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# Construction and Analysis of Two Baryon Correlation functions

Kostas Orginos

College of William and Mary

JLAB

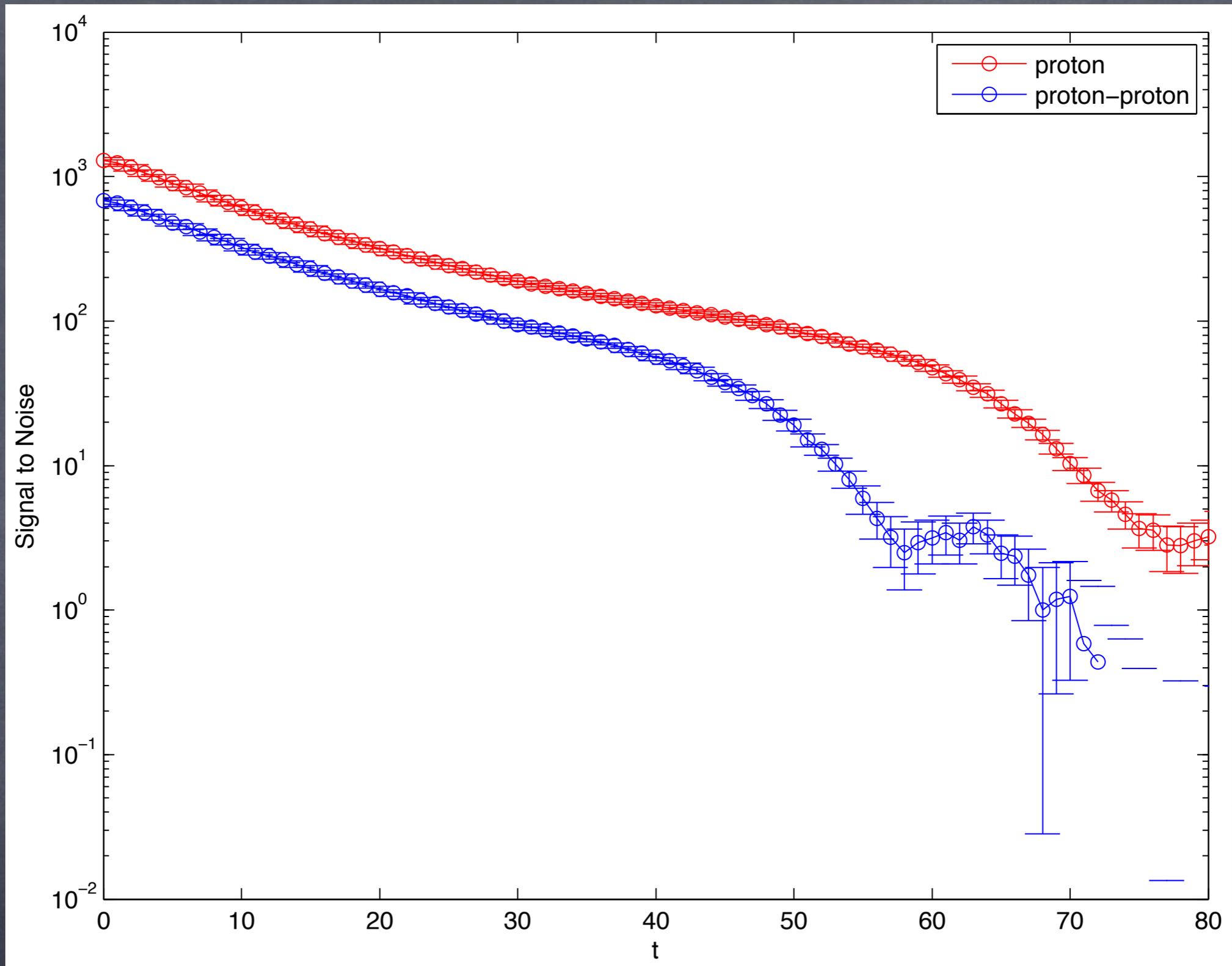


# Summary

- The problem
  - Hadron-Hadron scattering phase shifts
- Methods for extracting masses from correlators
  - The variable projection method
  - A new use of the generalized eigenvalue problem
  - Application to Baryon-Baryon spectrum

# Elastic Scattering Phases shifts

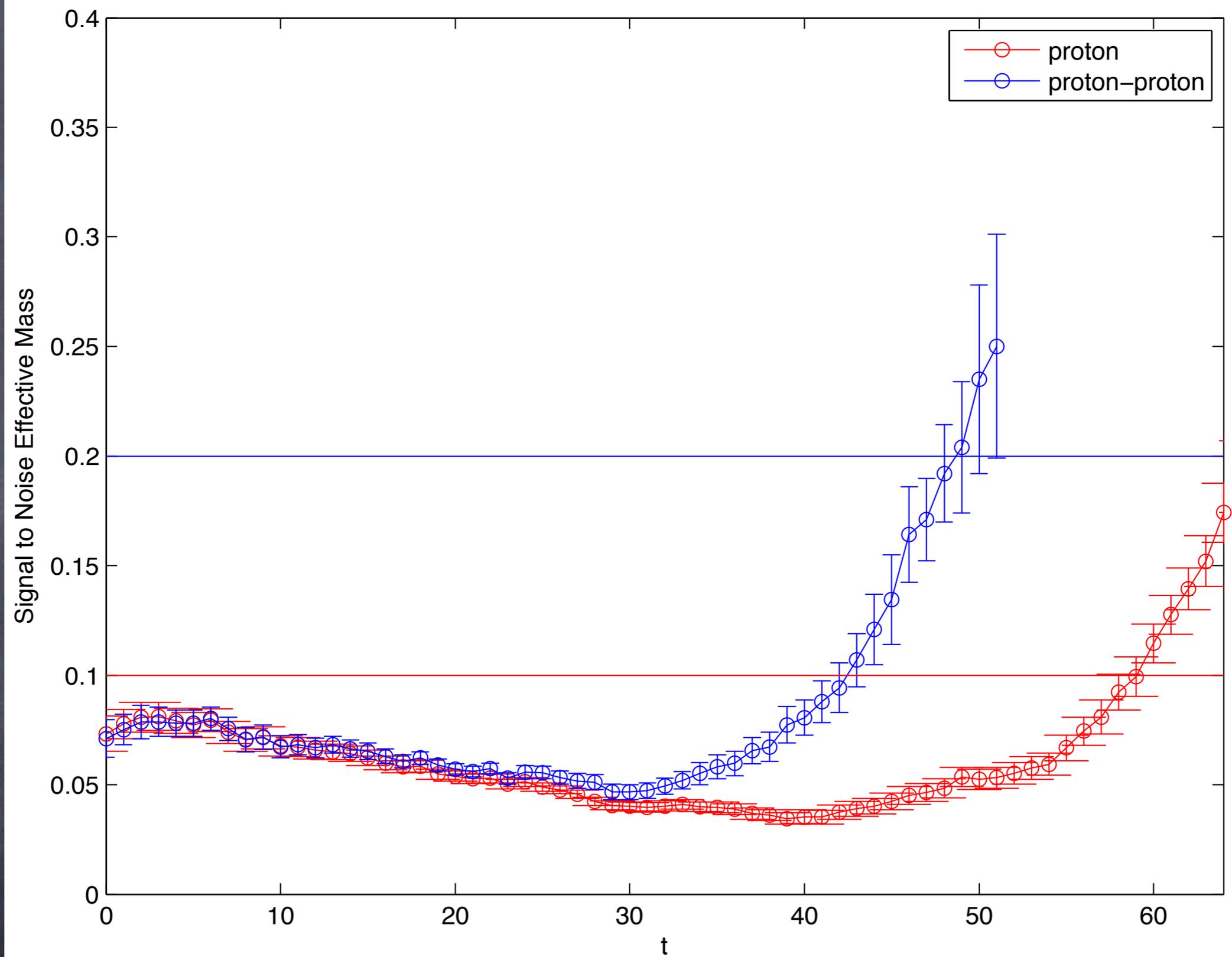
- Maiani-Testa no-go theorem
- Luscher: Finite volume two particle spectrum is related to elastic scattering phase shifts
- Computational problem: Calculate in Euclidean space and finite volume the two particle spectrum
- Extract energy levels from exponentially decaying correlation functions
- Baryons: Signal to noise ratio grows exponentially with Euclidean time



# Signal to Noise

$24^3 \times 128$   
 $M_\pi = 390 \text{ MeV}$

NPLQCD data



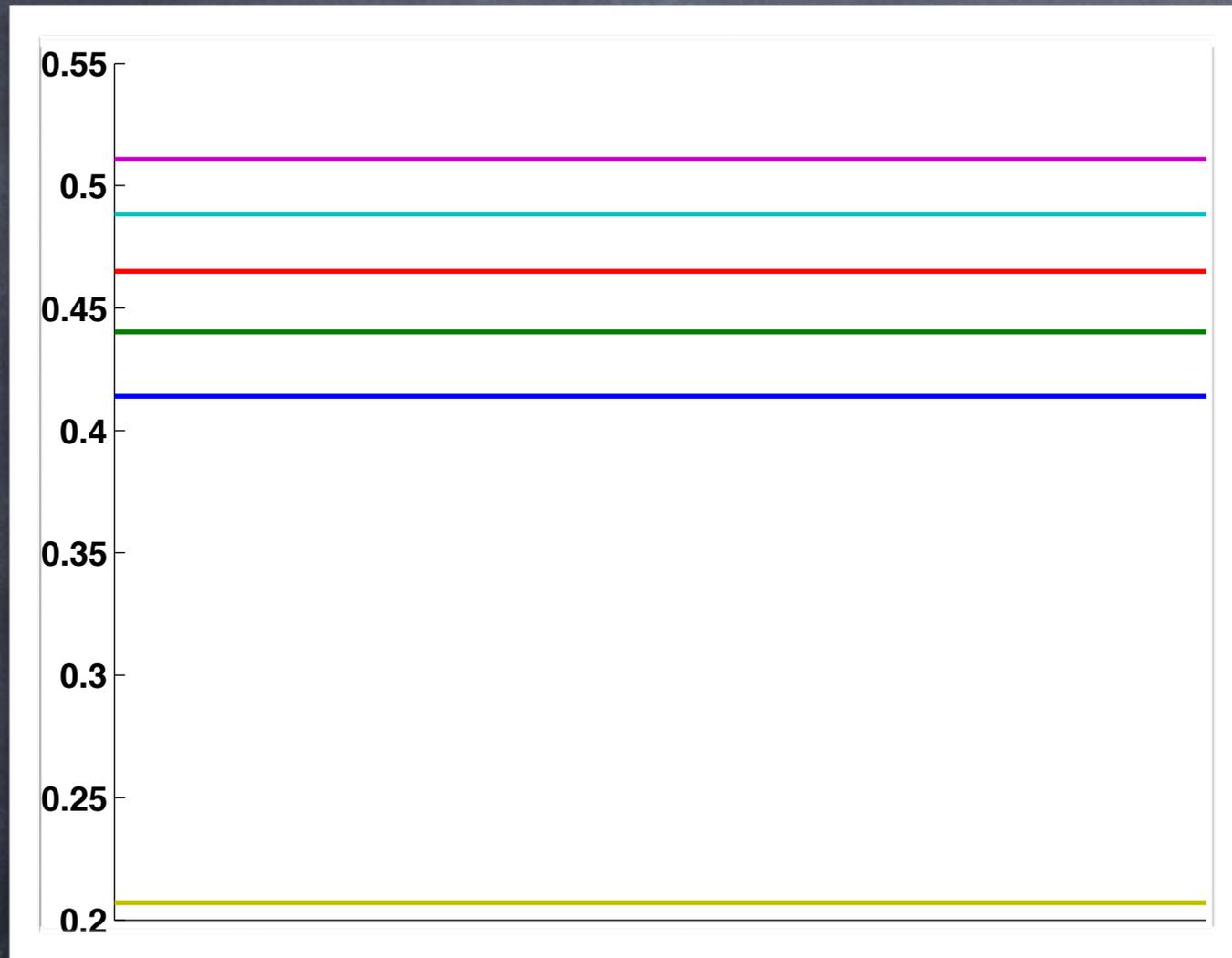
# Signal to Noise Effective Mass

$24^3 \times 128$   
 $M_\pi = 390 \text{ MeV}$

anisotropy factor 3.5

NPLQCD data

# Expected Two Nucleon spectrum



free 2 particle spectrum

$M_n$

$24^3$  box

anisotropy factor 3.5

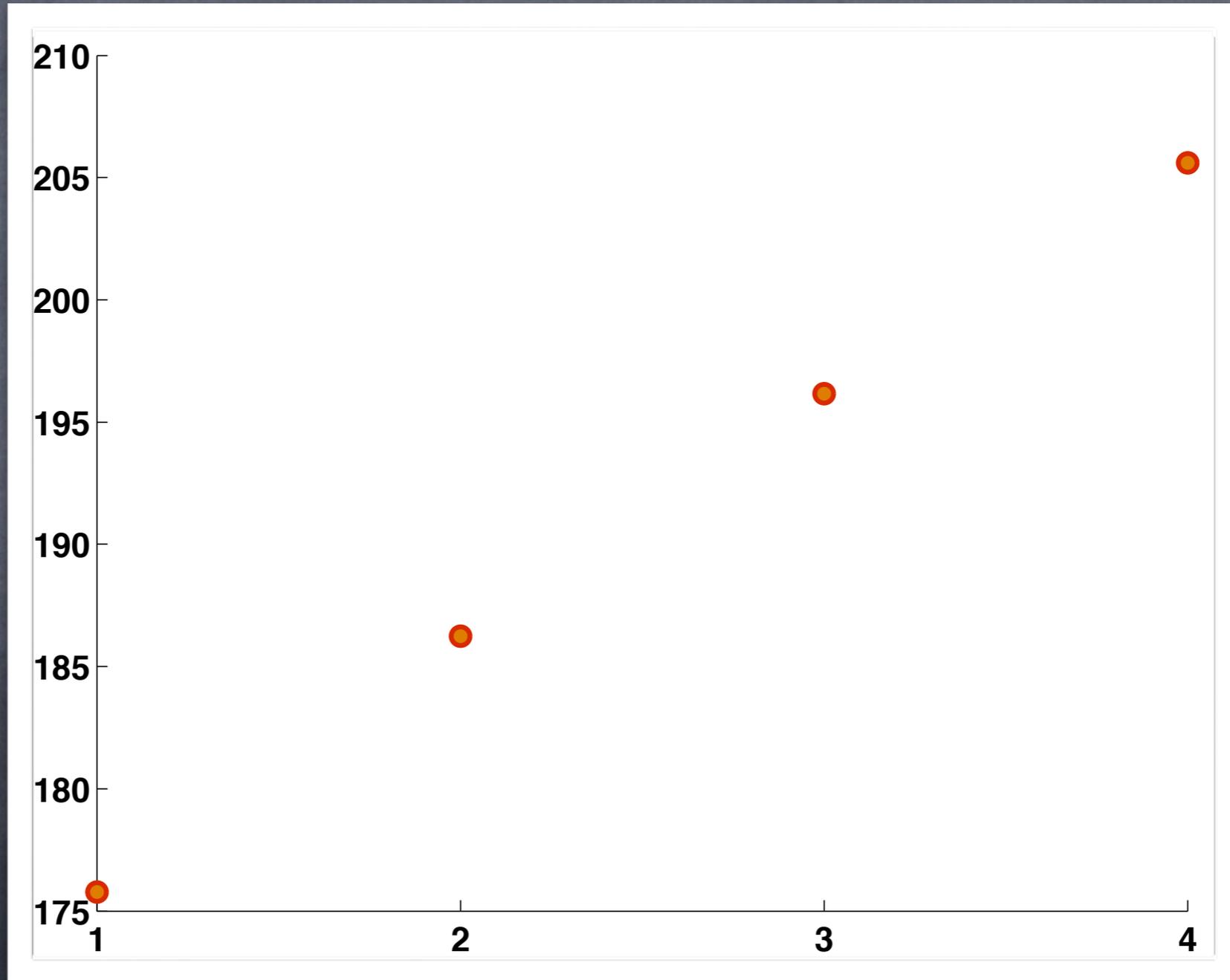
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# Needed Time Separation

$$e^{-\Delta E \delta t} \approx 10^{-2}$$

$24^3$  box

$\delta t$



anisotropy factor 3.5

Two particle state

# Conclusion

We need to fit for several low lying states for reliable estimation of the ground state of the two particle system in a finite box

# Fitting Exponentials

The Variable Projection method (VarPro)

Golub and Pereyra, SIAM J. Numer. Anal. Vol 10 No 2, 1973

$$t^2(Z, E) = \sum_{ij} [\bar{G}(t_i) - F(t_i, Z, E)] C_{ij}^{-1} [\bar{G}(t_j) - F(t_j, Z, E)]$$

$$F(t, Z, E) = \sum_{n=0}^N Z_n e^{-E_n t}$$

Separable Least Squares problem:

- Solve analytically for for the minimum for Z's at a given choice for E's
- Minimize numerically the resulting  $t^2$  as a function of E's

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$$F(t_i, Z, E) = \sum_{n=0}^N A_{in} Z_n$$

$$A_{in} = e^{-E_n t_i}$$

Solution for Z's:

$$Z(E) = [A^\dagger C^{-1} A]^{-1} A^\dagger C^{-1} Y$$

$$Y_i = \bar{G}(t_i)$$

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$$t^2(E) = \sum_{ij} d_i(E) C_{ij}^{-1} d_j(E)$$

$$d_i(E) = \bar{G}(t_i) - \sum_{n=1}^N A_{in} Z_n(E)$$

$$d(E) = Y - [A(E)^\dagger C^{-1} A(E)]^{-1} A(E)^\dagger C^{-1} Y$$

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# Fitting Exponentials

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- VarPro can be generalized for multiple correlation functions with full covariance
- Regular  $\chi^2$  error analysis is done after minimization
- Fitting exponentials is hard...

