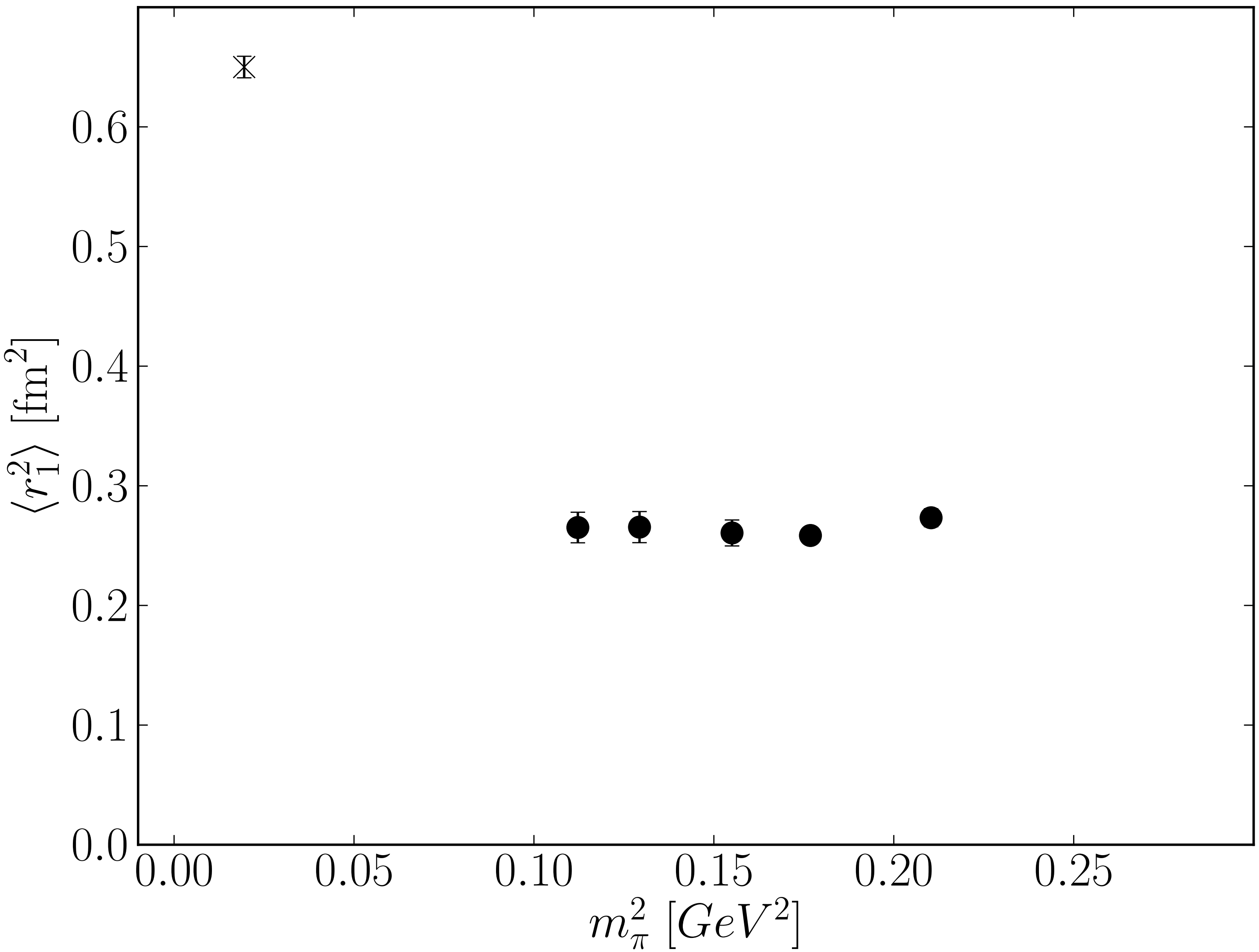
- *Form factor radii:*

$$r_i^2 = -6 \frac{dF_i(q^2)}{dq^2}\Big|_{q^2=0}$$

- *Magnetic moment* μ */anomalous magnetic moment*

$$\mu = 1 + \kappa = G_m(0)$$

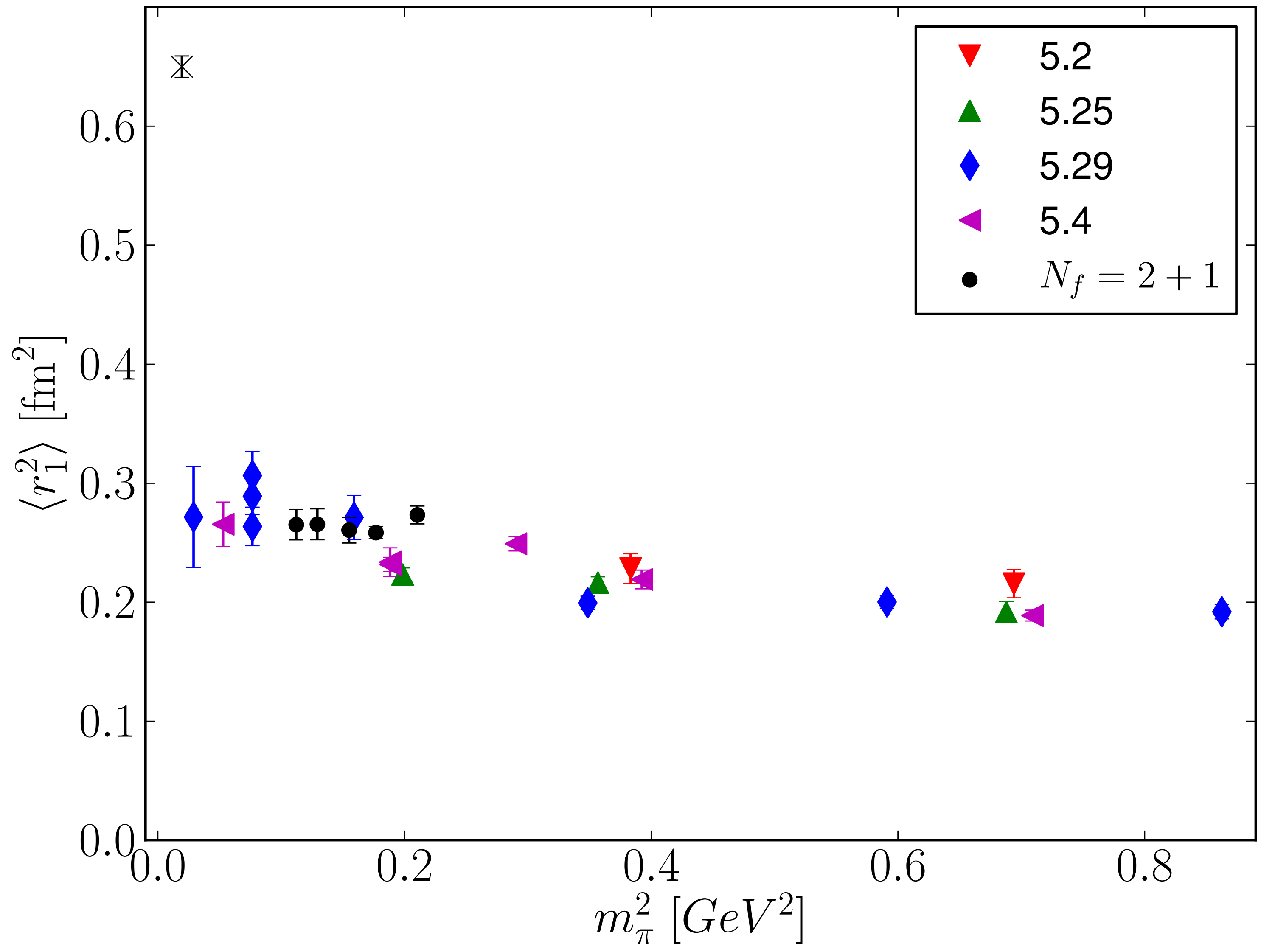
N (u-d)



