Form factors of the $D \to \pi$ and $D \to K$ semileptonic decays

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Outline

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Motivation

- recent accurate experimental determinations of the semileptonic rates (Belle, BaBar, BES, Cleo III, CLEO–c, FOCUS).

Two strategies:

Γ_{exp} + LQCD input
$\Rightarrow |V_{cq}|$

Γ_{exp} + CKM unitarity
$\Rightarrow$ LQCD results test,
then use for B physics

- Two *unquenched* determinations for $D \to \pi, K$ (FNAL–MILC, HPQCD), both use staggered dynamical quarks [Aubin et al. 2005, Bernard et al. 2009]

$D \to P \ell \nu_\ell$ ($P = \pi, K$) differential decay width

Approximate formula for $m_\ell \sim 0$

$$\frac{d\Gamma}{dq^2}(D \to P \ell \nu_\ell) = |V_{cq}|^2 \frac{G_F^2}{192\pi^3 m_D^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

Ingredients

- CKM matrix element squared
- Known, well measured factors
- Kinematic triangle function

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Form factors (scalar functions of $q^2 = (p-k)^2$, m_D, m_P)

$$\langle P(k)|\bar{q}\gamma_\mu c|D(p)\rangle = \left(p + k - q\frac{m_D^2 - m_P^2}{q^2}\right)_\mu f_+(q^2) + q_\mu \frac{m_D^2 - m_P^2}{q^2} f_0(q^2)$$

- Vector form factor
- Scalar form factor (negligible contribution $\propto m_\ell^2 \simeq 0$ if $\ell = e, \mu$)
- Kinematic constraint: $f_+(q^2 = 0) = f_0(q^2 = 0)$

ETMC simulation details

- Gauge sector $\Rightarrow$ Tree level Symanzik improved action

- Quarks sector $\Rightarrow$ Twisted Mass Wilson fermions at maximal twist $\Rightarrow$ automatic $O(a)$ improvement for physical quantities [Frezzotti and Rossi 2004]

- $\underline{N_f = 2}$ dynamical degenerate light quarks
- $a \simeq \{0.102,\ 0.086,\ \underline{0.068}\}\,\underline{\mathrm{fm}}$ (using f_π^{phys})
- $L^3 \times T = 24^3 \times 48$ and $32^3 \times 64$, $\underline{aL \sim 2.0 \div 2.7\,\mathrm{fm}}$
- $m_{ud}^{lat} \gtrsim m_s/6 \simeq 15 \div 20\,\mathrm{MeV} \Rightarrow \underline{m_\pi \simeq 270 \div 500\,\mathrm{MeV}}$
- $\underline{m_s^{lat} \simeq m_s} \simeq 90 \div 120\,\mathrm{MeV}$, $\underline{m_c^{lat} \simeq m_c} \simeq 1.2\,\mathrm{GeV}$

- twisted boundary conditions $\Rightarrow a\vec{p} = \frac{2\pi\vec{n}}{L} + \vec{\theta}$
[Bedaque 2004, Petronzio et al. 2004, Sachrajda and Villadoro 2005]

Extraction method (e.g. $D \to \pi$)

- reduce stat. fluctuations
- suitable double ratios of 2pts and 3pts $\Rightarrow$ no need for $\mathcal{Z}_V$ and $\mathcal{Z}_{\pi(D)}$
  [Hashimoto et al. 2000, Becirevic et al. 2005]
- exploit electric charge normalization
  $\langle H(-\vec{q})|V_4|H(\vec{q})\rangle = 2m_H \cdot f(q^2=0) = 2m_H \cdot 1$

Ratios and double ratios

$$\frac{C_4^{DV\pi}(\vec{0},t)\,C_4^{\pi VD}(\vec{0},t)}{C_4^{\pi V\pi}(\vec{0},t)\,C_4^{DVD}(\vec{0},t)} \xrightarrow{\text{plateau}} R_0' \,,$$

$$\frac{C_4^{\pi VD}(\vec{q},t)\,C^{\pi\pi}(\vec{0},t)}{C_4^{\pi VD}(\vec{0},t)\,C^{\pi\pi}(\vec{q},t)} \times \frac{C^{DD}(\vec{0},(t_{\text{source}}-t))}{C^{DD}(\vec{q},(t_{\text{source}}-t))} \xrightarrow{\text{plateau}} R_1'(q^2) \,,$$

$$\frac{C_i^{\pi VD}(\vec{q},t)}{C_4^{\pi VD}(\vec{q},t)} \xrightarrow{\text{plateau}} R_2'(q^2) \,,$$

