Chiral properties of light mesons in the N_f =2+1 overlap QCD

Jun Noaki (JLQCD+TWQCD Collaboration)

Coauthors

S.Aoki,

T.W. Chiu,

H. Fukaya,

S.Hashimoto,

T. H. Hsieh,

T. Kaneko,

H. Matsufuru,

T. Onogi,

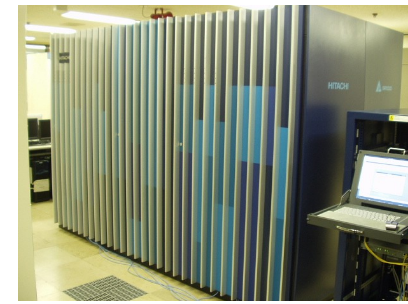
E. Shintani,

N. Yamada

Members

KEK: G. Cossu, S. Hashimoto, H. Ikeda, T. Kaneko, H. Matsufuru, K. Murano, K. Nagai, J. Noaki, N. Yamada

IBM BG/L, 57.3 Tflops



HITACHI, SR11000, 2.1 Tflops

Tsukuba: S. Aoki, N. Ishizuka, K. Kanaya, Y. Kuramashi, K. Takeda, Y. Taniguchi, A. Ukawa, T. Yoshie, T. Yamazaki

RIKEN(BNL): E. Shintani

Osaka: H. Fukaya, H. Ohki, T. Onogi

Hiroshima: K.I. Ishikawa, M. Okawa

Taipei (TWQCD): T.W. Chiu, T.H. Hsieh, K. Ogawa

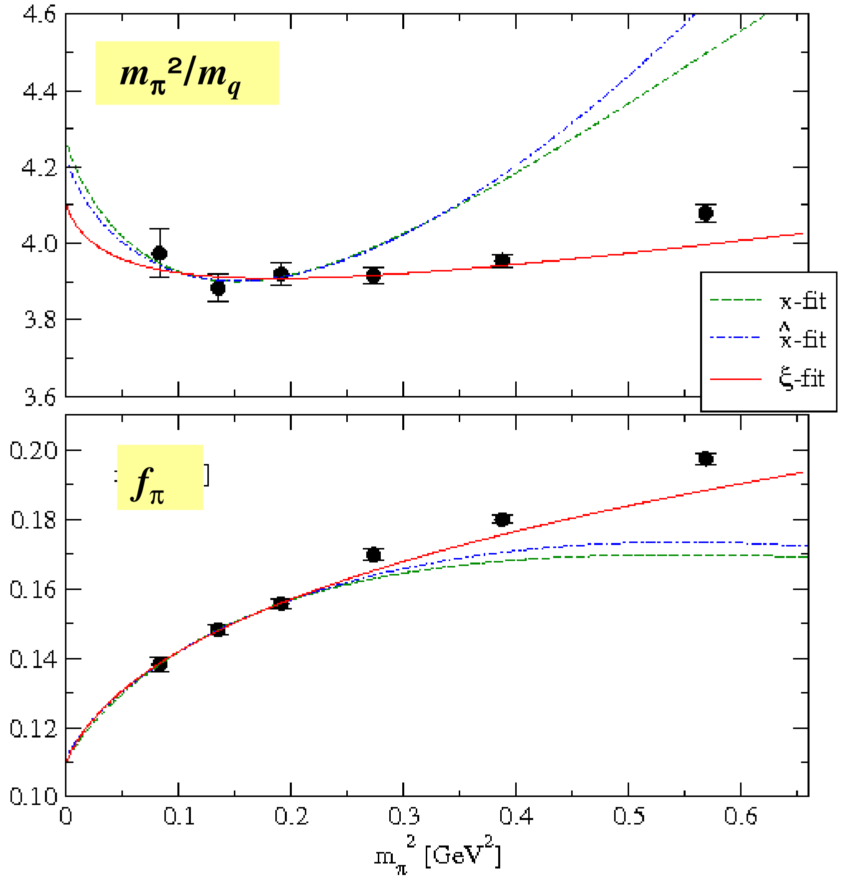
Test of Chiral Perturbation Theory (ChPT)

- Central question: "Safe to use ChPT for phenomenology in the real world?"
  - Must be OK in $m_q \to 0$. However, $m_s \sim 100$ MeV in reality.
  - Kaon must be treated ($m_K \sim 500$ MeV), in particular.
- Convergence study
  - "Does ChPT converge at ~ 500 MeV? Is NLO sufficient, or do we need NNLO, or ...?"
  - $N_f = 2$ theory studied JLQCD+TWQCD, 2008
  - Extension to $N_f = 2+1$ theory wanted
- $N_f = 3$ ChPT
  - Convergence study with $m_u = m_d = m_s$
  - Different from $N_f = 2$: $\Sigma(N_f = 2) \neq \Sigma(N_f = 3)$, $f(N_f = 2) \neq f(N_f = 3)$
  - Need to be studied independently

