5-dimensioal SU(2) lattice gage theory with Z_2orbifolding and its phase structure

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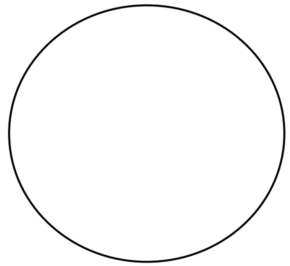
@ Lattice 2010

§ 1 Motivations

- Why 5—dimensional? 5th coordinate τ

▪Gauge-Higgs unification by extra-dimensions

$$A_5(x^\mu,\tau)=\sum_{n=-\infty}^{\infty} h_n(x^\mu)e^{\frac{in\tau}{R_5}} \quad \times\, V(h(x^\mu))$$

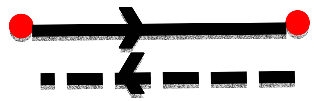


S^1

- Why orbifolding?

▪Symmetry breakings and fermion reductions

▪Center symmetry → helpless for Polyakov loop → Stick sym.



τ

- Why lattice gauge theory?

▪Gauge invariant formulation

▪Calculation of Gauge and Higgs masses → under calculation

§ 2 Setup and lattice Formulation

Irges and Knechtli('05,'07)

Gauge group → SU(2)

Gauge + Higgs → $A_\mu,\ A_5 \rightarrow U_{n,\mu}\ ,\ U_{n,5}$

Orbifolding for 5th dimension → S^1 → S^1/Z_2

$2\pi R$ → $2\pi R/2$

Lattice size → $L^4 \times N_5 \rightarrow L^4 \times N_5/2 = L^4 \times L_5$

With boundary conditions :