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$$
\begin{aligned}
R'_0 &= \frac{(m_\pi + m_D)^2}{4 m_\pi m_D} \left(f_0(q^2_{\mathrm{max}})\right)^2 , \\
R'_1(q^2) &= \frac{E_D + E_\pi}{m_\pi + m_D} \frac{f_+(q^2)}{f_0(q^2_{\mathrm{max}})} \left[1 + \frac{E_D - E_\pi}{E_D + E_\pi} \xi(q^2)\right] , \\
R'_2(q^2) &= \frac{2|\vec{q}|\xi(q^2)}{E_D + E_\pi + (E_D - E_\pi)\xi(q^2)} .
\end{aligned}
$$

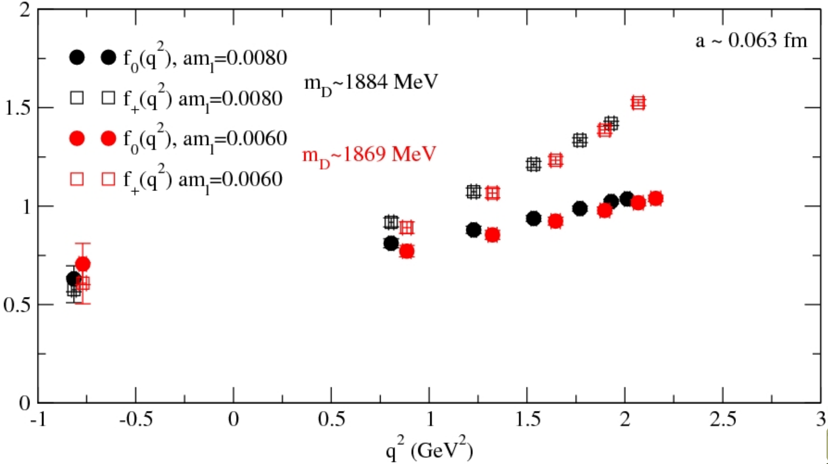
$$\xi(q^2) = f_-(q^2)/f_+(q^2)$$

Outline

Recipe

Simulated quark masses are not the physical ones. Moreover one wants $a \to 0$ limit.

- extract $f_{+,0}(q^2)$ at the several values of (a, m_l, m_s, m_c)



Recipe

Simulated quark masses are not the physical ones. Moreover one wants $a \to 0$ limit.

- extract $f_{+,0}(q^2)$ at the several values of (a, m_l, m_s, m_c)

- interpolate to physical charm quark mass [see F. Sanfilippo's talk]
- interpolate to physical strange quark mass by fixing the combination $2m_K^2 - m_\pi^2$ to its physical value ($\sim 476\,\mathrm{MeV}$)

- use HMChPT to fit the data in (m_π, q^2) and extrapolate to m_π^{phys}

- parametrically include $\mathcal{O}(a^2)$ effects in the chiral formulae

HMChPT formulae (1) [Becirevic et al. 2003, 2004]

for calculations on the lattice and in HQET it is more convenient to use the decomposition

$$\langle P|\bar{q}\gamma_\mu c|D\rangle = \sqrt{m_D}\left[v^\mu\ f_v(E_P) + p_T^\mu\ f_p(E_P)\right]$$

- $v^\mu = p_D^\mu/m_D$, 4–velocity of the D meson
- p_T^μ is the component of p_P^μ orthogonal to v^μ
- $E_P = v \cdot p_P = (m_D^2 + m_P^2 - q^2)/(2m_D)$,
  P meson energy in the D rest frame

$$f_0(q^2) = \frac{\sqrt{2m_D}}{m_D^2 - m_P^2}\left[(m_D - E_P)\, f_v(E_P) - p_T^2\, f_p(E_P)\right]$$

$$f_+(q^2) = \frac{1}{m_D}\left[f_v(E_P) + (m_D - E_P)\, f_p(E_P)\right]$$

(note that the kinematical constraint is automatically satisfied)

HMChPT formulae (2) [Becirevic et al. 2003, 2004]

Zeroth order in $1/m_D$, LO continuum formulae (chiral limit, soft pion)
$g = g_{D^* D\pi} \simeq 0.6$ from [Becirevic et al. 2009]

$$f_p^{LO}(m,E) = \frac{f_D\sqrt{m_D}}{f_\pi}\frac{g}{E+\Delta}, \qquad f_v^{LO}(m,E) = \frac{f_{D^*}\sqrt{m_D^*}}{f_\pi}$$

