Quark mass determination from pseudoscalar mesons

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ETM collaboration

Summary

Light quark mass determination

1. Average light quark mass determination
2. Scale setting

Strange quark mass determination

1. Determination from K
2. Determination from $s\bar{s}$ meson

Charm mass determination

1. Determination from D/D_s
2. Determination from η_c

Technical info: typical ETM setup

Lattice setup

- Regularization: Wilson Twisted Mass at maximal twist
- Gauge action: Symanzik Improved, tree level
- $O(a)$ improved
- Number of dynamical fermions: 2, lights degenerate

Configuration ensembles

- 4 lattice spacings, ranging from $0.05 - 0.10\,fm$
- Various sea masses in the range $10 - 50\,MeV$
- Pion mass: $280 - 500\,MeV$
- Two different volumes for some setups
- Typical statistics: $300 - 500$ independent configurations

Correlation functions

- Pseudoscalar operator chosen: $\bar{\psi}\gamma_5\psi$
- Evaluated stochastically using one-end trick

Light Quark
Mass Determination

Analysis strategy

1. Fit am_π and af_π with $SU(2)\chi - PT$ with discretization terms
2. Extrapolate fitted expressions to the physical point and to the continuum
3. Determine:
   - m_l^{phys} from the ratio $\frac{(aM_\pi)^{fit}}{(af_\pi)^{fit}}$
   - a from af_π

- Renormalization constants Z_P for quark masses obtained in RI-MOM scheme [arXiv:1004.1115]
- Twisted Quark mass renormalize as Z_P^{-1}

Motivation

Differences from recent ETMC determination of m_l (arXiv:0911.5061): analysis perfomed simultaneously at the four available lattice spacings

Analysis strategy

Global SU(2)$\chi - PT$ fit using all the 4 lattice spacings

$$
\begin{aligned}
m_\pi^2 &= 2B_0 m_l \left(1 + m_l \log \frac{2B_0 m_l}{\Lambda_3} + D_m a^2 + T_m^{NNLO}\right) \\
f_\pi &= f_0 \left(1 - 2m_l \log \frac{2B_0 m}{\Lambda_4} + D_f a^2 + T_f^{NNLO}\right)
\end{aligned}
$$

A) $NLO - SU(2)\chi PT$ without discretization effects $(D = T = 0)$

B) $NLO - SU(2)\chi PT$ with discretization effects $(T = 0)$

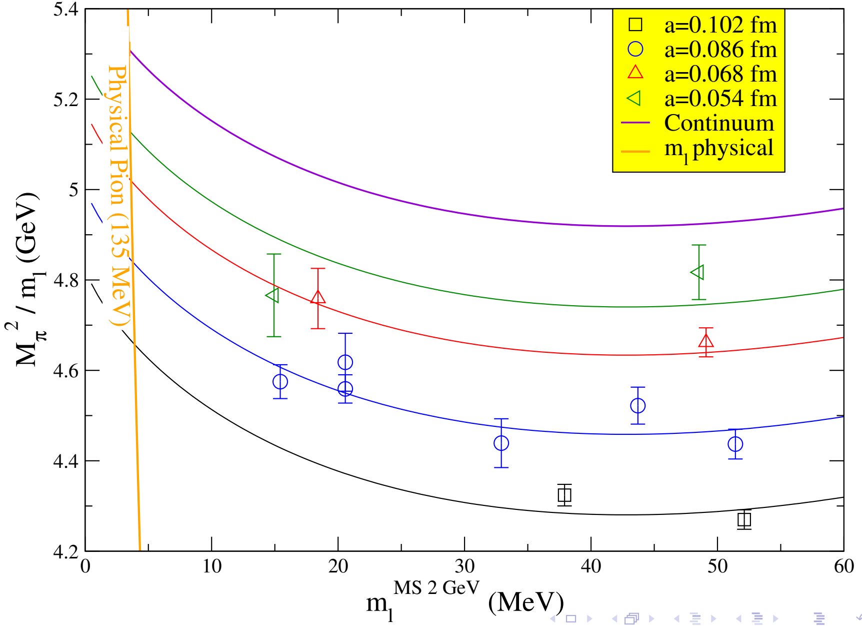
C) $NNLO - SU(2)\chi PT$ without discretization effects $(D = 0)$

D) $NNLO - SU(2)\chi PT$ with discretization effects

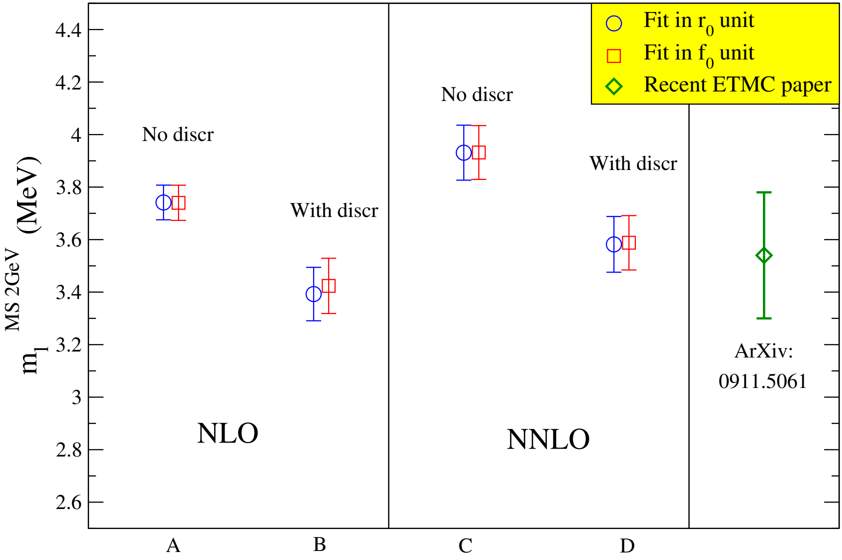
Finite volume effects treated analytically using χPT in finite box