# Alternative Methods

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Use multiple correlators and construct linear combinations that couple predominately to one state

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- “Variational”: Symmetric positive definite matrix of correlators [C. Michael, '85; Luscher&Wolf '90; ...]
- Prony methods: [Fleming '04; NPLQCD '08; Fleming et.al. '09 ]
- Matrix Prony [NPLQCD '08]

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# Generalized Pensile of Matrix

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# Generalized Pencil of Matrix

GPOF

Y. Hua and T. Sarkar

IEEE Transactions of Antennas and Propagation

Vol. 37 No. 2 p.229 '89

Consider

$$G_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle$$

$$K_{ij}^{lm}(t) = \langle \mathcal{O}_i(t + l\tau) \mathcal{O}_j^\dagger(-m\tau) \rangle = G_{ij}(t + (l + m)\tau)$$

Organized into a matrix

$$K(t) = \begin{pmatrix} G(t) & G(t + \tau) & G(t + 2\tau) & G(t + 3\tau) & \dots \\ G(t + \tau) & G(t + 2\tau) & G(t + 3\tau) & G(t + 4\tau) & \dots \\ G(t + 2\tau) & G(t + 3\tau) & G(t + 4\tau) & G(t + 5\tau) & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots \end{pmatrix}$$

If **N** operators and **M** shifts are used then K is an **NMxNM** matrix

$$G_{ij}(t) = \sum_n \langle 0 | e^{Ht} \mathcal{O}_i e^{-Ht} | n \rangle \langle n | \tilde{\mathcal{O}}_j^\dagger | 0 \rangle$$

Using the operators  $\mathcal{O}_i(t)$   $\tilde{\mathcal{O}}_i^\dagger(t)$  and

$$\begin{aligned} \mathcal{O}(t + m\tau) &= e^{m\tau H} \mathcal{O}(t) e^{-m\tau H} \\ \mathcal{O}^\dagger(t - m\tau) &= e^{-m\tau H} \mathcal{O}^\dagger(t) e^{m\tau H} \end{aligned}$$

The  $K(t)$  is the matrix of correlators resulting from

$\tilde{\mathcal{O}}_i^\dagger(t - m\tau)$  creation operators

and

$\mathcal{O}_i(t + m\tau)$  annihilation operators

In general  $K(t)$  is:

- A positive definite and symmetric if  $G(t)$  is positive definite and symmetric
- A rectangular matrix if the basic creation and annihilation operators are not the same set
- An  $(M_1 \ N_1) \times (M_2 \ N_2)$  matrix if  $M_1$  shifts with  $N_1$  basic annihilation operators and  $M_2$  shifts with  $N_2$  basic creation operators are used

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From now on I consider  $K(t)$  an  $N \times M$  matrix

# Estimating Energies

Assuming finite number ( $S$ ) of states contributing to the correlators with  $S < \min(N, M)$

$$K_{ij}(t) = \sum_n Z_i^n (\tilde{Z}_j^n)^* e^{-E_n t}$$

Look for vectors  $\omega^n$  and  $\tilde{\omega}^n$  that

$$\omega^{n\dagger} Z^m = \sum_{i=1}^N \omega_i^{n*} Z_i^m = \delta_{nm} \quad \tilde{Z}^{n\dagger} \tilde{\omega}^m = \sum_{i=1}^M \tilde{Z}_i^{n*} \tilde{\omega}_i^m = \delta_{nm}$$

$$K(t) \tilde{\omega}^n = Z^n e^{-E_n t} = e^{-E_n(t-t_0)} K(t_0) \tilde{\omega}^n$$

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$$K(t)\tilde{\omega}^n = \lambda K(t_0)\tilde{\omega}^n$$

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$$K(t) \tilde{\omega}^n = \lambda K(t_0) \tilde{\omega}^n$$

$$\lambda = e^{-E_n(t-t_0)}$$

$$K(t)\tilde{\omega}^n = \lambda K(t_0)\tilde{\omega}^n$$

$$\omega^{n\dagger}K(t) = \lambda\omega^{n\dagger}K(t_0)$$

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Generalized eigenvalue problem but  $K$  is not symmetric  
and not positive

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Generalized eigenvalue problem but  $K$  is not symmetric  
and not positive

Consider the singular value decomposition

$$K(t_0) = U\Sigma V^\dagger$$

$$K(t)\tilde{\omega}^n = \lambda K(t_0)\tilde{\omega}^n$$

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Generalized eigenvalue problem but  $K$  is not symmetric  
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Consider the singular value decomposition

$$K(t_0) = U\Sigma V^\dagger$$

Let  $M < N$       Then we have  $M$  singular values

But only  $S$  (the number of states contributing to the  
correlator) of them are non-zero

$$K(t) = \begin{pmatrix} G(t) & G(t + \tau) & G(t + 2\tau) & G(t + 3\tau) & \cdots \\ G(t + \tau) & G(t + 2\tau) & G(t + 3\tau) & G(t + 4\tau) & \cdots \\ G(t + 2\tau) & G(t + 3\tau) & G(t + 4\tau) & G(t + 5\tau) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{pmatrix}$$

The  $S+1$  column is a linear combination for the first  $S$  columns given that the signal is a sum of exponentials

Let  $\sigma_i$  be the non-zero singular values of  $K(t_0)$

$$\Sigma' = \begin{pmatrix} \sigma_1 & 0 & \cdots \\ 0 & \sigma_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad S \times S \text{ matrix}$$

$$U' = U(:, 1 : S) \quad N \times S \text{ matrix} \quad V' = V(:, 1 : S) \quad M \times S \text{ matrix}$$

$$K(t_0) = U' \Sigma' V'^{\dagger}$$

$$V'^{\dagger} V' = \mathbf{1}_{S \times S}$$

$$U'^{\dagger} U' = \mathbf{1}_{S \times S}$$

Let

$$A = \Sigma'^{-\frac{1}{2}} U' K(t) V' \Sigma'^{-\frac{1}{2}}$$

SxS matrix

Then

$$A \tilde{q}^n = \lambda_n \tilde{q}^n$$

$$q^{n\dagger} A = \lambda_n q^{n\dagger}$$

$$q^{n\dagger} \tilde{q}^m = \delta_{nm}$$

with

$$\tilde{\omega}^n = V' \Sigma'^{-\frac{1}{2}} \tilde{q}^n$$

$$\omega^{n\dagger} = q^{n\dagger} \Sigma'^{-\frac{1}{2}} U'^{\dagger}$$

is an eigenvalue problem of a non-symmetric matrix

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is an eigenvalue problem of a non-symmetric matrix

Z factor reconstruction

$$\tilde{Z}^{\dagger} = Q^{\dagger} \Sigma'^{\frac{1}{2}} V'^{\dagger} \quad Z = U' \Sigma'^{\frac{1}{2}} \tilde{Q}$$

# In practice...

Keep singular values that  $\frac{\sigma_k}{\max(\sigma_i)} > tol$

The tolerance can be taken to be of the order of the signal to noise ratio

$$tol \sim \frac{\text{err}[G(t_0)]}{\bar{G}(t_0)}$$

Typical tolerance values  $10^{-2}$  to  $10^{-3}$

# In practice...

$$A(t) = \Sigma'^{-\frac{1}{2}} U' K(t) V' \Sigma'^{-\frac{1}{2}}$$

$$A(t)\tilde{q}^n = \lambda_n(t)\tilde{q}^n$$

$$\lambda_n(t) = \left(1 - \sum_i z_i\right) e^{-E_n(t-t_0)} + \sum_i z_i e^{-E'_i(t-t_0)}$$

# In practice...

$$A(t) = \Sigma'^{-\frac{1}{2}} U' K(t) V' \Sigma'^{-\frac{1}{2}}$$

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principal correlator



# In practice...

- Scan over several  $t_0$  values
- Chose tolerance based on signal to noise ratio
- Fit the principal correlators in  $(t_0+1, t_{\max})$

$$\lambda_n(t) = e^{-E_n(t-t_0)}$$

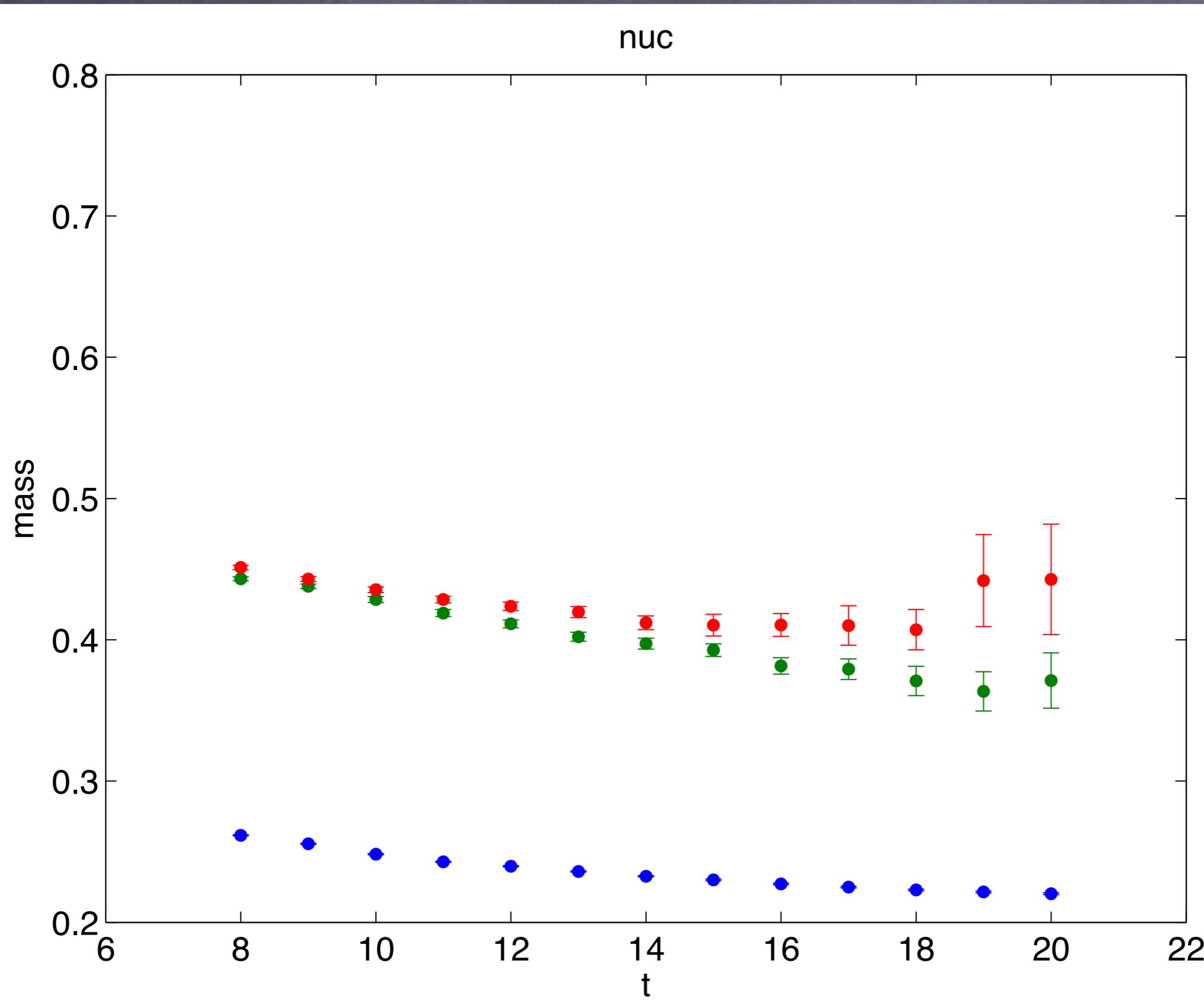
- $t_{\max}$  as large possible with  $\lambda(t)$  resolved
- Confidence level is checked

# Single Baryon spectrum

- Use smeared-smeared correlation functions
- Use a basis of all local independent interpolating fields
- JLab anisotropic lattices at 390MeV pion mass
- Compare GPOF with regular "variational"