($\Delta = m_{D^*} - m_D$ correctly accounts for the pole, ($\Delta \simeq 145\,\mathrm{MeV}$, from PDG). The fit formulae, including $\mathcal{O}(a^2)$ discretization effects

$$f_p(m,E) = \frac{C_0}{E+\Delta}\left(1 + \underbrace{\frac{3}{4}(1+3g^2)\xi\log\xi}_{\text{chiral logs}} + \underbrace{C_1(E)m^2 + C_2(E)}_{\text{analytic terms}} + C_3 a^2\right)$$

$$f_v(m,E) = D_0\left(1 + \underbrace{\frac{9g^2-5}{4}\xi\log\xi - 2E^2F(m,E)}_{\text{chiral logs}} + \underbrace{D_1(E)m^2 + D_2(E)}_{\text{analytic terms}} + D_3 a^2\right)$$

($\xi = m^2/(4\pi f)^2$, $f \simeq 0.121 MeV$, [Baron et al. 2009])

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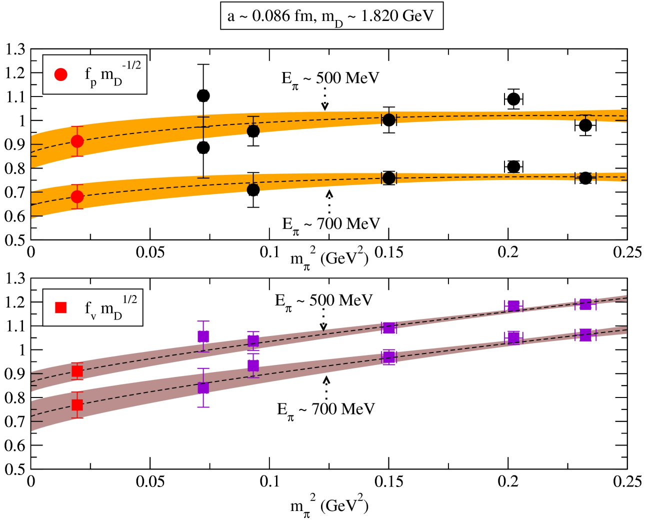
$$F(m, E) = \begin{cases} \log\xi + 2\sqrt{1-(m/E)^2}\log\left[E/m(1+\sqrt{1-(m/E)^2})\right] & E \geq m \\ \log\xi - 2\sqrt{(m/E)^2-1}\left[\pi/2 - \arctan\left(1/(\sqrt{(m/E)^2-1})\right)\right] & E < m \end{cases}$$

Outline

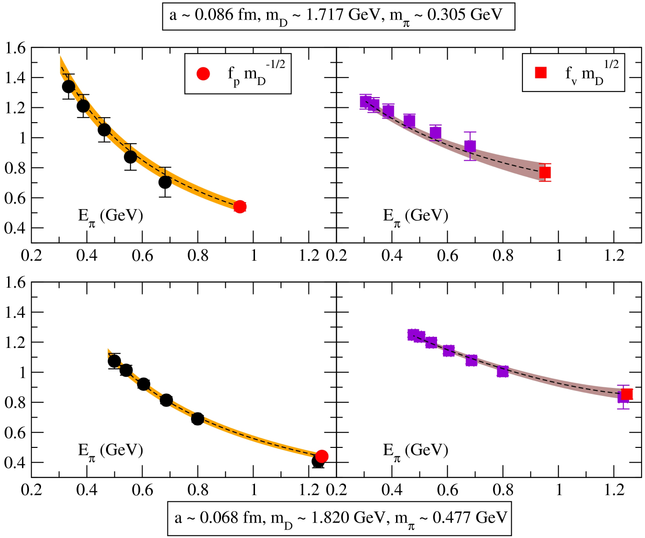
1. Motivation and simulation details
2. Steps towards physical results
3. Results