- Colangelo-Durr-Haefeli [hep-lat/0503014] formulas
- Constraints:
  - $L \gtrsim 2fm$ (p-regime of χPT)
  - $M_\pi L \gg 1$ (finite size effects are small)

M_π^2/m_l fit in $SU(2)\chi PT$ at NLO with disc effects



m_l from different fits



Strange Quark

Mass Determination

Ansatze for fits

Two kinds of fits:

NLO $SU(2)$ partially quenched:

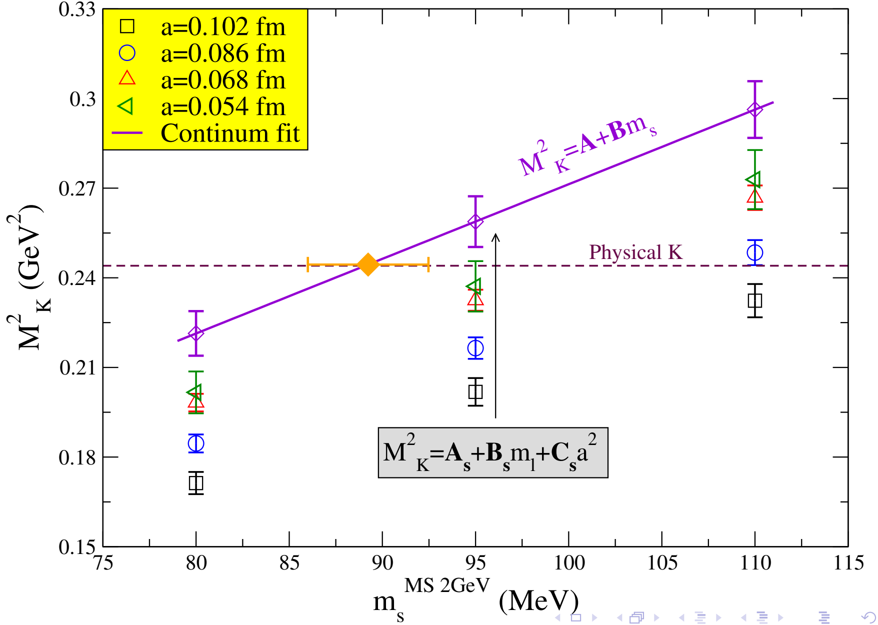
$$M_K^2 = A_s + B_s m_l + C_s a^2$$

NNLO $SU(3)$ with **some** (but not all) higher order terms:

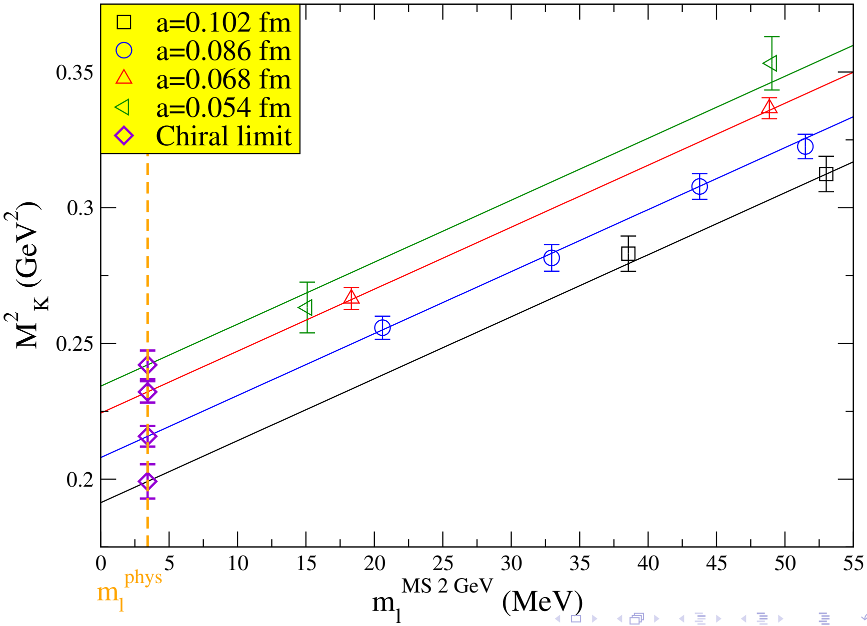
$$M_K^2 = 2B_0 \frac{m_l + m_s}{2} \Big(1 + \frac{2B_0}{(4\pi f_0)^2} m_s \log m_s \quad + A m_l + B m_s + C m_s^2 + $$
$$+ D m_l m_s + (F + G m_s) a^2\Big)$$

- Finite volume effects are very small
- a, f_0, m_l taken from the 4 different fits A B C D of m_π, in f_0 unit

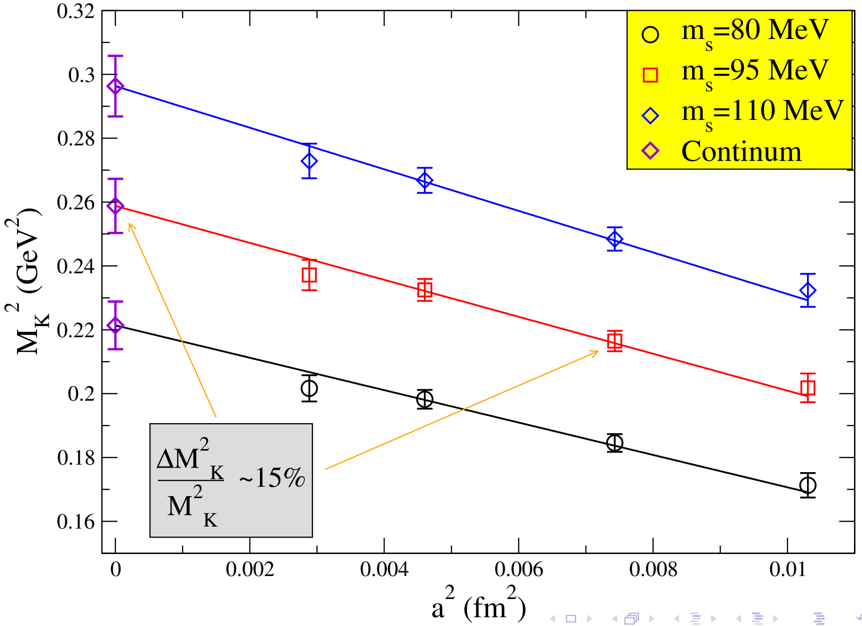
m_s from K in $SU(2)$ fit



Chiral extrapolation at ref mass $m_s = 95\,MeV$



Discretization effects on M_K^2 (chirally extrapolated)



Determination of m_s from $s\bar{s}$ meson

What is the $s\bar{s}$ meson?

