N, Δ and Ω excited state spectra in N_f=2+1 QCD

* Subcollaboration:

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- arXiv:1004.5072
- Submitted to Physical Review D
- Long-term goal: Solve QCD to determine the mass spectrum of QCD: baryons, mesons, hybrids, $\cdots$

Lattice parameters

- N_f = 2+1 QCD (PRD 79, 034502)
  - Gauge action: Symanzik-improved
  - Fermion action: Clover-improved Wilson

- Anisotropic: a_s= 0.122 fm, a_t = 0.035 fm

ensemble	1	2	3
m_ℓ	$-$.0840	$-$.0830	$-$.0808
m_s	$-$.0743	$-$.0743	$-$.0743
Volume	$16^3\times$ 128	$16^3\times$ 128	$16^3\times$ 128
$N_{\rm cfgs}$	344	570	481
$t_{\rm sources}$	4	4	4
m_π	0.0691(6)	0.0797(6)	0.0996(6)
m_K	0.0970(5)	0.1032(5)	0.1149(6)
m_Ω	0.2951(22)	0.3040(8)	0.3200(7)
m_π (MeV)	392(4)	438(3)	521(3)

Analyses for N, Δ and Ω spectra

- Many interpolating field operators in each IR of octahedral group: Prune to ≈ 10
- "Distillation" technology for smearing: Use 32 eigenvectors of Laplacian
- Matrices of correlation functions: Diagonalize them at $t^* \approx 8$, Fix eigenvectors at t^*.
- Diagonal correlation functions: Fit them & extract six energies
- Lattice spectra: Compare patterns with experimental resonance spectra.

Limitations

- **Three-quark operators:**
  - No multiparticle operators
  - Scant evidence for scattering states

- **One volume:** No extrapolations or δ's

- m_π **large:** Energies generally are high.

- **Spins:** $J^P = \frac{5}{2}^-$ **seen,** higher spins ambiguous.

Computational Resources

- USQCD allocations
- Jefferson Laboratory clusters
- Fermi National Accelerator Lab clusters
- and the Chroma software system (Edwards *et al.*)

Thanks to all for their support.

Matrices of correlation functions & smearing of quark fields

$$C_{ij}(t,t') = \sum_{\mathbf{xy}} \langle B_i(\mathbf{x},t) B_j^\dagger(\mathbf{y},t') \rangle$$

$$B_i(\mathbf{x},t) = C_i^{\alpha\beta\gamma} \epsilon^{abc} q_\alpha^{af_1}(\mathbf{x},t) q_\beta^{bf_2}(\mathbf{x},t) q_\gamma^{cf_3}(\mathbf{x},t).$$

Smearing: Project to eigenvectors of Laplacian (PRD 80, 054506)

$$q_\alpha^a(\mathbf{x},t) \longrightarrow \sum_k v_{a\mathbf{x}}^{(k)} \widetilde{q}_\alpha^{(k)}(t).$$

$$\left(-\nabla^2 \right)_{\mathbf{xy}}^{ab} v_{b,\mathbf{y}}^{(k)} = \lambda_k v_{a\mathbf{x}}^{(k)}$$

$$C_{ij}(t,t') = \Phi_{i,k\ell m}^{\alpha\beta\gamma}(t) \left\langle \widetilde{q}_\alpha^{(k)}(t) \widetilde{q}_\beta^{(\ell)}(t) \widetilde{q}_\gamma^{(m)}(t) \right.$$
$$\left. \bar{\widetilde{q}}_{\bar{\alpha}}^{(\bar{k})}(t') \bar{\widetilde{q}}_{\bar{\beta}}^{(\bar{\ell})}(t') \bar{\widetilde{q}}_{\bar{\gamma}}^{(\bar{m})}(t') \right\rangle \Phi_{j,\bar{k}\bar{\ell}\bar{m}}^{\bar{\alpha}\bar{\beta}\bar{\gamma}\dagger}(t')$$

Determine energies

Calculate eigenvectors at $t^* = t_0 + 1$

$$\overline{C}(t^*)V(t^*) = \overline{C}(t_0)V(t^*)\Lambda(t^*)$$

Rotate matrices to fixed basis, calculate diagonal elements

$$\widetilde{\lambda}_n(t) = \Big(V^\dagger(t^*)C(t)V(t^*)\Big)_{nn}$$

Fit diagonal correlation elements

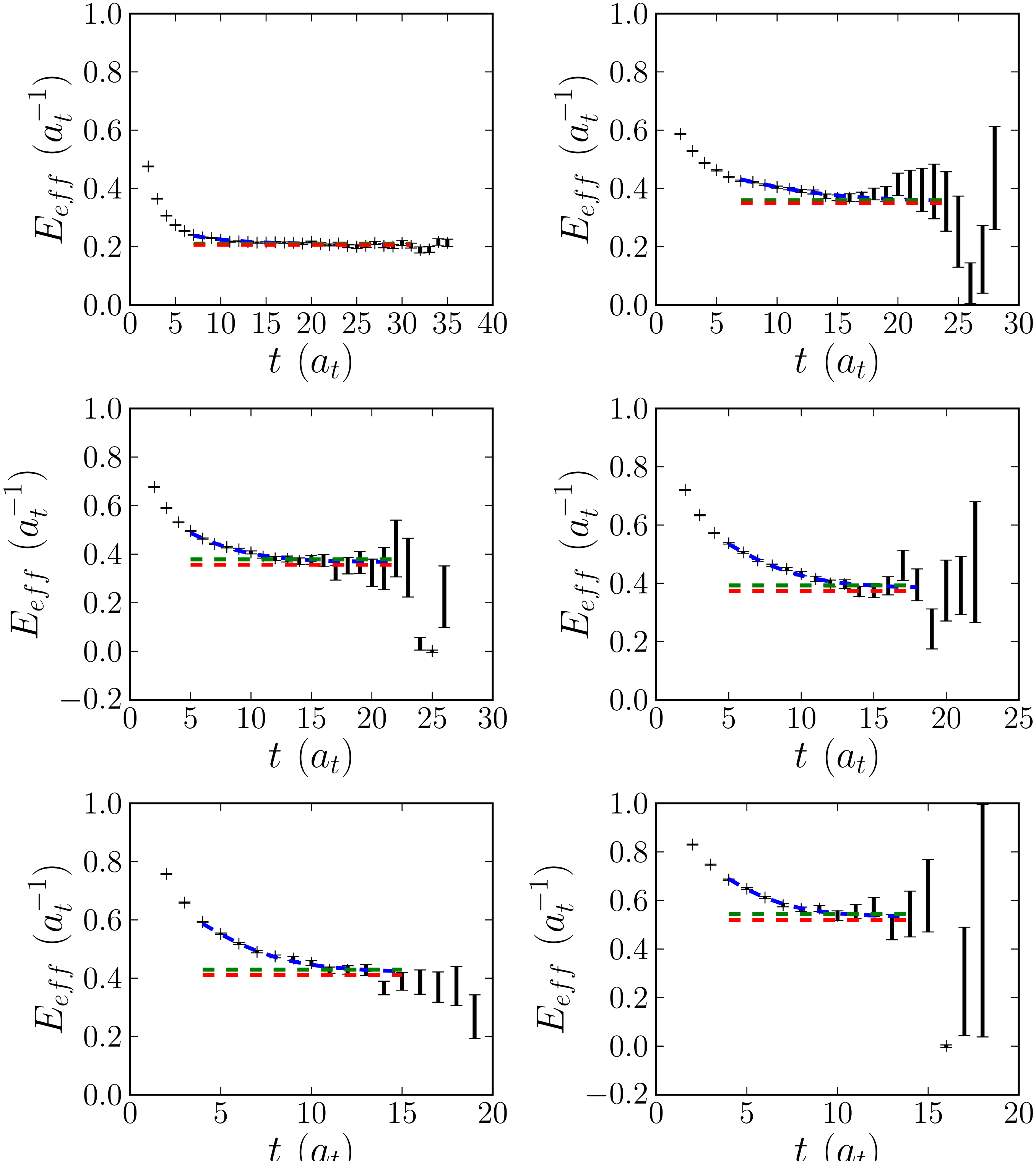
$$\lambda_{fit}(t) = (1-A)e^{-E(t-t_0)} + Ae^{-E'(t-t_0)}$$

Extract E.