on Four dimensions → Periodic BC

on 5th dimension → see next page

Topological space and Lattice coordinate

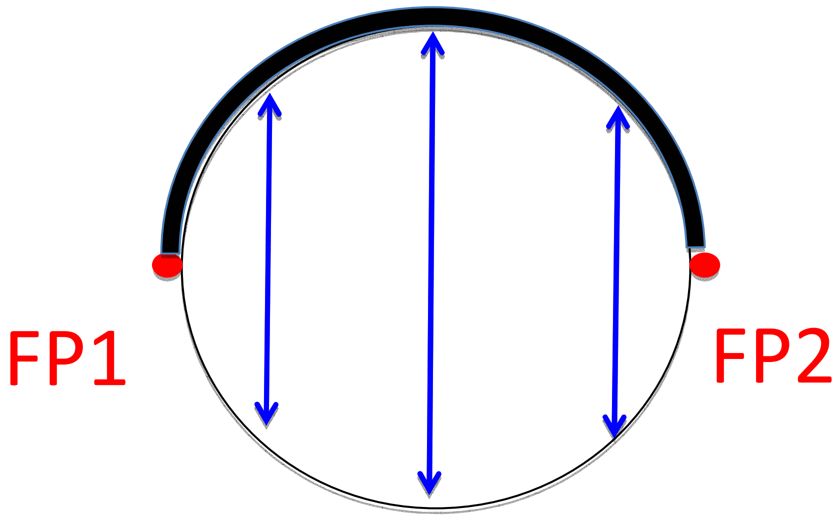
$$S^1 \to S^1 / Z_2$$

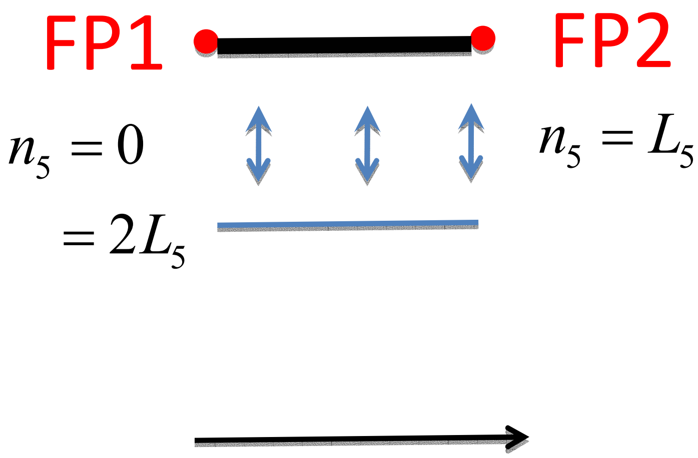
$$S^1 \to 2\pi R = N_5 a$$

$$S^1 / Z_2 \to 2\pi R / 2 = N_5 a / 2 = L_5 a$$

$$set \quad a = 1$$

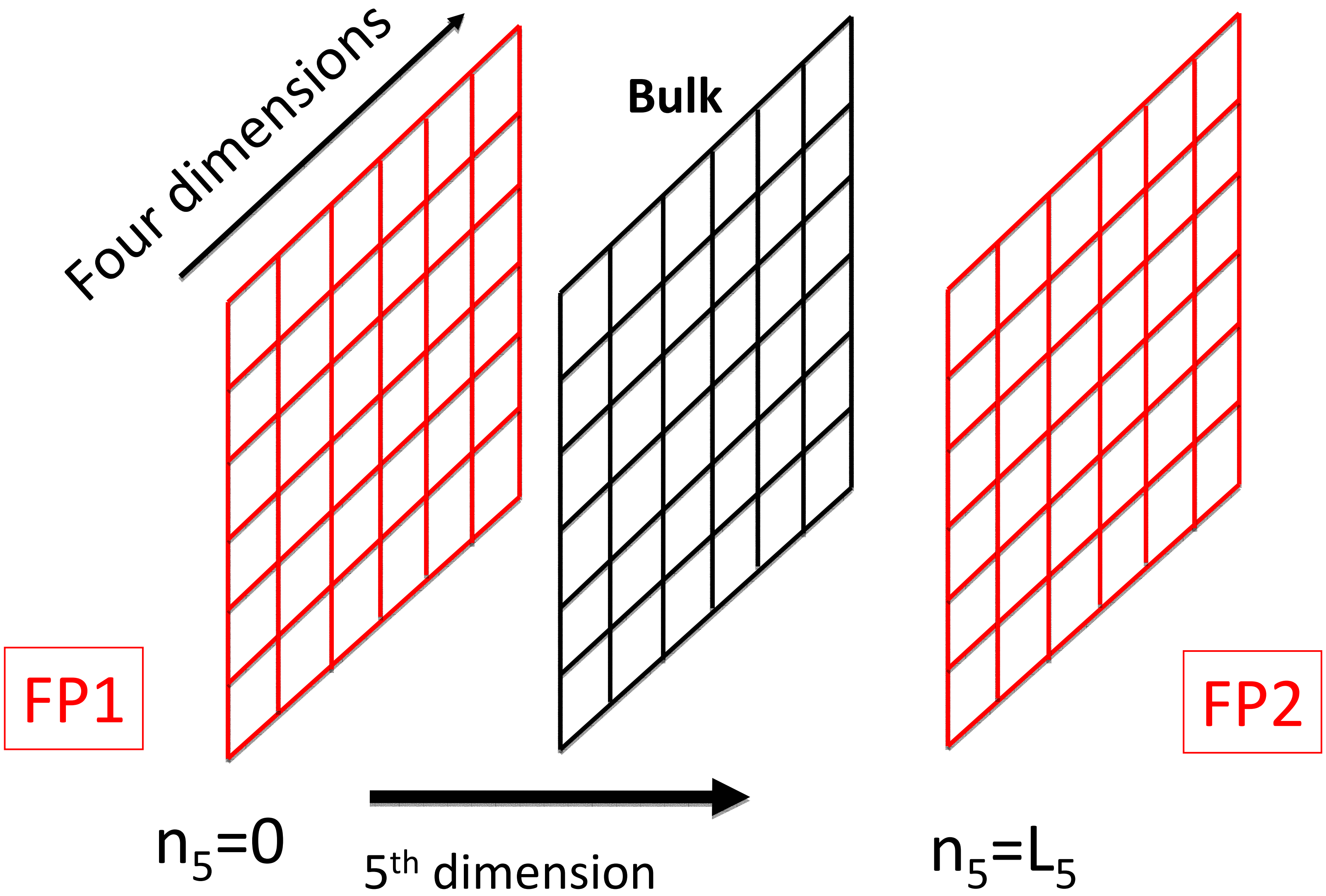
$$n_5 = 0, \ldots, L_5$$





5^{th} dimension

5-dimensional lattice



Z_2 orbifolding on gauge fields

- On an S^1/Z_2(Bulk),

$$\left.\begin{aligned} U_{\{n_\mu,n_5\},\mu} &= gU_{\{n_\mu,2L_5-n_5\},\mu}g^\dagger, \\ U_{\{n_\mu,n_5\},5} &= gU^\dagger_{\{n_\mu,2L_5-n_5-1\},5}g^\dagger, \end{aligned}\right\} \rightarrow no\ constraint$$

$$g = i\sigma_3$$

- At FP1 and FP2,

$$U_{\{n_\mu,n_5\},\mu} = gU_{\{n_\mu,n_5\},\mu}g^\dagger \rightarrow U_{\{n_\mu,n_5\},\mu} = e^{iA^3_\mu(n)\frac{\sigma^3}{2}}$$ **U(1)!**

$$U_{\{n_\mu,n_5\},5} = gU^\dagger_{\{n_\mu,2L_5-n_5-1\},5}g^\dagger \rightarrow no\,constraint$$

$$g = i\sigma_3$$

Our action and measure

$$S_{S^1} = \beta \sum_{P \in M^4 \times S^1} [1 - \frac{1}{2} \mathrm{Tr}\ U_P]$$