- We take into account only connected correlation functions
- In the real world the $s\bar{s}$ meson mixes with $(u\bar{u} + d\bar{d})$
- This would exist in a world with two different quarks with the same mass

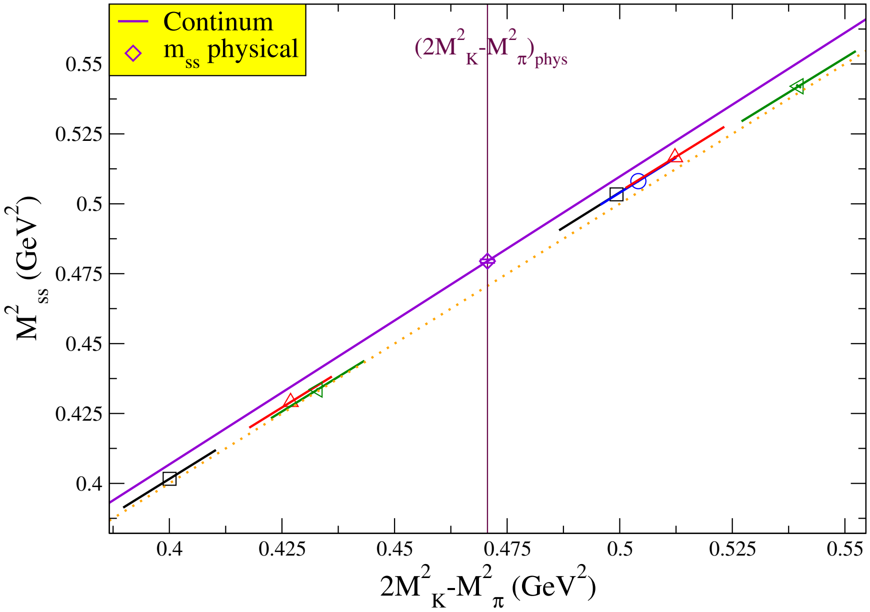
The $s\bar{s}$ meson defined by χPT and the experimental values of M_π and M_K
At LO in χPT: $M^2_{s\bar{s}} = 2M^2_K - M^2_\pi$

Main motivation

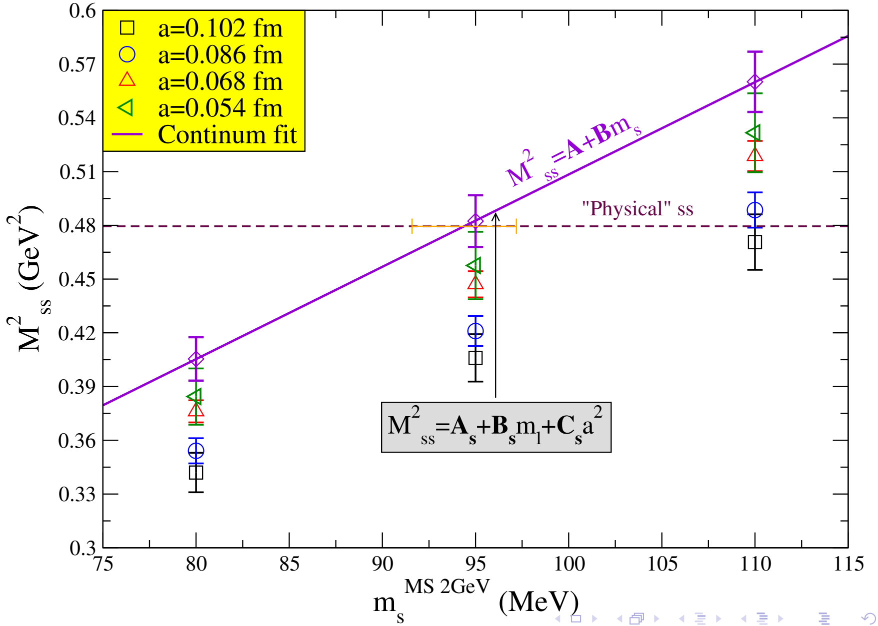
- The $s\bar{s}$ meson mass depend from m_l only through virtual loops
- One can expect to do little chiral extrapolation

This is the advantage of the tortuous procedure

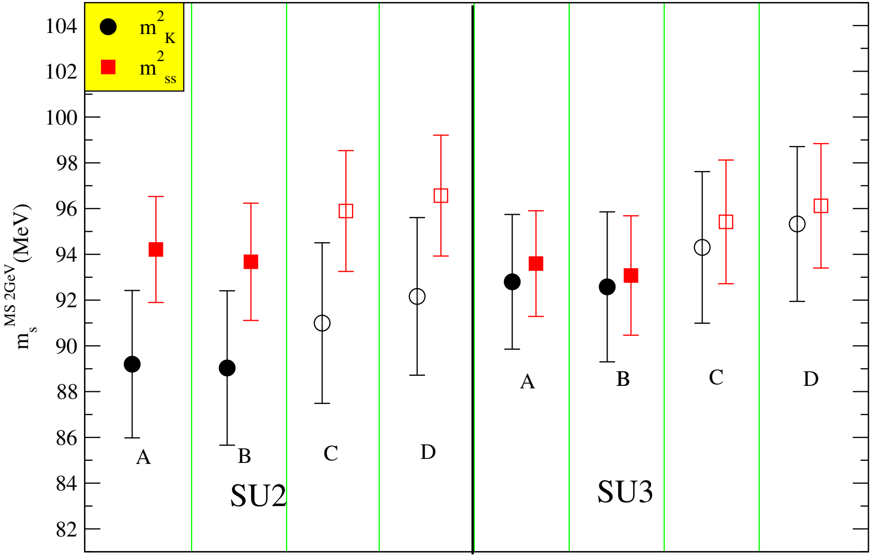
Definition of physical $M_{s\bar{s}}$ in $SU(2)$ fit