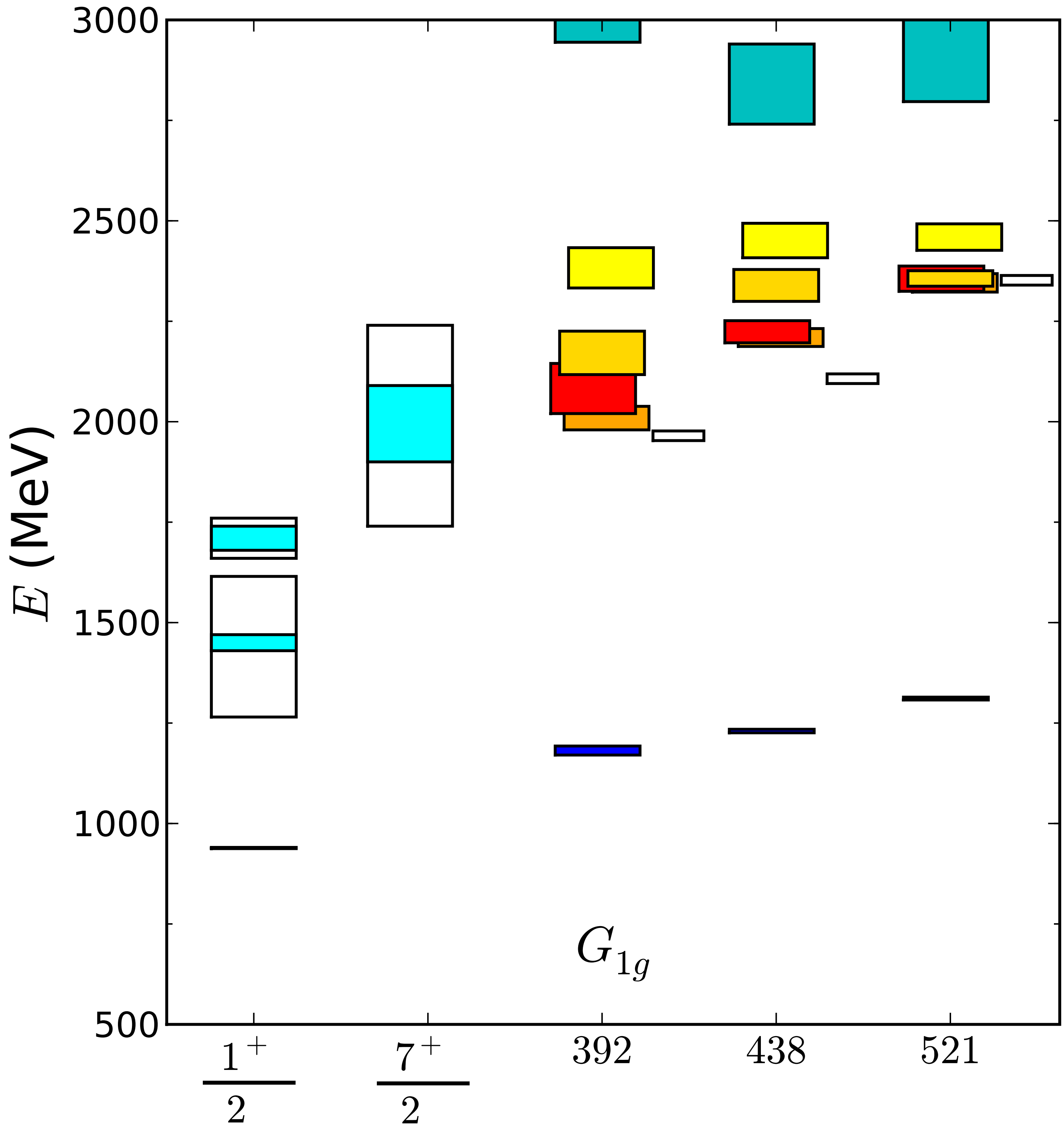
Subduction of J to $\mathcal{O}_D$

IR	Parity	Dimension	J $\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$
G_{1g}	+1	2	1			1
H_g	+1	4		1	1	1
G_{2g}	+1	2			1	1
G_{1u}	-1	2	1			1
H_u	-1	4		1	1	1
G_{2u}	-1	2			1	1

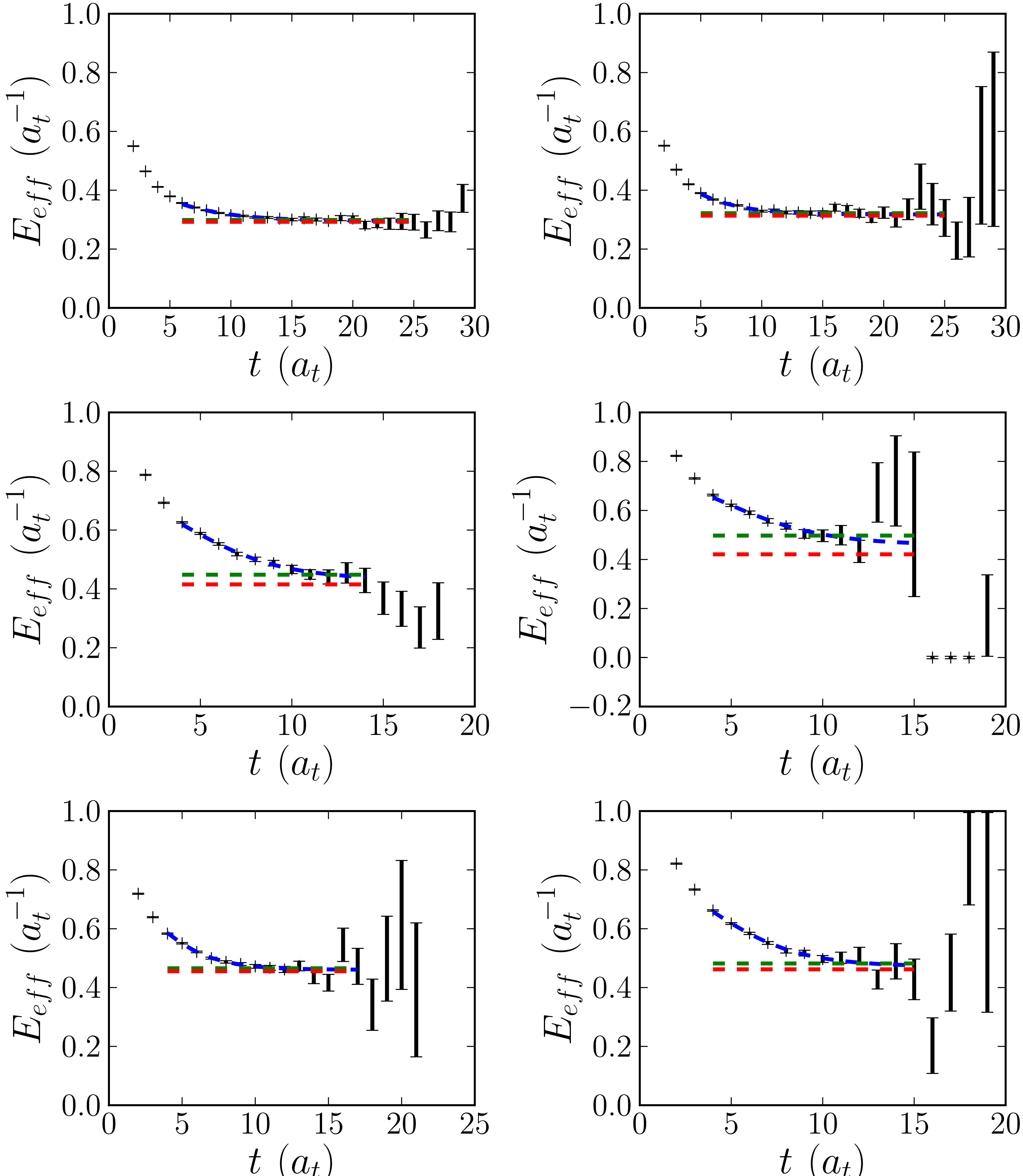
- **Spin $\frac{1}{2}$: Isolated G_1 state,**
- **Spin $\frac{3}{2}$: isolated H state.**
- **Spin $\frac{5}{2}$: degenerate G_2 and H states**
- **Spin $\frac{7}{2}$: degenerate G_1, H and G_2 states**