0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\langle r_1^2 \rangle$ [fm^2]
0.00
0.05
0.10
0.15
0.20
0.25
m_π^2 $[GeV^2]$

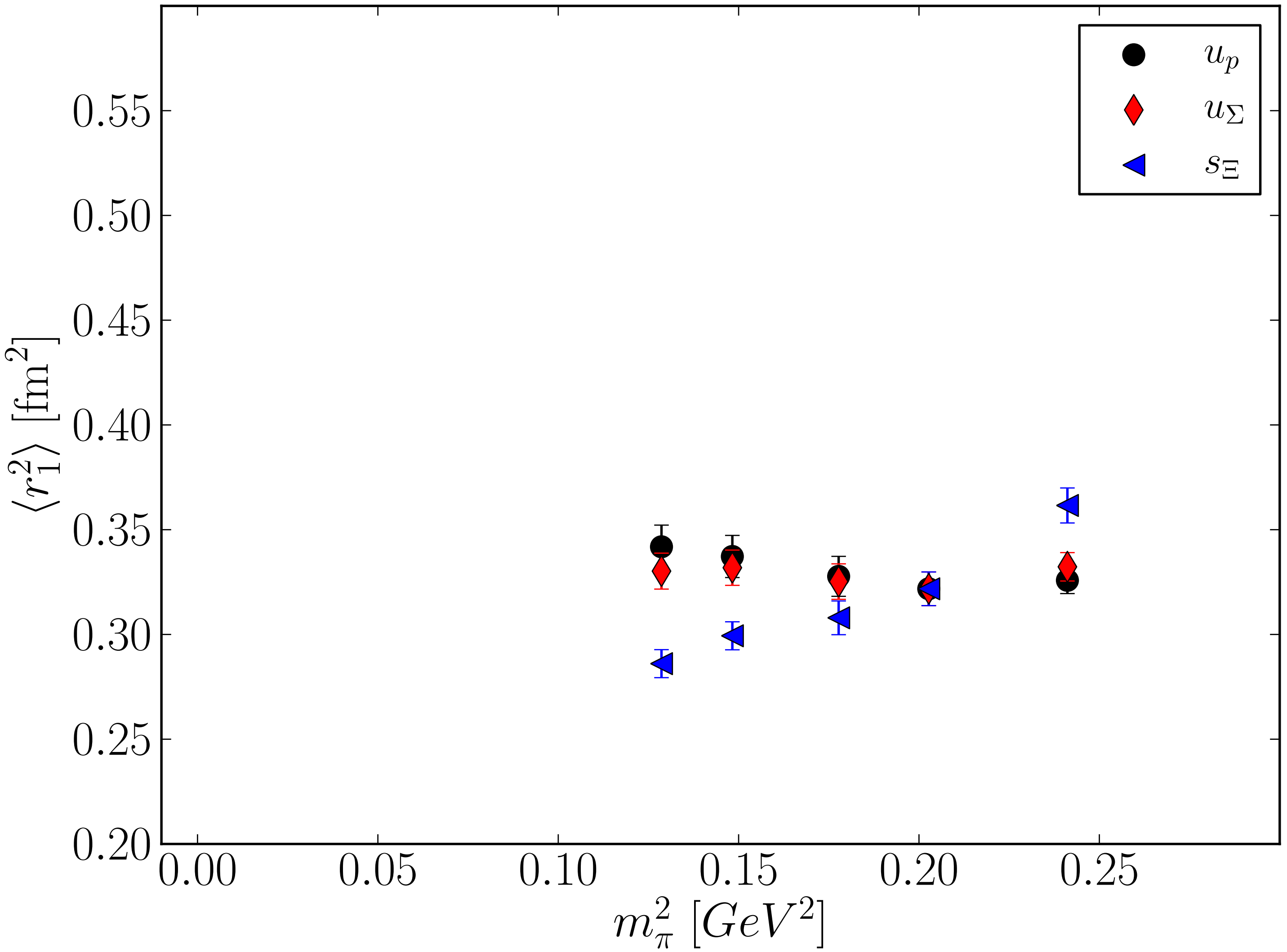
N (u-d)

[nf=2, D.Pleiter]