m_s from $s\bar{s}$ with $SU(2)$ fit



Results for m_s



Charm Quark
Mass Determination

Analysis strategy

Three different mesons: D, D_s, η_c

Ansatze for chiral and continuum limit:

$$
\begin{aligned}
M_D &= A_c + B_c m_l + C_c a^2 \\
M_{D_s} &= A_c + B_c m_l + C_c m_s + D m_s m_l + (E_c + F m_S) a^2 \\
M_{\eta_c} &= A_c + B_c m_l + C_c a^2
\end{aligned}
$$

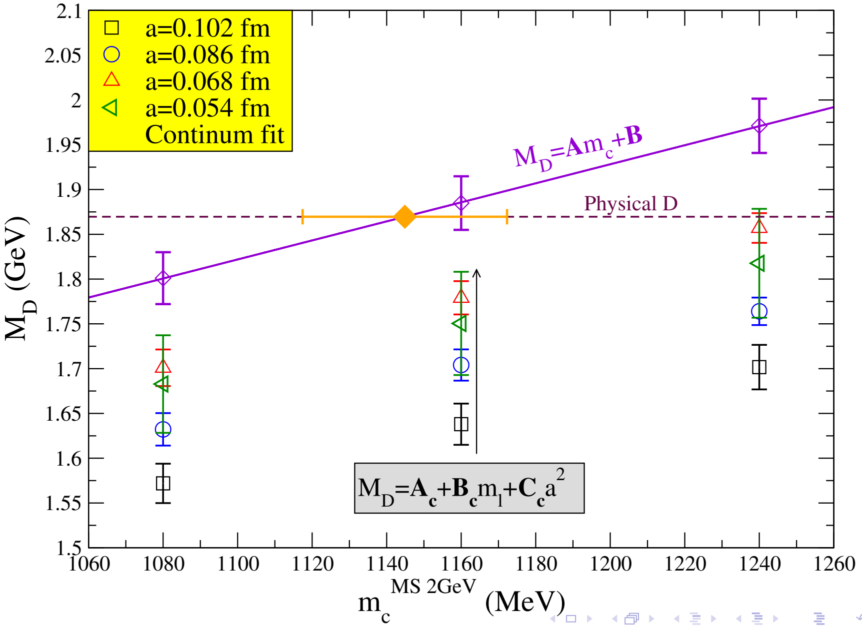
Ansatze formulas for fit in m_c

Taking inspiration from *HQET*:

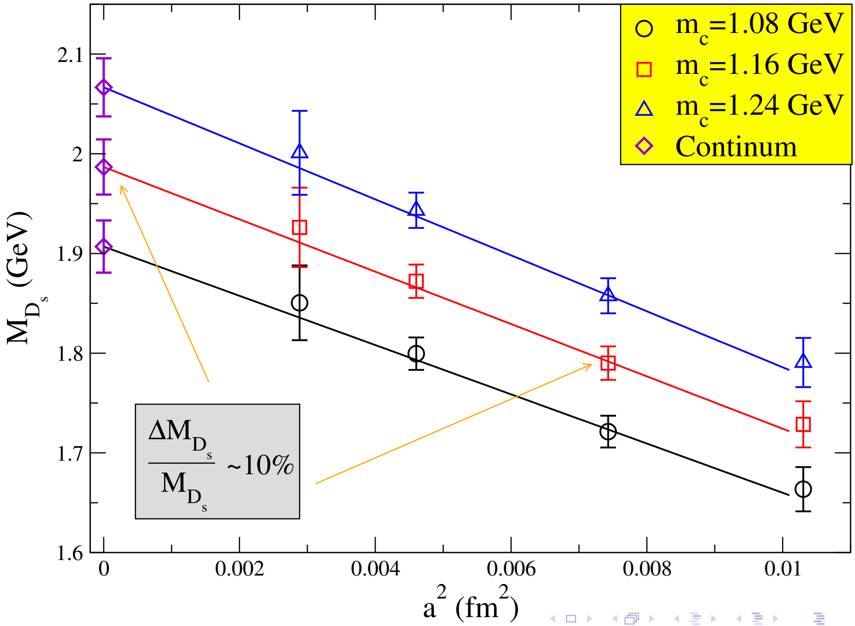
$$M_{D/D_s/\eta_c} = A m_c + B + C/m_c$$

Either A or C can be put to 0

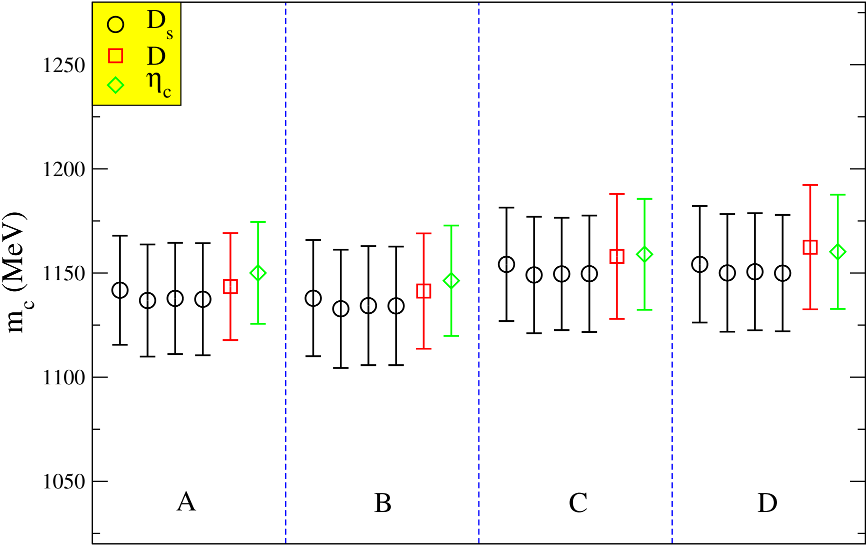
Determination from D meson



Discretization effects on M_{D_s} (m_l and m_s physical)