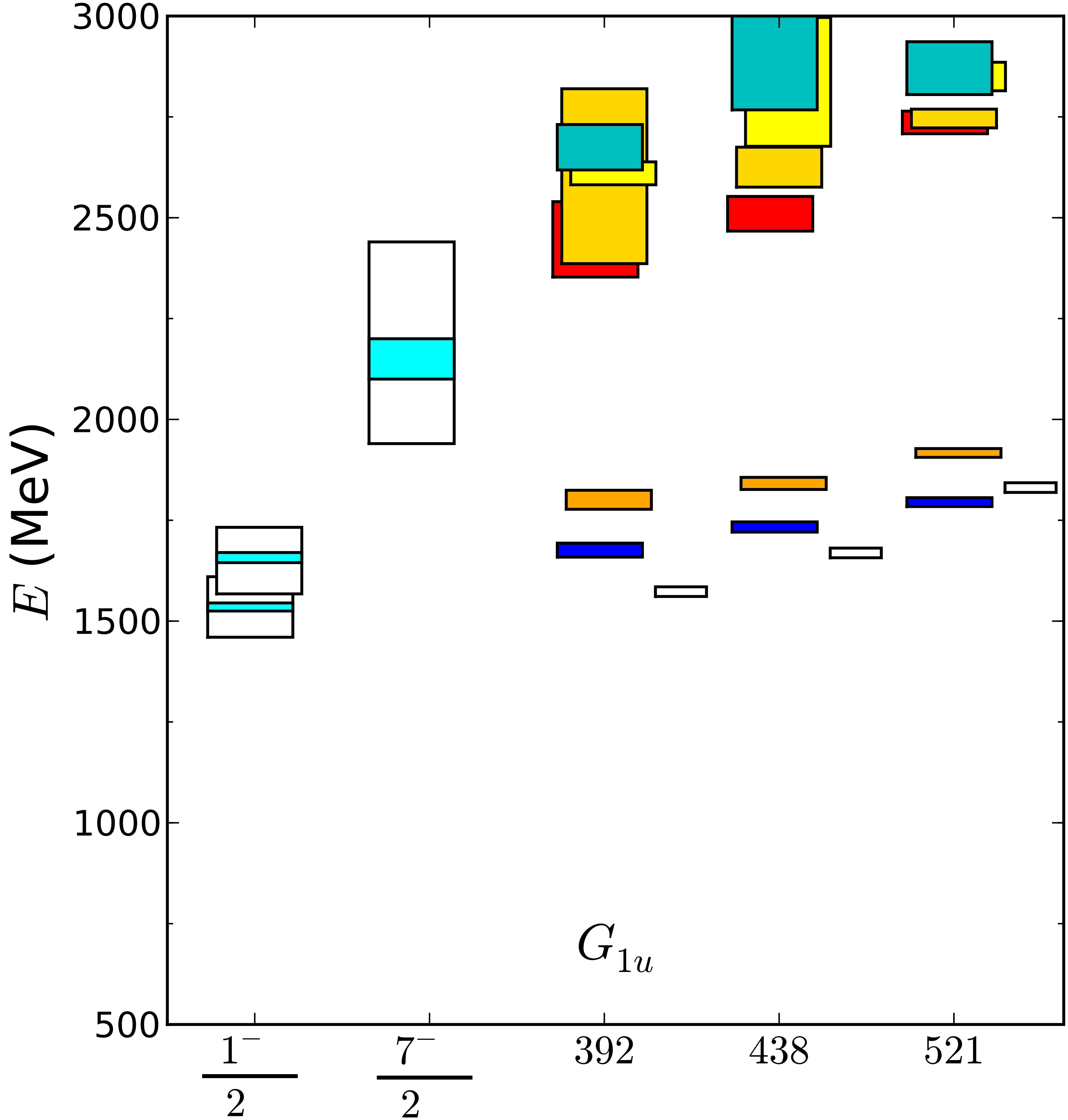
Nucleon G_{1g} effective energies: m_π = 392(4) MeV



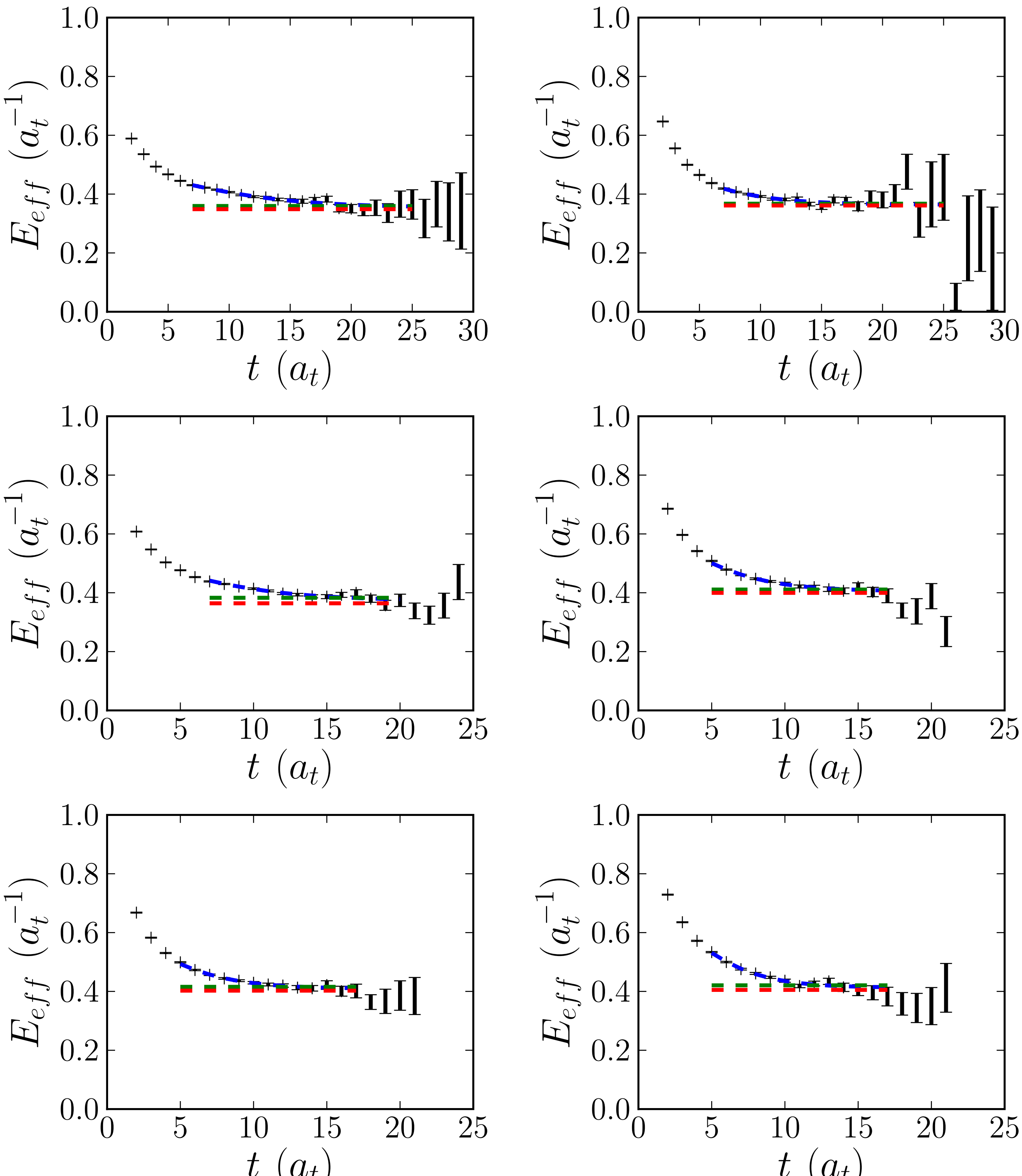
3000
2500
2000
1500
1000
500
E (MeV)
G_{1g}
$\frac{1^+}{2}$
$\frac{7^+}{2}$
392
438
521