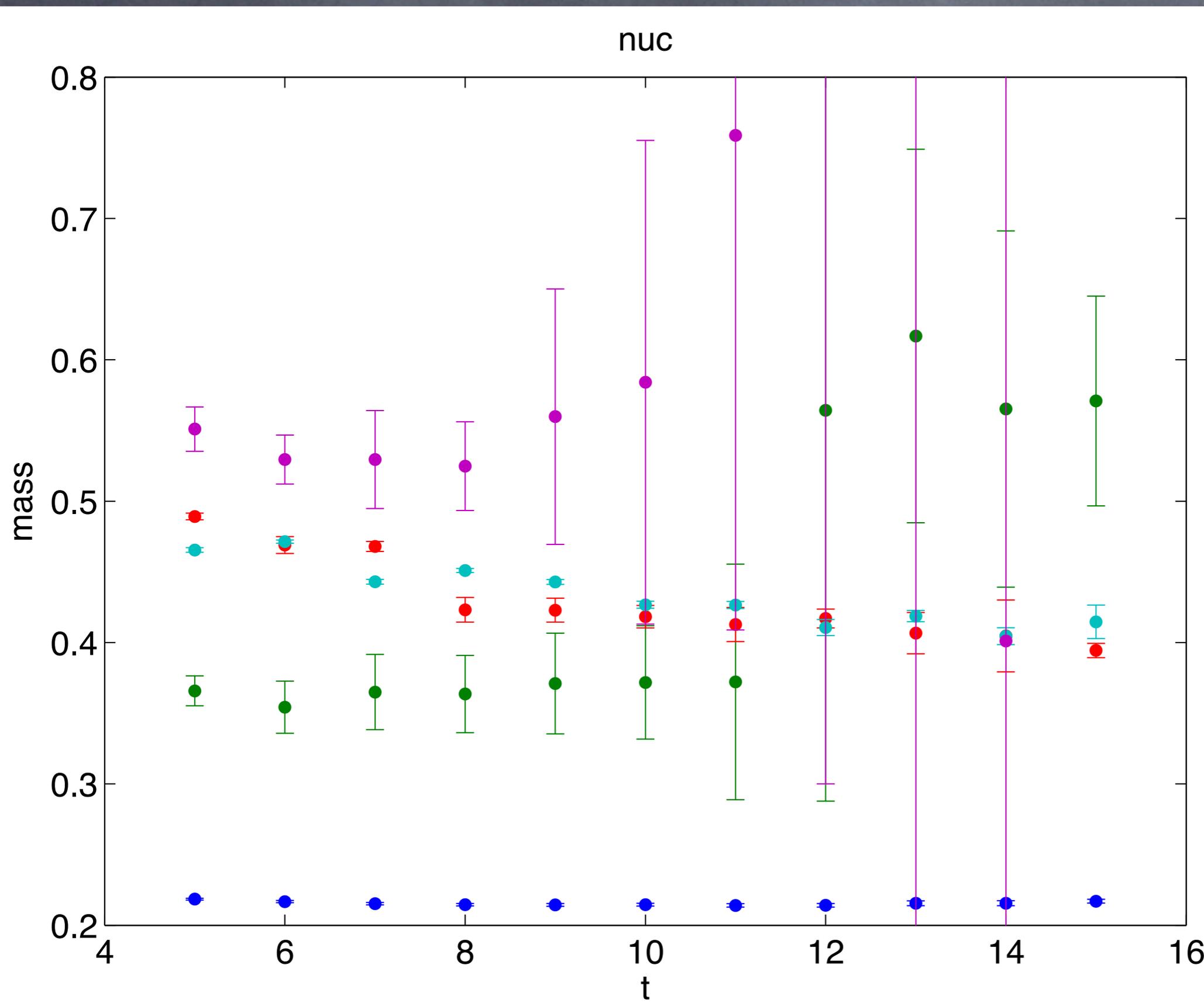
# Nucleon Mass

$16^3 \times 128$

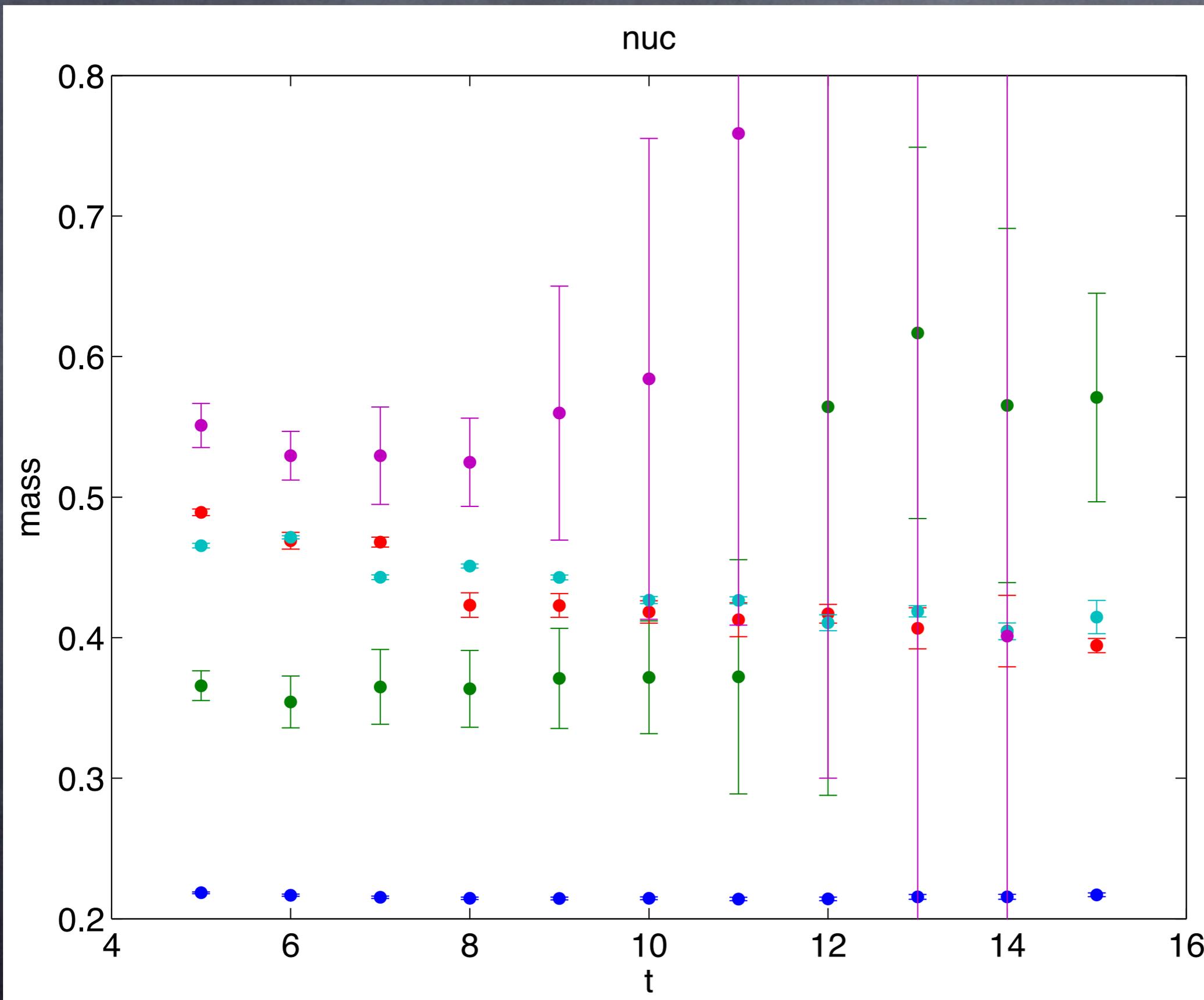


# Nucleon Mass

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# Nucleon Mass



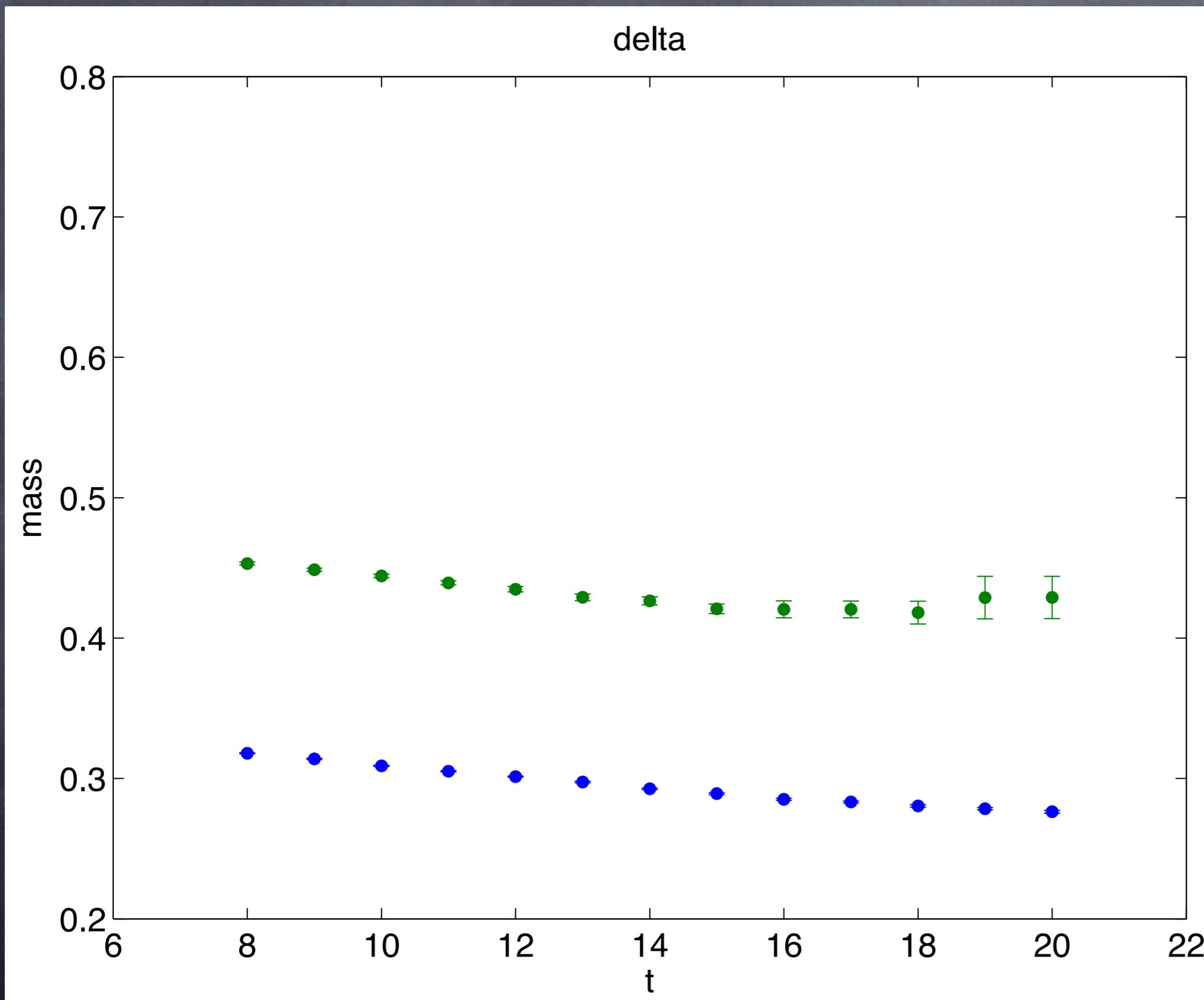
$16^3 \times 128$

2 source shifts  
2 sink shifts  
3 local operators

6x6 matrix

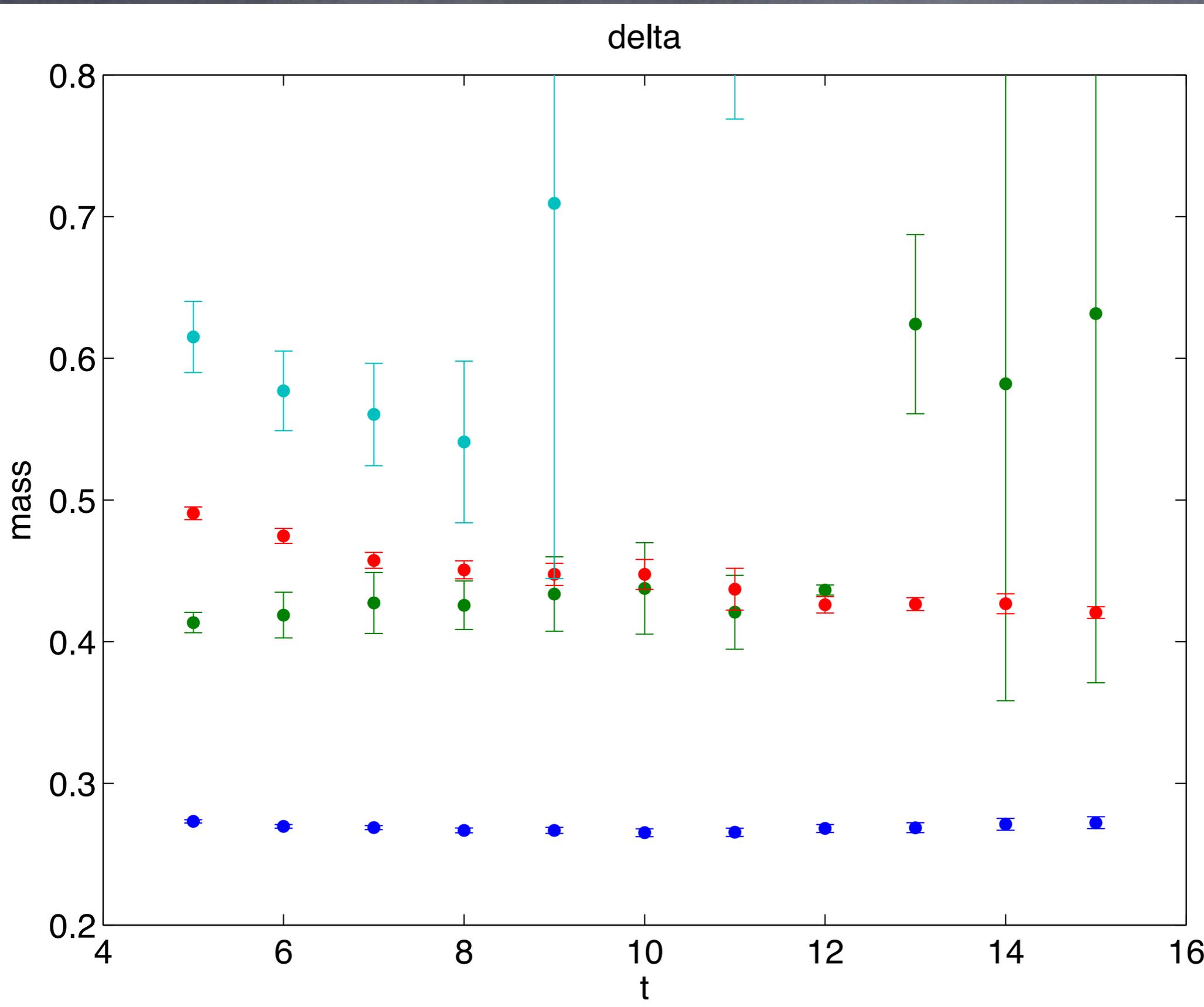
# Delta Mass

$16^3 \times 128$

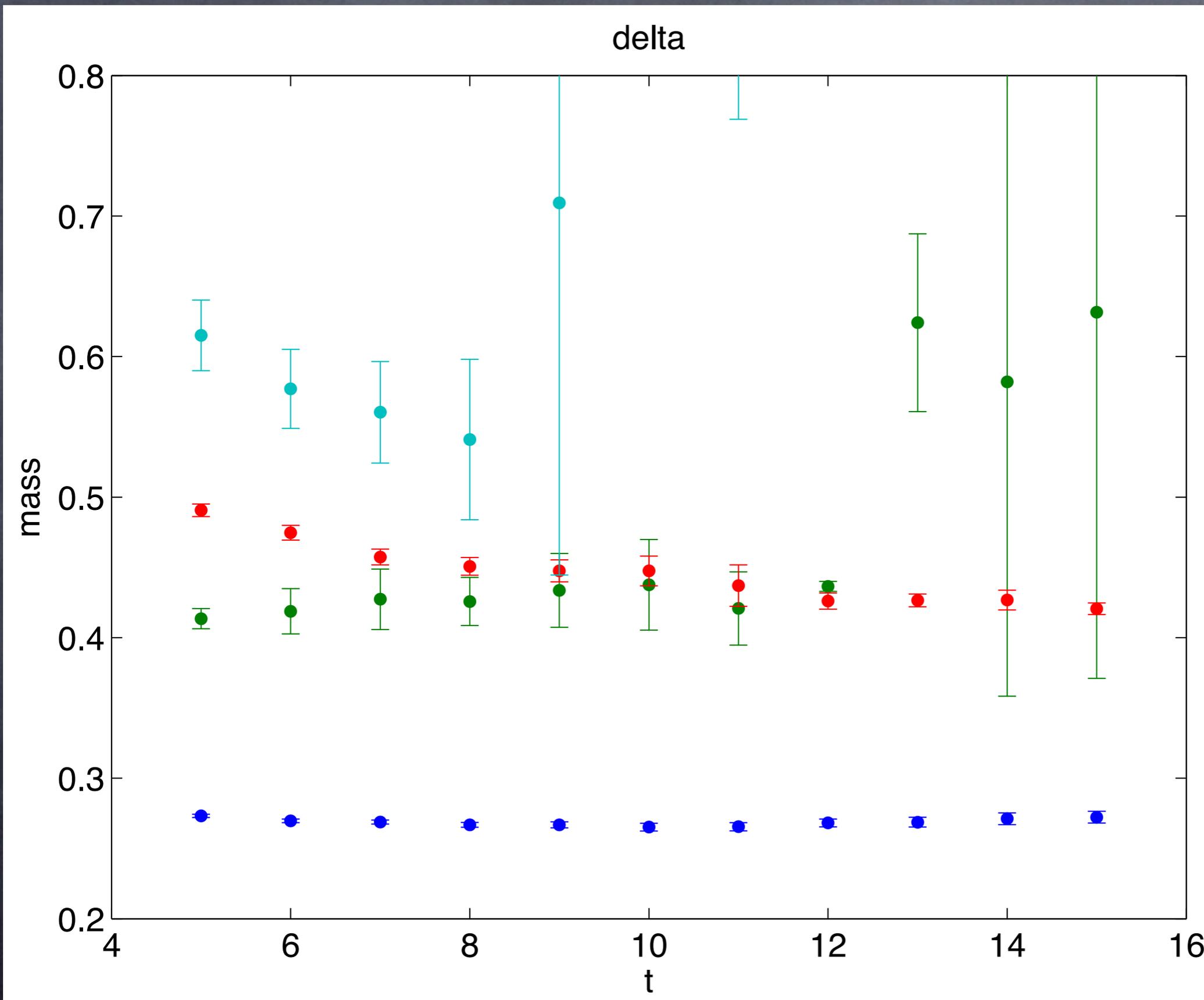


# Delta Mass

$16^3 \times 128$



# Delta Mass



$16^3 \times 128$

2 source shifts  
2 sink shifts  
2 local operators

4x4 matrix

# Two Baryon Correlation functions

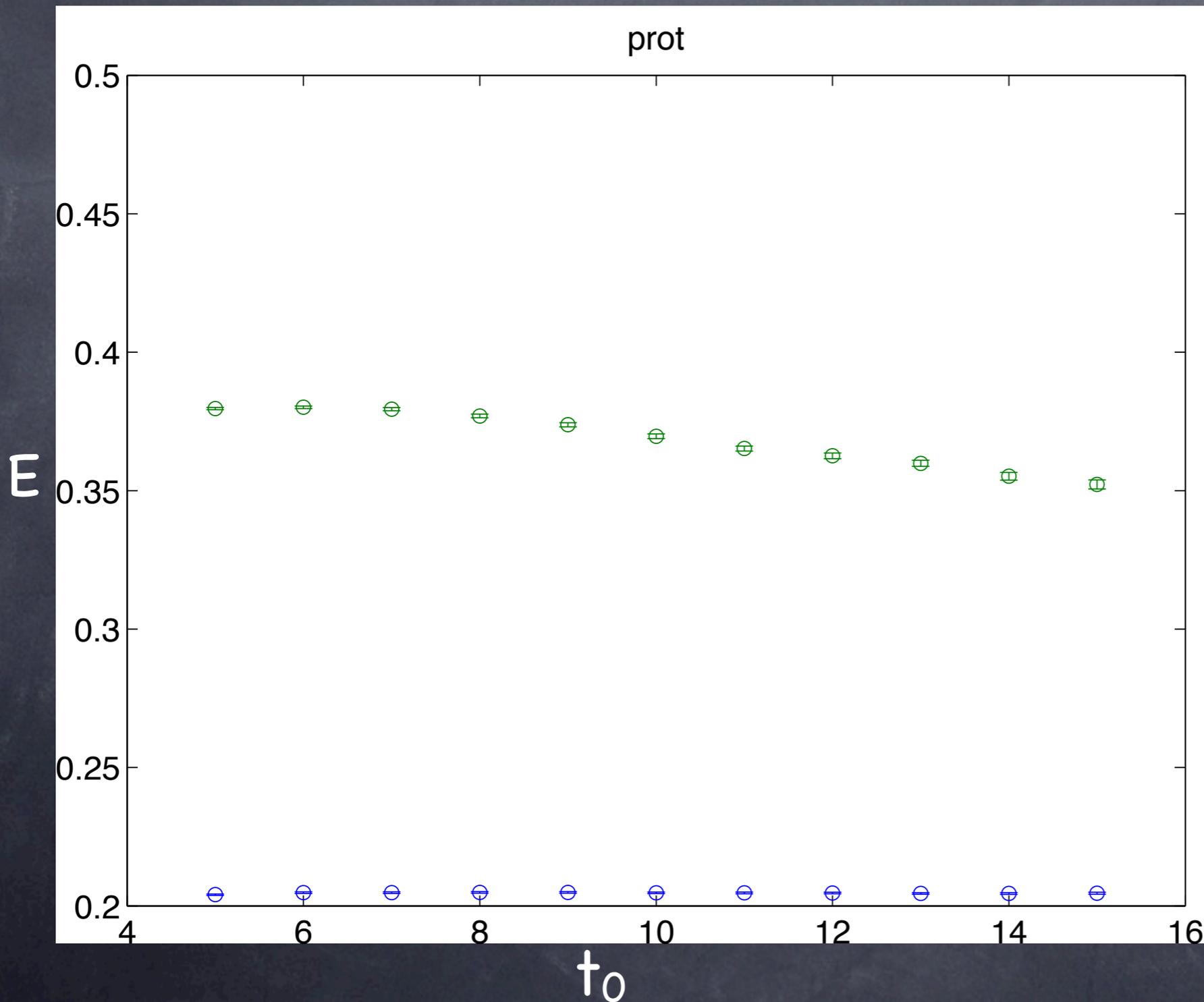


- Single smeared quark source
- Multiple sink interpolating fields
  - Smeared, Point and Smeared-Point
- Resulting  $G$  is a  $3 \times 1$  matrix
- No-need for all-to-all propagators
- Very high statistics (300K correlation functions on 2K lattices)