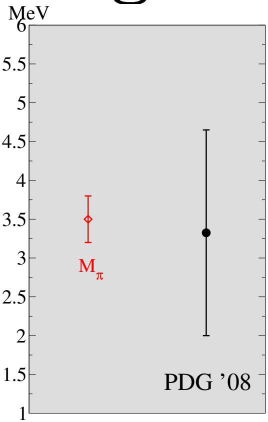


Results for m_c



Preliminary results

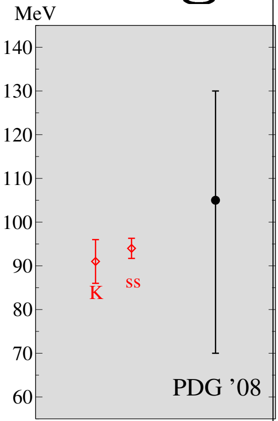
Light



$3.5(3)\ MeV$

at $2\ GeV$

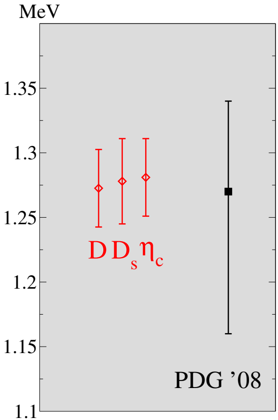
Strange



$91(5)\ MeV$

at $2\ GeV$

Charm

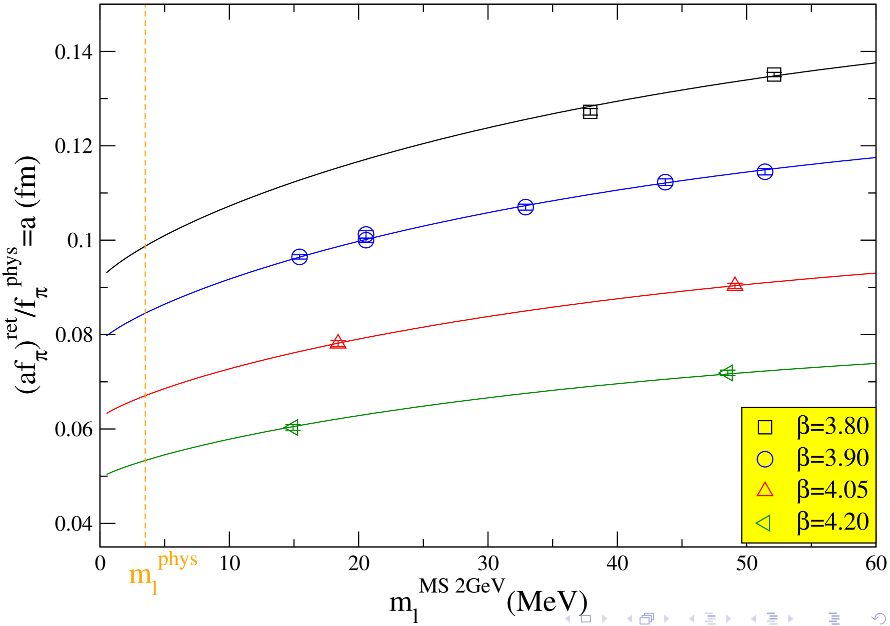


$1.27(3)\ GeV$

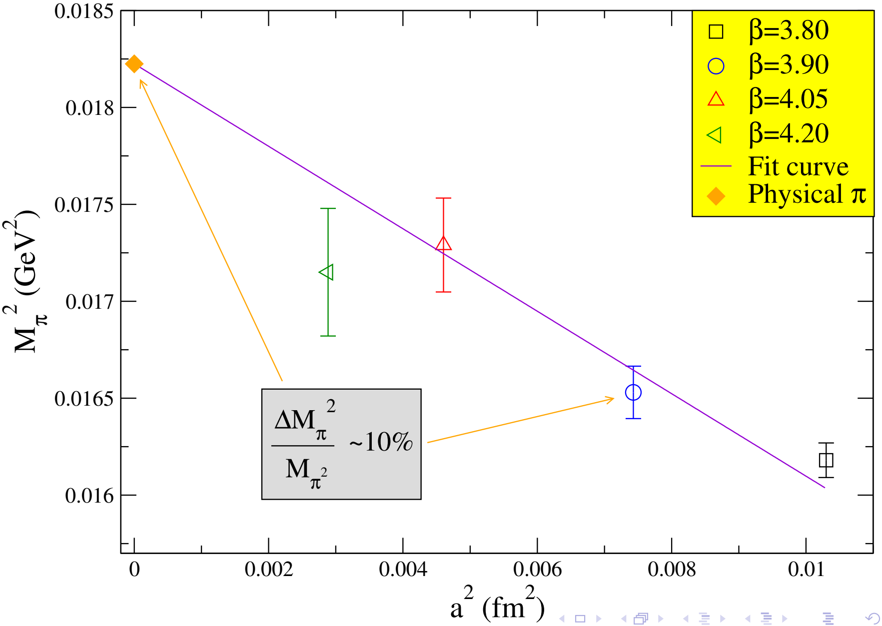
at m_c

Secret Backup slides

Lattice spacings from $(af_\pi)^{lat} / f_\pi^{phys}$



Discretization effects for M_π^2



Determination of m_s from K

Analysis strategy

1. Preliminary interpolation to fixed reference m_s^{ref}
   Procedure advantage: the analysis can be divided in two step
2. For each separate reference mass extrapolate M_K^2:
   - to the continuum limit $a \to 0$
   - to the physical light quark mass m_l^{phys}
3. Fit the continuum and chiral value of M_K^2
4. Obtain physical strange