Results

Hyperon radii



Ratios

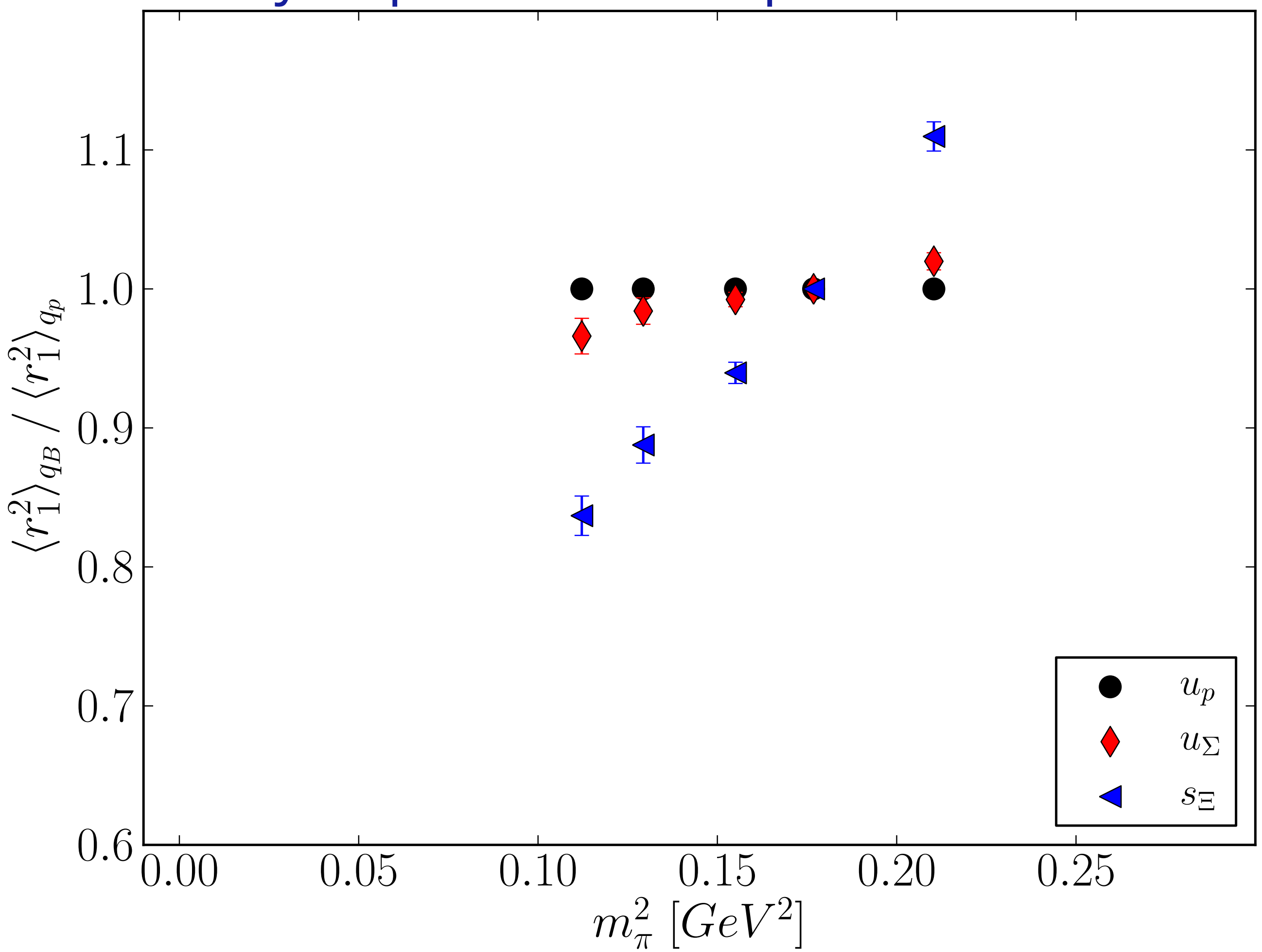
- Consider ratios to highlight SU(3) flavour symmetry breaking effects, e.g.

$$\frac{\langle r_1^2 \rangle_{u_\Sigma}}{\langle r_1^2 \rangle_{u_p}}$$

Results

Hyperon radii

Doubly represented quarks



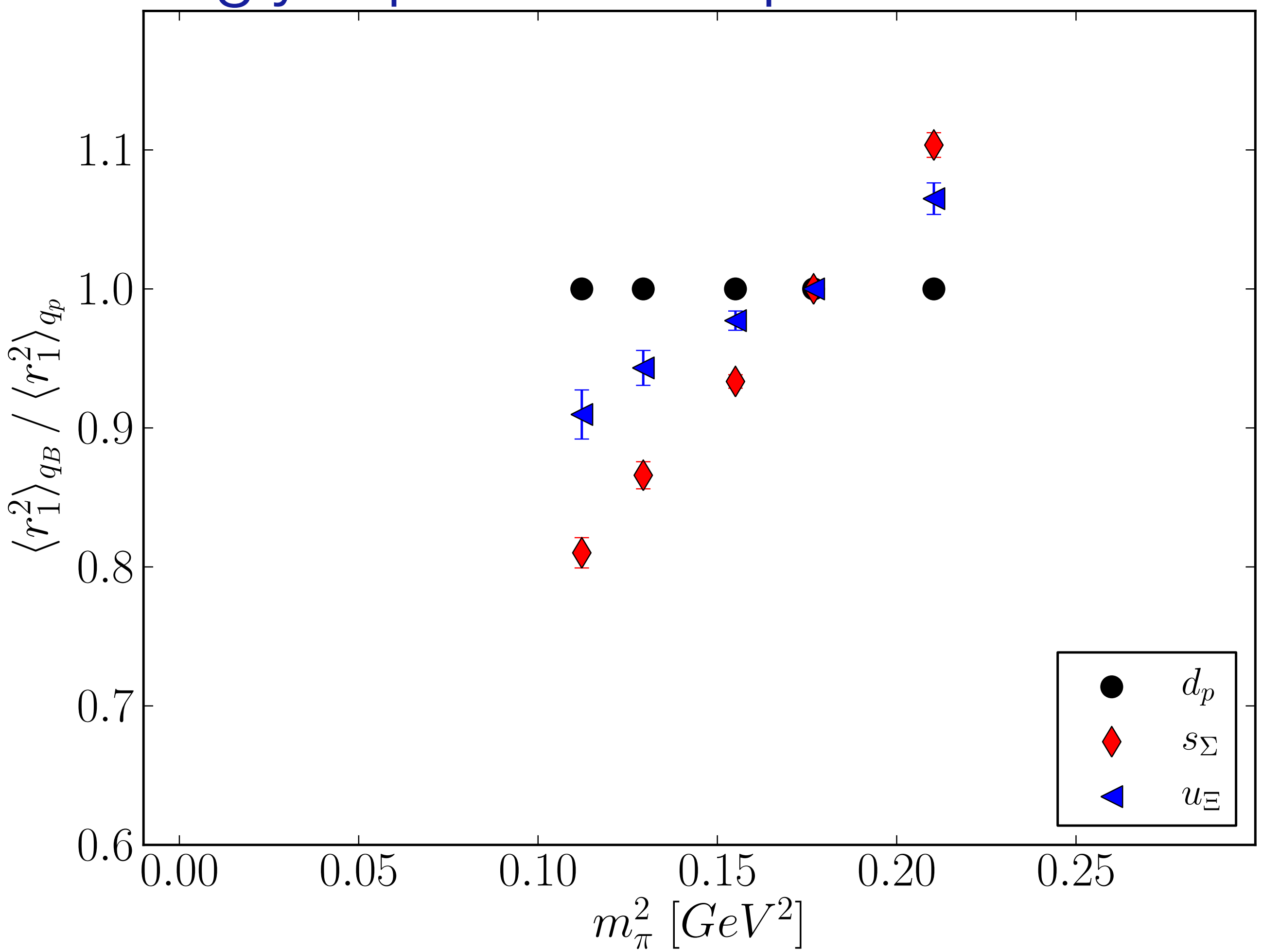
✳u quark is more broadly distributed in the proton than Sigma