NPLQCD data

# Single Baryon

NPLQCD data



single operator

one source smearing

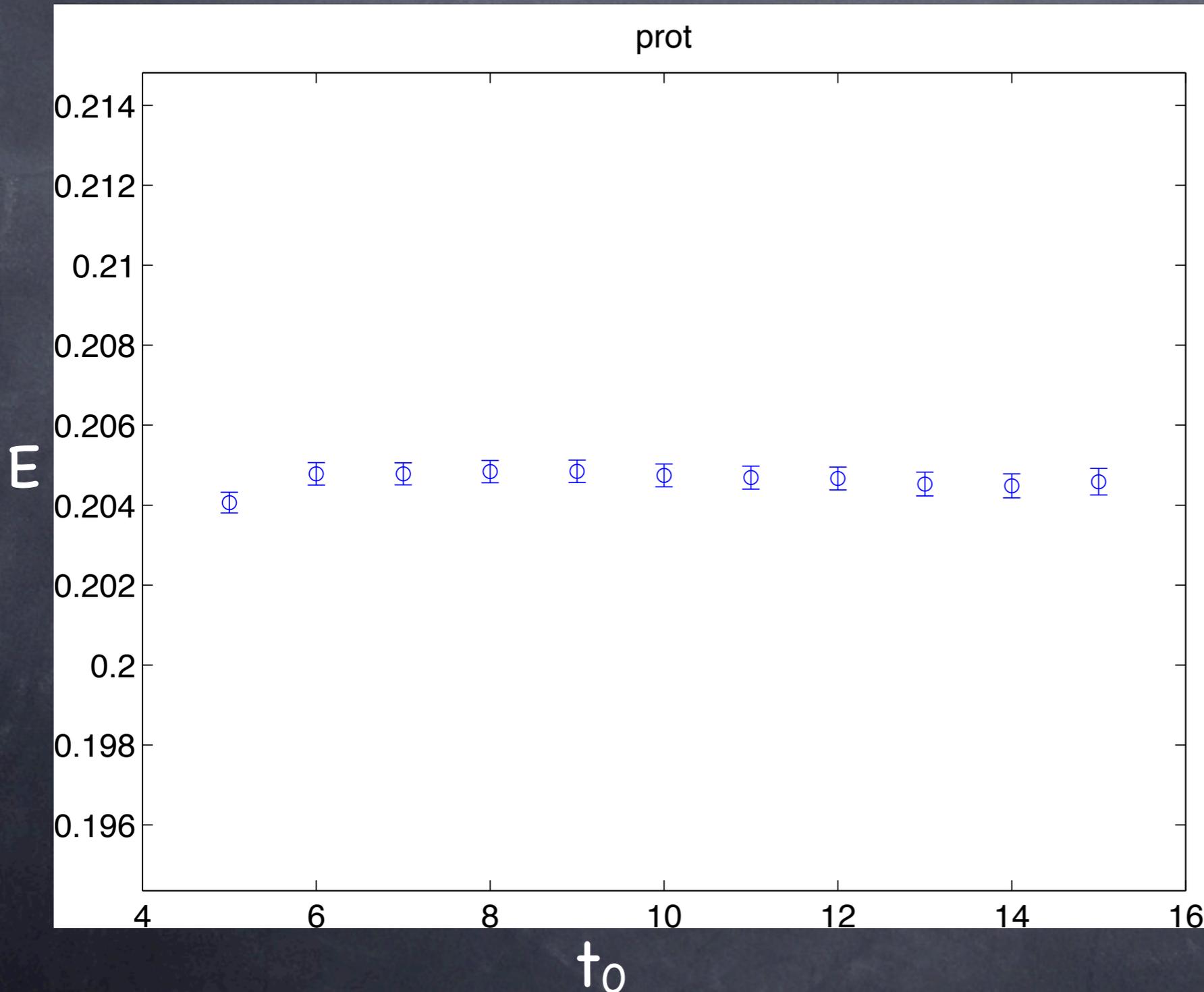
smeared and point sink

two source shifts

2x2 matrix

# Single Baryon

NPLQCD data



single operator

one source smearing

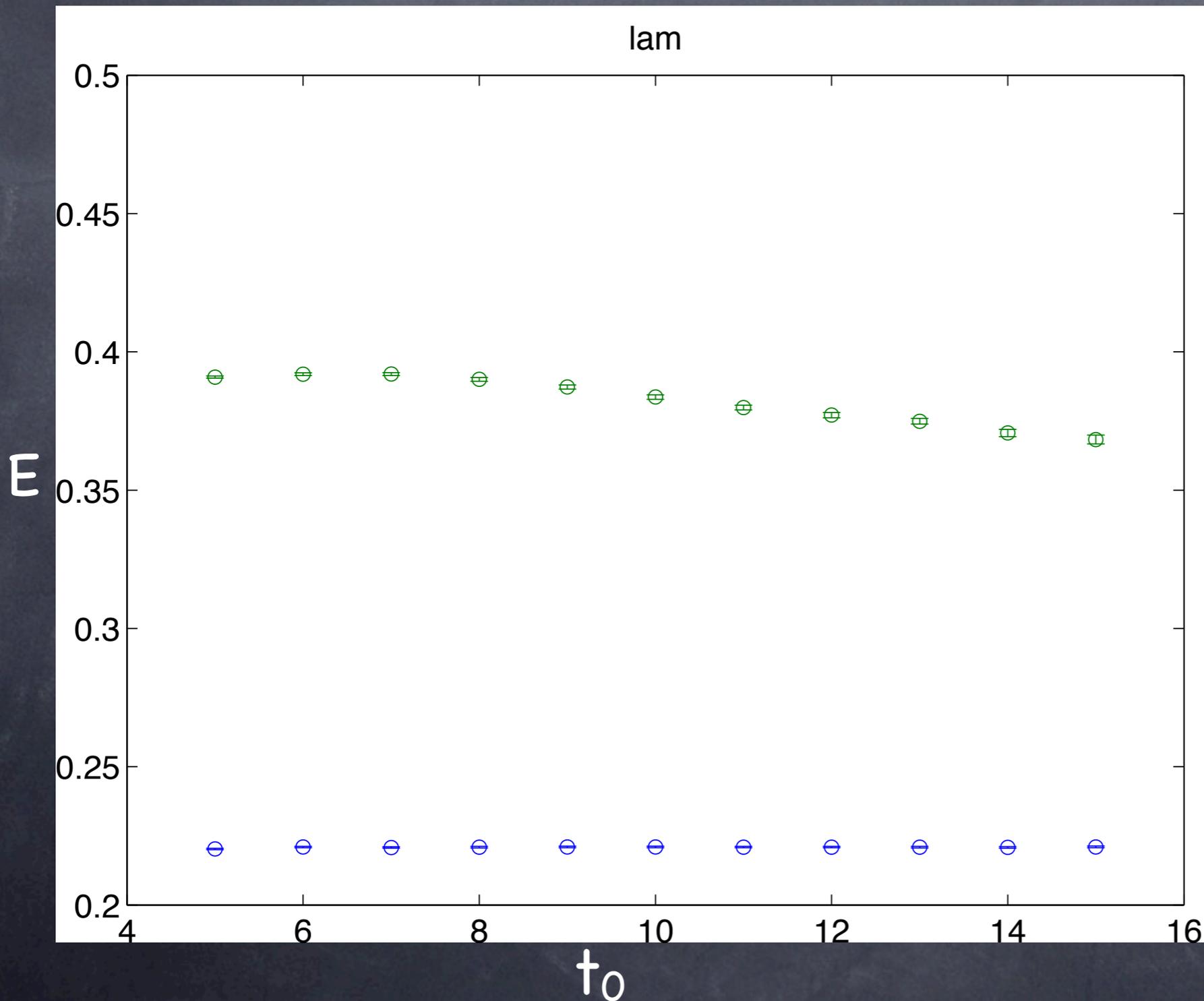
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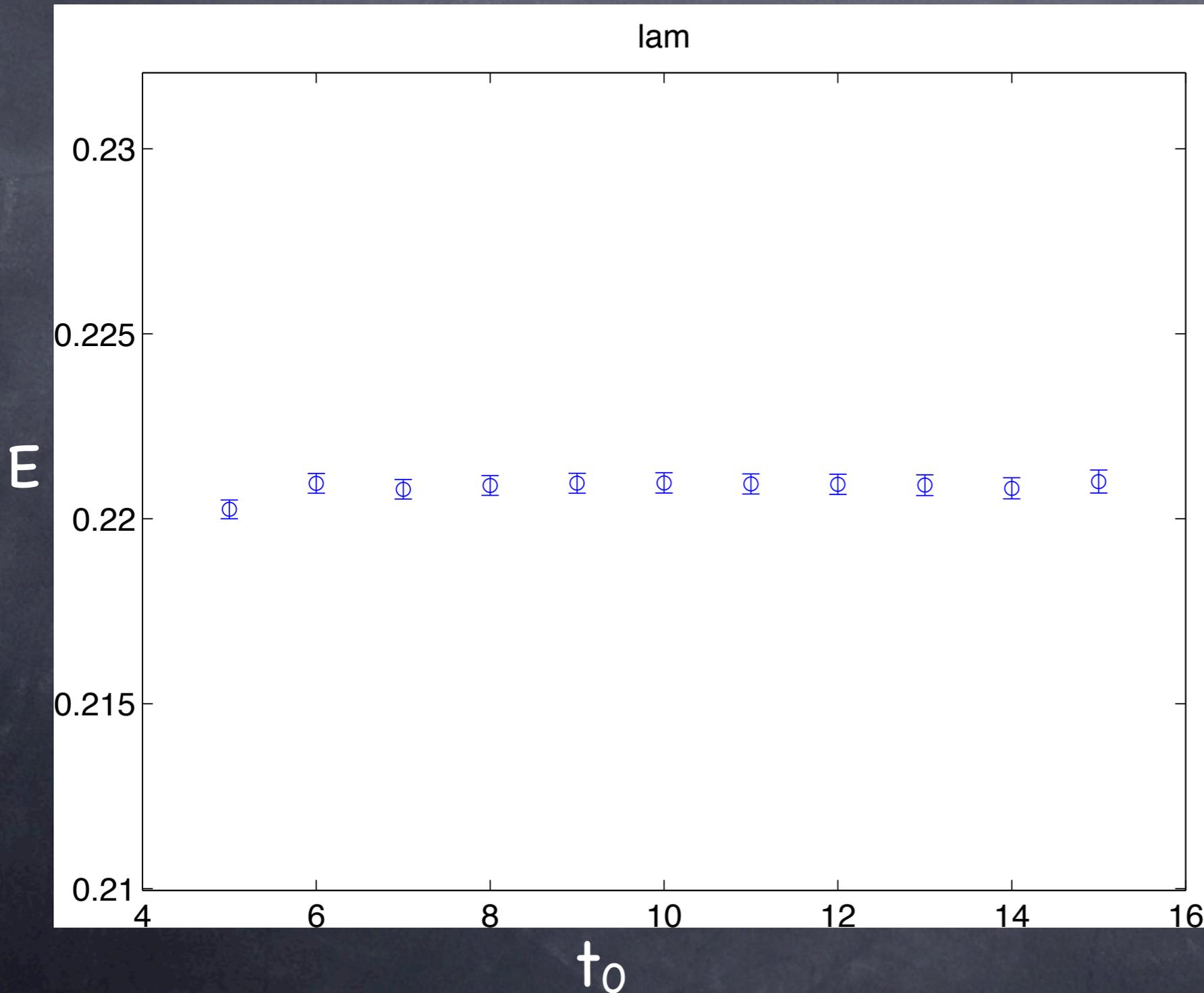
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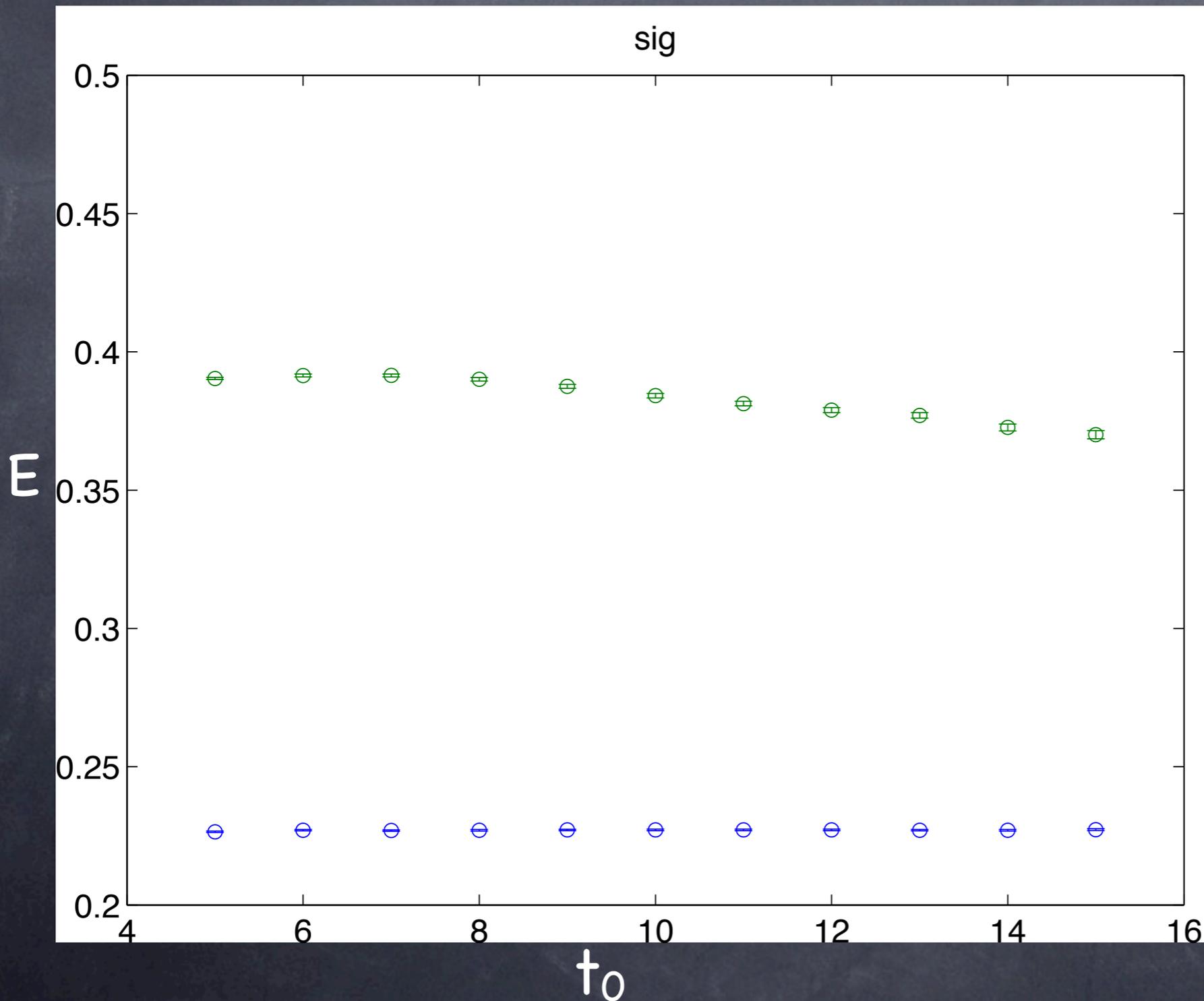
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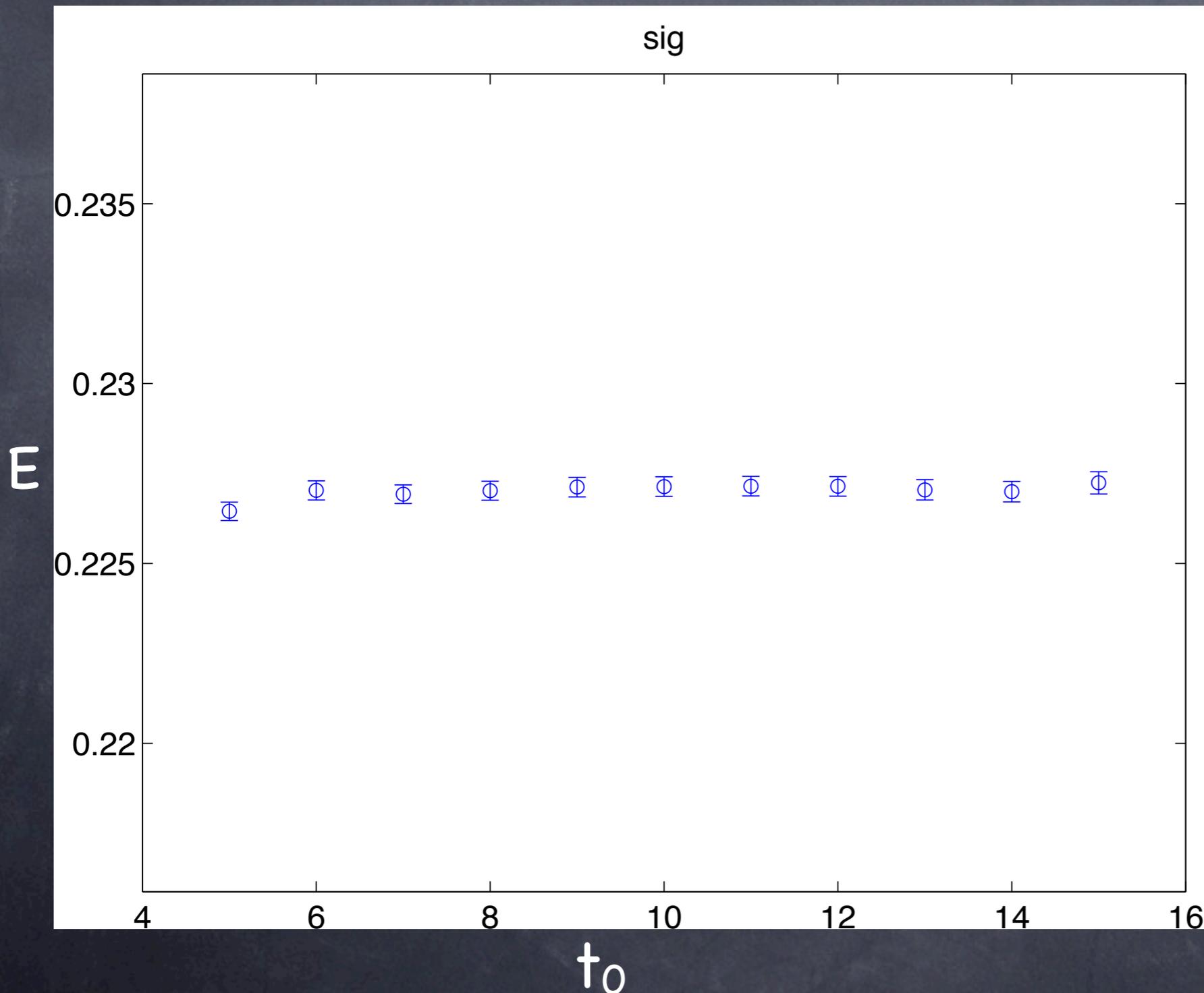
smearing and point sink