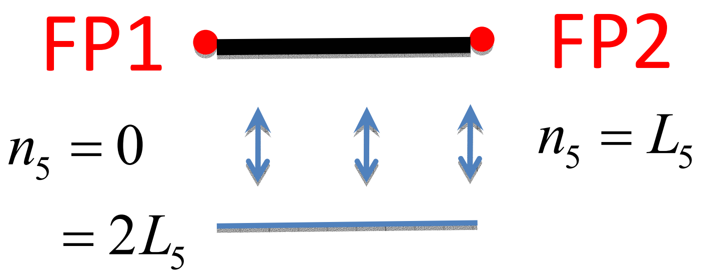
$$\rightarrow S_{S^1/Z_2} = \underline{\beta} \sum_{P \in bulk} [1 - \frac{1}{2} \mathrm{Tr}\ U_P] + \frac{\beta}{\underline{2}} \sum_{P \in FP1, FP2} [1 - \frac{1}{2} \mathrm{Tr}\ U_P]$$

$$\int \prod_{L^4 \times N_5} dU_{n,M} \rightarrow \int \prod_{Bulk} dU_{n,M} \int \prod_{FP1, FP2} dU_{n,\mu}$$

$SU(2)$ $\qquad$ $SU(2)$ $\qquad$ $U(1) \times U(1)'$

§ 3 Symmetry and phase structure

- 4D,5th gauge symmetry → ○
- Center symmetry → × (invisible)



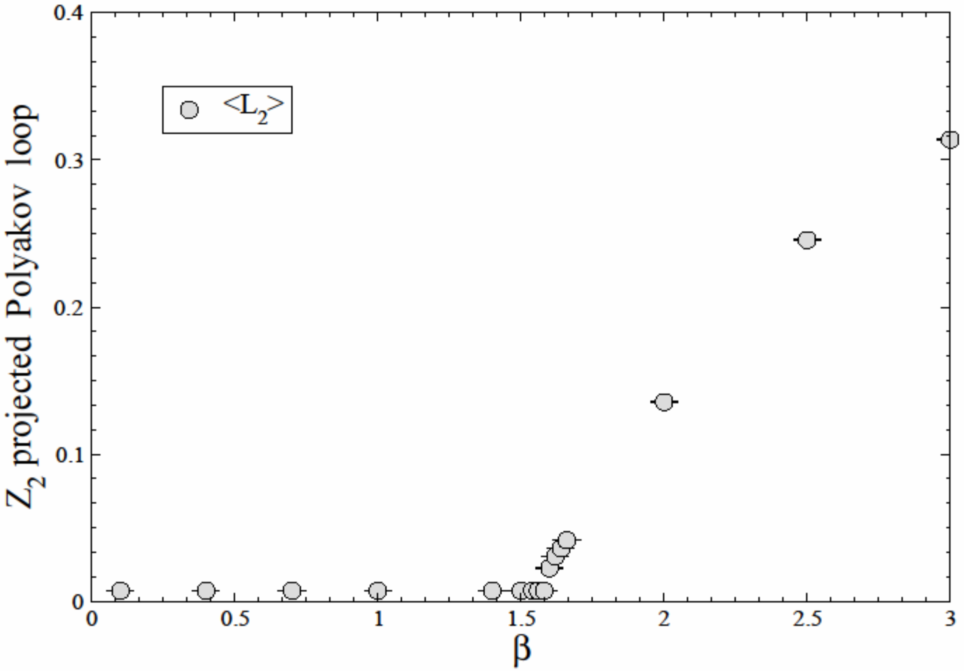
Twice $(-)^2$ appears for Polyakov loop, L_2

A half of the loop is gauge-non-invariant!

$$L_2(n_\mu) \equiv \mathrm{Tr}\, U_{\{n_\mu,0\},5} U_{\{n_\mu,1\},5} \cdots U_{\{n_\mu,L_5-1\},5}\ g U^\dagger_{\{n_\mu,L_5-1\},5} \cdots U^\dagger_{\{n_\mu,1\},5} U^\dagger_{\{n_\mu,0\},5}\, g^\dagger$$

Calculate the Polyakov loop(Z_2 projected loop)

$\langle L_2 \rangle$



β

8^4x8

→ **8^4x4**

Order parameter !?

