Flavor dependence of the S parameter in $SU(3)$ gauge theory

David Schaich

Department of Physics and Center for Computational Science, Boston University

for the Lattice Strong Dynamics Collaboration

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Lattice Strong Dynamics (LSD) Collaboration

Argonne	James Osborn
Boston	Adam Avakian, Ron Babich, Richard Brower, Saul D. Cohen, Claudio Rebbi, DS
Harvard	Mike Clark
Livermore	Michael Cheng, Pavlos Vranas
MIT	Fu-Jiun Jiang
UC Davis	Joseph Kiskis
Maryland	Mike Buchoff
Yale	Thomas Appelquist, George Fleming, Meifeng Lin, Ethan Neil, Gennady Voronov

Formed in 2007 to pursue non-perturbative studies
of strongly interacting theories likely to produce observable signatures
at the Large Hadron Collider.

Outline

1. The S parameter
2. Lattice methods (briefly)
3. (Preliminary) Analysis and results
4. Conclusions and outlook

Motivation

What is the mechanism behind electroweak symmetry breaking?

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

- Many possibilities: standard model, technicolor, etc.
- In the absence of direct detection

  exploit precision measurements of electroweak observables
- Consider vacuum polarization (oblique) corrections,

  parameterize effects of physics beyond the standard model

$$\gamma \sim\!\bullet\!\sim \gamma \;=\; i\, e^2\, \Pi_{QQ}\, g^{\mu\nu} + \cdots$$

$$Z \sim\!\bullet\!\sim \gamma \;=\; i\, \frac{e^2}{cs} \left(\Pi_{3Q} - s^2\, \Pi_{QQ}\right) g^{\mu\nu} + \cdots$$

$$Z \sim\!\bullet\!\sim Z \;=\; i\, \frac{e^2}{c^2 s^2} \left(\Pi_{33} - 2s^2\, \Pi_{3Q} + s^4\, \Pi_{QQ}\right) g^{\mu\nu} + \cdots$$

$$\Pi_{XY}^{\mu\nu}(q) = \sum_x e^{iq\cdot x} \left\langle J_X^\mu(x) J_Y^\nu(0) \right\rangle = g^{\mu\nu} \Pi_{XY}^\perp(q^2) + (q^\mu q^\nu \text{ terms})$$

Definition of S

Peskin and Takeuchi, PRD **46**, 381 (1992)

$$= i\, e^2\, \Pi_{QQ}\, g^{\mu\nu} + \cdots$$

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$$\Pi_{VV} = 2\Pi_{3Q}$$
$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3Q}$$

$$S \equiv -4\pi \frac{N_f}{2} \frac{d}{dq^2} \left[\Pi^{\perp}_{VV}(q^2) - \Pi^{\perp}_{AA}(q^2)\right]_{q^2=0}$$

- In terms of spectral functions $\rho(s) = -12\pi \mathrm{Im}\Pi'(s)$
- Subtract the standard model contribution

so that S measures deviations from the SM

$$S = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \frac{N_f}{2} \left[\rho_V(s) - \rho_A(s)\right]$$

(m_H is SM Higgs mass)

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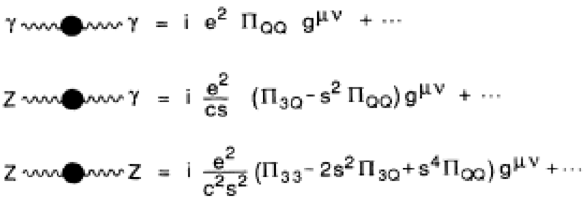

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$$S = \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ \frac{N_f}{2} \left[\rho_V(s) - \rho_A(s) \right] - \frac{1}{4} \left[1 - (1 - m_H^2/s)^3 \theta(s - m_H^2) \right] \right\}$$

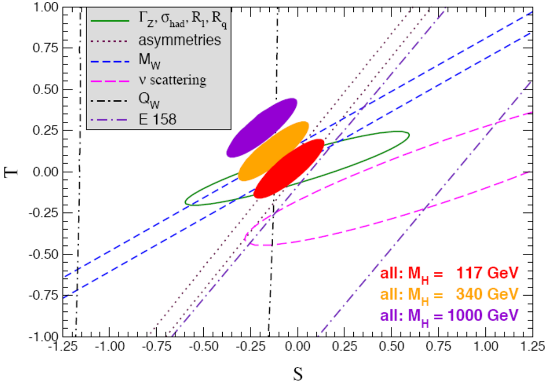
(m_H is SM Higgs mass)