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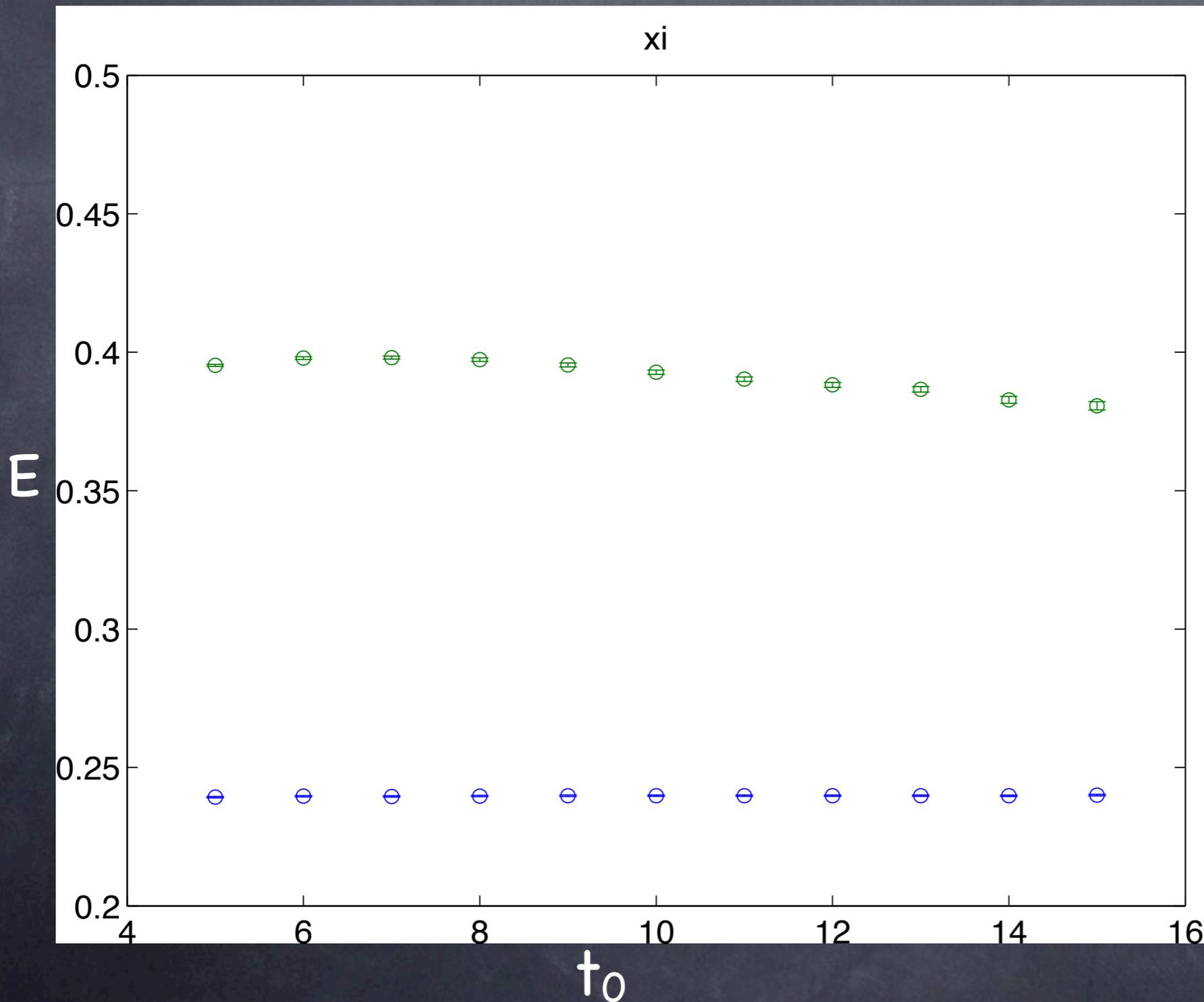
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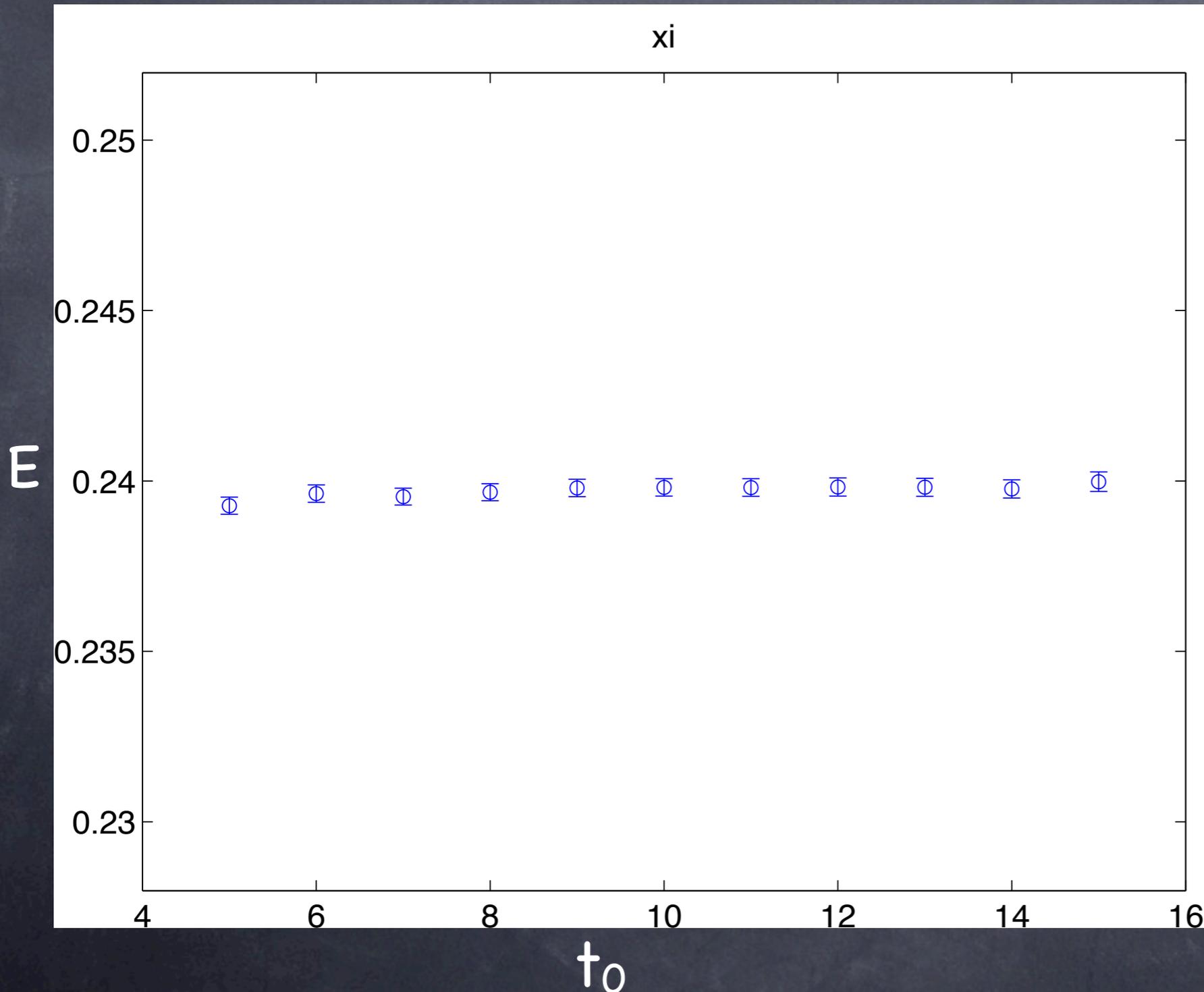
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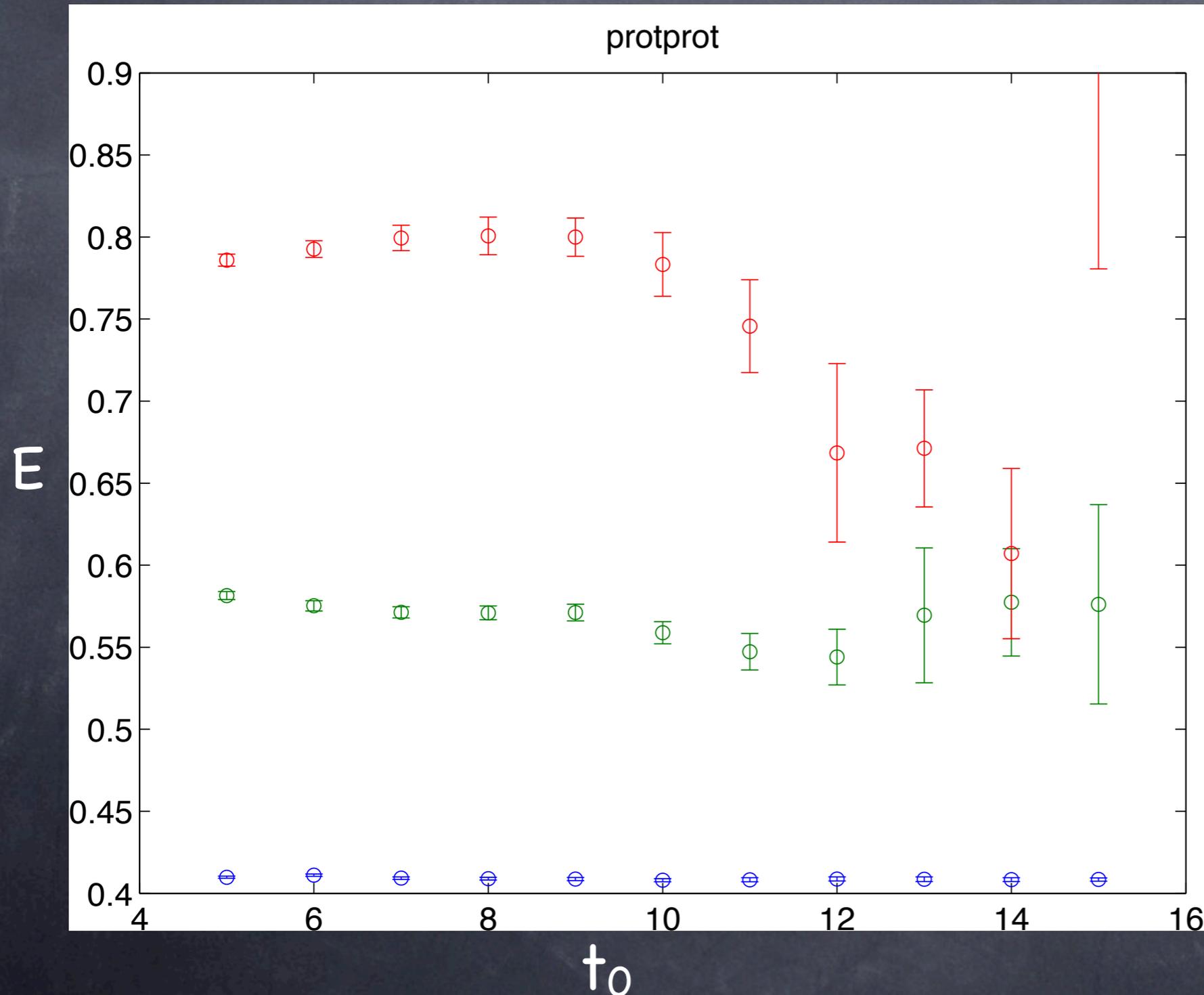
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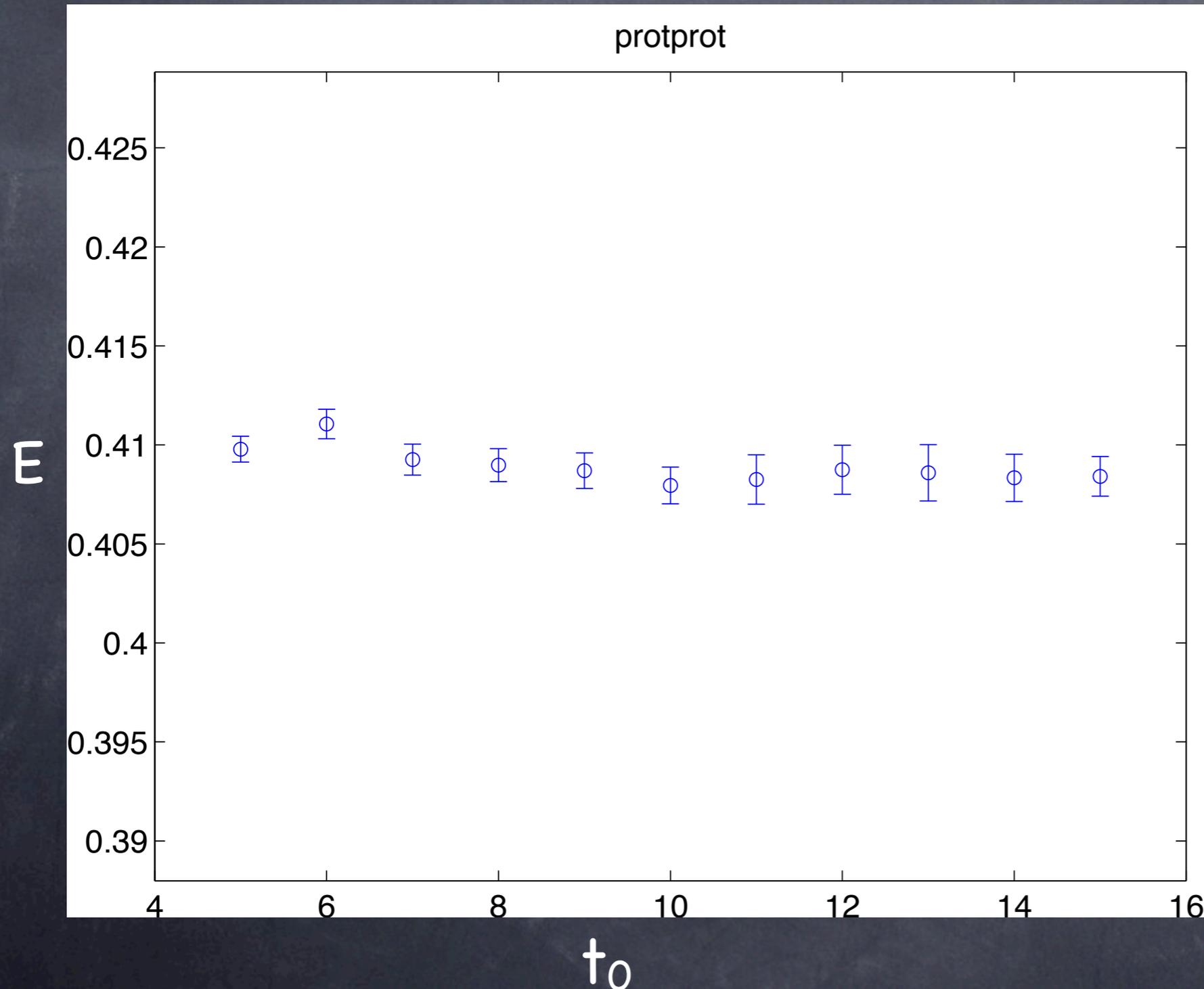
smeared, point and  
smeared-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