Fit quality: m_π dependence

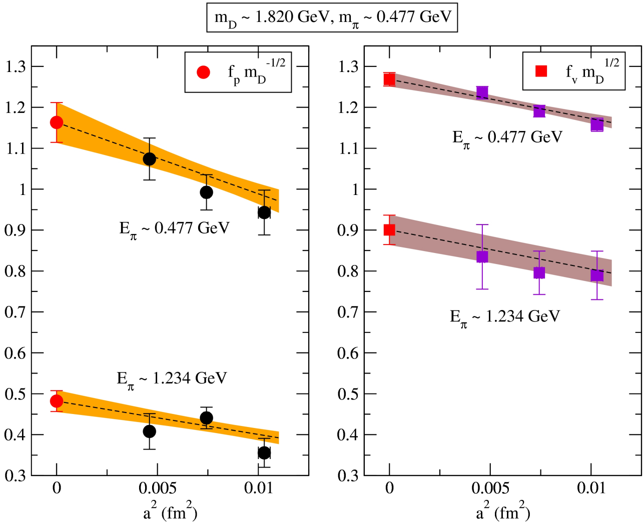


- chiral behaviour for fixed a and at a given value of E_π

Fit quality: dependence on E_π



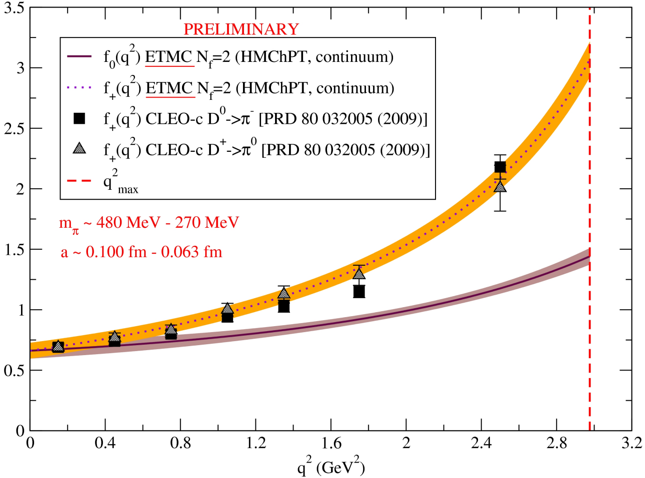
- dependence on E_π, for fixed m_π and fixed a
- $C_1(E)$, $C_2(E)$ and $D_1(E)$, $D_2(E)$
- simple polynomial ansatz for E dependence
- pole dominates f_p
- more terms in f_v



- dependence on a^2, for fixed m_π and at diffent values of E_π

well described by a term linear in a^2

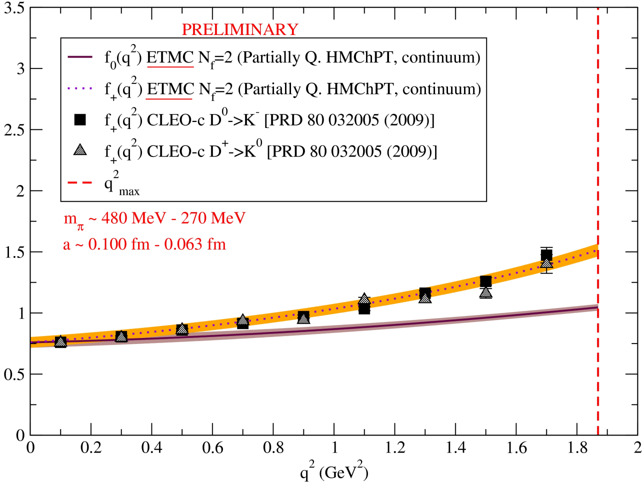
$N_f = 2$ form factors at the physical point: $D \to \pi$



- CKM unitarity is assumed
- $|V_{cd}| \simeq \lambda = 0.2258$ [UTFIT]
- good agreement with bin per bin exp data

$N_f = 2$ form factors at the physical point: $D \to K$ (1)

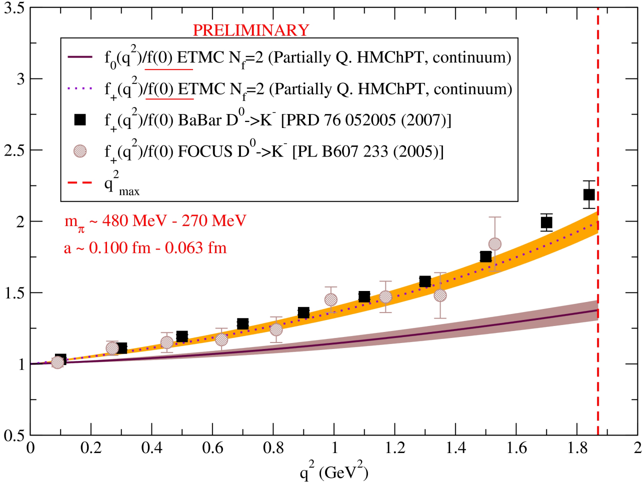
$f_{+,0}(q^2)$: lattice vs CLEO–c



- CKM unitarity is assumed
- $|V_{cs}| \simeq 1 - \lambda^2/2 = 0.9745$ [UTFIT]
- good agreement with bin per bin exp data