✳s quark in Xi least broadly distributed

Results

Hyperon radii

Singly represented quarks

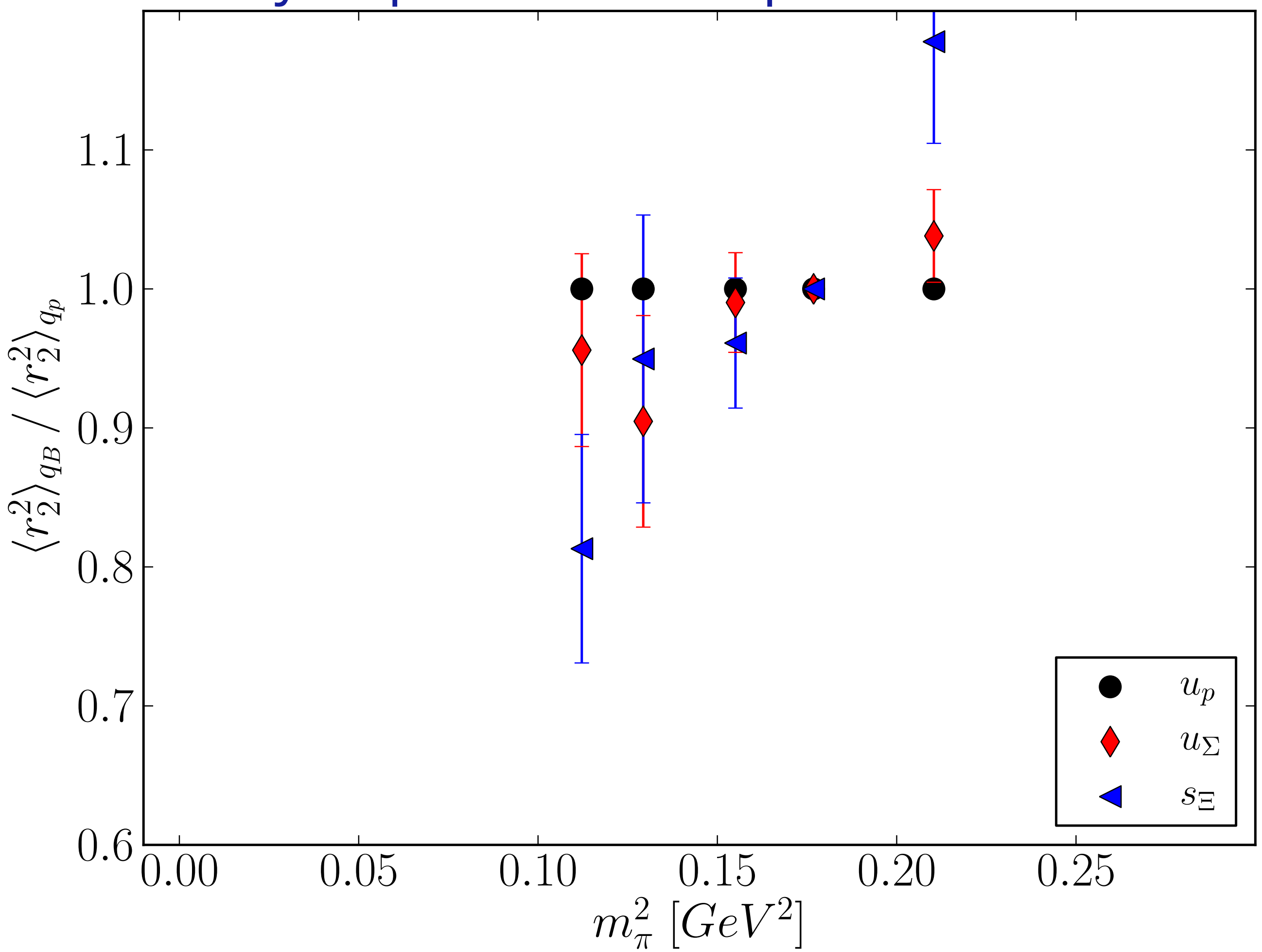


- ✳d quark is more broadly distributed in the proton than Xi
- ✳s quark in Sigma least broadly distributed