Definition of $s\bar{s}$ meson

Operative definition of $s\bar{s}$

- Take $M_{s\bar{s}}$, M_K and M_π for each lattice spacing
- Perform a fit of $M_{s\bar{s}}(M_K, M_\pi, \mathfrak{a})$ over $M_{s\bar{s}}^{Lat}$ in term of M_K^{Lat} and M_π^{Lat}
- Define $M_{s\bar{s}} \equiv M_{s\bar{s}}\left(M_K^{phys}, M_\pi^{phys}, \mathfrak{a}=0\right)$

Ansatze for $M_{s\bar{s}}$ expression in terms of M_π and M_K

$$SU(2): M_{s\bar{s}}^2 = A + B\,M_\pi^2 + C\,M_K^2 + D\,a^2$$

$$SU(3): M_{s\bar{s}} = \left(2M_K^2 - M_\pi^2\right)\left(1 + (\xi_s - \xi_l)\log(2\xi_s) + (a_V/2+1)(\xi_s - \xi_l) + \right.$$
$$\left. + A\xi_s^2 + B\xi_s\xi_l\right) - M_\pi^2\left(-\xi_l\log(2\xi_l) + \xi_s\log(2\xi_s) + a_V(\xi_{ls} - \xi_l)\right) + C\,a^2$$

where $\xi_l = \frac{M_\pi^2}{(4\pi f_0)^2}$ and $\xi_s = \frac{2M_K^2 - M_\pi^2}{(4\pi f_0)^2}$

Correlation between data

- $M_{s\bar{s}}$, M_K are strongly correlated
- It is important to consider this correlation when performing the fit

Determination of m_s from $s\bar{s}$ meson

Analysis strategy

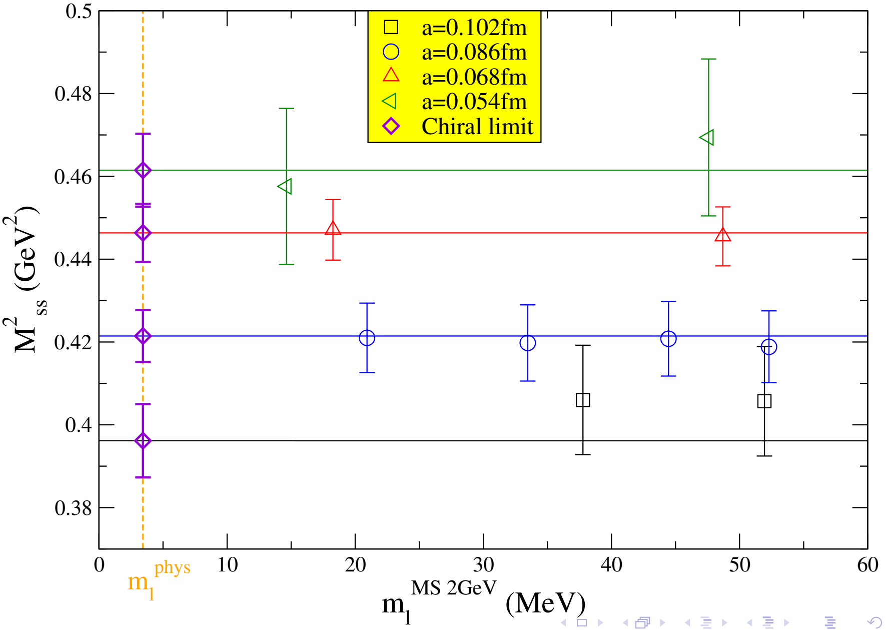
- Reference mass are important also for this analysis
- $M_{s\bar{s}}$ mass is almost flat in m_l
- Discretization effects are not larger than in K

Ansatze for $M_{s\bar{s}}$ expression in term of m_l and m_s

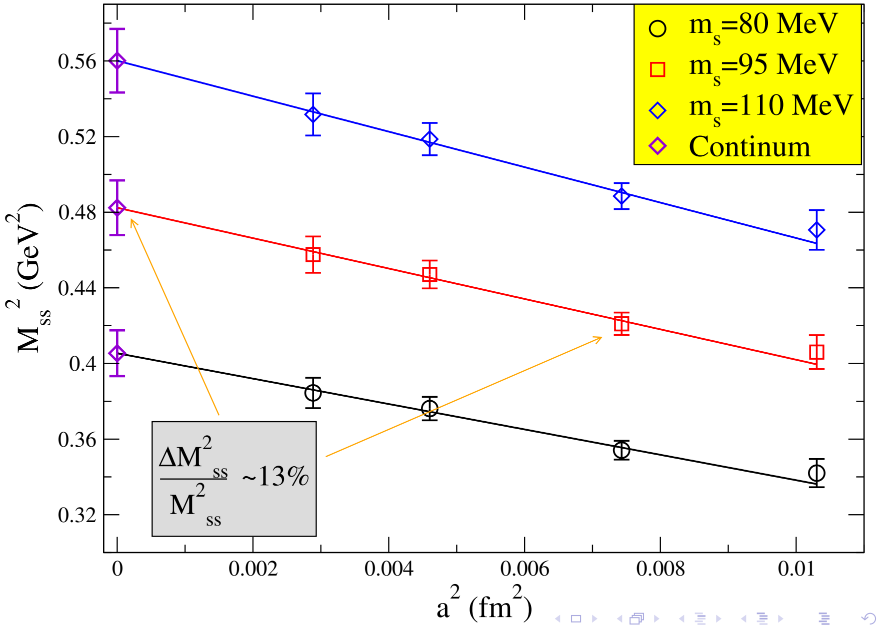
$$SU(2): M^2_{s\bar{s}} = A_s m_s + B_s m_s m_l + C_s a^2$$
$$SU(3): M_{s\bar{s}} = A m_s (1 + (\xi_s - \xi_l) \log(2\xi_s) + (a_V/2+1)(\xi_s - \xi_l) - \\ - m_l(-\xi_l \log(2\xi_l) + \xi_s \log(2\xi_s) + a_V(\xi_{ls} - \xi_l)) + (B + C m_s) a^2)$$