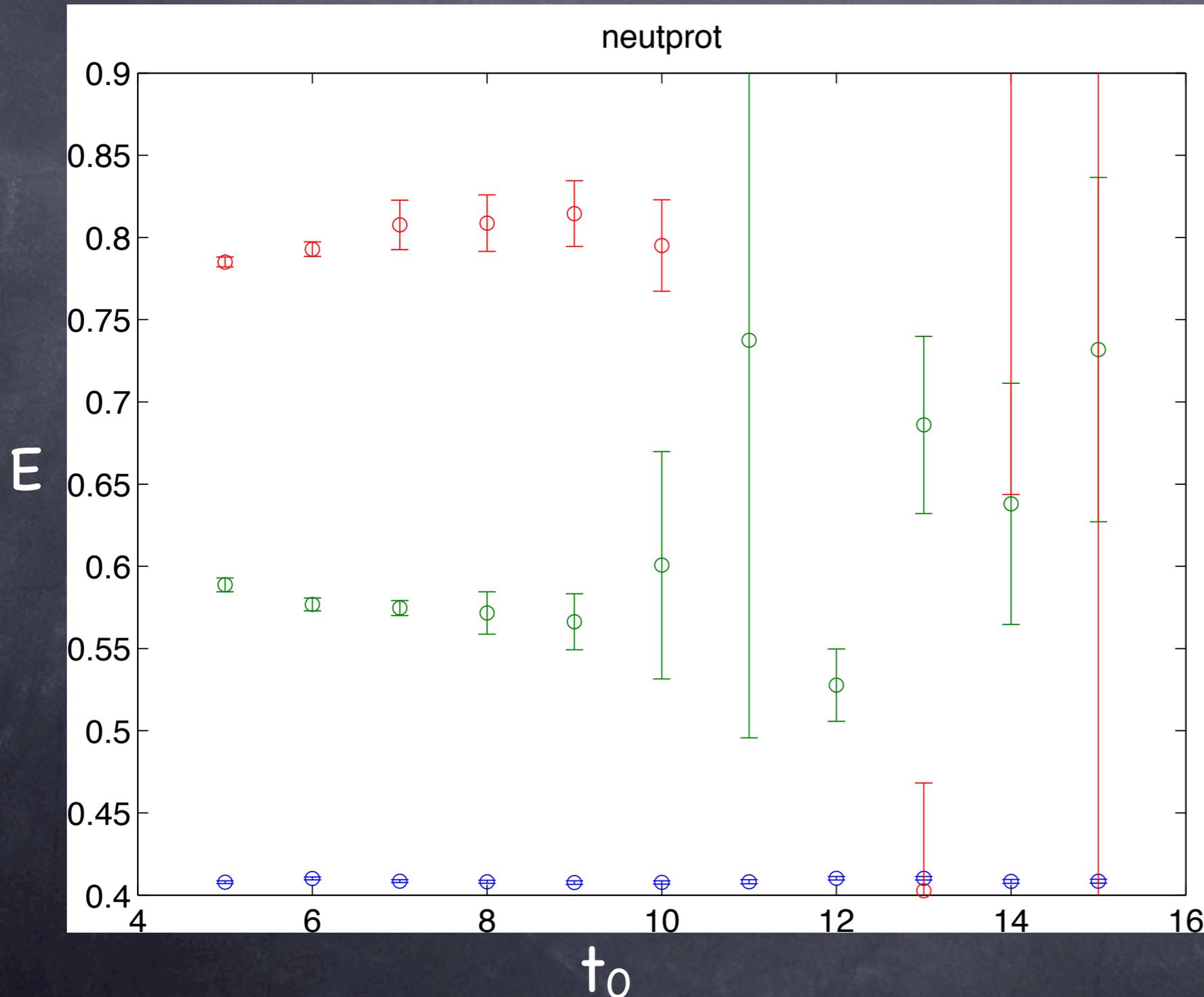
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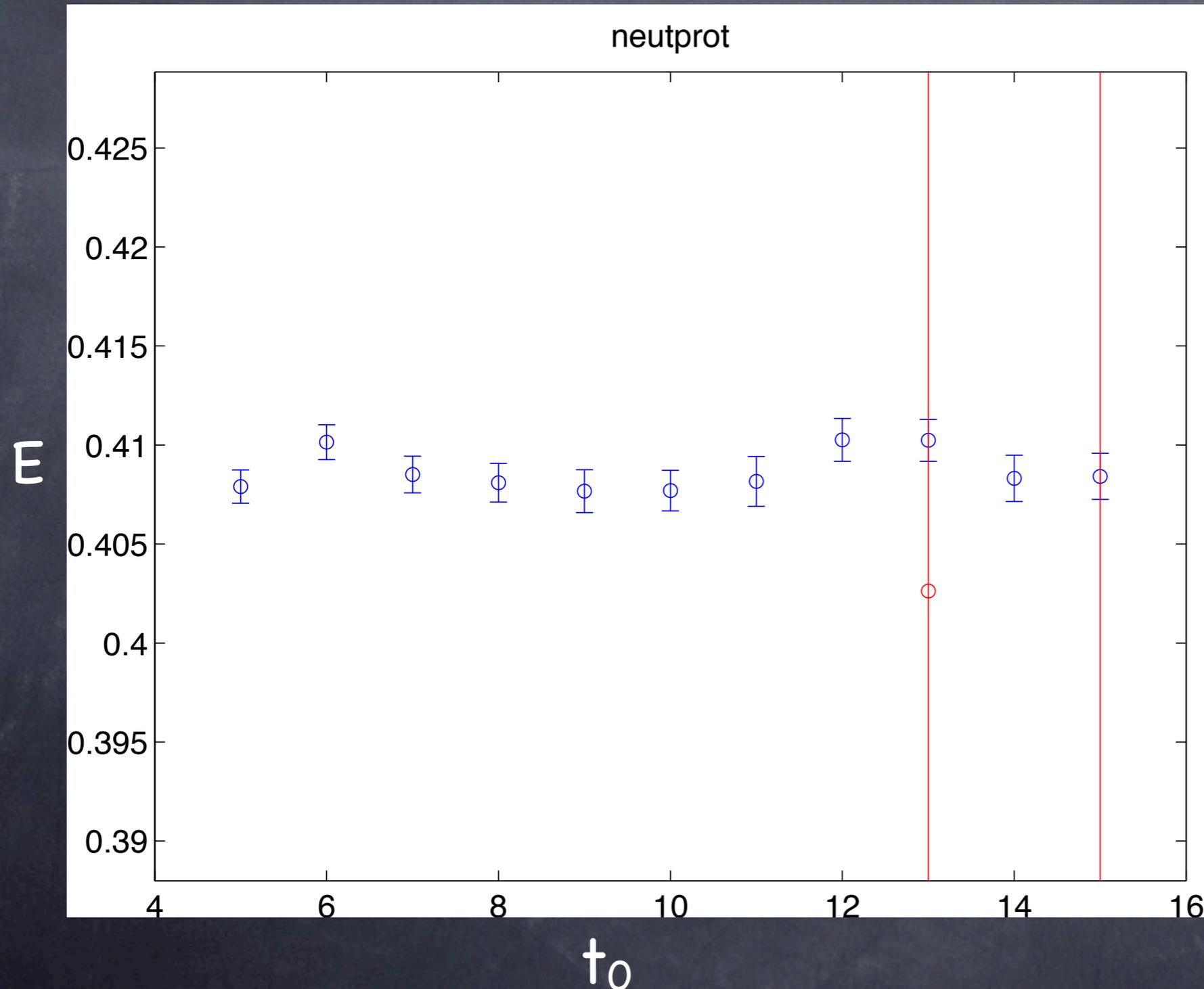
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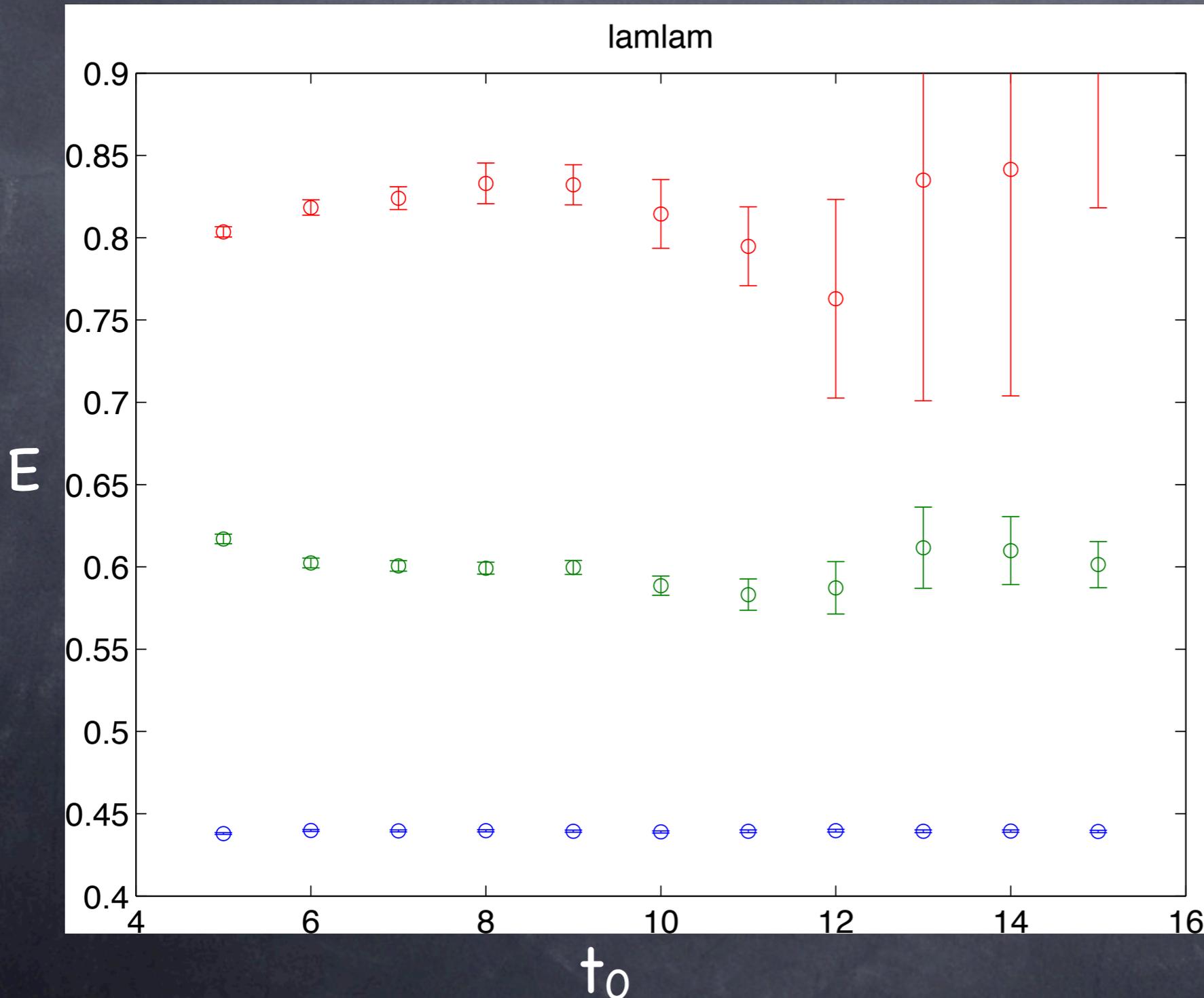
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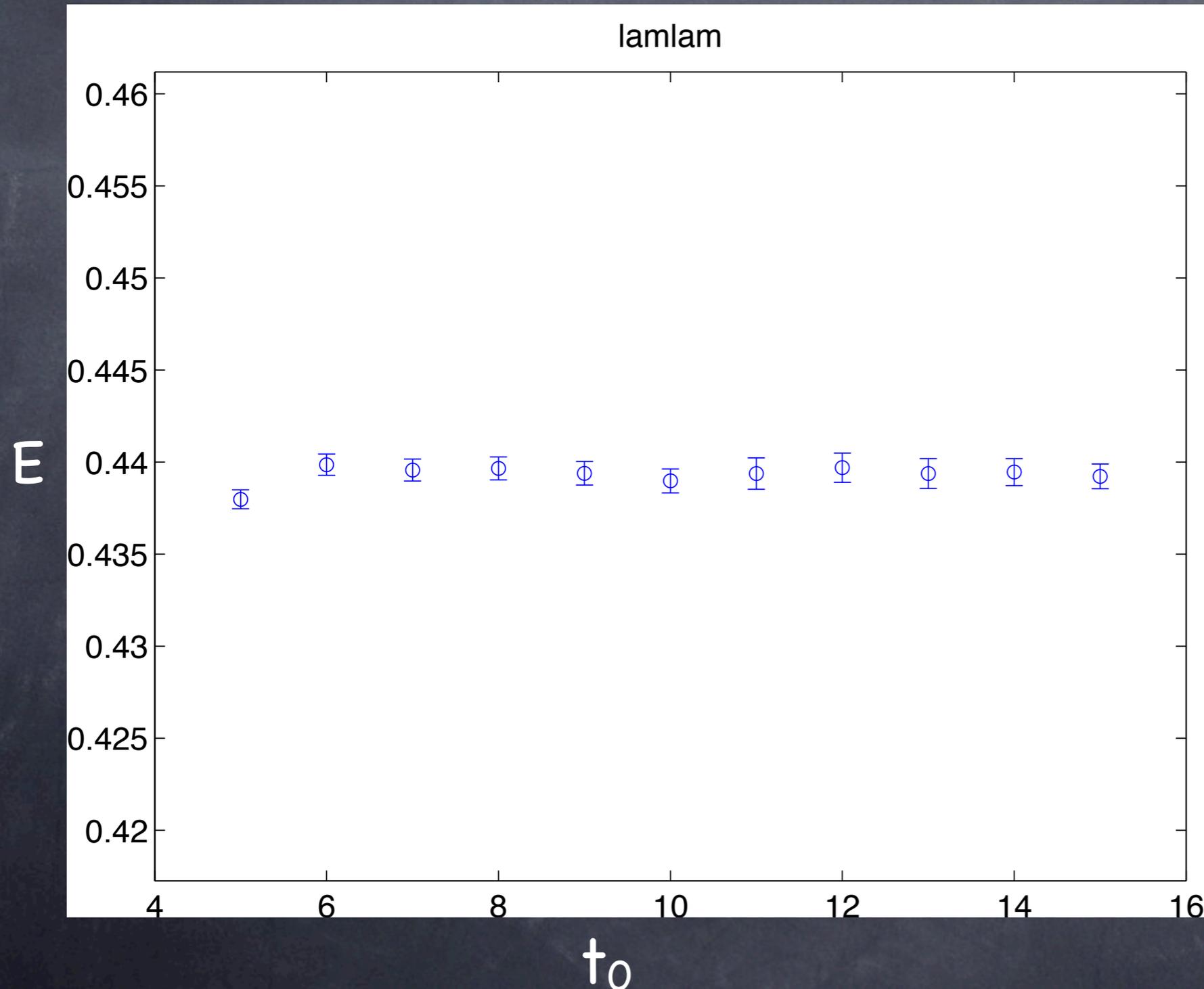
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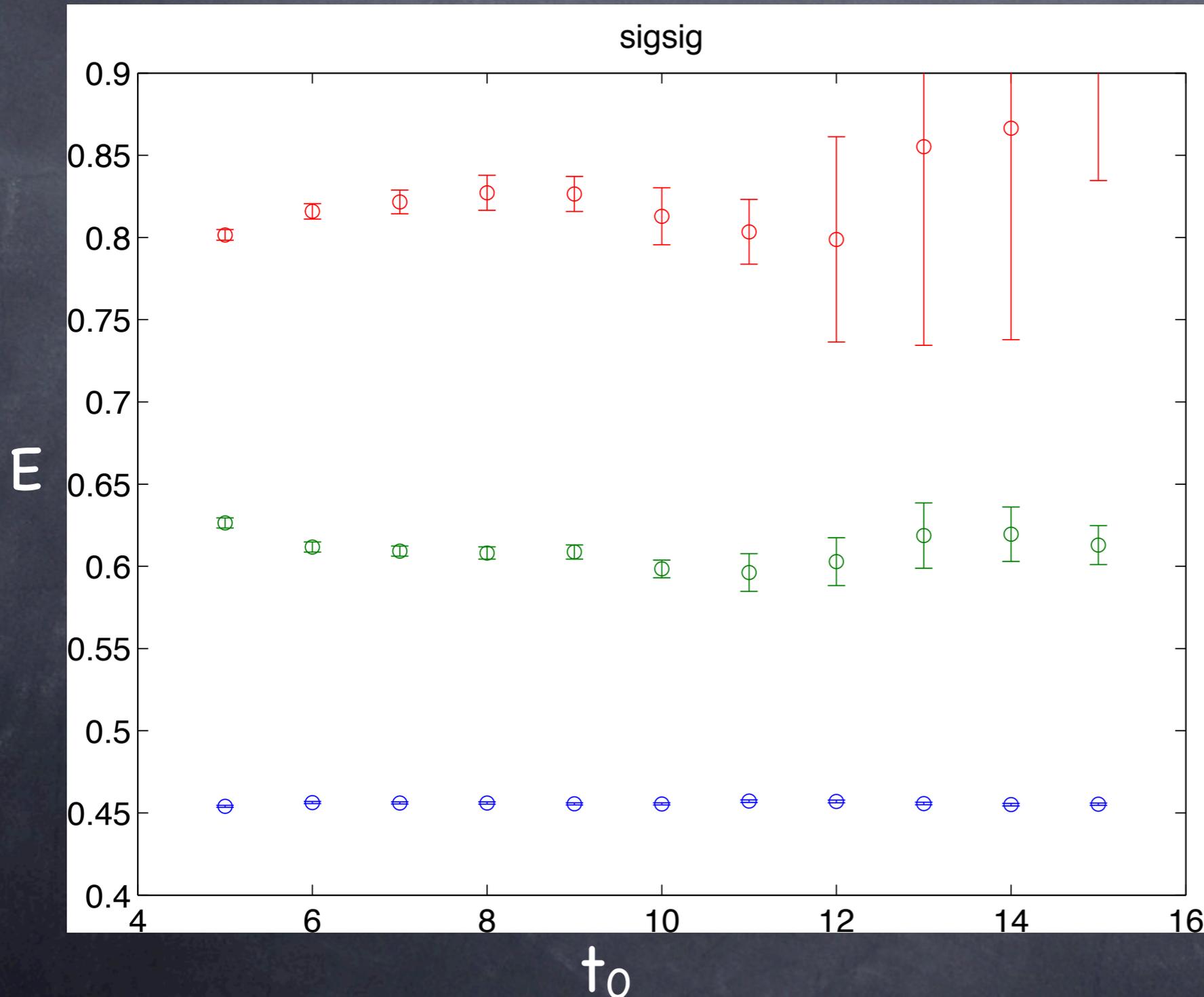
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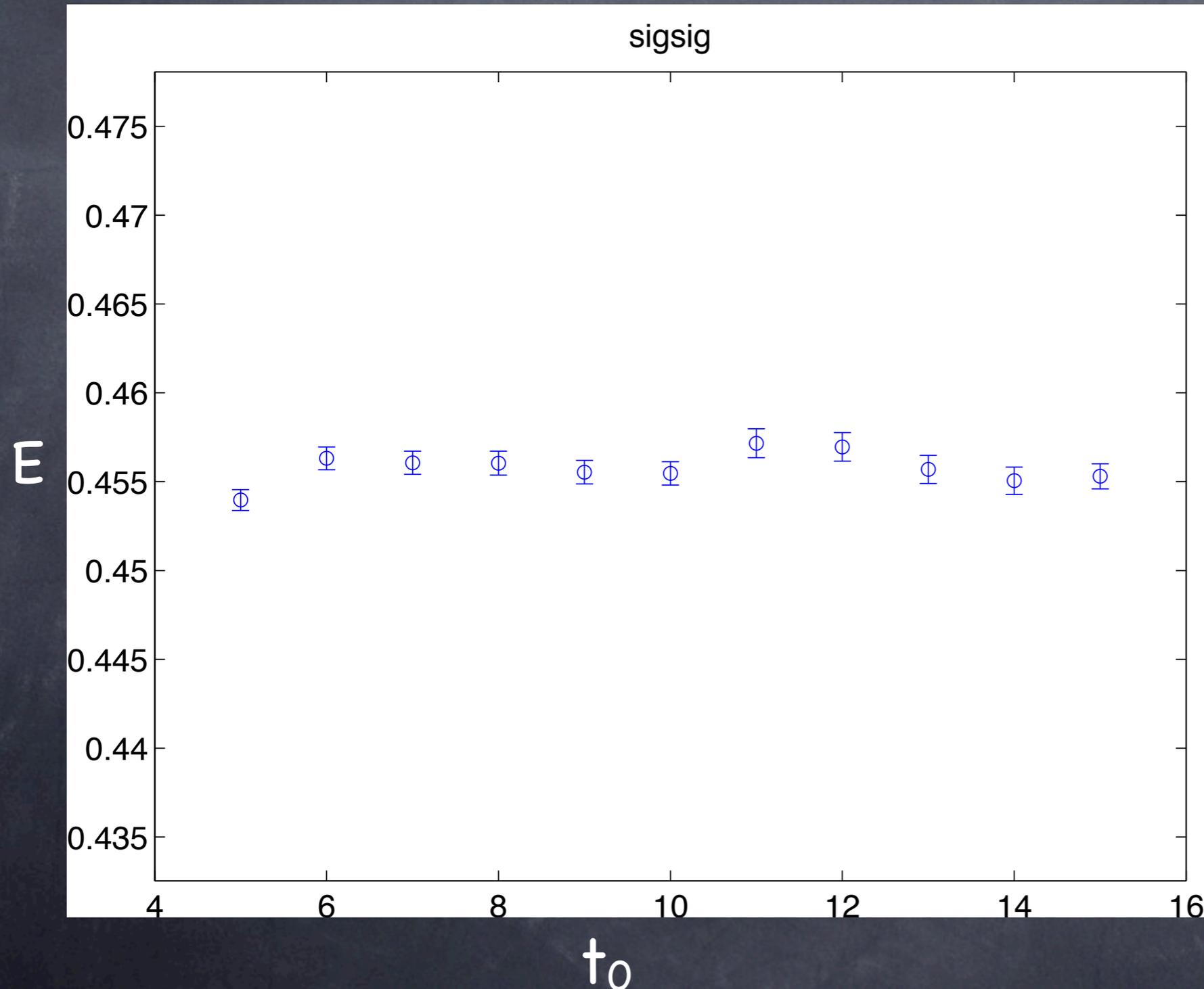
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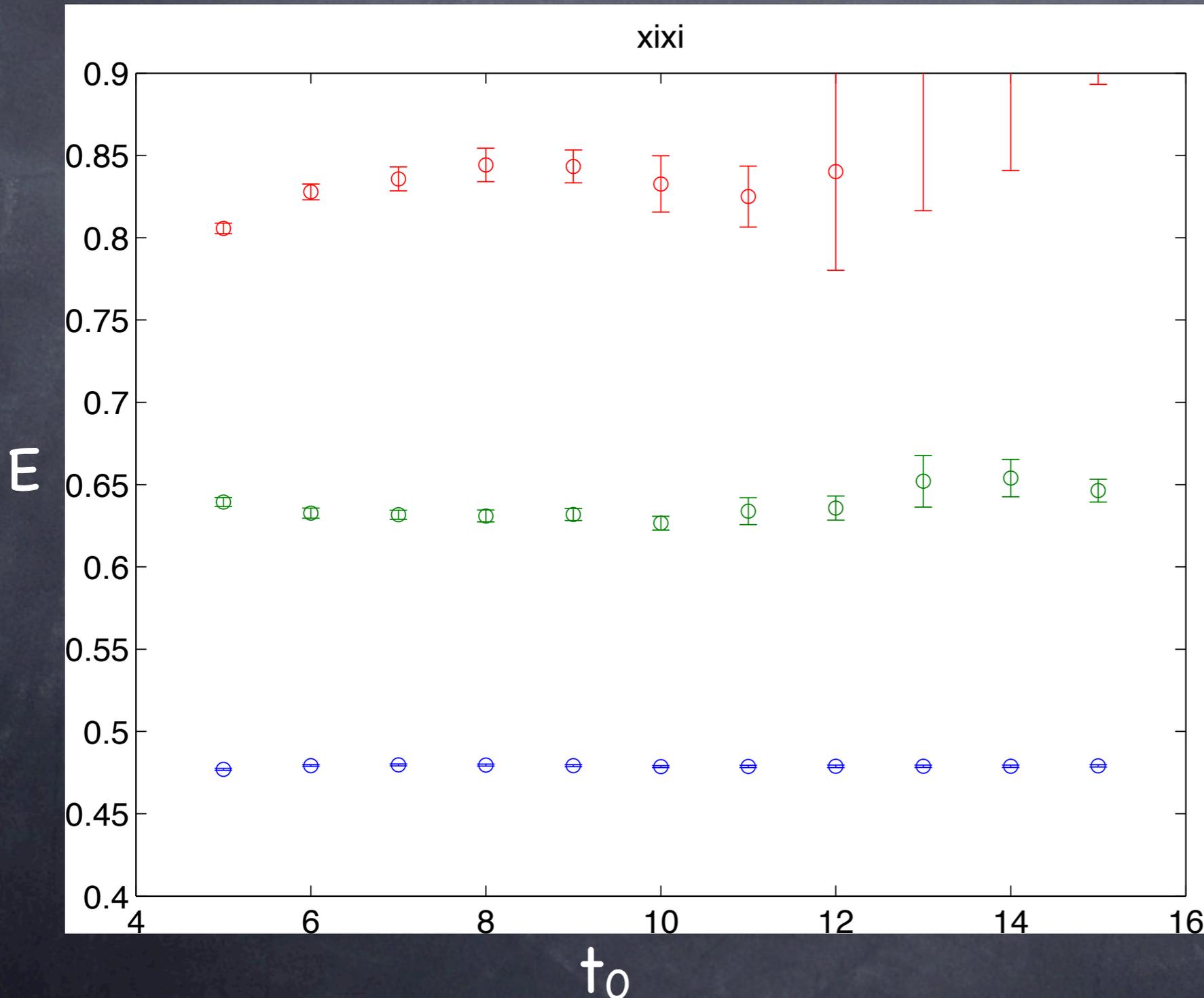
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# Two Baryons

NPLQCD data



single operator  
one source smearing

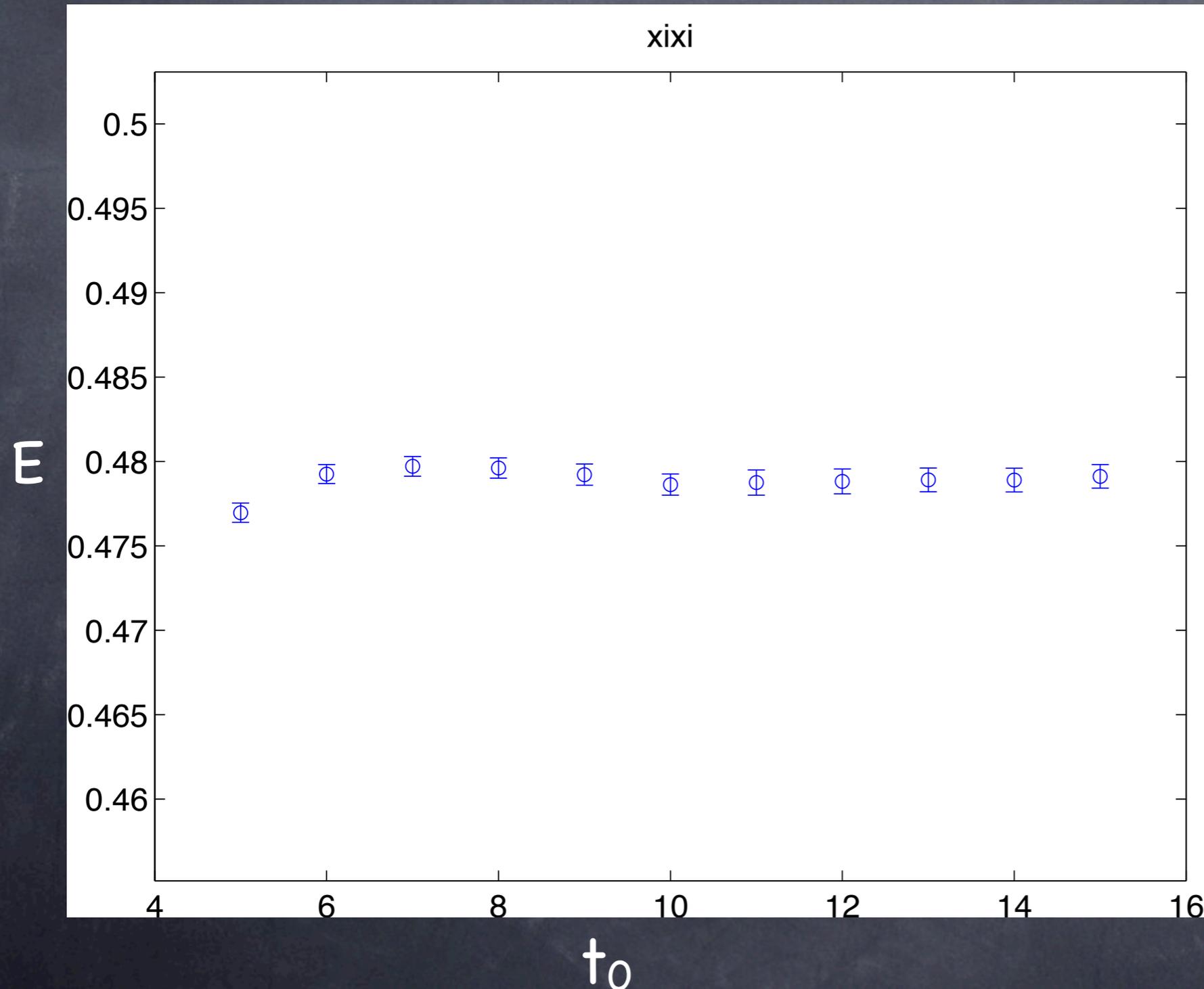
smeared, point and  
smeared-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