Results

Hyperon radii

Doubly represented quarks



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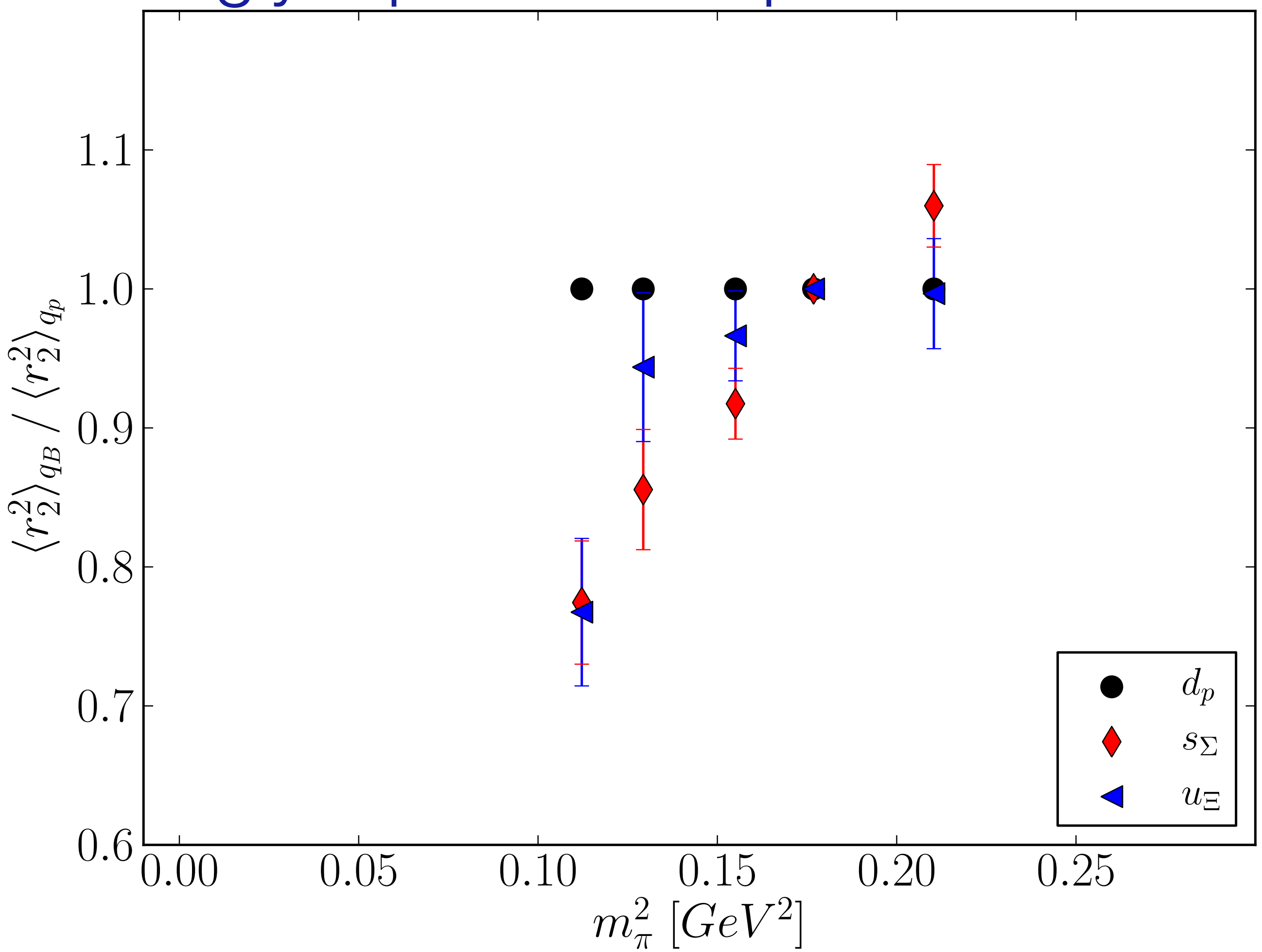


- ✳d quark is more broadly distributed in the proton than Xi
- ✳s quark in Sigma least broadly distributed

Hyperon semi-leptonic form factors

$$\langle p', s' | V_\mu | p, s \rangle = \overline{u}(p', s') \left(F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_{B'} + M_B} + F_3(q^2) i \frac{q_\mu}{M_{B'} + M_B} \right) u(p, s)$$