For what?

$\beta_c \fallingdotseq 1.6$

Stick transformation ('09, Ishiyama, Murata, So, Takenaga)

$$U_\mu(2) \to i\sigma_2 U_\mu(2)(-i\sigma_2) = U^*{}_\mu(2) \quad at \;\; FP2$$

$$U_{\{n_\mu, L_5-1\},5} \to U_{\{n_\mu, L_5-1\},5}(-i\sigma_2)$$

$$U^\dagger_{\{n_\mu, L_5-1\},5} \to (i\sigma_2) U^\dagger_{\{n_\mu, L_5-1\},5}$$

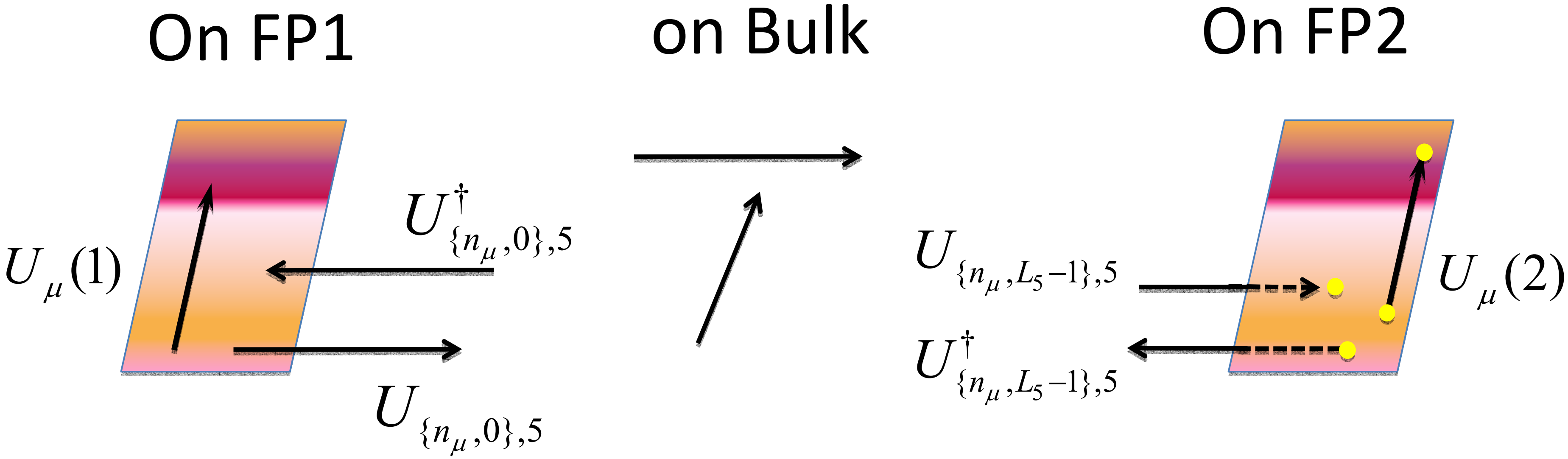
Other link variables $\to$ unchanged

Tr (Plaquette), path integral measure
→ invariant under this stick trans.

Consistent with Z_2 orbifolding!

Stick symmetry

• Yellow $= \pm i\sigma_2$



Consistency between Z_2 orbifolding and Stick trans. at FP2

- (A) SU(2) →U(1) : $U_\mu(2) = gU_\mu(2)g^\dagger$, $g = i\sigma_3$
- (B) Stick trans. : $U_\mu(2) \rightarrow i\sigma_2 U_\mu(2)(-i\sigma_2) = U^*{}_\mu(2)$
- (A) → (B) is same as (B) → (A)

∵)

$$i\sigma_3 i\sigma_2 U_\mu(2)(-i\sigma_2)(-i\sigma_3) = i\sigma_2 i\sigma_3 U_\mu(2)(-i\sigma_3)(-i\sigma_2)$$

About Path integral measure

At FP2, after Stick trans.