$N_f = 2$ form factors at the physical point: $D \to K(2)$

$f_{+,0}(q^2)$ normalized to $f(0)$: lattice vs BaBar and FOCUS



- independent on $|V_{cs}|$
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Conclusions and outlook

- Obtained: $N_f = 2$ form factors for both $D \to \pi$ and $D \to K$ at the physical point
  - HMChPT for extrapolating towards physical pion mass, down to $q^2 = 0$
  - HMChPT coefficients of analytic terms depend on energy $\Rightarrow$ with simple polynomial ansatzs the formulae describe the q^2 dependence
  - Maximally twisted tmLQCD $\Rightarrow$ only $\mathcal{O}(a^2)$ discr. effects in phys. obs.
  - FSE under control ($M_\pi L \gtrsim 4$ or $M_\pi L \gtrsim 3.7$)
- Outlook:
  - Momentum dependence through different paramterizations (*BK, z–expansion*)
  - $|V_{cd}|$ and $|V_{cs}|$
  - What about B physics?

Conclusions and outlook

Thanks everybody!

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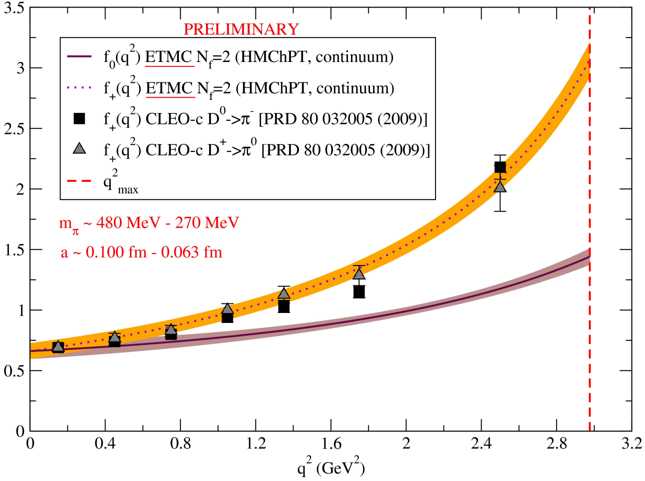

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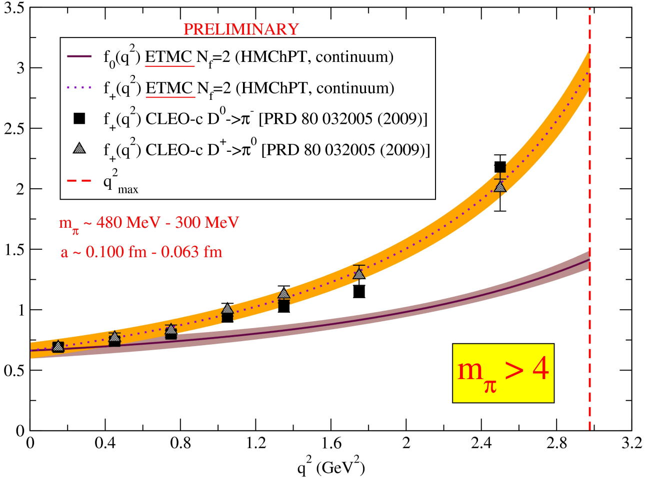

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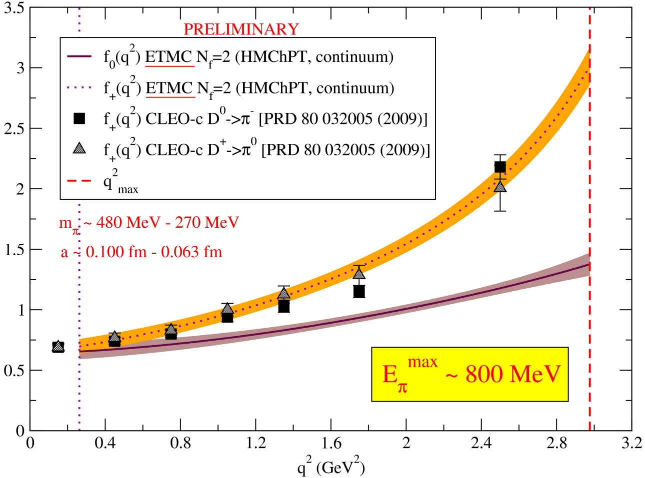

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