where $\xi_q = \frac{2B_0 m_q}{(4\pi f_0)^2}$

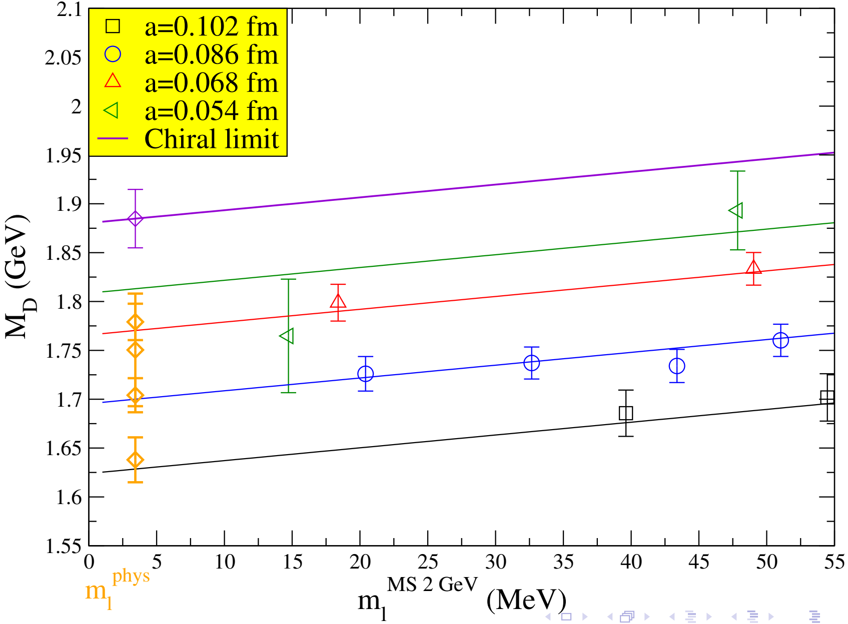
Chiral extrapolation of $M^2_{s\bar{s}}$ at $m_s = 95\, MeV$



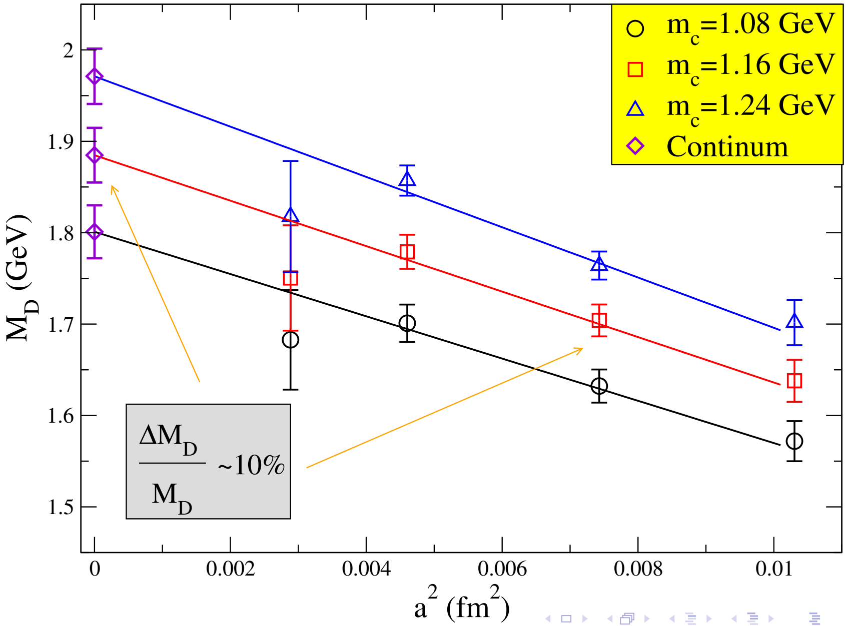
Discretization effects on M_{ss}^2 (chirally extrapolated)



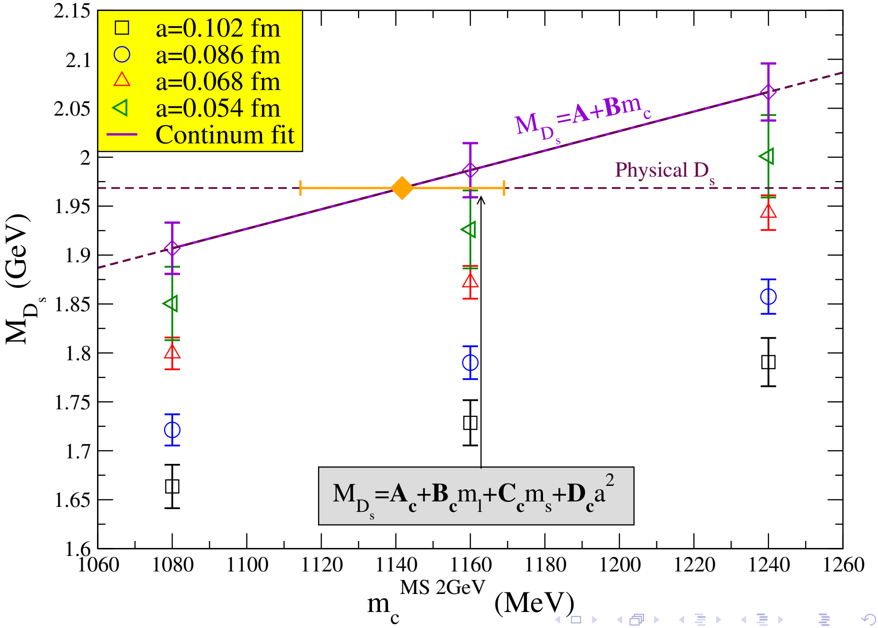
Chiral extrapolation of M_D at $m_c = 1.16\,GeV$