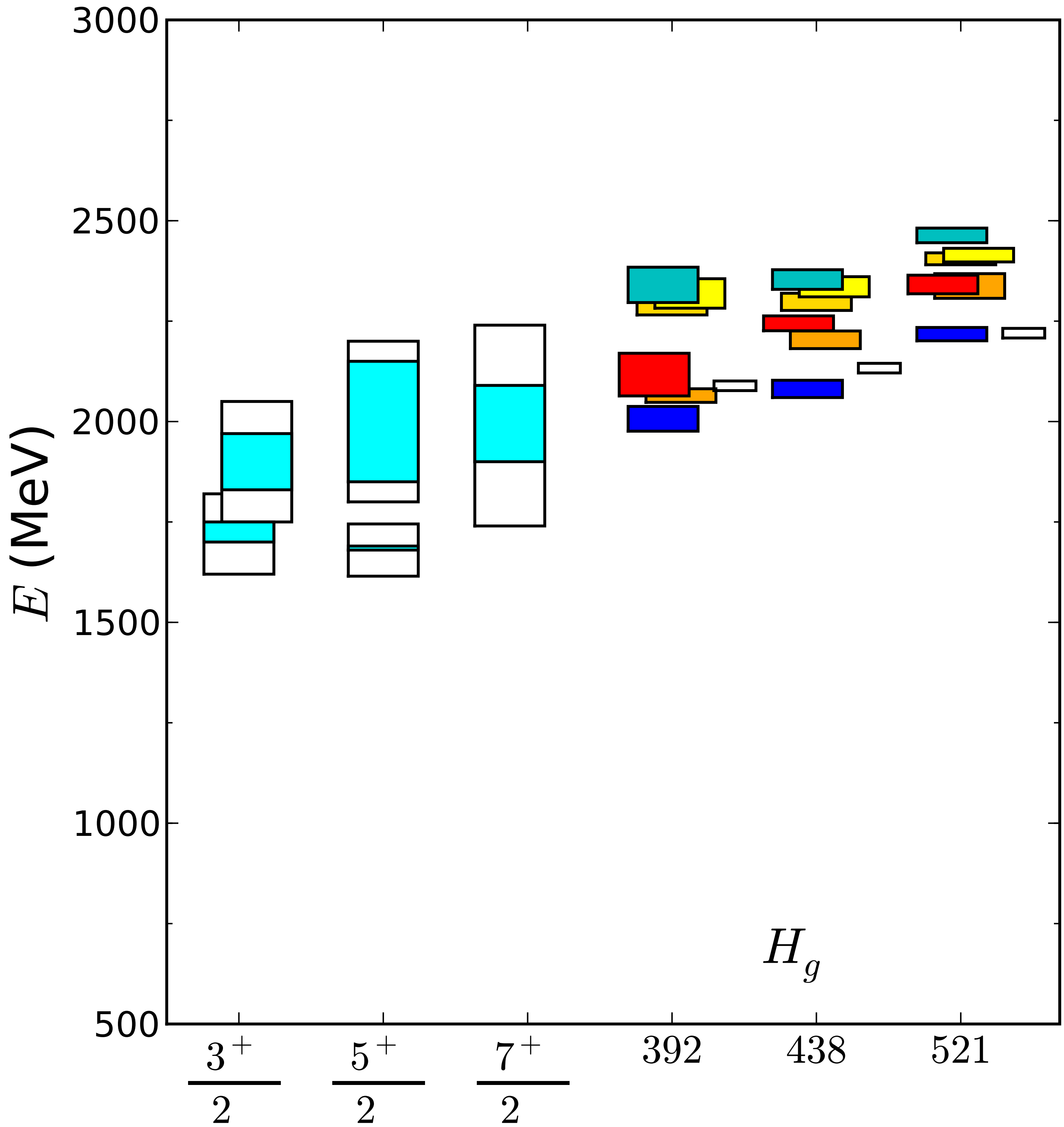
Nucleon G_{1u} effective energies: m_π = 392(4) MeV

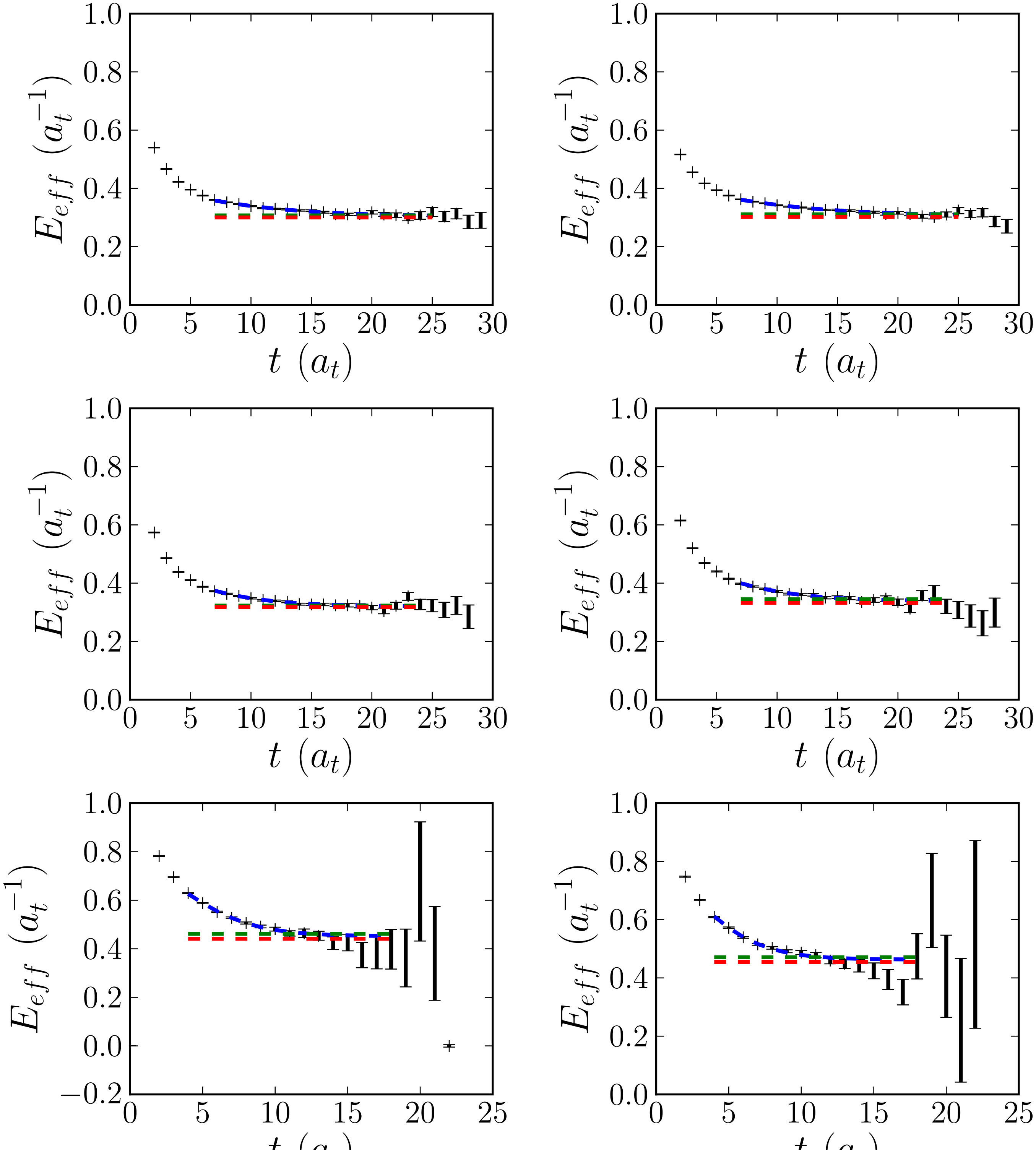


E (MeV)
3000
2500
2000
1500
1000
500
G_{1u}
$\frac{1^-}{2}$
$\frac{7^-}{2}$
392
438
521