Experiments find $S \approx 0$

Extract S from global fit to experimental data for

- Z decay partial widths and asymmetries
- Deep inelastic neutrino scattering
- M_W / M_Z
- Atomic parity violation

Result: S consistent with zero



(PDG)

Conventional wisdom for S

Peskin and Takeuchi highlight two contributions to S:

1. Single-pole approximation for ρ_A and ρ_V
   assuming QCD-like spectrum and Weinberg sum rules

$$0.25 \frac{N_f}{2} \frac{N_c}{3}$$

2. χPT for pseudo-Nambu–Goldstone bosons at energies below M_ρ

$$\frac{1}{48\pi} \left(N_f^2 - 4\right) \log \left[\frac{M_\rho^2}{M_{PNGB}^2}\right]$$

Both contributions positive and grow with N_f

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3. Parity doubling in walking/conformal theories reduces S?

These arguments are widely used in model building
but need to be investigated from first principles.

S on the lattice

- Chiral symmetry is crucial $\Rightarrow$ use domain wall fermions
- Explore dependence on N_f and N_c, starting from QCD
  $SU(3)$ gauge theory; fundamental $N_f = 2$ & 6; scales matched
- Exploratory calculations $\Rightarrow$ short runs $\lesssim$ 1000 configurations
  300 million core hours on LLNL BG/L + USQCD clusters + NSF Teragrid
  $32^3 \times 64$ lattices; $L_s = 16$; $0.005 \leq m_f \leq 0.03$

$$m_{res} = 2.59(1) \times 10^{-5} \text{ for } N_f = 2$$
$$m_{res} = 82.6(4) \times 10^{-5} \text{ for } N_f = 6$$

- Previous lattice calculations of S by
  JLQCD Collaboration ($N_f = 2$ overlap) PRL **101**, 242001 (2008) [0806.4222]
  RBC-UKQCD Collaboration ($N_f = 2+1$ DWF) PRD **81**, 014504 (2010) [0909.4931]

Currents and correlators

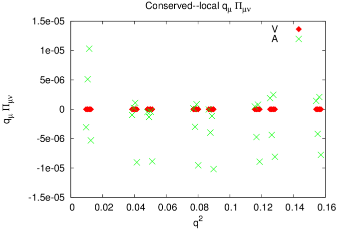
We measure

$$\Pi^{\mu\nu}_{V-A}(q) = \sum_x e^{iq\cdot x}\left[\langle \mathcal{V}^\mu(x) V^\nu(0)\rangle - \langle \mathcal{A}^\mu(x) A^\nu(0)\rangle\right]$$

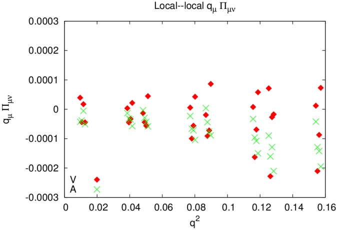
- $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$ are conserved domain wall currents (point-split, summed over the fifth dimension)
- V^ν and A^ν are local currents defined on the domain walls
- Conserved currents ensure that lattice artifacts cancel, needed for clean signal RBC-UKQCD
- $\langle \mathcal{V}^\mu(x)\mathcal{V}^\nu(0)\rangle$ and $\langle \mathcal{A}^\mu(x)\mathcal{A}^\nu(0)\rangle$ require $\mathcal{O}(L_s)$ inversions
- Suffices to use $\langle \mathcal{V}^\mu(x) V^\nu(0)\rangle$

Ward identities and violations

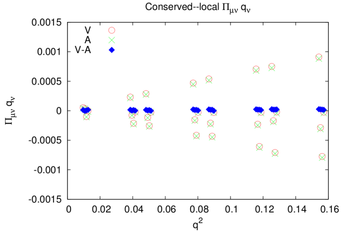
$$\widehat{q}_\mu \left[\sum_x e^{iq\cdot x} \langle \mathcal{V}^\mu(x) V^\nu(0)\rangle\right] = 0$$