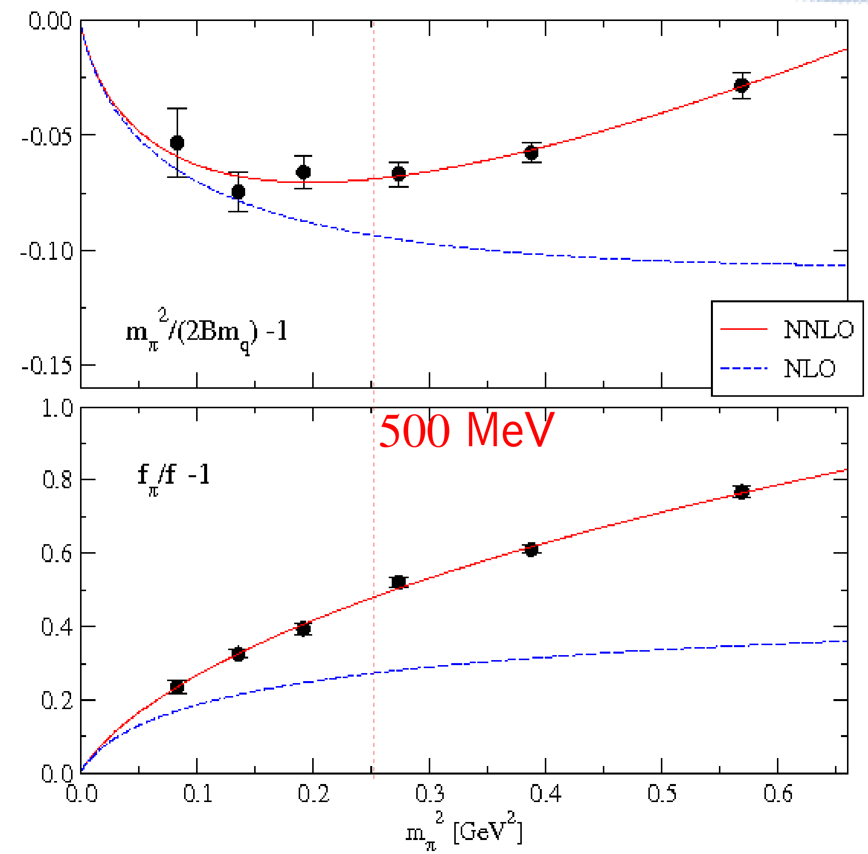
Study for $N_f = 2$

PRL101(2008)202004

- Fit to NLO ChPT with different expansion params



- Convergence property



- NLO is valid for $< m_K$
- Useful expansion parameter:

$$\xi = \left(\frac{m_\pi^2}{4\pi f_\pi} \right)^2$$

What happens in $N_f = 2+1$?

Overlap fermion

- Exact chiral symmetry on the lattice = theoretically clean
  - No need to modify ChPT,

    i.e. no need of SchPT, WchPT,... evan at finite a^{-1}
  - Any N_f is possible to study.
- Caveat
  - Numerically expensive
    - Volume is relatively small → finite size effect
  - Topology is fixed → finite size effect

This talk: $N_f = 2+1$

- An extension of the previous works ($V = 16^3$x48)
- Plan
  - Introduction
  - Study of finite size effects with new data
    - $V = 24^3$x48 for the two lightest masses → direct check of FSE
    - $Q = 1$ for the lightest mass → test of Q dependence
  - Study of SU(3) chiral limit with new data
    - $m_{ud} = m_s$ covering relevant mass range → comparison with $N_f = 2$
  - Update of the physics results Σ_0, f_π, f_K, m_{ud}, m_s, LECs
  - Summary and outlook

Setup

- Main ensembles [$V = 16^3$x48, $Q = 0$, 2,500 HMC trajectories]
  - Quark masses: 5 light x 2 strange (290 MeV $< m_\pi <$ 780 MeV)
  - Signal improvements with low-lying modes (low-mode-averaging)
  - Non-perturbative renormalization (through RI/MOM) for Z_m PRD81(2010)034502

- Lattice scale from Omega-baryon mass
  - Linear fit in the lattice unit

$$am_\Omega = M_0 + \alpha (a m_\pi)^2 + \beta (a m_K)^2$$

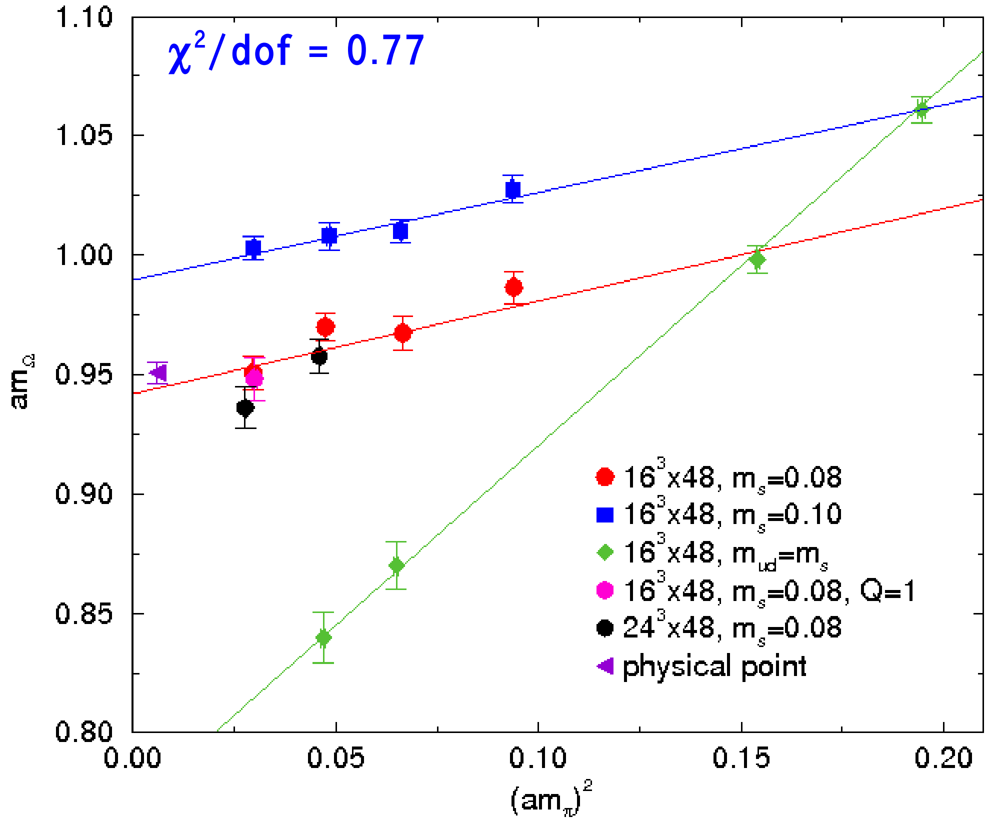
  - At the physical point:

$$m_\Omega^{\text{phys}} = 1.672 \text{ GeV}$$

$$m_\pi^{\text{phys}} = 135 \text{ MeV}$$

$$m_K^{\text{phys}} = 495 \text{ MeV}$$

$$a^{-1} = 1.759(8)(5) \text{ GeV}$$



- Plan
  - Introduction
  - Study of finite size effects with new data
    - $V = 24^3\text{x}48$ for the two lightest masses
    - $Q = 1$ for the lightest mass
  - Study of SU(3) chiral limit with new data
  - Update of the physics results $\Sigma_0, f_\pi, f_K, m_{ud}, m_s$, LECs
  - Summary and outlook

Finite size effect

- Conventional FSE
  - Effects of wrap-around π and K : $\sim e^{-m_\pi L}$
  - Estimated analytically in ChPT by re-summed Lüscher's formula

Colangelo, Dürr and Haefeli, 2005

$am_{ud} = 0.015$:	$m_\pi L$	m_π	m_K	f_π	f_K
16^3	2.75	-3.3%	-0.6%	+8.2%	+3.3%
24^3	4.00	-0.3%	-0.04%	+0.8%	+0.3%

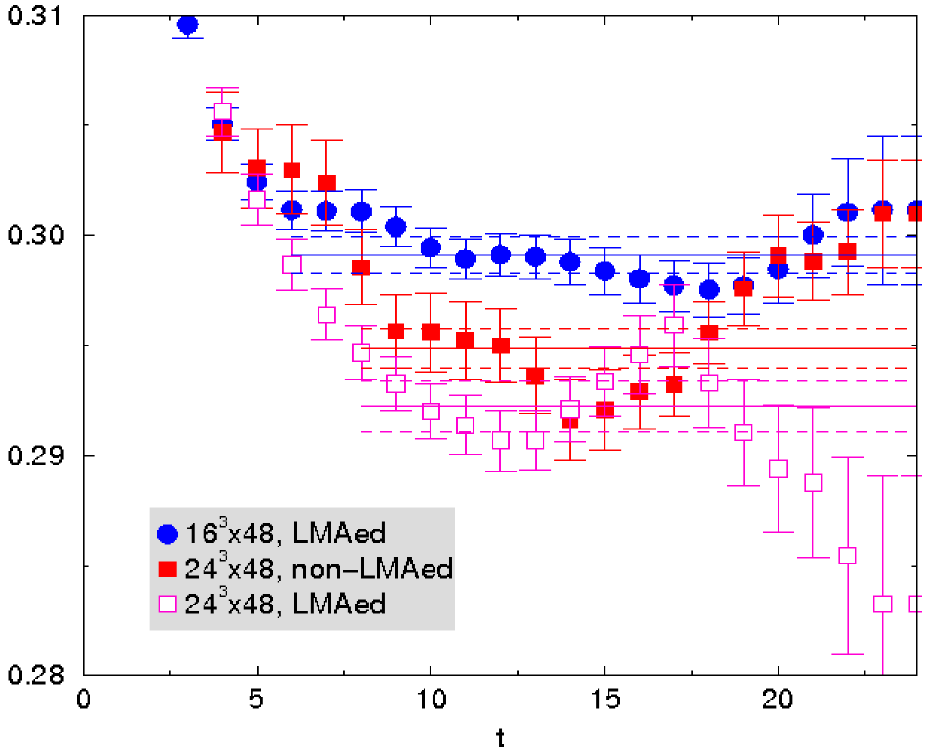
- Fixed topology effect: another type of FSE
  - Global topology is irrelevant in the infinite volume; At finite V, we expect a effect of $O(1/V)$.

