Electromagnetic corrections to light hadrons masses

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Preliminary work based on a subset of BMW collaboration QCD ensembles

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Isospin symmetry breaking

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Mass splitting allows crucial processes like **neutron β decay**.

Dashen's theorem

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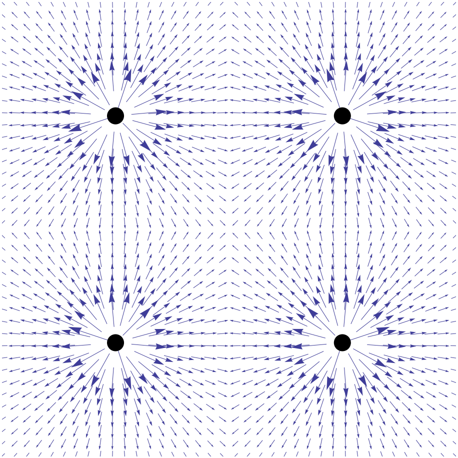
How large are the corrections ?

$$\begin{aligned} \Delta_A D &= \Delta_{\mathrm{EM}} M_K^2 - \Delta_{\mathrm{EM}} M_\pi^2 \\ \Delta_R D &= \frac{\Delta_{\mathrm{EM}} M_K^2}{\Delta_{\mathrm{EM}} M_\pi^2} - 1 \end{aligned}$$

Dashen's theorem corrections

	$\Delta_A D$ (MeV^2)	$\Delta_R D$	
phenomenology	1230	**0.80**	[Donoghue'1993]
	1300 ± 400	1.02 ± 0.30	[Bijnens'1993]
	360	0.26	[Baur'1995]
	1060 ± 320	0.87 ± 0.39	[Bijnens'1996]
	1080	**0.68**	[Gao'1997]
	1070	0.74	[Bijnens'2007]
lattice	526	0.39	[Duncan'1996]
	340 ± 92	0.30 ± 0.08	[RBC'2007]
	1250 ± 550	?	[MILC'2008]

Electromagnetism on $\mathbb{T}^4$



Electromagnetic field generated by a static point charge cannot be made periodic and continuous.

Electromagnetism on $\mathbb{T}^4$

On $\mathbb{T}^4$, the Maxwell-Gauss equation:

$$\partial_\mu F_{\mu\nu} = j_\nu$$

imposes global electric neutrality :

$$Q_{\text{total}} = \int_{\mathbb{T}^3} \mathrm{d}^3\mathbf{x}\, j_0(x) = \int_{\mathbb{T}^3} \mathrm{d}^3\mathbf{x}\, \partial_k F_{k0}(x) = 0$$

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A possible solution : modify Maxwell equations

$$\partial_\mu F_{\mu\nu} = j_\nu - \frac{1}{V} L_\nu c_\nu \quad \text{with} \quad c_\nu \doteq \int_{\mathbb{T}^3} \mathrm{d}^3 x_\nu^\perp j_\nu(x)$$

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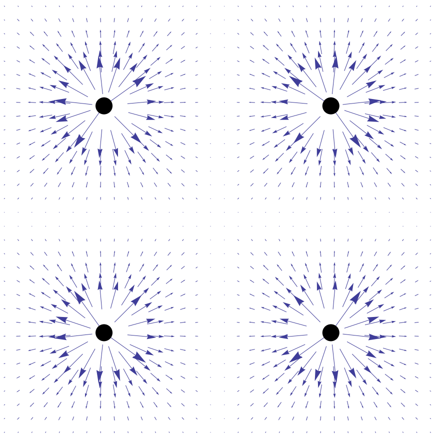

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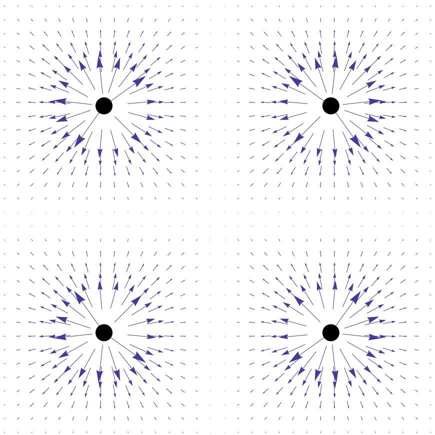

- Electromagnetic field is now periodic and continuous.
- In a large volume compared to relevant distances, physics is almost the same as in infinite volume.

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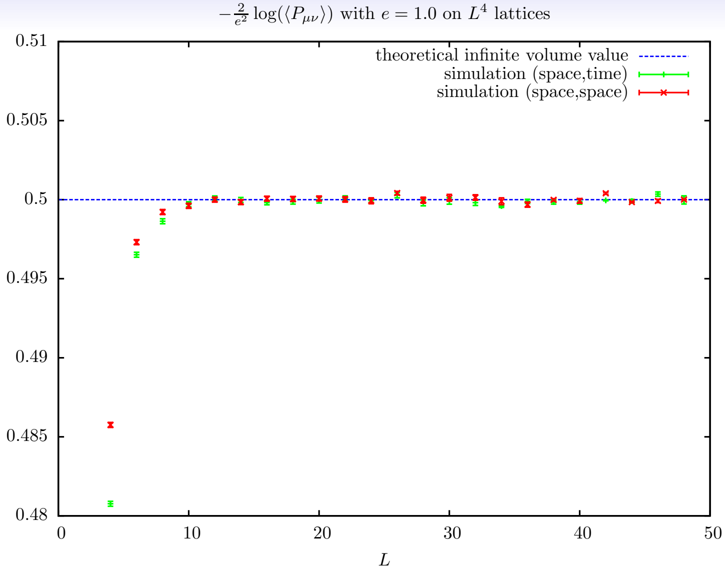
- Boltzmann weight $e^{-S^{(\mathrm{DF})}_{\mathrm{Maxwell}}[A]}$ is **normal**. Electromagnetic fields are simple and cheap to generate.