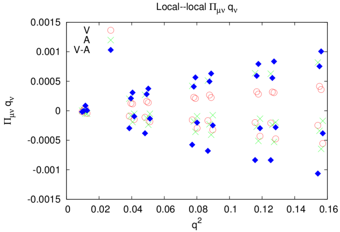
$$\widehat{q}_\mu \left[\sum_x e^{iq\cdot x} \langle V^\mu(x) V^\nu(0)\rangle\right] \neq 0$$



$$\left[\sum_x e^{iq\cdot x} \left(\langle \mathcal{V}^\mu V^\nu\rangle - \langle \mathcal{A}^\mu A^\nu\rangle\right)\right] \widehat{q}_\nu \approx 0$$



$$\left[\sum_x e^{iq\cdot x} \left(\langle V^\mu V^\nu\rangle - \langle A^\mu A^\nu\rangle\right)\right] \widehat{q}_\nu \neq 0$$

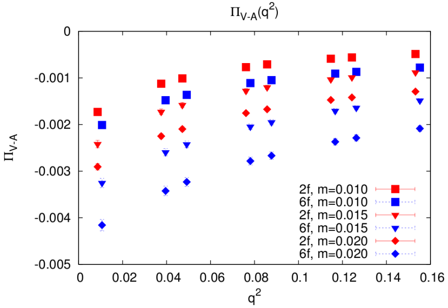


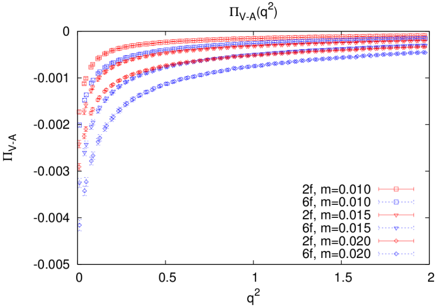
Polarization function

We have information from both the correlators and the spectrum

$$\Pi^{\perp}_{V-A}(q^2) = -F_\pi^2 + \frac{q^2}{12\pi^2}\int_0^\infty ds \frac{\rho_V(s) - \rho_A(s)}{s + q^2}$$

$$S = 4\pi \frac{d}{dq^2}\left[\Pi^{\perp}_{V-A}(q^2)\right]_{q^2=0}$$





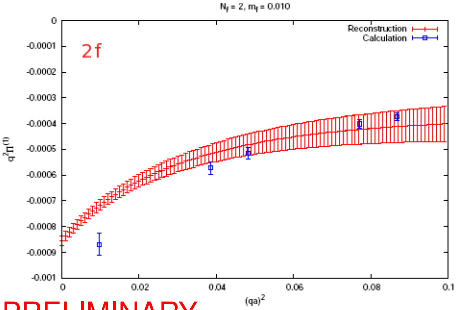
What are the best ways to extract S from $\Pi^{\perp}_{V-A}(q^2)$?

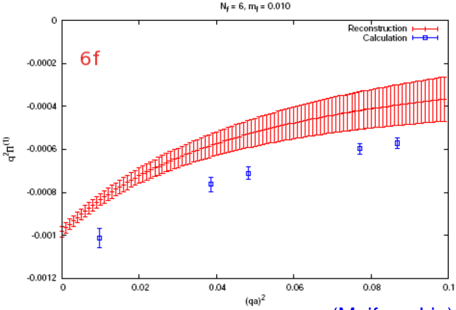
Single-pole dominance model

$$\rho_A(s) = 12\pi^2 F_{a_1}^2 \delta\left(s - M_{a_1}^2\right) \qquad \rho_V(s) = 12\pi^2 F_\rho^2 \delta\left(s - M_\rho^2\right)$$

$$\Pi_{V-A}^{\perp}(q^2) = -F_\pi^2 + q^2\left(\frac{F_\rho}{M_\rho^2 + q^2} - \frac{F_{a_1}}{M_{a_1}^2 + q^2}\right)$$

- Reconstruct $\Pi_{V-A}^{\perp}(q^2)$ using results summarized in previous talk
- Compare against direct calculation