$$dU_\mu(2) \to dU_\mu^*(2) = dU_\mu(2)$$

$$dU_{\{n_\mu, L_5-1\},5} \to dU_{\{n_\mu, L_5-1\},5}(-i\sigma_2)$$

$$= dU_{\{n_\mu, L_5-1\},5} \quad \because) \ \text{right invariant}$$

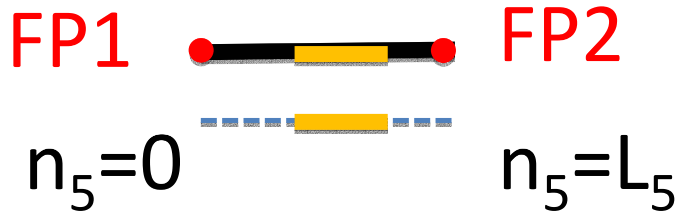
Other link variables → unchanged

Center symmetry and stick symmetry

- Center symmetry for Polyakov loop, L_2

  → invisible! i.e. $L_2 \rightarrow L_2$

∵ twice $(-)^2 L_2$



- Stick symmetry of Polyakov loop

  → visible! i.e. $L_2 \rightarrow -L_2$

∵ only one at FP2, $(-)^1 L_2$

$$U_{\{n_\mu,L_5-1\},5}\, i\sigma_3 U^\dagger_{\{n_\mu,L_5-1\},5} \rightarrow U_{\{n_\mu,L_5-1\},5}(-i\sigma_2)\, i\sigma_3 (i\sigma_2) U^\dagger_{\{n_\mu,L_5-1\},5}$$

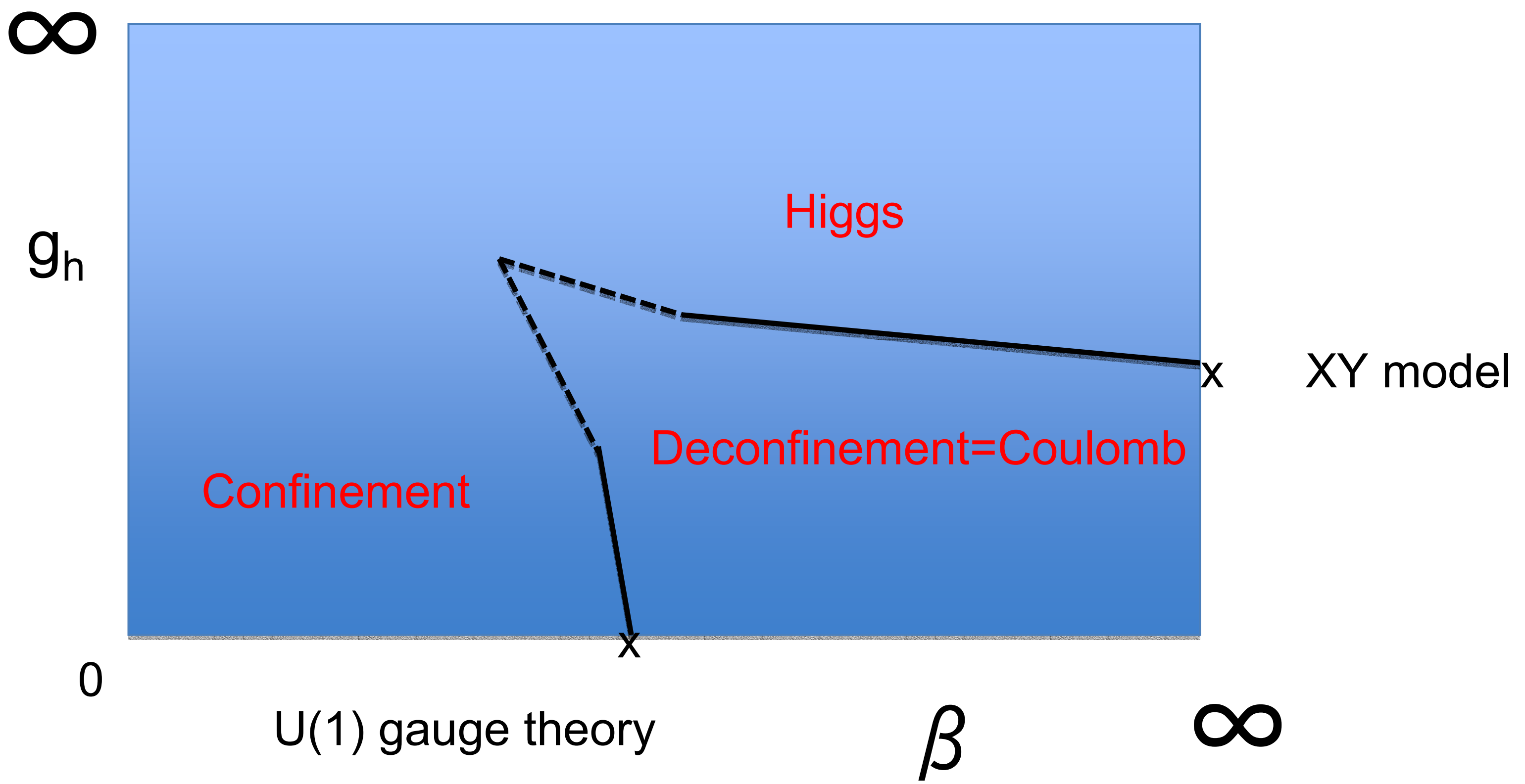
$$= U_{\{n_\mu,L_5-1\},5}(-1)\, i\sigma_3 U^\dagger_{\{n_\mu,L_5-1\},5}$$