Numerical check



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- Thus, simulation is **quenched** in QED.

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Considering the parametrization :

$$m_u^{\mathrm{val},0} = m_u^{\mathrm{sea},0} + 4\delta_c, \quad m_d^{\mathrm{val},0} = m_d^{\mathrm{sea},0} + \delta_c, \quad m_s^{\mathrm{val},0} = m_s^{\mathrm{sea},0} + \delta_c$$

we search a value of δ_c where mass isospin symmetry is restored.

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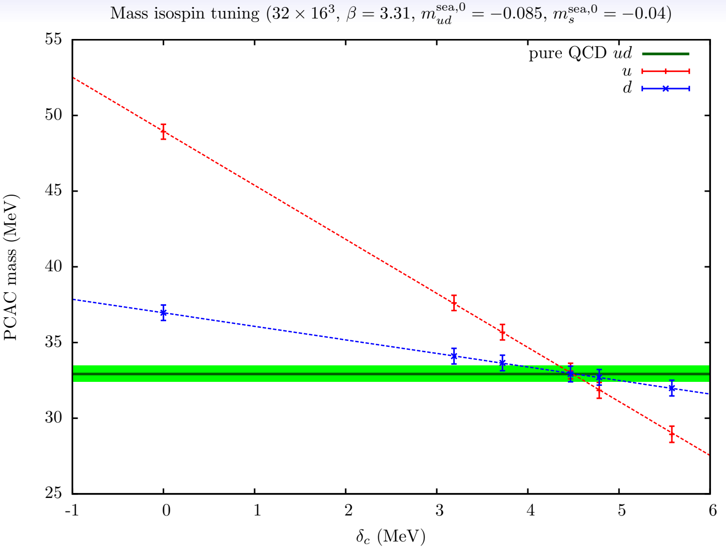

Going to the physical point

To compute the physical value f^{ϕ} of a quantity f we extrapolate it by a Taylor expansion in $M_{\pi^+}^2$:

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Extrapolated quantities are :

$$M_{\pi^+}^2,\ \Delta M_{\pi}^2,\ M_{K^0}^2,\ M_{K^+}^2 \text{ and } \Delta M_K^2$$

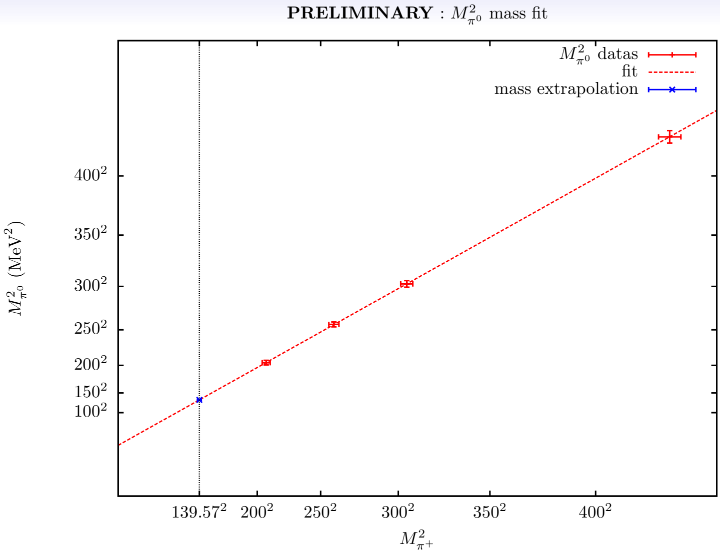
QCD configurations

The following configurations have been analyzed :

β	m^0_{ud}	m^0_s	size	N_{conf}	M_π (MeV)	$M_\pi L$
3.31	-0.08500	-0.04	32×16^3	218	420	8.1
3.31	-0.09300	-0.04	48×24^3	128	300	8.6
3.31	-0.09530	-0.04	48×24^3	210	250	7.2
3.31	-0.09756	-0.04	48×24^3	130	200	5.8

$a^{-1} \simeq 1697$ MeV $= 0.11628$ fm and $m_s \simeq m_s^\phi$
$e = 0.302822$

Chiral extrapolation



Preliminary results

At $M_{\pi^+} = M^{\phi}_{\pi^+} = 139.57018\ \mathrm{MeV}$:

$$\begin{aligned}
M_{\pi^0} &= 134.5 \pm 1.1\ \mathrm{MeV}\\
\Delta_{\mathrm{EM}} M_{\pi} &= 5.1 \pm 1.1\ \mathrm{MeV}\\
\Delta_{\mathrm{EM}} M_{\pi}^2 &= 1380 \pm 50\ \mathrm{MeV}^2\\
M_{K^+} &= 501.3 \pm 2.0\ \mathrm{MeV}\\
M_{K^0} &= 499.0 \pm 2.0\ \mathrm{MeV}\\
\Delta_{\mathrm{EM}} M_{K} &= 2.2 \pm 0.2\ \mathrm{MeV}\\
\Delta_{\mathrm{EM}} M_{K}^2 &= 2200 \pm 180\ \mathrm{MeV}^2\\
\Delta_{A} D &= 830 \pm 180\ \mathrm{MeV}^2\\
\Delta_{R} D &= 0.60 \pm 0.14
\end{aligned}$$

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Thank you for listening !