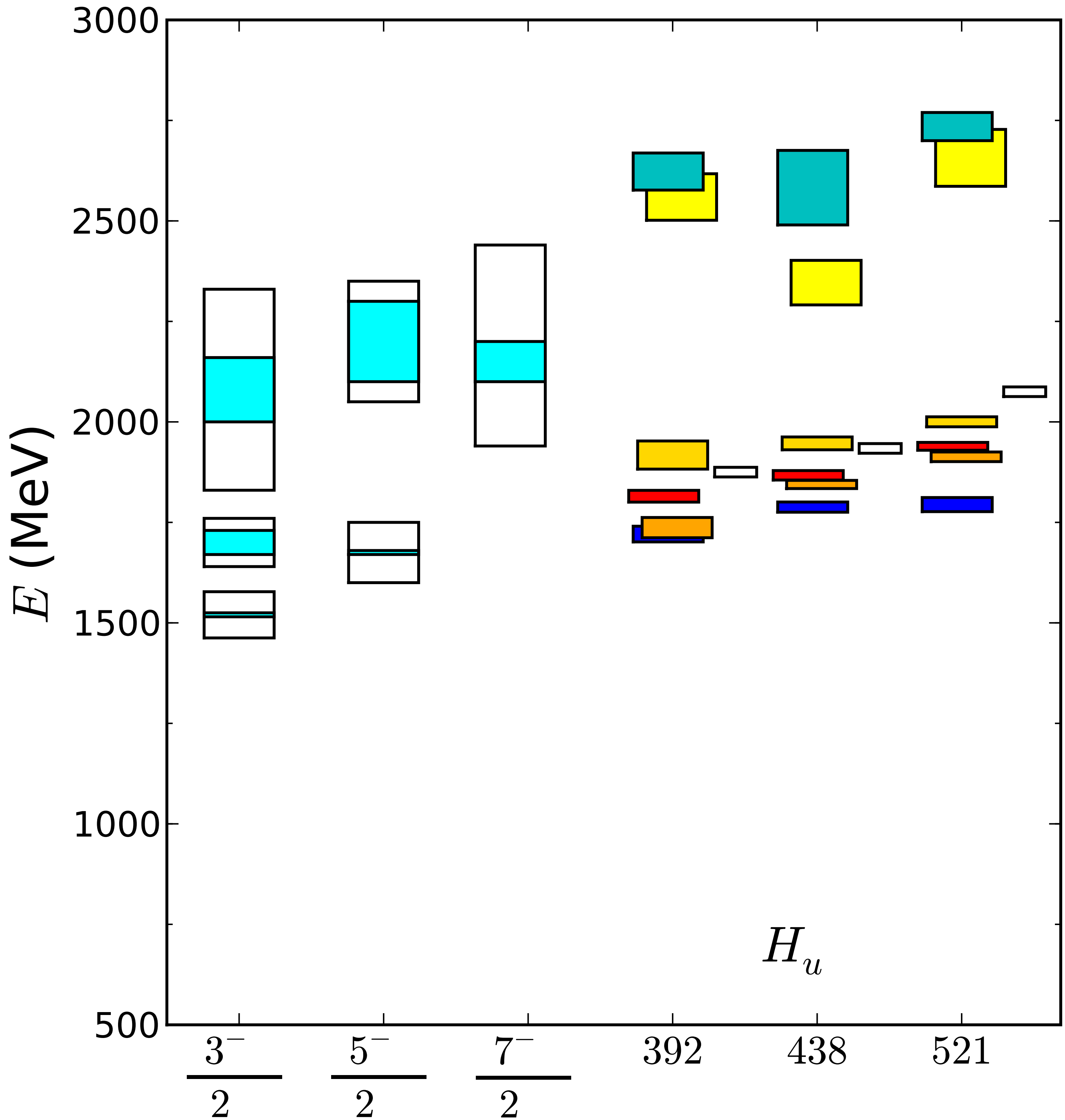


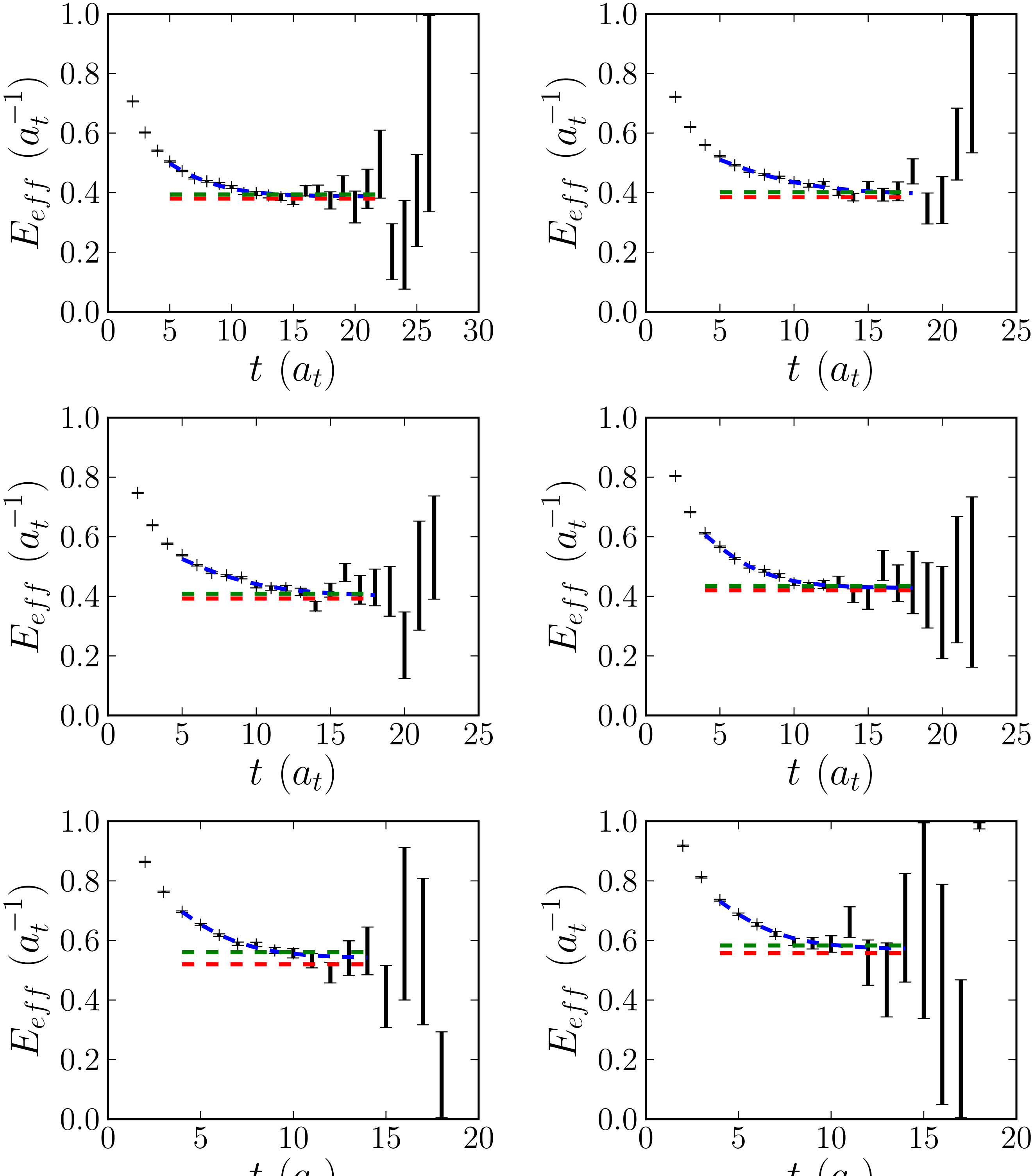
Nucleon H_g effective energies: $m_\pi = 392(4)$ MeV


3000
2500
2000
1500
1000
500
E (MeV)
H_g
$\frac{3^+}{2}$
$\frac{5^+}{2}$
$\frac{7^+}{2}$
392
438
521