§ 4　Gauge-Higgs　System

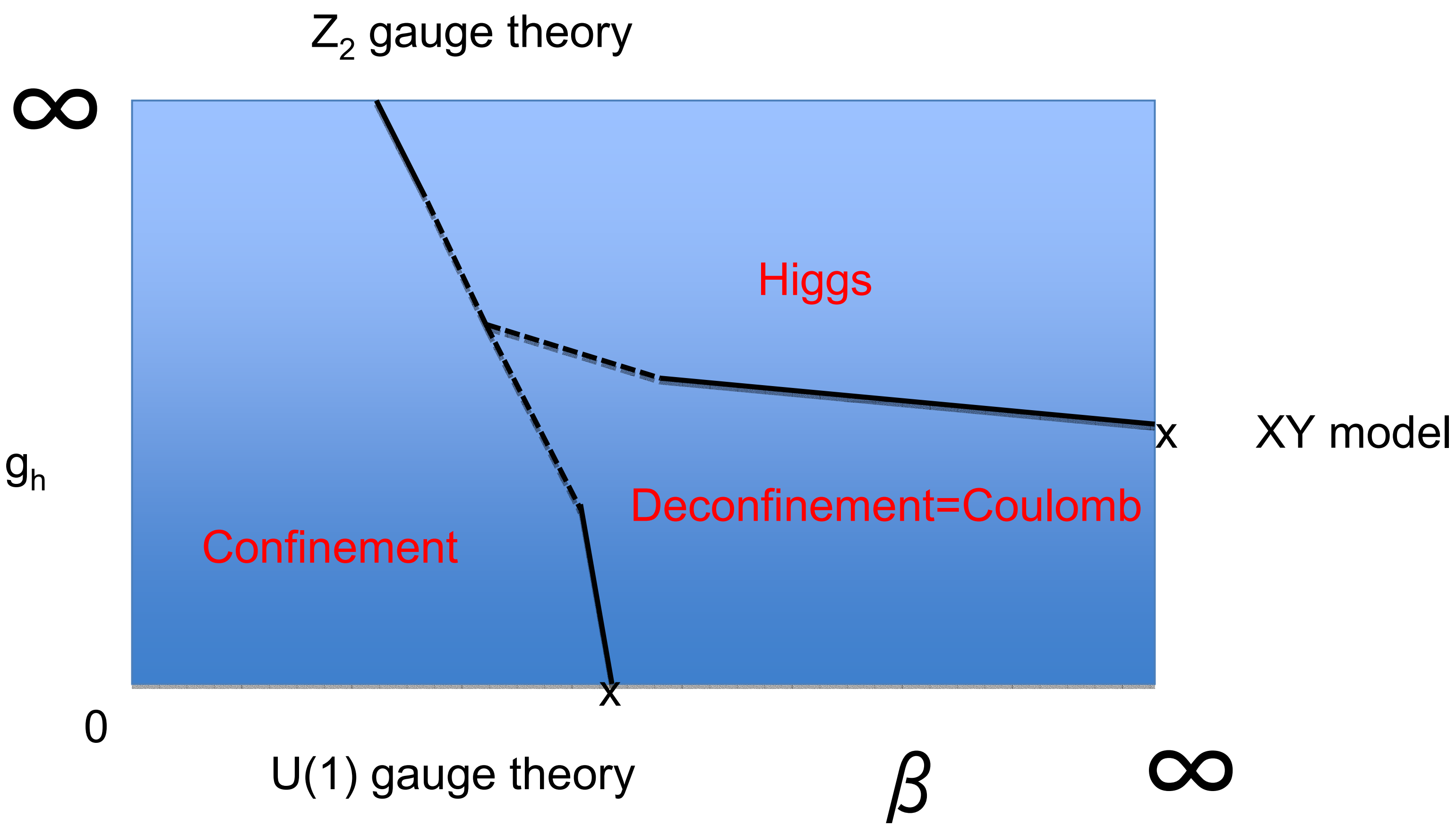
- 5-dimensional gauge Theory

→　Four-dimensional Gauge-Higgs System on FPs, U(1)

- Fradkin-Shenker（'79）U(1) gauge + unit charge Higgs



U(1) gauge + two units charge Higgs



Which case is our model?