Brower et al., 2003; Aoki et al., 2007; Aoki and Fukaya, 2010

	m_π	m_K	f_π	f_K
16^3	+1.8%	+0.3%	0%	+0.6%
24^3	+0.5%	+0.1%	0%	+0.2%

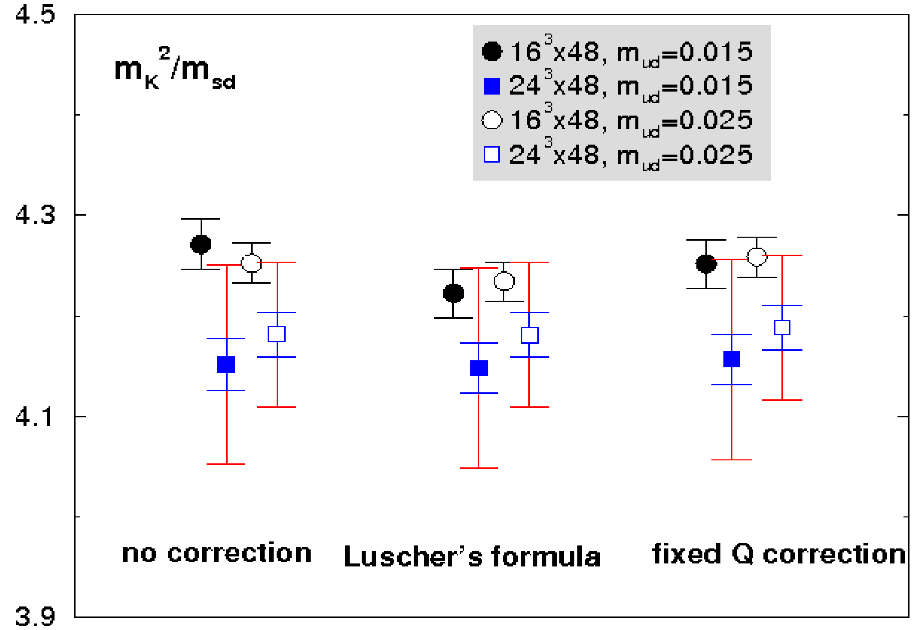
Direct comparison

- eg. kaon for the lightest



- Statistical error underestimated
- $\lambda_{\mathrm{low}} < am_s$ for 24^3, dulls LMA
- Additional error ~ |(LMAed) -(non LMAed)|

- Consistency after FSE correction?

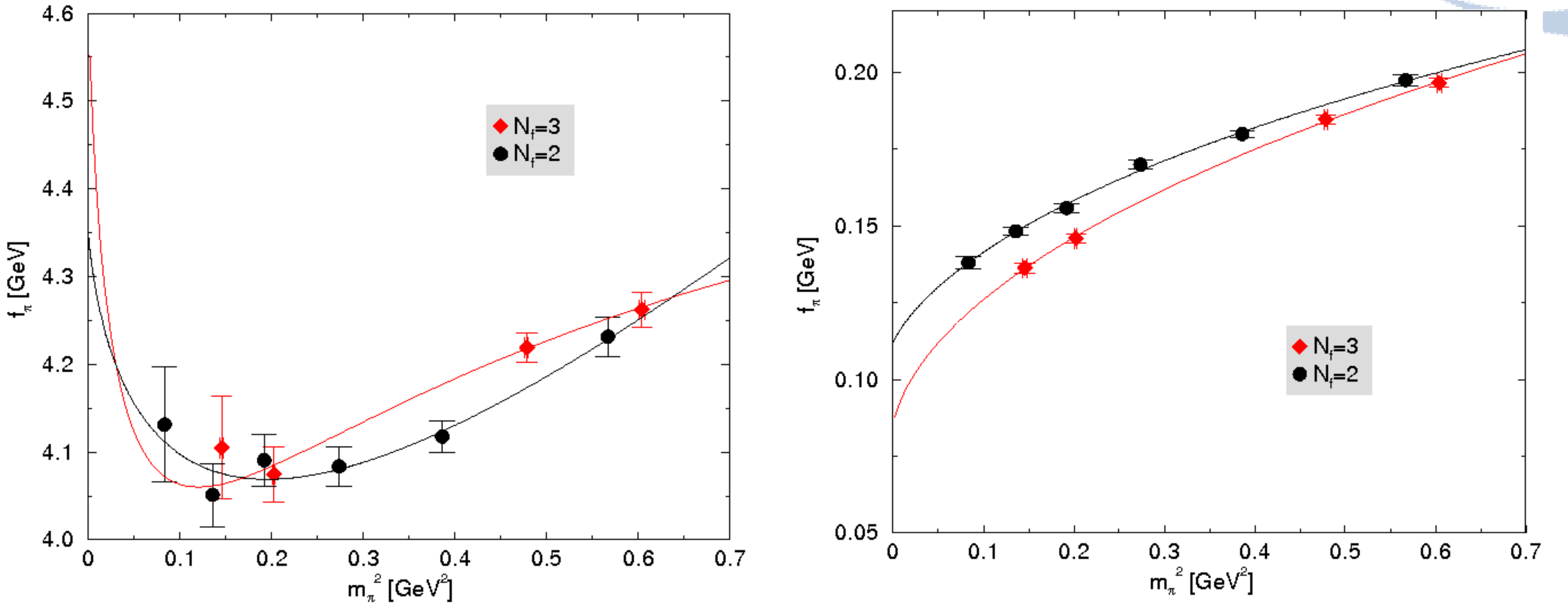


- Statistic+systematic error obscures the difference between 16^3 and 24^3
- The same goes for the $Q = 1$ data.
- Hereafter, analysis contains all data points after the FSE corrections.