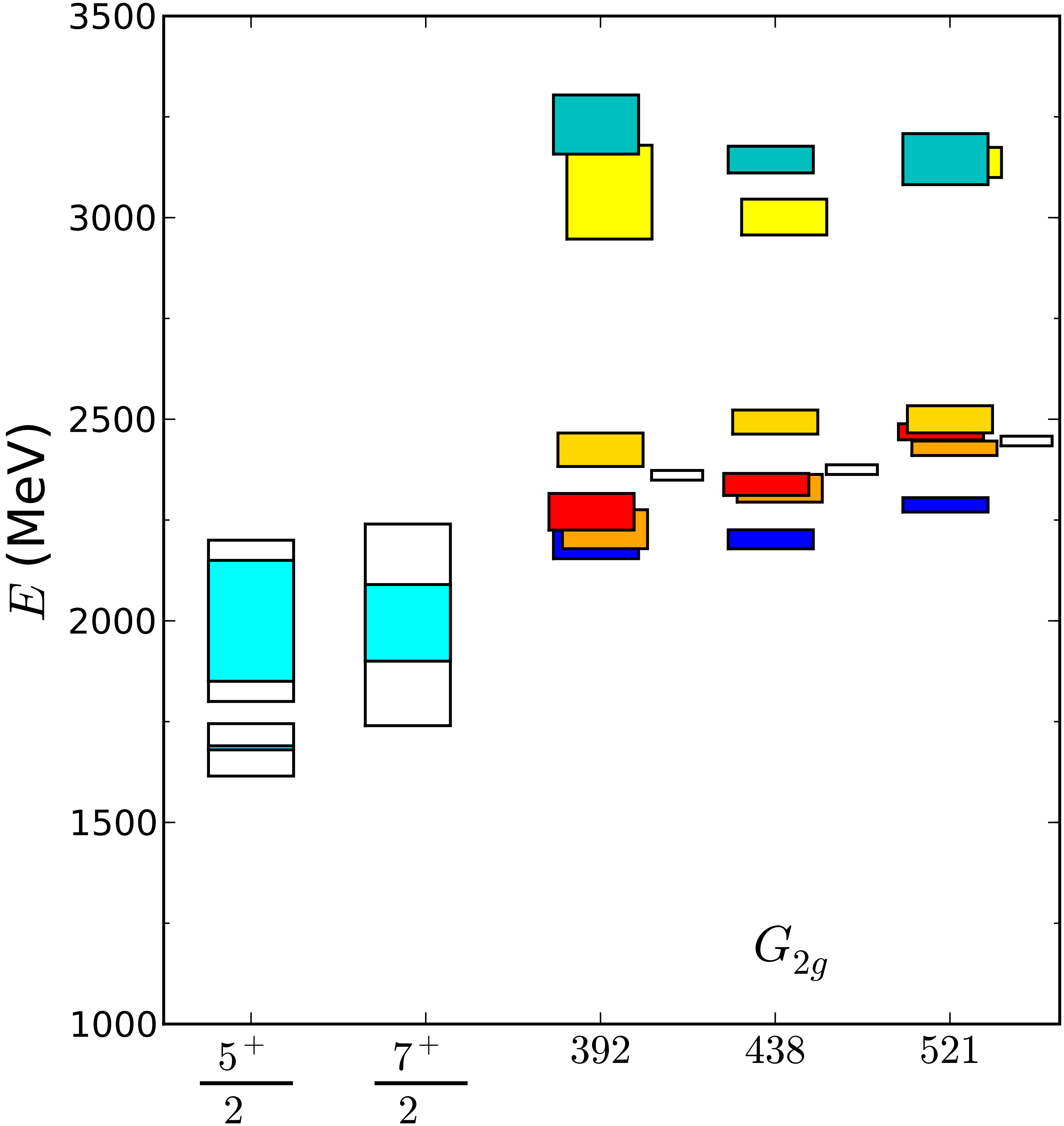


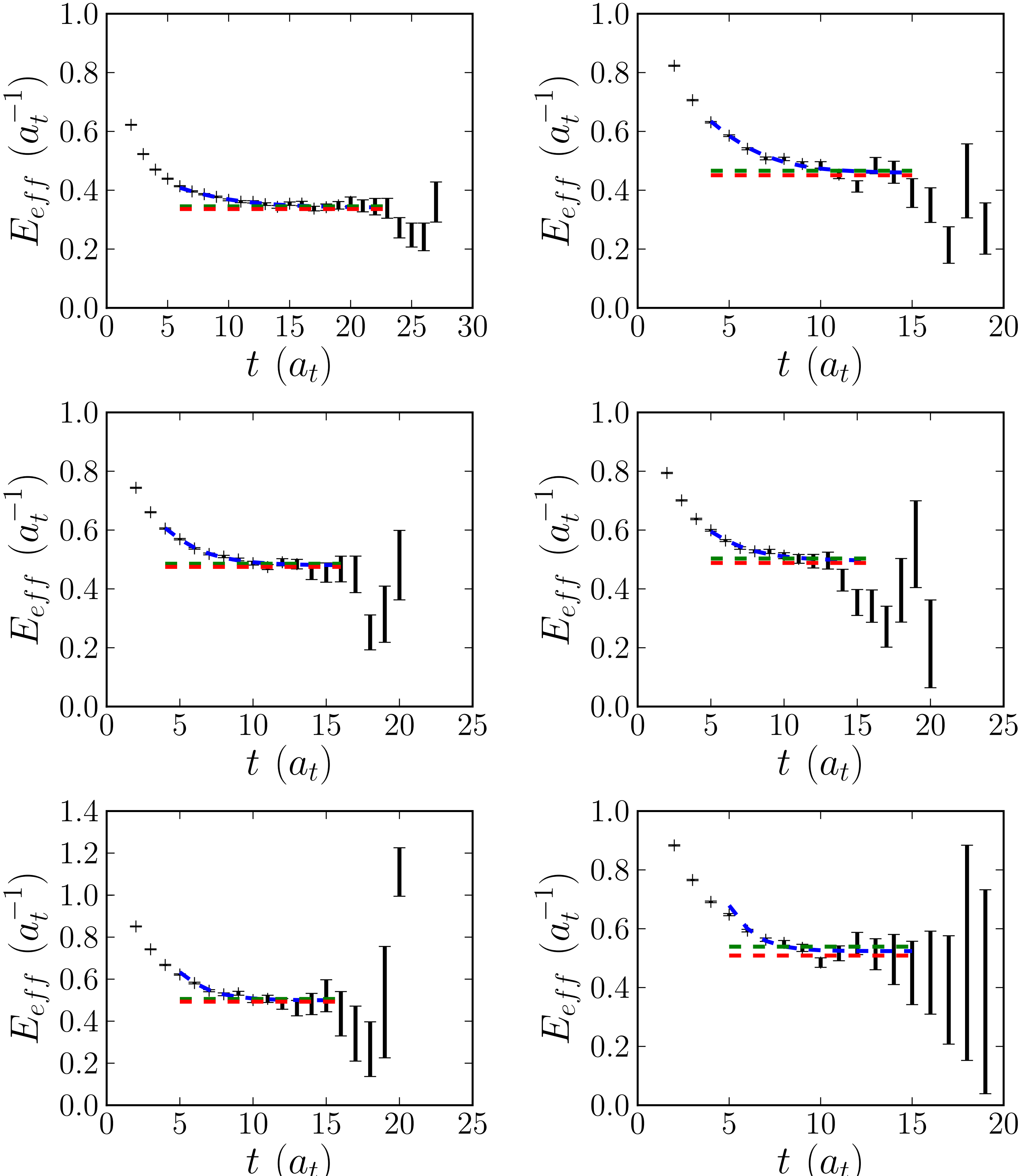
Nucleon H_u effective energies: m_π = 392(4) MeV