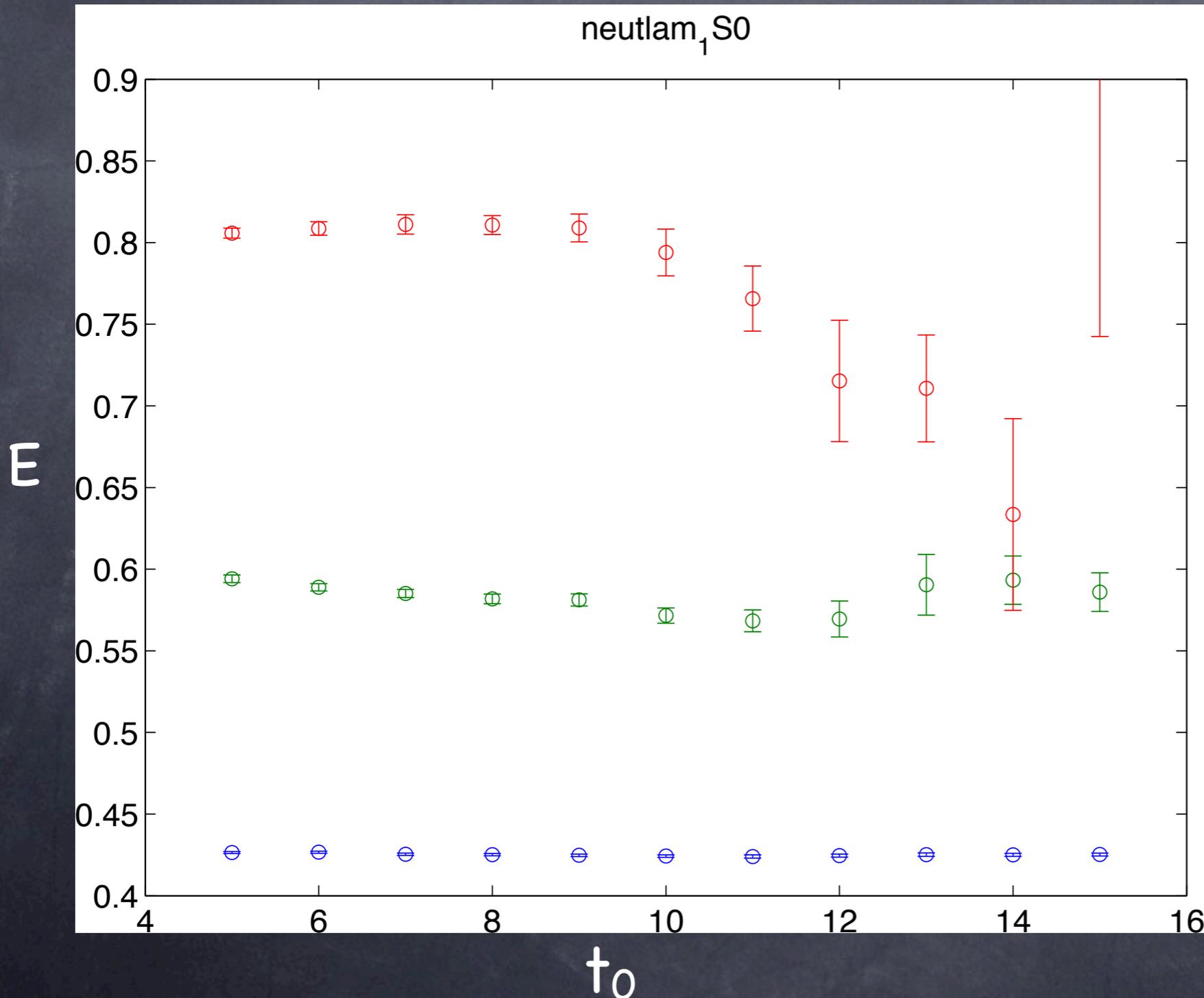
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

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NPLQCD data



single operator

one source smearing

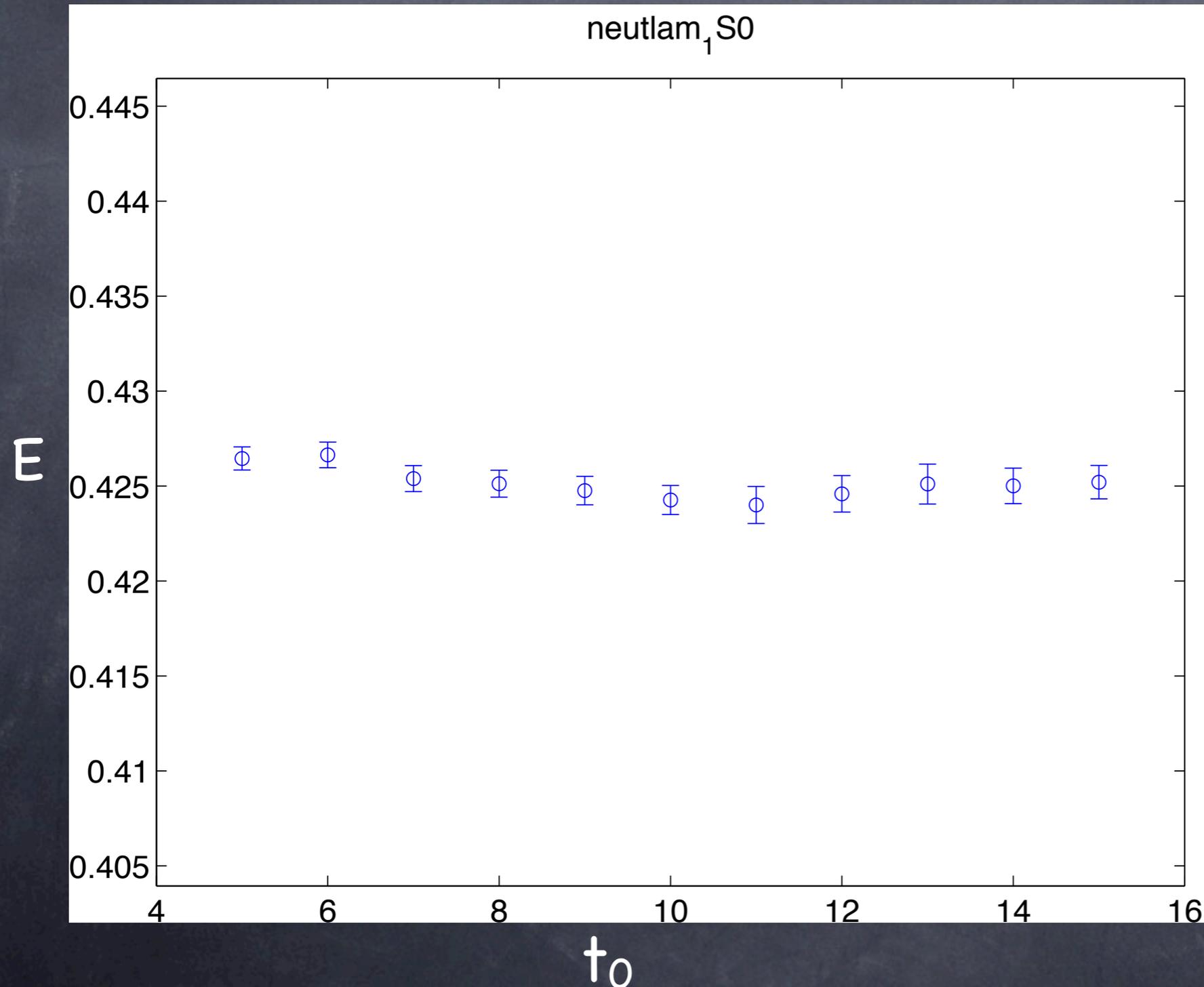
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

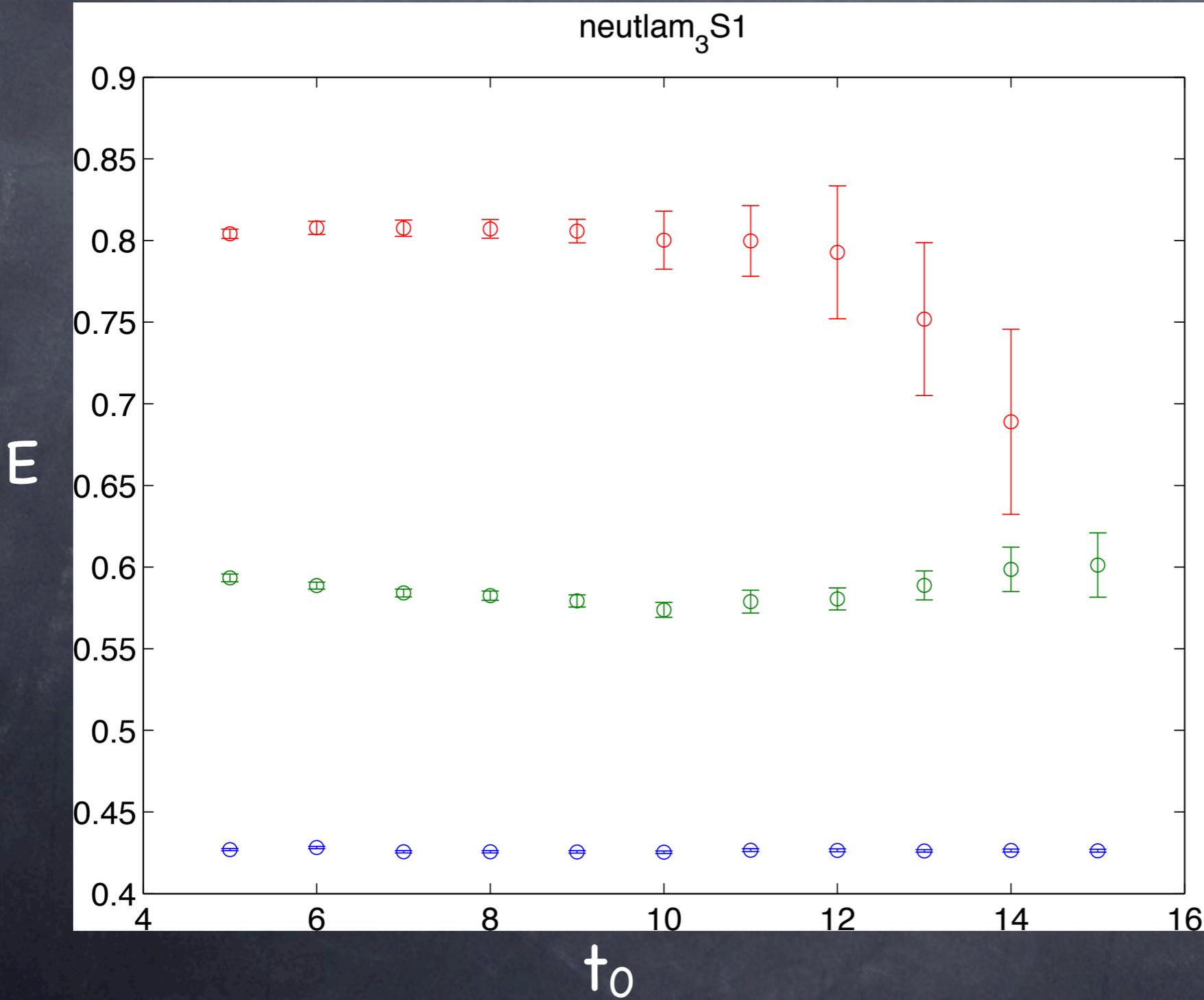
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Two Baryons

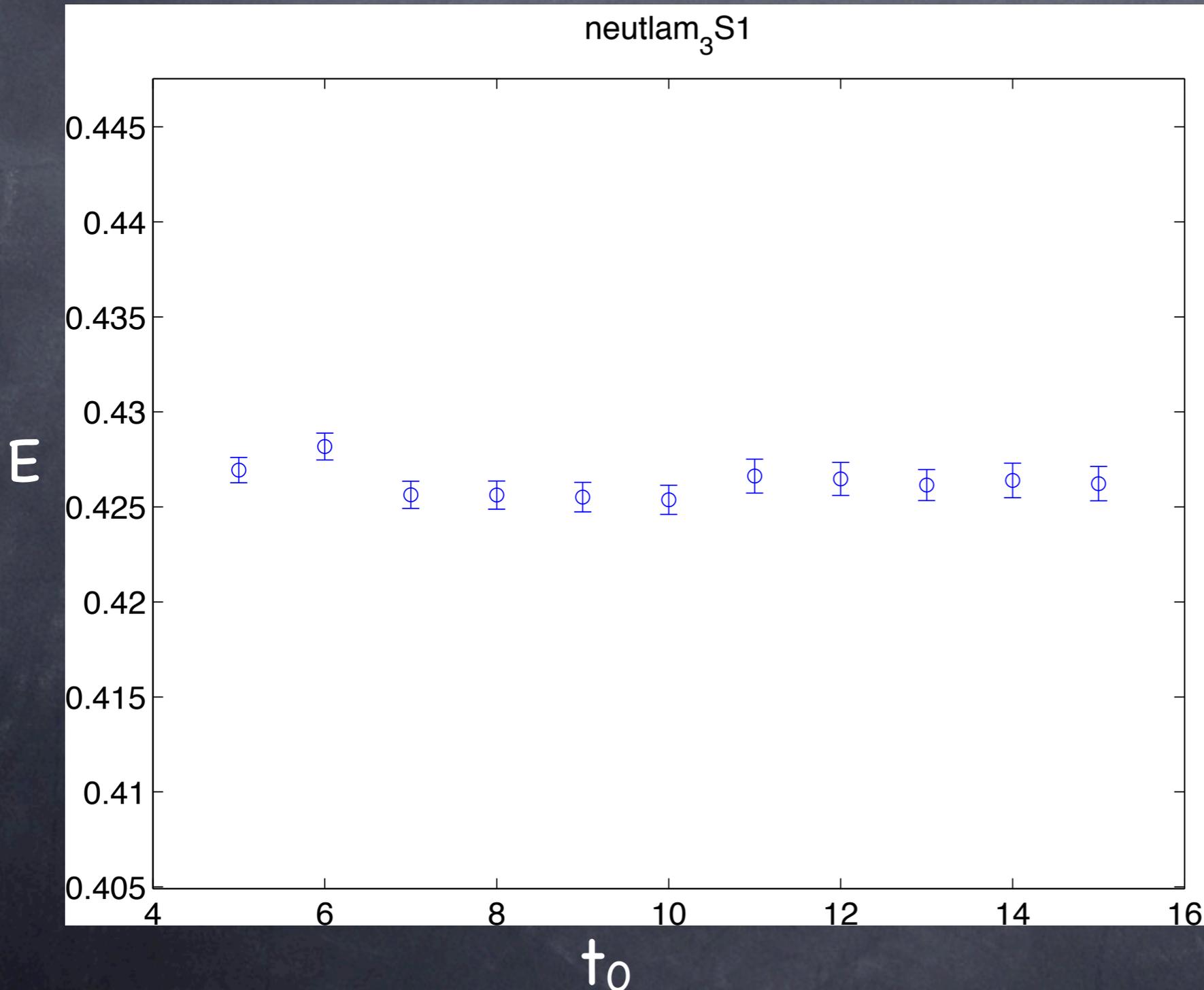
NPLQCD data



single operator  
one source smearing  
smeared, point and  
smeared-point sink  
three source shifts  
3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

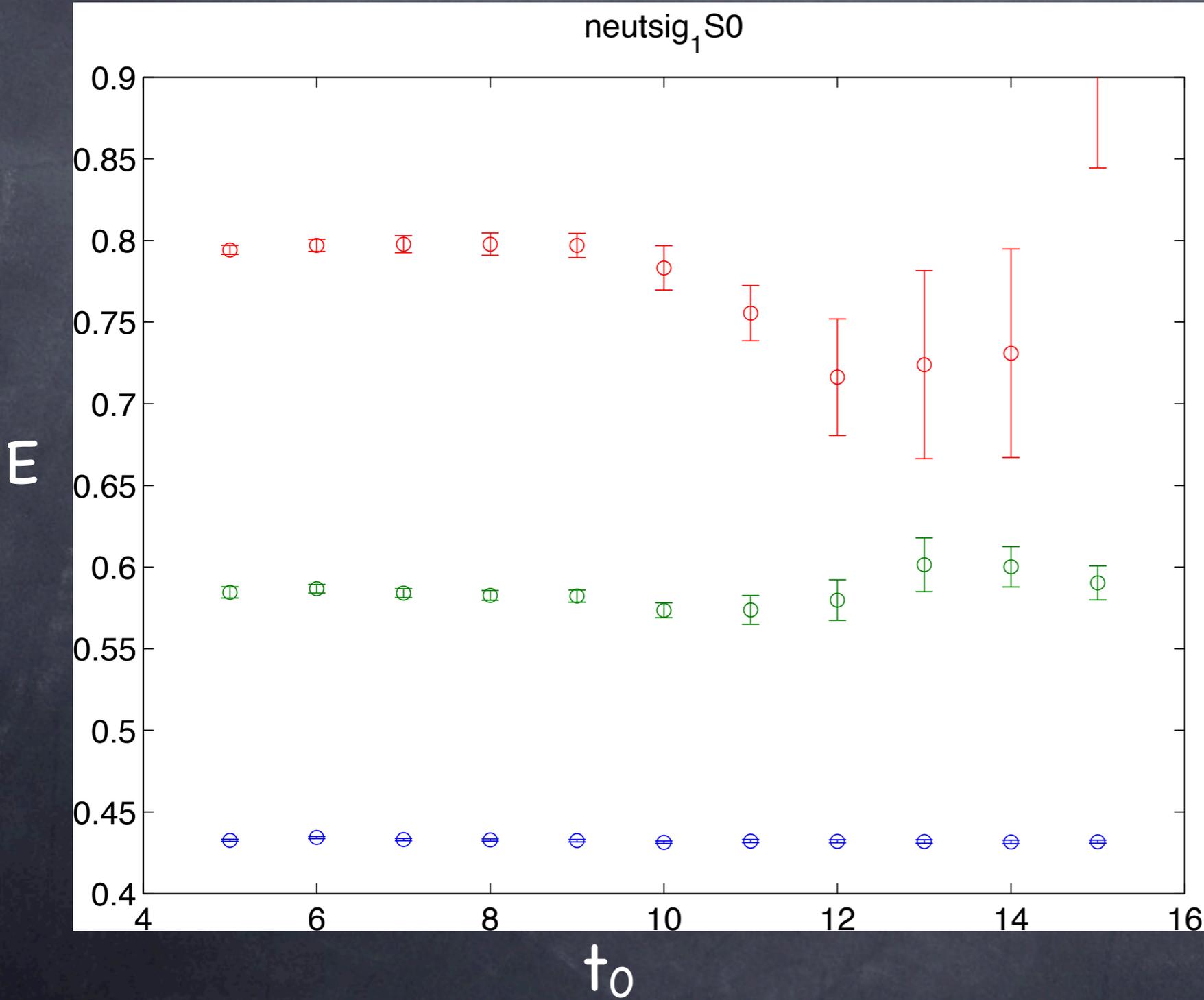
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