Double Ratio

E.g. $\Sigma^- \to n \ell \nu_\ell$

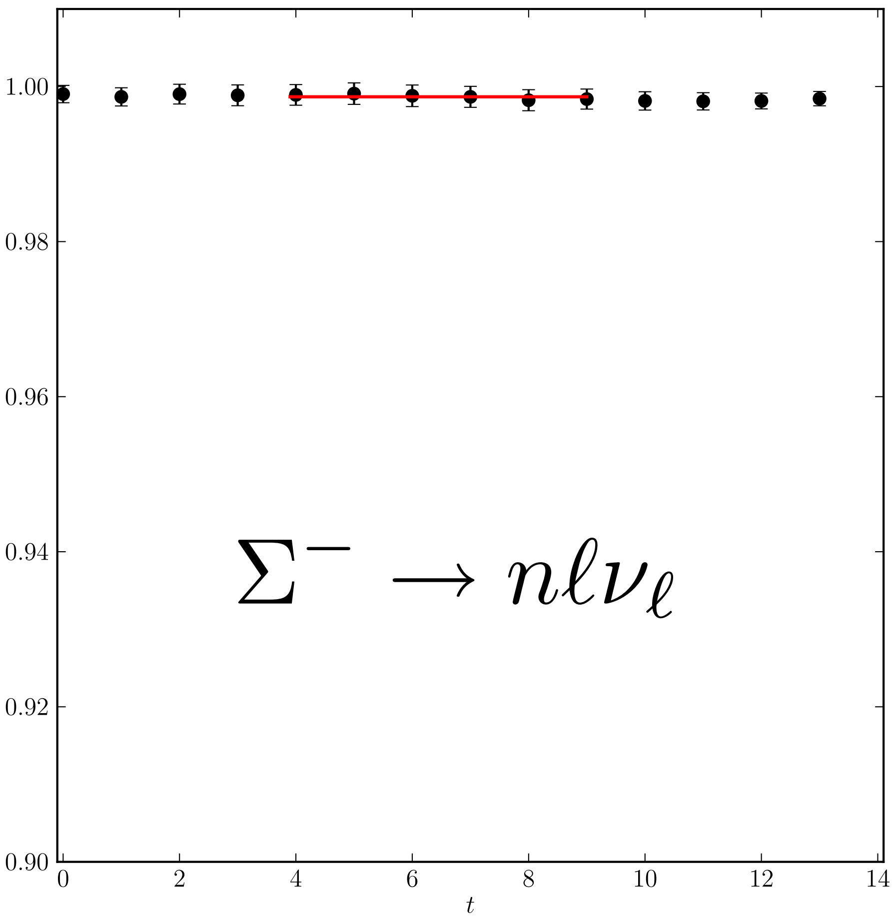
- Extract scalar form factor

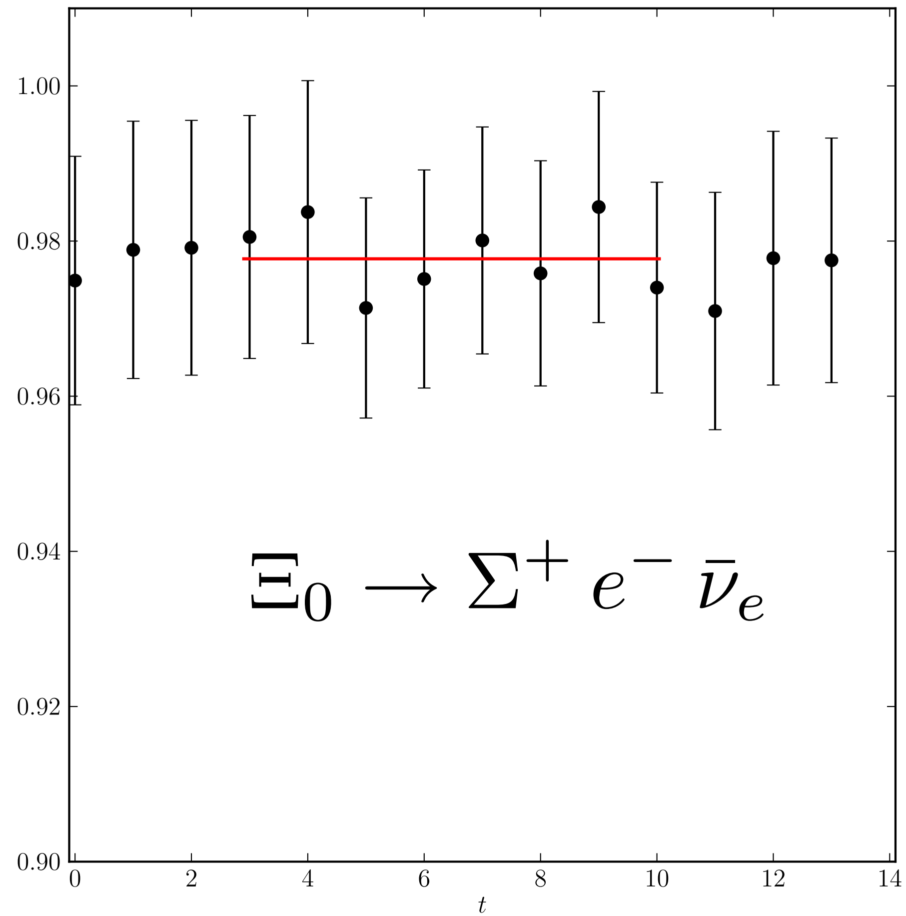
$$f_0(q^2) = f_1(q^2) + \frac{q^2}{m_\Sigma^2 - m_n^2} f_3(q^2)$$

- at $q^2_{\rm max} = (m_\Sigma - m_n)^2$ with high precision via

$$\begin{aligned} R(t', t) &= \frac{G_4^{\Sigma n}(t', t; \vec{0}, \vec{0}) G_4^{n\Sigma}(t', t; \vec{0}, \vec{0})}{G_4^{\Sigma\Sigma}(t', t; \vec{0}, \vec{0}) G_4^{nn}(t', t; \vec{0}, \vec{0})} \\ &\longrightarrow \frac{\langle n | \bar{s}\gamma_4 u | \Sigma \rangle \langle \Sigma | \bar{u}\gamma_4 s | n \rangle}{\langle \Sigma | \bar{s}\gamma_4 s | \Sigma \rangle \langle n | \bar{u}\gamma_4 u | n \rangle} \\ &= |f_0(q^2_{\rm max})|^2 \end{aligned}$$

Double Ratio





$m_\pi = 358$ MeV, $m_K = 453$ MeV

$f_0(q^2_{\rm max}) = 0.99934(66)$

$m_\pi = 337$ MeV, $m_K = 460$ MeV

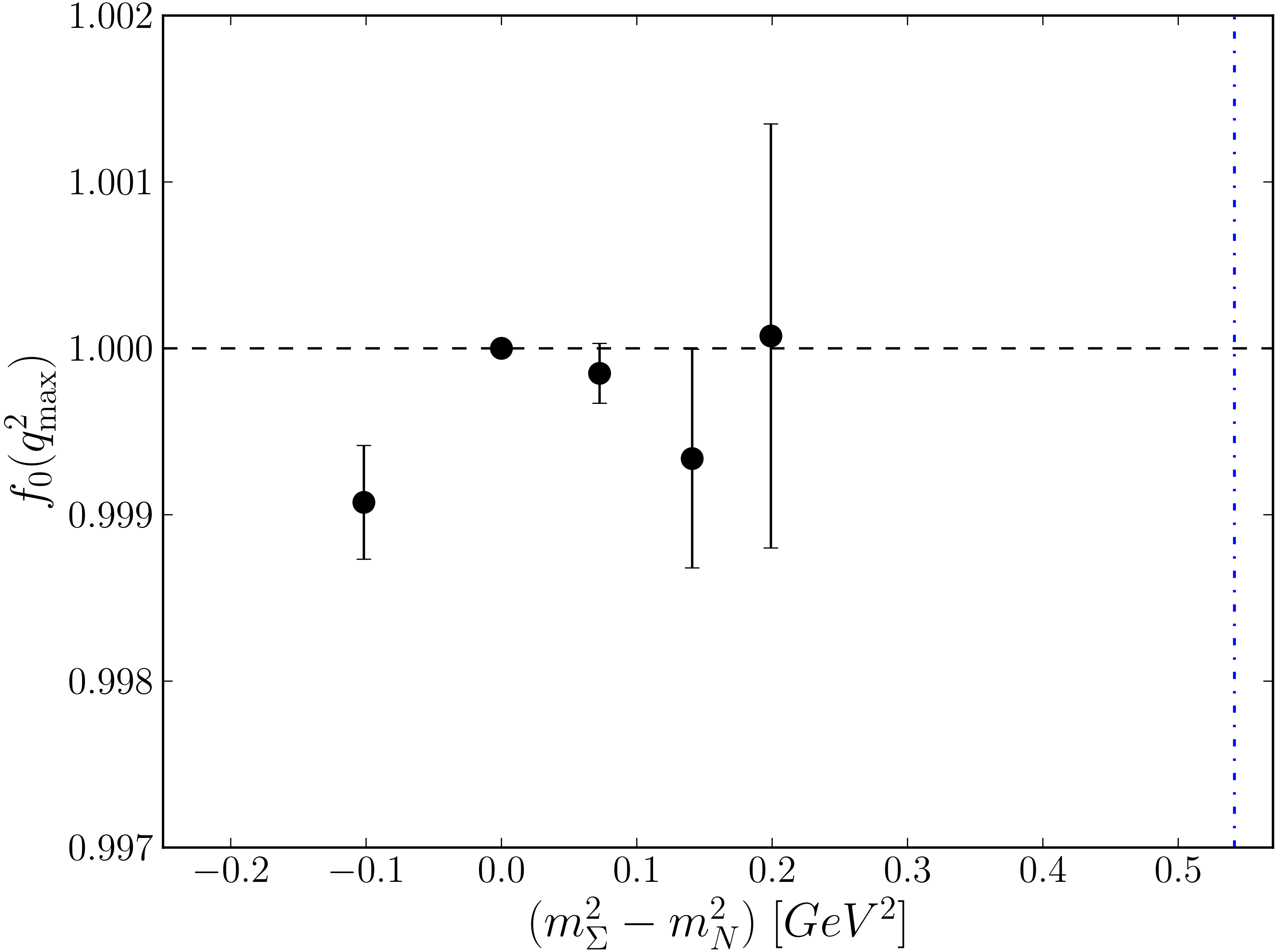
$f_0(q^2_{\rm max}) = 0.9888(75)$

[very preliminary: only 95 measurements]