- Plan
  - Introduction
  - Study of finite size effects with new data
  - Study of SU(3) chiral limit with new data
    - $m_{ud} = m_s$ covering relevant mass range
  - Update of the physics results $\Sigma_0, f_\pi, f_K, m_{ud}, m_s$, LECs
  - Summary and outlook

SU(3) chiral extrapolation

- At a glance: flavor dependence



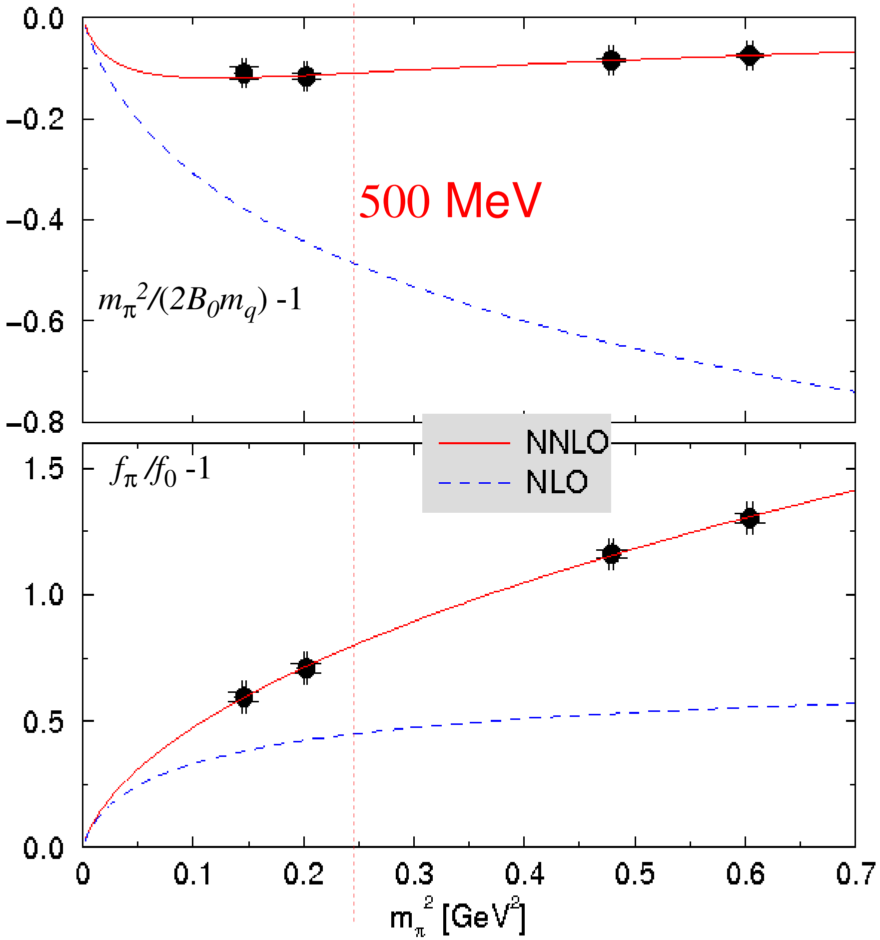
  - Look at the curvature:

$$m_\pi^2/m_{ud} = 2B_{(0)}\left(1+\frac{2}{N_f}\xi_\pi \ln\xi_\pi+\dots\right) \qquad f_\pi = f_{(0)}(1-N_f\xi_\pi \ln\xi_\pi+\dots)$$

  - N_f dependence of the curvature is seen in data.
  - Decay constant is small in the SU(3) chiral limit.

Convergence of SU(3) ChPT

- Convergence ratio



NNLO formula

$$m_\pi^2/m_q = 2\,B_0\cdot\left[M(\xi) - 8(4\pi)^2(3L_4^r + L_5^r)\cdot\xi\left(1-\frac{25}{3}\xi\ln\xi\right)\right.$$
$$\left. + 48(4\pi)^2 L_6^r\cdot\xi\left(1-\frac{19}{3}\xi\ln\xi\right) + 16(4\pi)^2 L_8^r\cdot\xi(1-8\xi\ln\xi)\right] + \alpha\cdot\xi^2,$$
$$f_\pi = f_0\cdot\left[F(\xi) + 4(4\pi)^2(3L_4^r + L_5^r)\cdot\left(1-\frac{15}{2}\xi\ln\xi\right)\right] + \beta\cdot\xi^2$$

	cf. $N_f = 2$
$\frac{[m_\pi^2/m_q]_{\mathrm{NLO}} - [m_\pi^2/m_q]_{\mathrm{LO}}}{[m_\pi^2/m_q]_{\mathrm{LO}}} = -(49 \pm 37)\%$	$-(8.3 \pm 2.0)\%$
$\frac{[m_\pi^2/m_q]_{\mathrm{NNLO}} - [m_\pi^2/m_q]_{\mathrm{NLO}}}{[m_\pi^2/m_q]_{\mathrm{NLO}}} = +(72 \pm 106)\%$	$+(1.9 \pm 0.6)\%$
$\frac{[f_\pi]_{\mathrm{NLO}} - [f_\pi]_{\mathrm{LO}}}{[f_\pi]_{\mathrm{LO}}} = +(45 \pm 17)\%$	$+(25.4 \pm 4.9)\%$
$\frac{[f_\pi]_{\mathrm{NNLO}} - [f_\pi]_{\mathrm{NLO}}}{[f_\pi]_{\mathrm{NLO}}} = +(24 \pm 14)\%$	$+(22.6 \pm 0.9)\%$

Large stat. error, but comparable convergence with $N_f = 2$ case.