3000
2500
2000
1500
1000
500
E (MeV)
H_u
3⁻/2
5⁻/2
7⁻/2
392
438
521

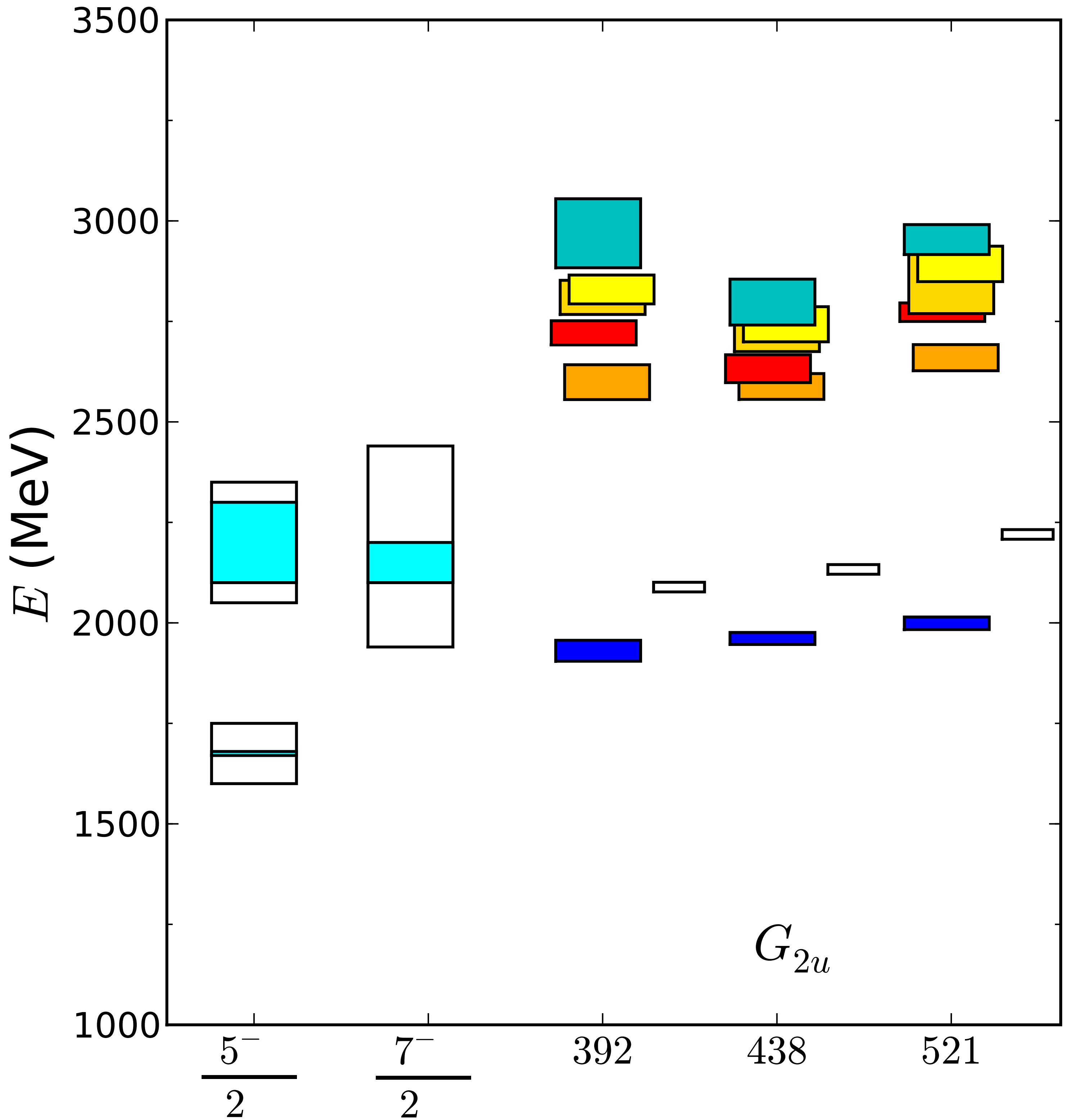


Nucleon G_{2g} effective energies: m_π = 392(4) MeV

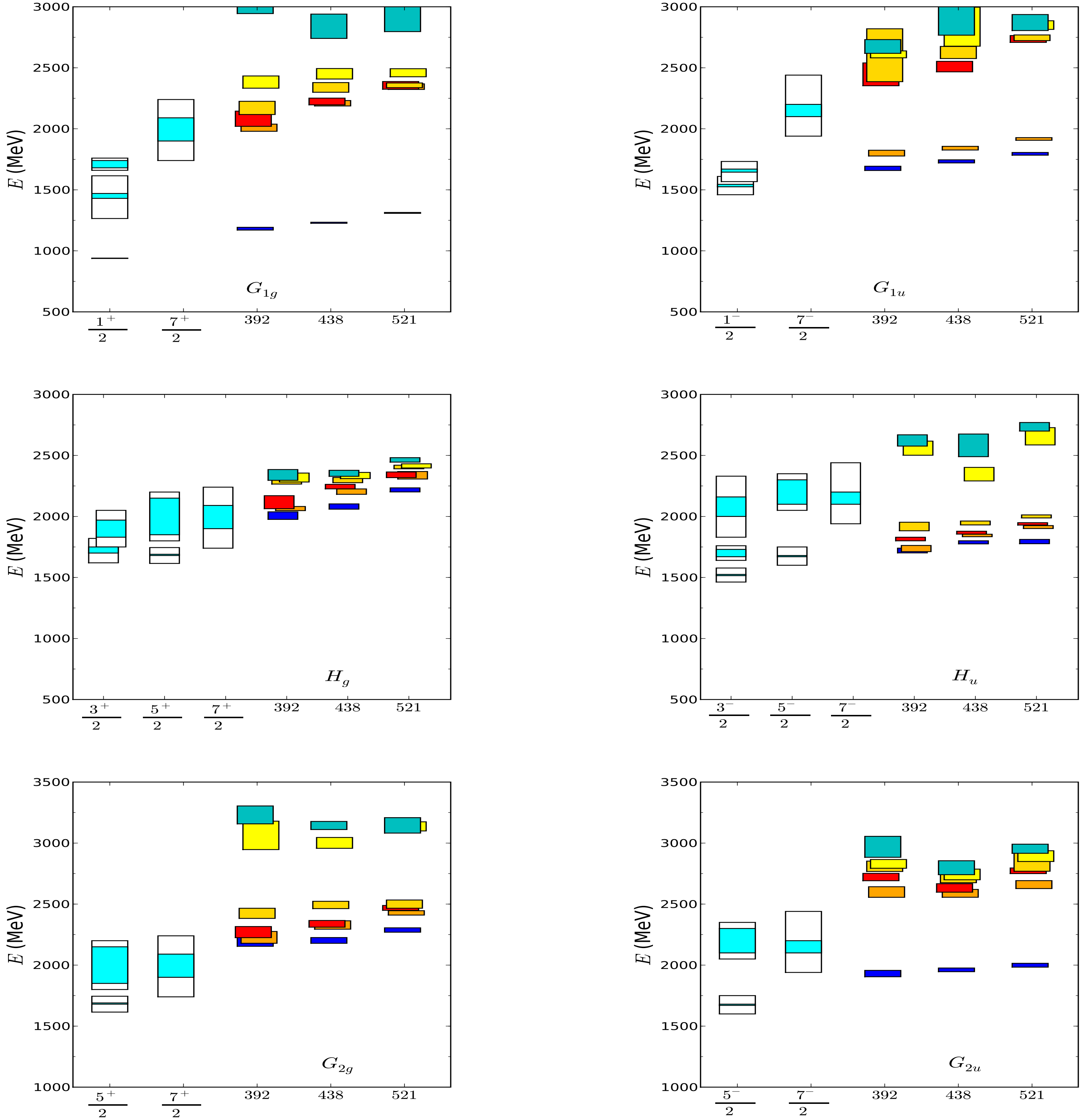

3500
3000
2500
2000
1500
1000
E (MeV)
G_{2g}
$\frac{5^+}{2}$
$\frac{7^+}{2}$
392
438
521