Gauge invariant effective theory

Elitzur's Theorem → Only gauge invariant quantities are essential!

Zero mode along 5th dimension is not invariant! Stick is applicable.

$$X(n_\nu) \equiv U_{\{n_\nu,0\},5} U_{\{n_\nu,1\},5} \cdots U_{\{n_\nu,L_5-1\},5} \rightarrow X(n_\nu)(-i\sigma_2)$$

$$L_2(n_\nu) = \mathrm{Tr}\ X(n_\nu) g X^\dagger(n_\nu) g^\dagger \rightarrow -L_2(n_\nu)$$

$$\overline{U}_\mu(n_\nu, I) \equiv e^{iA_\mu^3(n_\nu,I)},\ U_\mu(n_\nu, I) \equiv e^{iA_\mu^3(n_\nu,I)\sigma^3},\ g = i\sigma_3$$

$$S_{eff} = -\beta \sum_{4D} (\overline{U}(P,1) + \mathrm{C.C.}) - \beta \sum_{4D} (\overline{U}(P,2) + \mathrm{C.C.})$$

U(1)xU(1)'

$$-g_h \sum_{4D} \mathrm{Tr}\, X^\dagger(n_\nu) U_\mu(n_\nu,1) X(n_\nu + \hat{\mu}) U_\mu^\dagger(n_\nu,2)$$

Gauge-Higgs coupling

$$+\sum_{4D} V(\mathrm{Tr}\ X(n_\nu) g X^\dagger(n_\nu) g^\dagger)$$

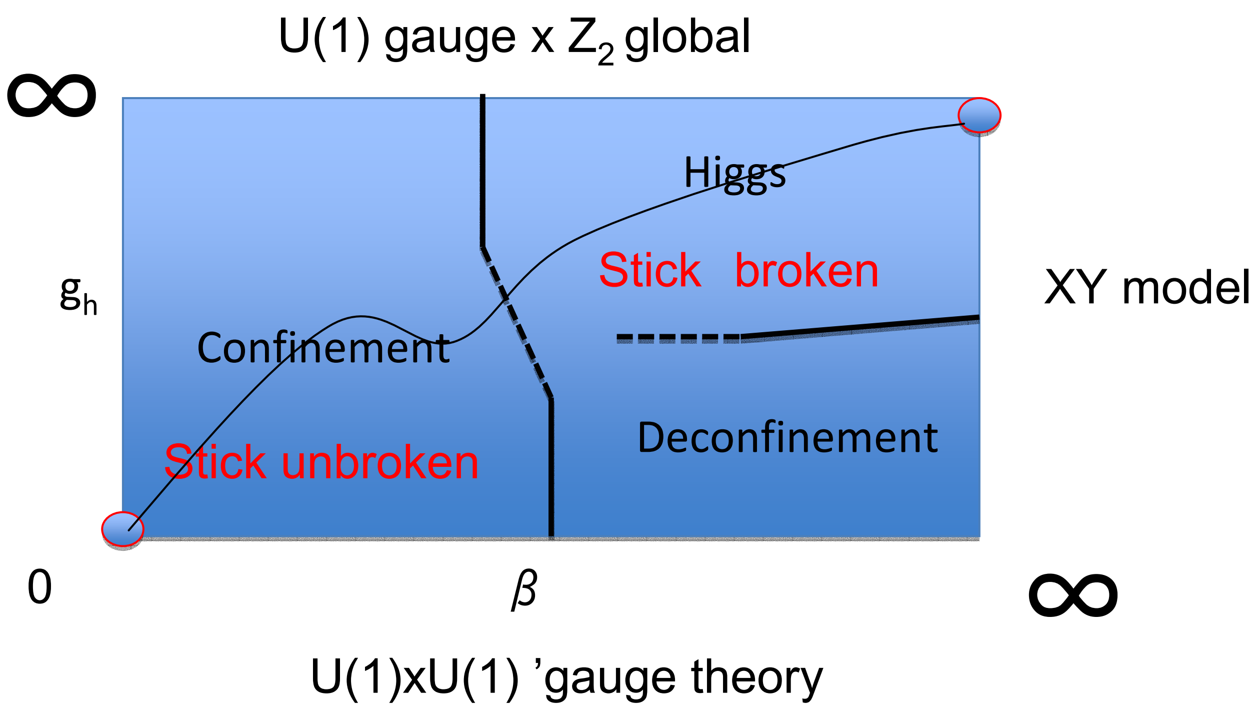
Potential for L_2

$g_h(\beta)$, $V(\beta)$

- $g_h=0$ → $U(1)\times U(1)'$ gauge theory
- $g_h=\infty$ → $U(1)$ gauge theory;