- Plan
  - Introduction
  - Study of finite size effects with new data
  - Study of SU(3) chiral limit with new data
  - Update of the physics results $\Sigma_0, f_\pi, f_K, m_{ud}, m_s$, LECs
  - Summary and outlook

Chiral extrapolation to NNLO

- Treat pion and kaon on an equal footing
  - Expansion parameters $\xi_\pi = \left(\frac{m_\pi^2}{4\pi f_\pi}\right)^2,\ \xi_K = \left(\frac{m_K^2}{4\pi f_K}\right)^2$ from measured values
  - NNLO formulae G.Amorós, J. Bijnens and P. Talavera, 2000

$$m_\pi^2/m_{ud} = 2B_0 M_\pi(\xi_\pi, \xi_K;\ L_4^r, L_5^r, L_6^r, L_8^r) + \alpha_1^\pi \xi_\pi^2 + \alpha_2^\pi \xi_\pi \xi_K + \alpha_3^\pi \xi_K^2$$

$$m_K^2/m_{sd} = 2B_0 M_K(\xi_\pi, \xi_K;\ L_4^r, L_5^r, L_6^r, L_8^r) + \alpha_1^K \xi_\pi(\xi_\pi - \xi_K) + \alpha_2^K \xi_K(\xi_K - \xi_\pi) + (\alpha_1^\pi + \alpha_2^\pi + \alpha_3^\pi)\xi_\pi \xi_K$$

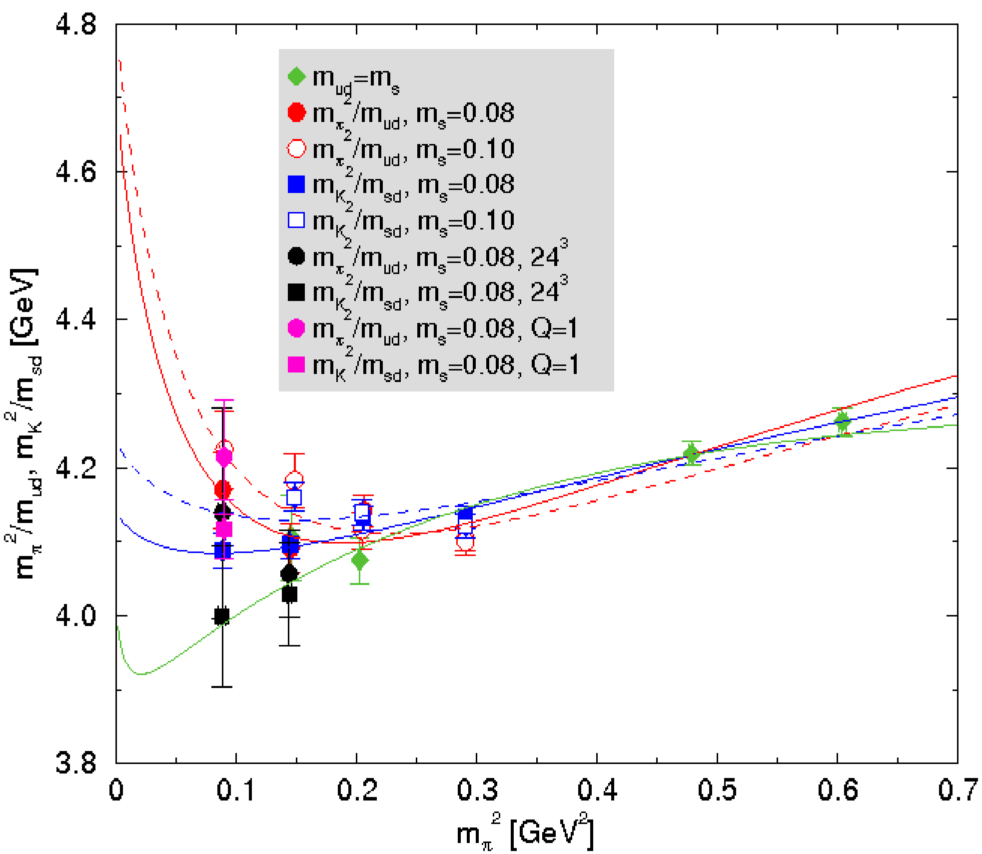
$$f_\pi = f_0 F_\pi(\xi_\pi, \xi_K;\ L_4^r, L_5^r) + \beta_1^\pi \xi_\pi^2 + \beta_2^\pi \xi_\pi \xi_K + \beta_3^\pi \xi_K^2$$