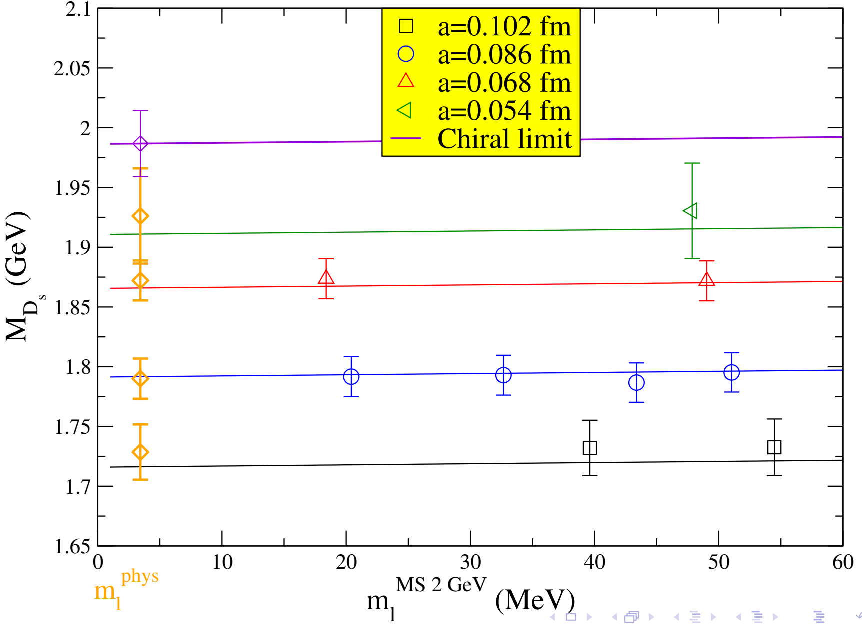
Discretization effects on M_D (chirally extrapolated)



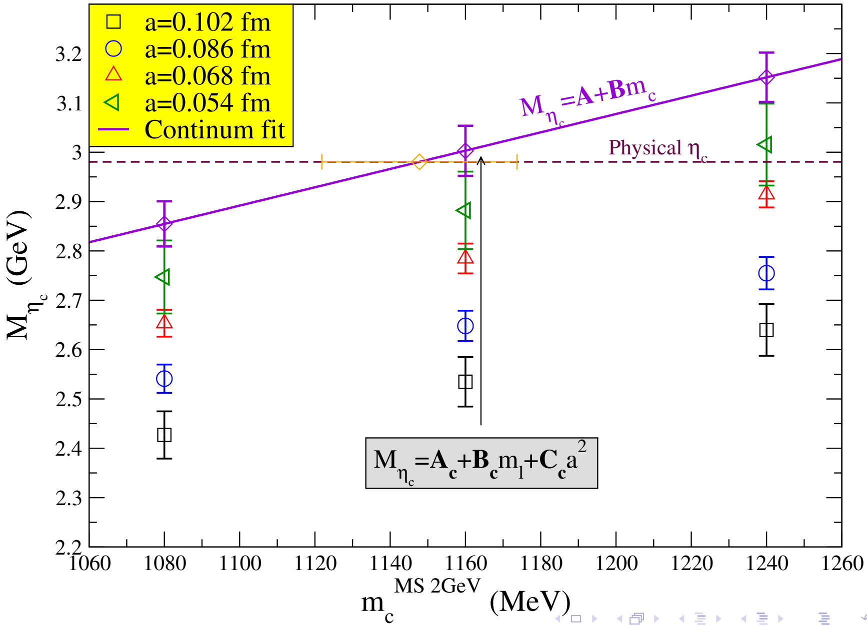
Determination from D_s meson



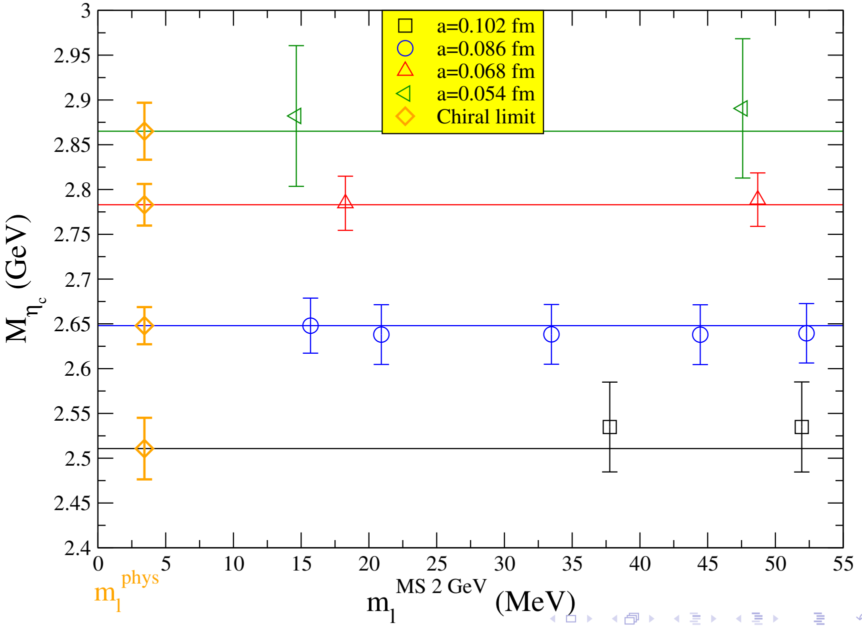
Chiral extrapolation of M_{D_s} at $m_c = 1.16\ GeV$



Determination from η_c meson



Chiral extrapolation of M_{η_c} at $m_c = 1.16\ GeV$



Discretization effects on M_{η_c} (chirally extrapolated)