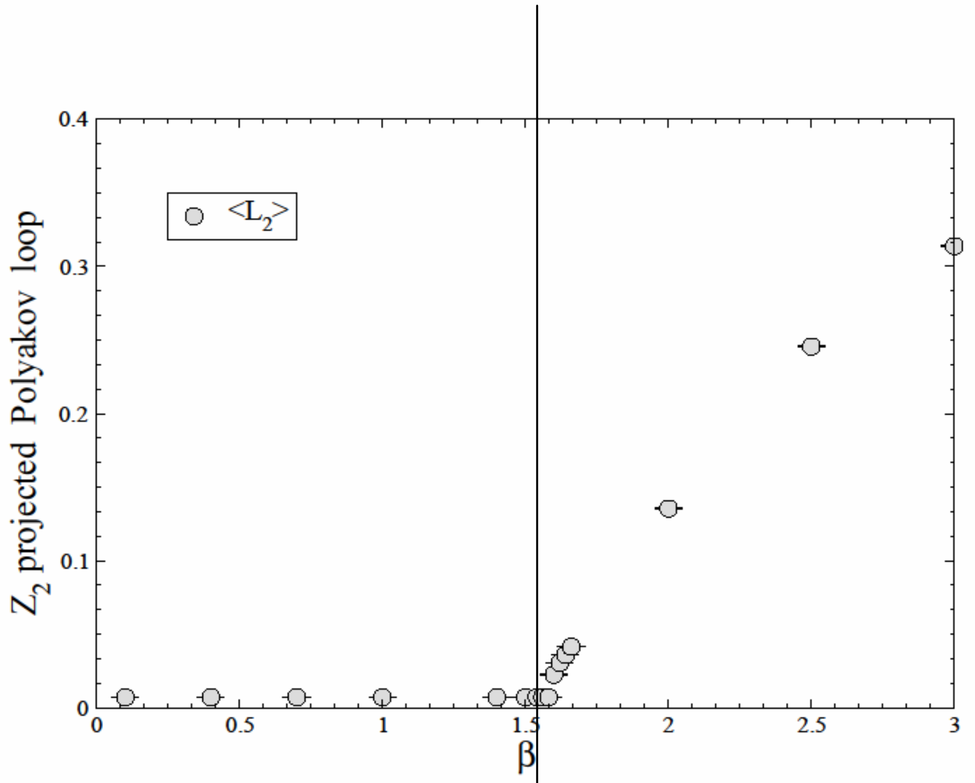
$$U_\mu(1) = \pm U_\mu(2),$$

$$X(n+\hat{\mu}) = \pm X(n)$$



Similar to two-unit charge Higgs system

<L_2>



β

Stick unbroken

Stick broken

§５ Generalization of Stick sym.

- G=SU(2) ⇒ G
- H=U(1) ⇒ H={ghg^{-1}=h}
- Stick ⇒ N_G(H)/ C_G(H)
- L_2 ⟶ −L_2 ⇒ L_2 ⟶ zL_2

  z=±1 z ∈ a subroup of C_G(H)

$$N_G(H) = i\sigma_2 e^{i\alpha\sigma_3}$$

$$C_G(H) = e^{i\alpha\sigma_3}$$

N_G(H)=normalizer with H of G

C_G(H)=centralizer with H of G

- Two series of the generalization

SU(2) $\rightarrow$ SO(2N), SU(2N),Sp(2N),…. even series

$\rightarrow$ SU(2N+1),SO(2N+1),G_2,…. odd series

- Additional condition for orbifolding,

$$\mathrm{Tr}\ g = 0$$

$$\because)\quad < L_2 > \propto (\mathrm{Tr}\ g)^2$$

In the strong coupling limit! Stick unbroken!

§ 6 Summaries

- 5-D SU(2) LGT with Z_2orbifolding
- Order parameter, Polyakov loop L_2
- Stick symmetry instead of center is useful
- Phase structure and effective theory

Further Studies for

- Calculation of Higgs mass
- Generalization and realistic model buildings
- Continuum limit vs Cut off theory

Refs.
K. Ishiyama, M. Murata, H.S and K. Takenaga, **Prog.Theor.Phys.123:257-269,2010**
N. Irges and F. Knechtli, **Nucl.Phys.B719:121-139,2005**:**B775:283-311,2007**