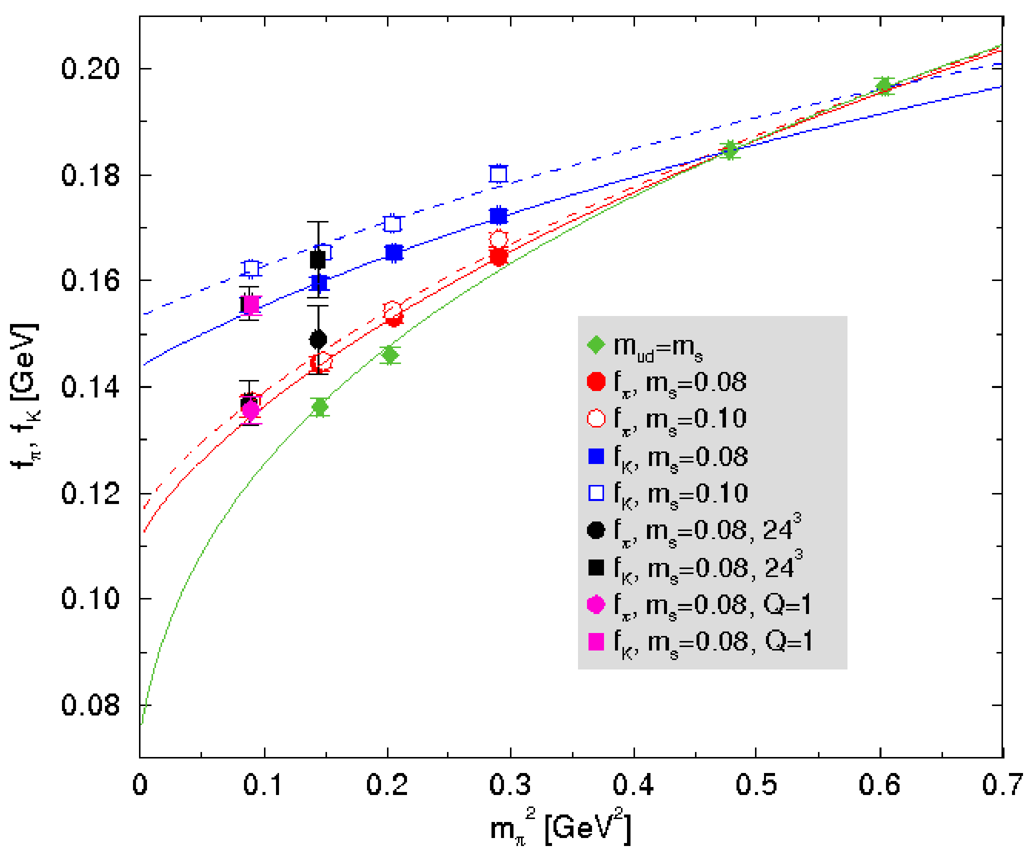
$$f_K = f_0 F_K(\xi_\pi, \xi_K;\ L_4^r, L_5^r) + \beta_1^K \xi_\pi(\xi_\pi - \xi_K) + \beta_2^K \xi_K(\xi_K - \xi_\pi) + (\beta_1^\pi + \beta_2^\pi + \beta_3^\pi)\xi_\pi \xi_K$$

  - Loop functions: M_π, M_K, F_π, F_K to two-loop level (thanks to J. Binens for the fortran code)
  - LECs $B_0, f_0, L_4^r, L_5^r, L_6^r, L_8^r$ are determined.
  - Input: $L_1^r = 4.3\times10^{-4},\ L_2^r = 7.3\times10^{-4},\ L_3^r = -2.53\times10^{-3},\ L_7^r = -3.1\times10^{-4}$
  - Remove non-linearity of fit parameters by setting $2B_0/f_0 = R$ = const., chosen to minimize χ^2/dof
  - Analytic strange-sea dependence dropped: $\alpha_3^\pi = \beta_3^\pi = 0$ will be discussed later.

Fit curves

- Simultaneous fit of m_π^2/m_{ud} , m_K^2/m_{sd} , f_π , f_K
  - Correlation among them taken into account: $\chi^2/\mathrm{dof} = 2.30$