Results

Hyperon semi-leptonic form factor

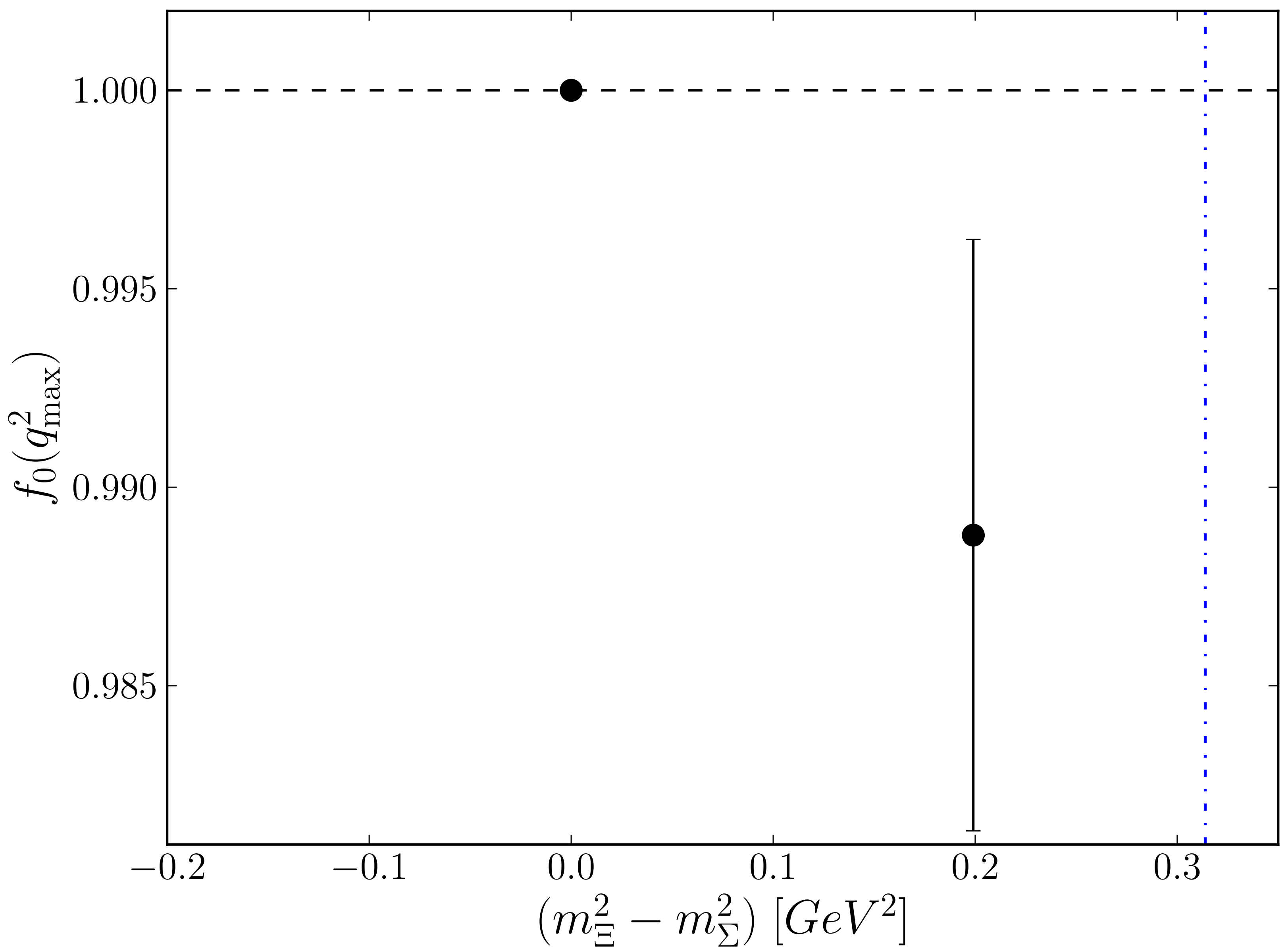
$$\Sigma^- \to n\ell\nu_\ell$$



Results

Hyperon semi-leptonic form factor

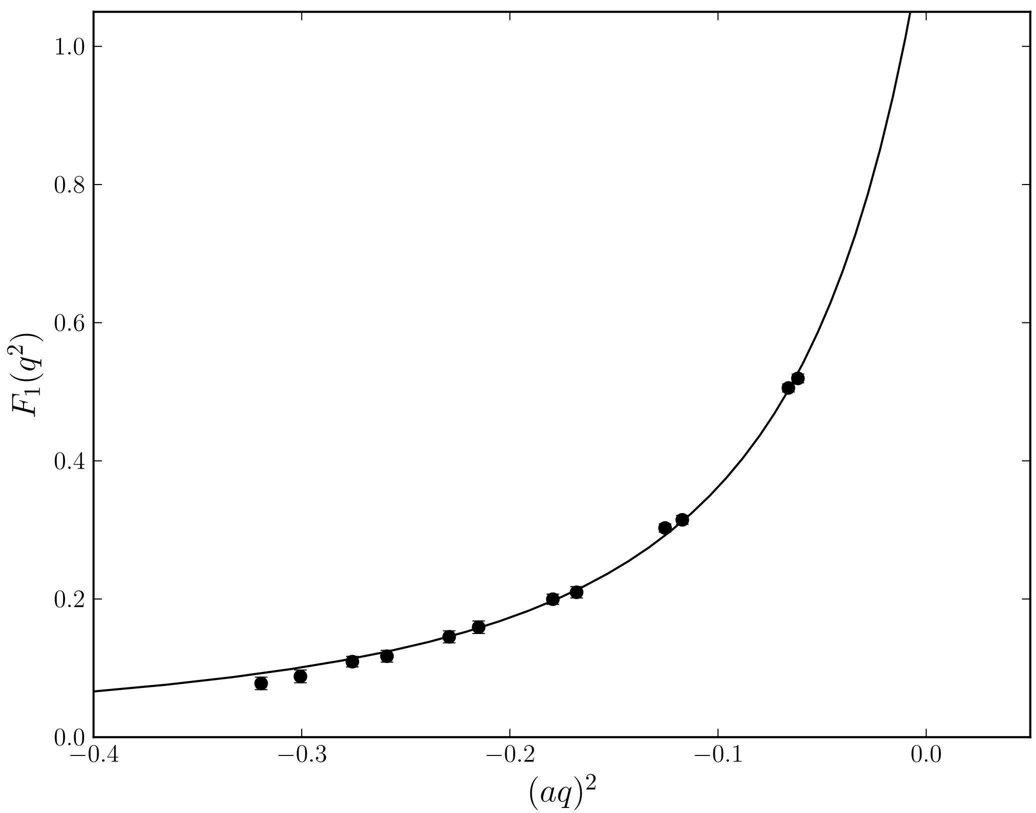
$$\Xi_0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$$