PRELIMINARY

(Meifeng Lin)

Chiral perturbation theory for $N_f = 2$

$$S = \frac{1}{12\pi}\left(\bar{\ell}_5 + \log\left[\frac{m_\pi^2 \frac{v^2}{f_\pi^2}}{m_H^2}\right] - \frac{1}{6}\right)$$

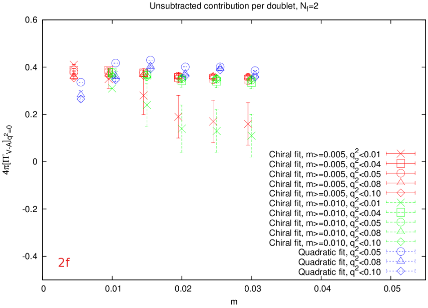
$\bar{\ell}_5$ is extracted from Gasser and Leutwyler, Ann. Phys. **158**, 142 (1984)

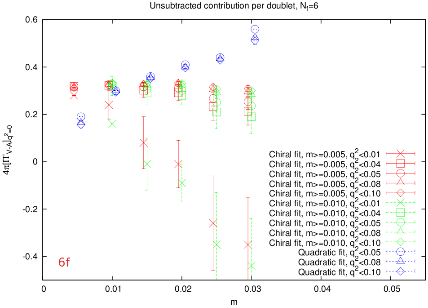
$$\Pi_{V-A}^{\perp}(q^2) = -F_\pi^2 + q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + \frac{2}{3}(1+x)\bar{J}(x)\right]$$

$$\bar{J}(x) = \frac{1}{16\pi^2}\left(\sqrt{1+x}\log\left[\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right] + 2\right), \qquad x \equiv 4M_\pi^2/q^2$$

- As discussed in previous talks, expect χPT marginal for $N_f = 2$, inapplicable for $N_f = 6$
- General-N_f analog not yet known
- Must take only two flavors to the chiral limit, any others remain massive

Comparing $N_f = 2$ and $N_f = 6$ VERY PRELIMINARY





- Compare contribution to S per electroweak doublet
- Don't yet subtract standard model contribution
- Fits with and without using χPT converge for smaller m_f
- $\sim 25\%$ reduction when N_f increases from 2 to 6

Conclusions and outlook

- We calculate S on the lattice using conserved DWF currents
- Preliminary results for $N_f = 2$
  agree with expectations and previous studies
- Preliminary signs of reduction in contribution per doublet
  as N_f increases from 2 to 6.

Next steps

1. Extracting final results and systematics from data presented above
2. Implementing twisted boundary conditions to explore smaller q^2
3. Continue exploring how S varies with N_f, N_c

Bonus slides!

Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^\mu(x) = \sum_{s=0}^{L_s-1} j^\mu(x,s) \qquad \mathcal{A}^\mu(x) = \sum_{s=0}^{L_s-1} \text{sign}\left(s - \frac{L_s-1}{2}\right) j^\mu(x,s)$$

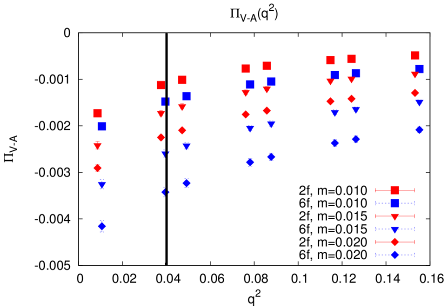
$$j^\mu(x,s) = \overline{\Psi}(x+\widehat{\mu},s)\frac{1+\gamma^\mu}{2}U^\dagger_{x,\mu}\Psi(x,s) - \overline{\Psi}(x,s)\frac{1-\gamma^\mu}{2}U_{x,\mu}\Psi(x+\widehat{\mu},s)$$

Local currents:

$$V^\mu(x) = \overline{q}(x)\gamma^\mu q(x) \qquad A^\mu(x) = \overline{q}(x)\gamma^\mu\gamma^5 q(x)$$

$$q(x) = P_L\Psi(x,0) + P_R\Psi(x,L_s-1)$$

$m_\rho^2 = 0.04$



Limited range of q^2 for χPT