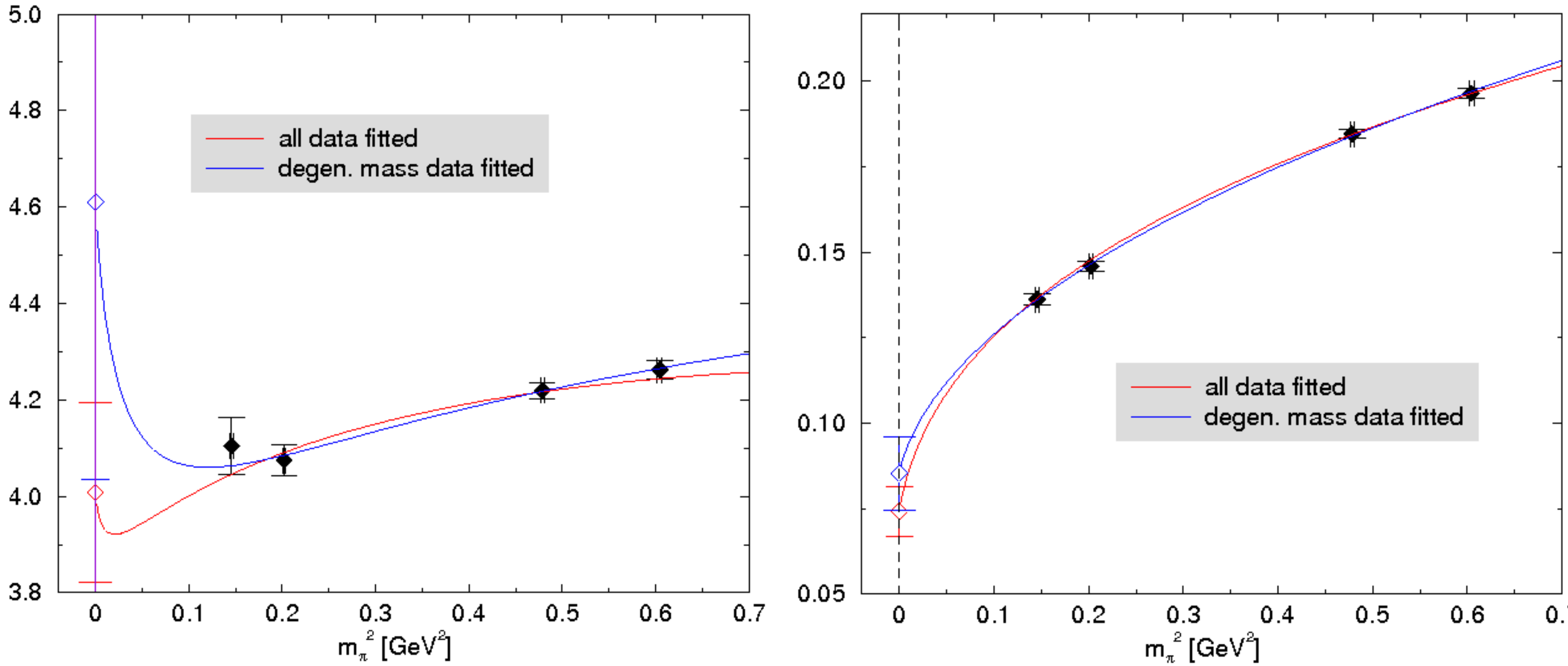




Horizontal axis is rescaled as $\xi_\pi x(4\pi f_\pi)^2$

Consistency check

- Comparison with the degenerate case
  - Coefficient of analytic term: $\alpha = \alpha_1^\pi + \alpha_2^\pi + \alpha_3^\pi$, $\beta = \beta_1^\pi + \beta_2^\pi + \beta_3^\pi$



  - Consistency at current precision level justifies the omission of α_3^π, β_3^π *i.e.* the strange-sea quark dependence in the analytic term.

Physics results

- From fit parameters (note f_π=130 MeV normalization)

$$f_0 = 74.2(7.2)\text{ MeV},\quad 2B_0 = 4.01(19)\text{ MeV},\quad \Sigma_0^{1/3} = 177(13)\text{ MeV}$$

$$L_4^r(770\text{ MeV}) = 6.2(2.1)\times10^{-5},\quad L_5^r(770\text{ MeV}) = -6.5(5.9)\times10^{-5},$$

$$L_6^r(770\text{ MeV}) = 2.7(1.7)\times10^{-5},\quad L_8^r(770\text{ MeV}) = -2.8(2.6)\times10^{-5},$$

- Extrapolation to physical points: $[m_\pi^2/m_{ud}]^{\text{phys}},\ [m_K^2/m_{sd}]^{\text{phys}},\ f_\pi^{\text{phys}},\ f_K^{\text{phys}}$

$$f_\pi = 118.4(3.6)\text{ MeV},\quad f_K = 146.8(2.5)\text{ MeV},\quad f_K/f_\pi = 1.240(19)$$

- Quark mass: $m_{ud}^{\text{phys}} = \dfrac{(m_\pi^{\text{phys}})^2}{[m_\pi^2/m_{ud}]^{\text{phys}}},\quad m_{sd}^{\text{phys}} = \dfrac{(m_K^{\text{phys}})^2}{[m_K^2/m_{sd}]^{\text{phys}}},\quad m_s^{\text{phys}} = 2m_{sd}^{\text{phys}} - m_{ud}^{\text{phys}}$

$$m_{ud} = 4.080(58)\text{ MeV},\quad m_s = 115.1(1.2)\text{ MeV},\quad m_s/m_{ud} = 28.20(24)$$

- Plan
  - Introduction
  - Study of finite size effects with new data
  - Study of SU(3) chiral limit with new data
  - Update of the physics results $\Sigma_0, f_\pi, f_K, m_{ud}, m_s$, LECs
  - Summary and outlook

Summary & outlook

- Dynamical overlap fermions enable consistent study of ChPT depending on N_f.
- Study of finite size effect by direct comparison
  - $m_\pi L > 4.0$ is practically free from FSE.
  - No strong inconsistency with analytic FSE estimations
- Convergence study of SU(3) ChPT as an extension of $N_f = 2$ study
  - NNLO formulae provide reasonable fit up to $\sim m_s$.
  - Comparable convergence with $N_f = 2$ is observed.
- Update of physical results using all available data points
  - Relatively reliable SU(3) LECs thanks to the $m_{ud} = m_s$ data points.
- Future
  - Interesting to investigate the convergence for other phenomenologically important quantities such as the kaon form factor, $K \to \pi\pi$ matrix elements,
  - Exact chiral symmetry provides the theoretically clean framework.