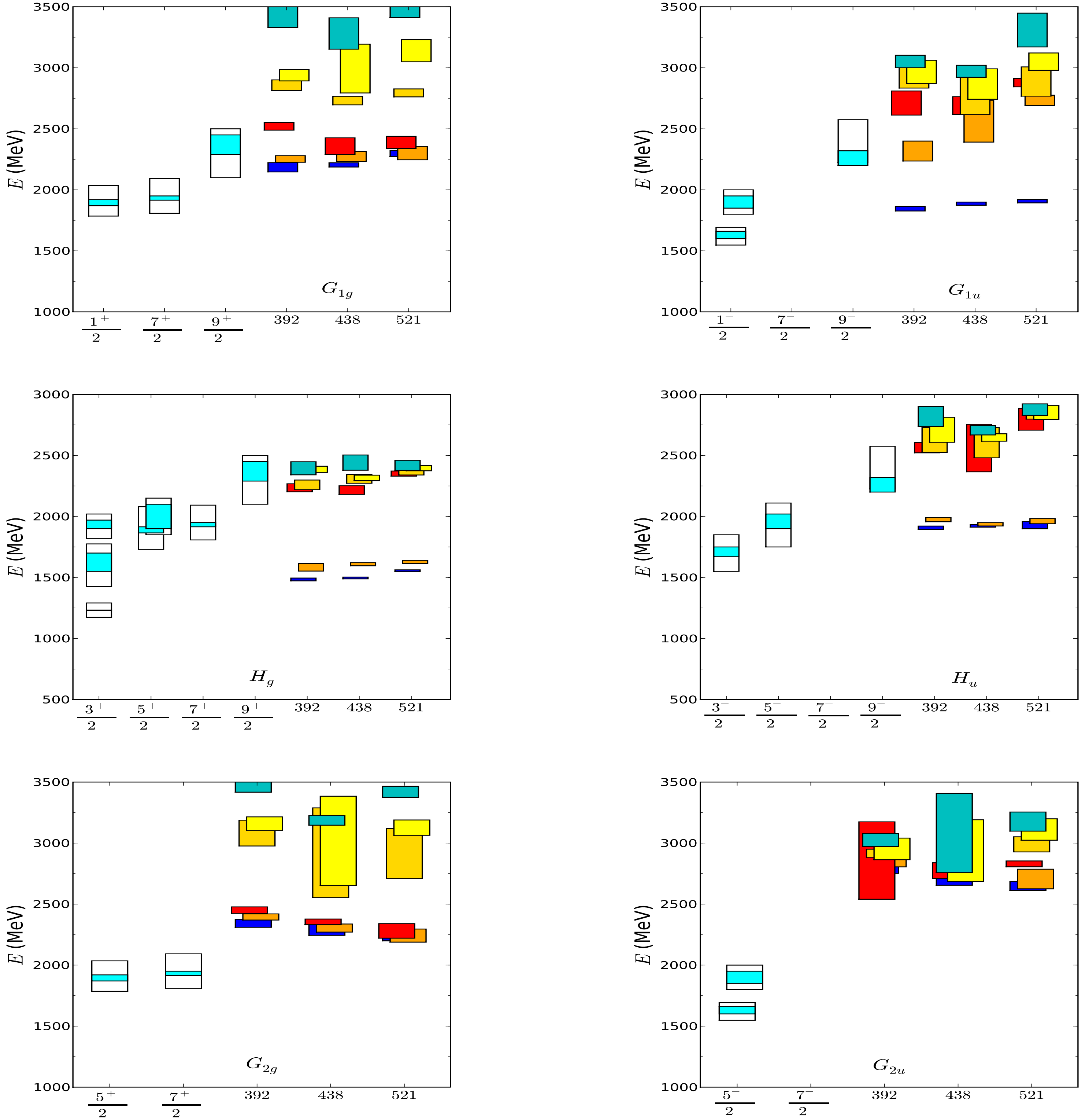
Nucleon G_{2u} effective energies: m_π = 392(4) MeV



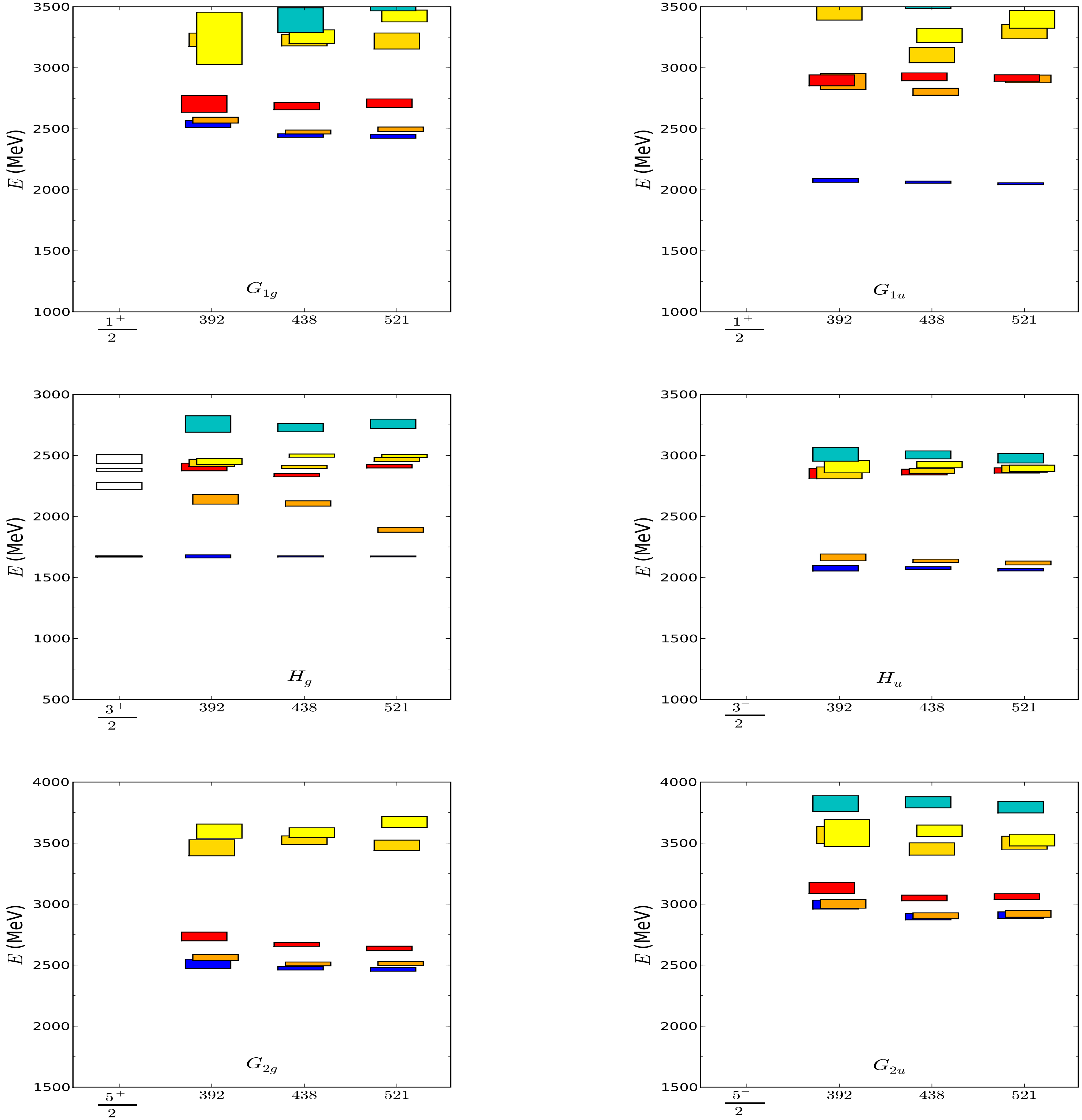
3500
3000
2500
2000
1500
1000
E (MeV)
G_{2u}
$\frac{5^-}{2}$
$\frac{7^-}{2}$
392
438
521



E (MeV)
G_{1g}
G_{1u}
H_g
H_u
G_{2g}
G_{2u}
392
438
521
$\frac{1}{2}^+$
$\frac{7}{2}^+$
$\frac{1}{2}^-$
$\frac{7}{2}^-$
$\frac{3}{2}^+$
$\frac{5}{2}^+$
$\frac{3}{2}^-$
$\frac{5}{2}^-$



E (MeV)
G1g
Gₗᵤ
Hg
Hu
G2g
G2u
392
438
521
1/2+
7/2+
9/2+
1/2−
7/2−
9/2−
3/2+
5/2+
3/2−
5/2−
3500
3000
2500
2000
1500
1000
500



E (MeV)
G_{1g}
G_{1u}
H_g
H_u
G_{2g}
G_{2u}
$\frac{1}{2}^+$
$\frac{3}{2}^+$
$\frac{3}{2}^-$
$\frac{5}{2}^+$
$\frac{5}{2}^-$
392
438
521

Summary

- 6 lowest energy N, Δ and Ω states in each IR for m_π = 392(4), 438(3) and 521(3) MeV.

- First excited baryon spectrum based on N_f = 2+1 QCD using anisotropic lattices

- Patterns of lowest energies are similar to the patterns of lowest physical resonance states.

- Good evidence for $J^P = \frac{5}{2}^-$ state.

- Quark field smearing using lowest eigenmodes of Laplace operator works well

- Program is on track to produce reasonable spectra. Next steps: lower m_π, larger and more volumes, multiparticle operators and operators subduced from continuum J's.

"This is not the end. It is not even the beginning of the end.

But, perhaps, it is the end of the beginning"