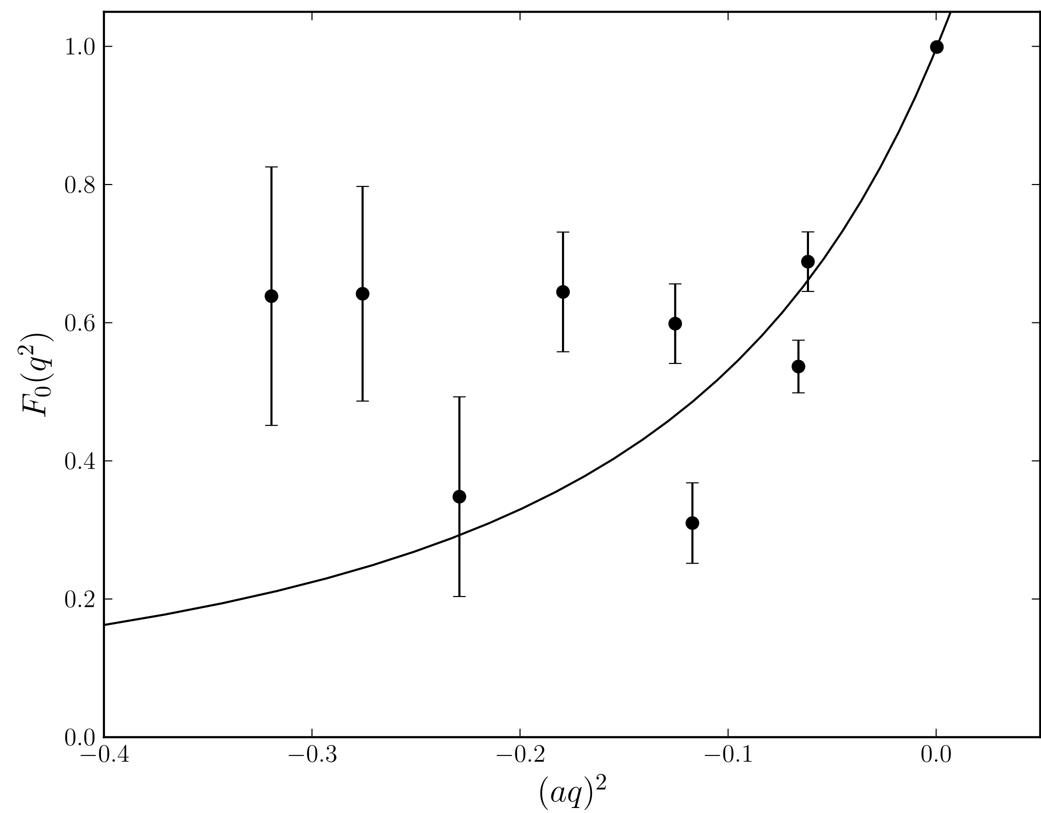


Results

Hyperon semi-leptonic form factor

$$\langle p', s' | V_\mu | p, s \rangle = \overline{u}(p', s') \left(F_1(q^2)\gamma_\mu + F_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_{B'} + M_B} + F_3(q^2) i \frac{q_\mu}{M_{B'} + M_B} \right) u(p, s)$$





no point at q^2_{max} to constrain fit

$$f_0(q^2) = f_1(q^2) + \frac{q^2}{m_\Sigma^2 - m_n^2} f_3(q^2)$$

poorly determined

Double Ratio

[Becirevic et al., hep-lat/0411016]

- Determine f_0/f_1 using a second double ratio

$$
\begin{aligned}
R_i(t', t) &= \frac{G_i^{\Sigma n}(t', t; \vec{p}, \vec{p}\,')\,G_4^{\Sigma\Sigma}(t, t'; \vec{p}, \vec{p}\,')}{G_4^{\Sigma n}(t, t'; \vec{p}, \vec{p}\,')\,G_i^{\Sigma\Sigma}(t, t'; \vec{p}, \vec{p}\,')} \\
&\longrightarrow \frac{f_0(q^2)}{f_1(q^2)}
\end{aligned}
$$

Double Ratio

[Becirevic et al., hep-lat/0411016]

- Determine f_0/f_1 using a second double ratio

$$R_i(t',t) = \frac{G_i^{\Sigma n}(t',t;\vec{p},\vec{p}\,')G_4^{\Sigma\Sigma}(t,t';\vec{p},\vec{p}\,')}{G_4^{\Sigma n}(t,t';\vec{p},\vec{p}\,')G_i^{\Sigma\Sigma}(t,t';\vec{p},\vec{p}\,')}$$

$$\longrightarrow \frac{f_0(q^2)}{f_1(q^2)}$$

Work In Progress.....

Summary and Outlook

- 32^3 at lighter masses
- run more $\Xi^0 \longrightarrow \Sigma^+ \ell \nu_\ell$
- axial transition form factors $\frac{g_1(q^2)}{f_1(q^2)}, \quad g_{\Sigma n}, \; g_{\Xi\Sigma}, \; \cdots$
- double ratio to obtain precise results for $\frac{f_0(q^2)}{f_1(q^2)} \longrightarrow f_0(q^2)$
- $\Rightarrow |V_{us}|$
- Twisted boundary conditions [a la RBC/UKQCD arXiv:1004.0886]