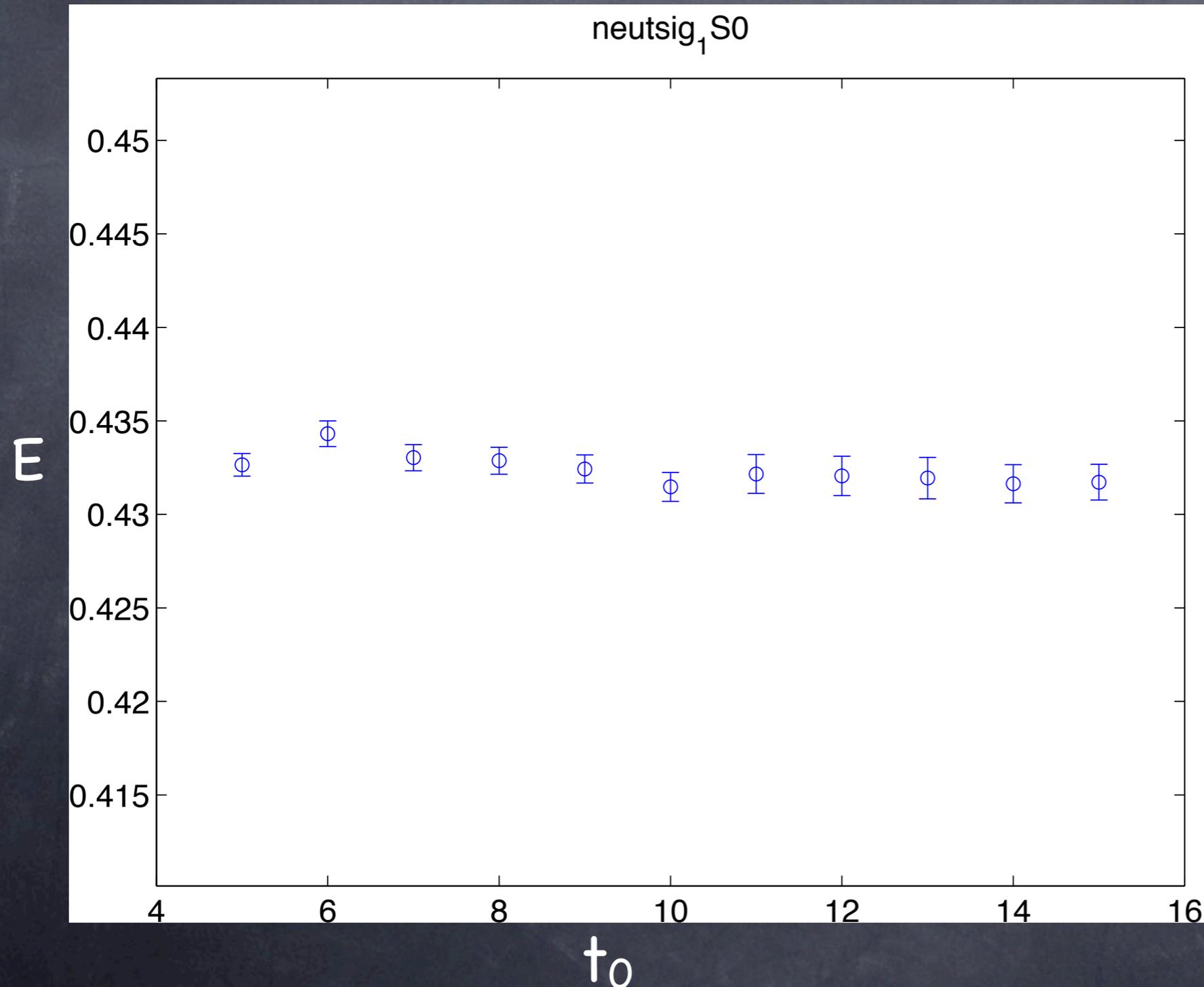
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

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NPLQCD data



single operator

one source smearing

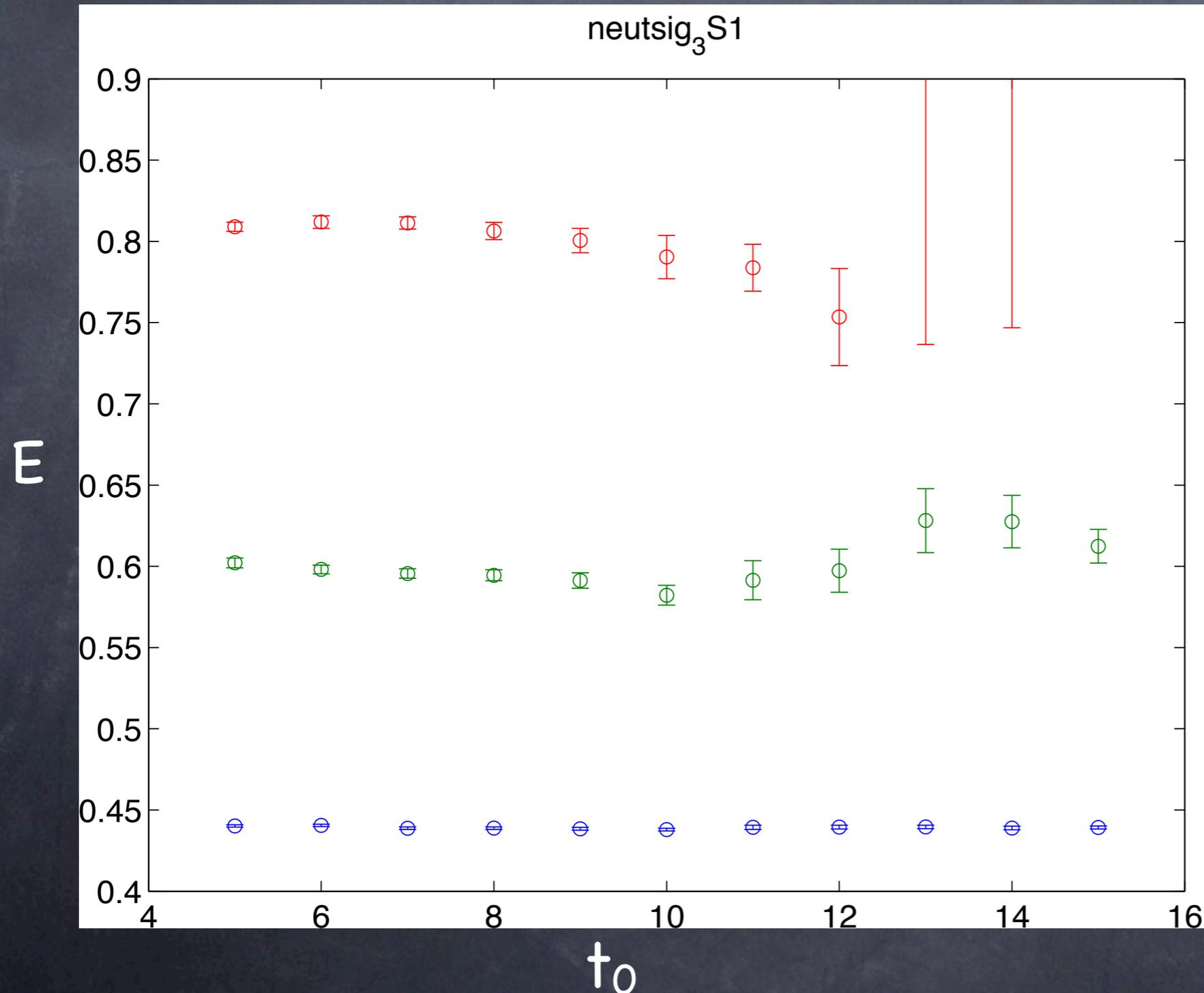
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

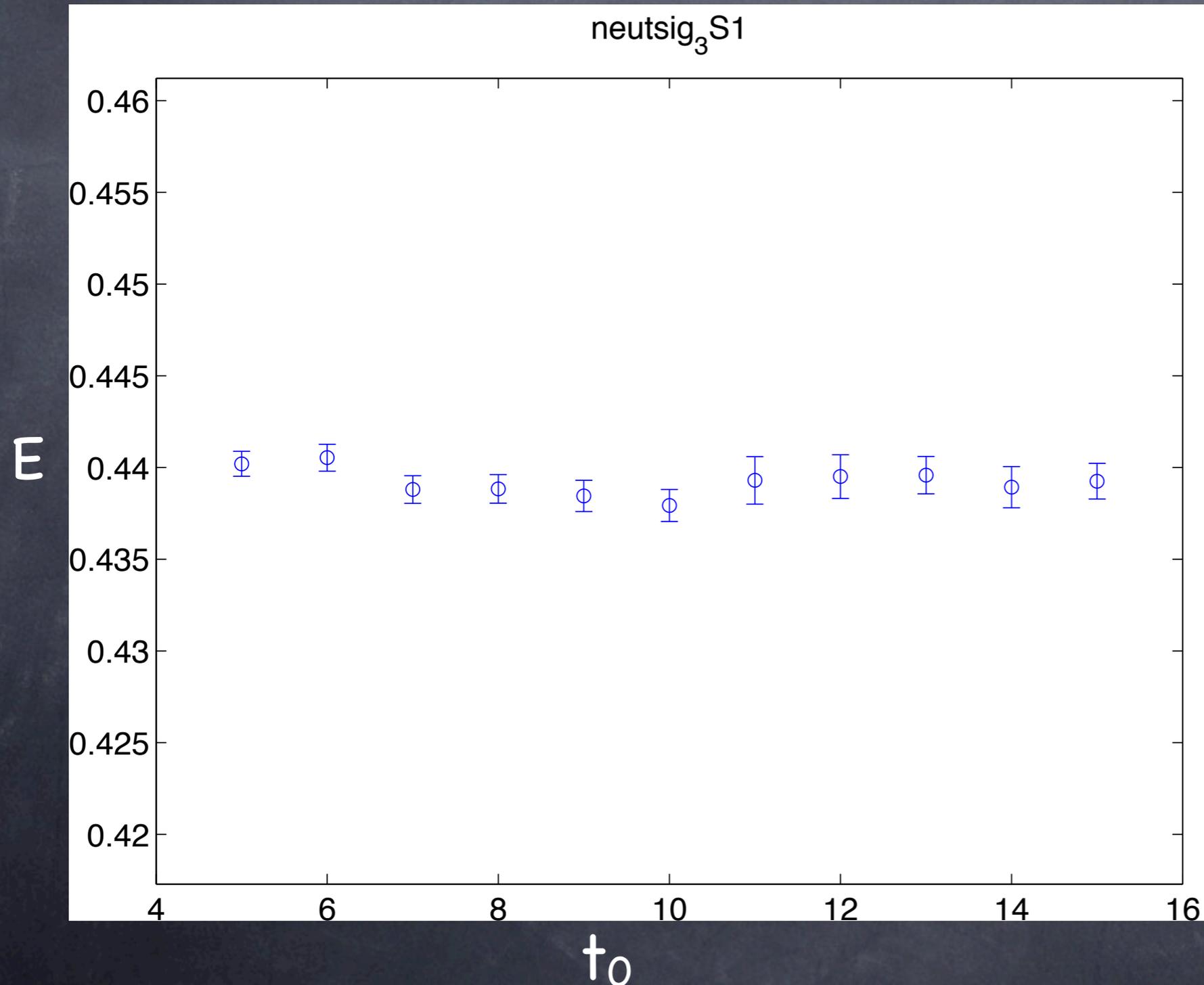
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Two Baryons

NPLQCD data



single operator

one source smearing

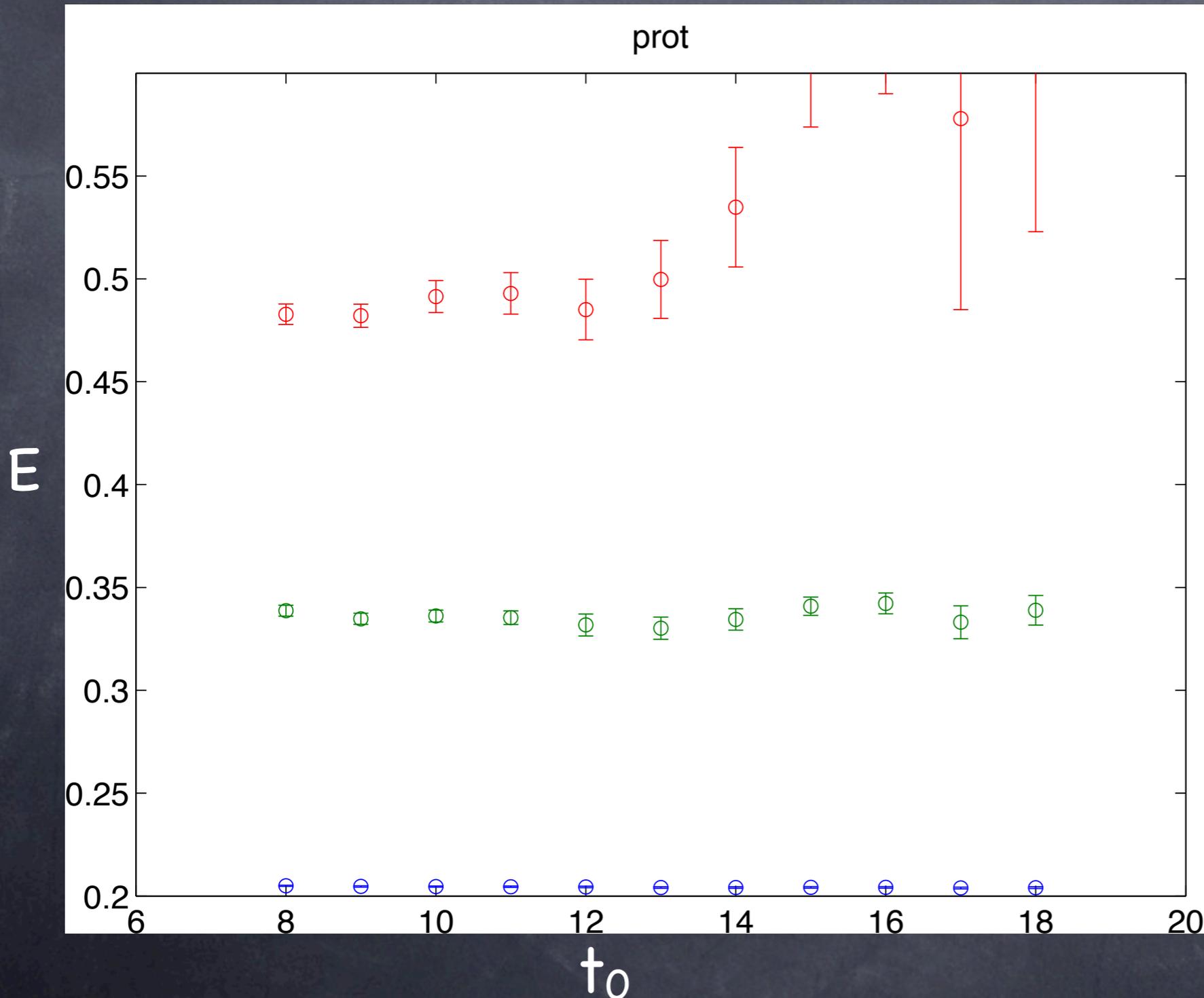
smearing, point and  
smearing-point sink

three source shifts

3x3 matrix

# Single Baryon

NPLQCD data



single operator

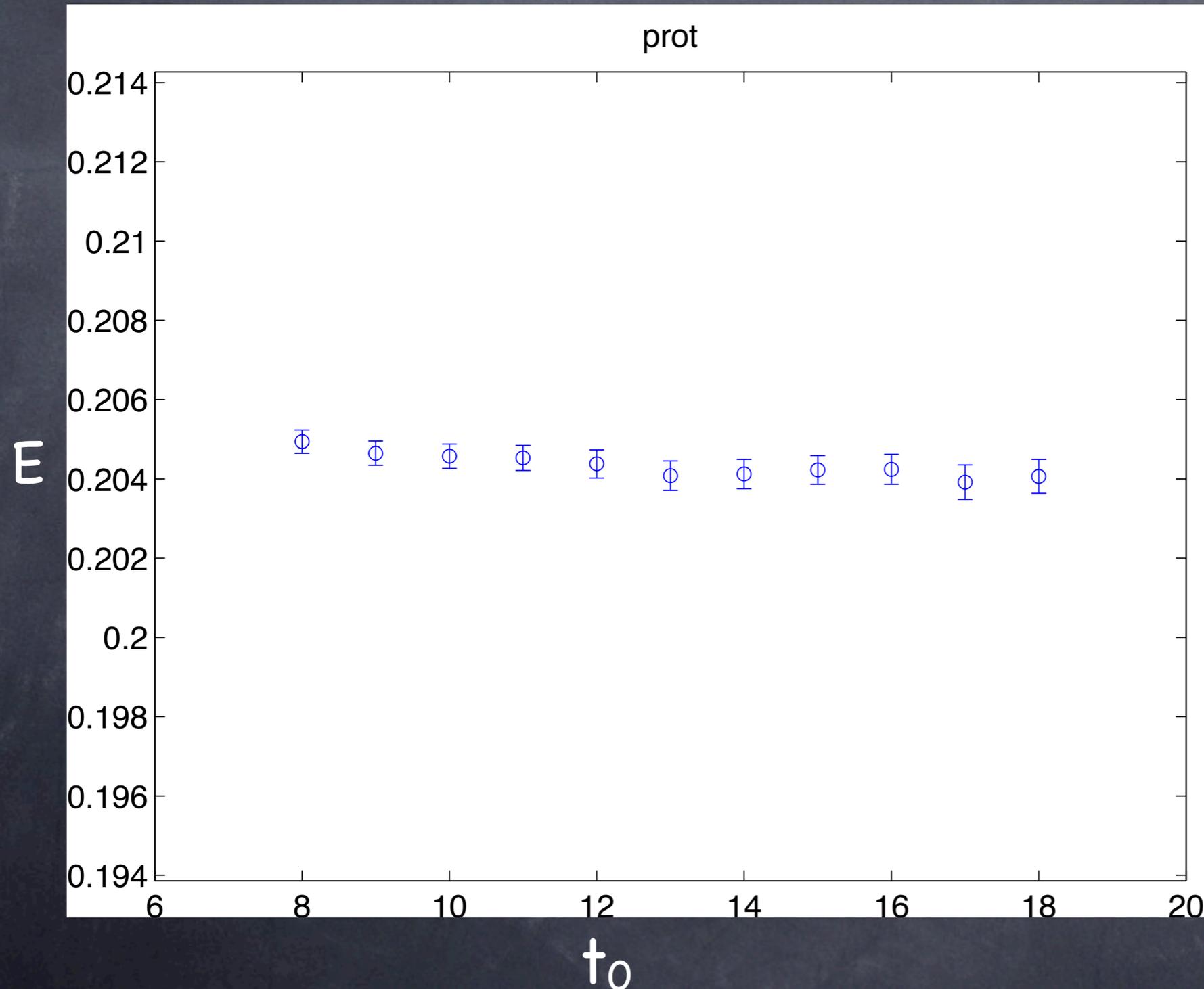
one source smearing

smearing and point sink

VarPro 3 state fits

# Single Baryon

NPLQCD data



single operator

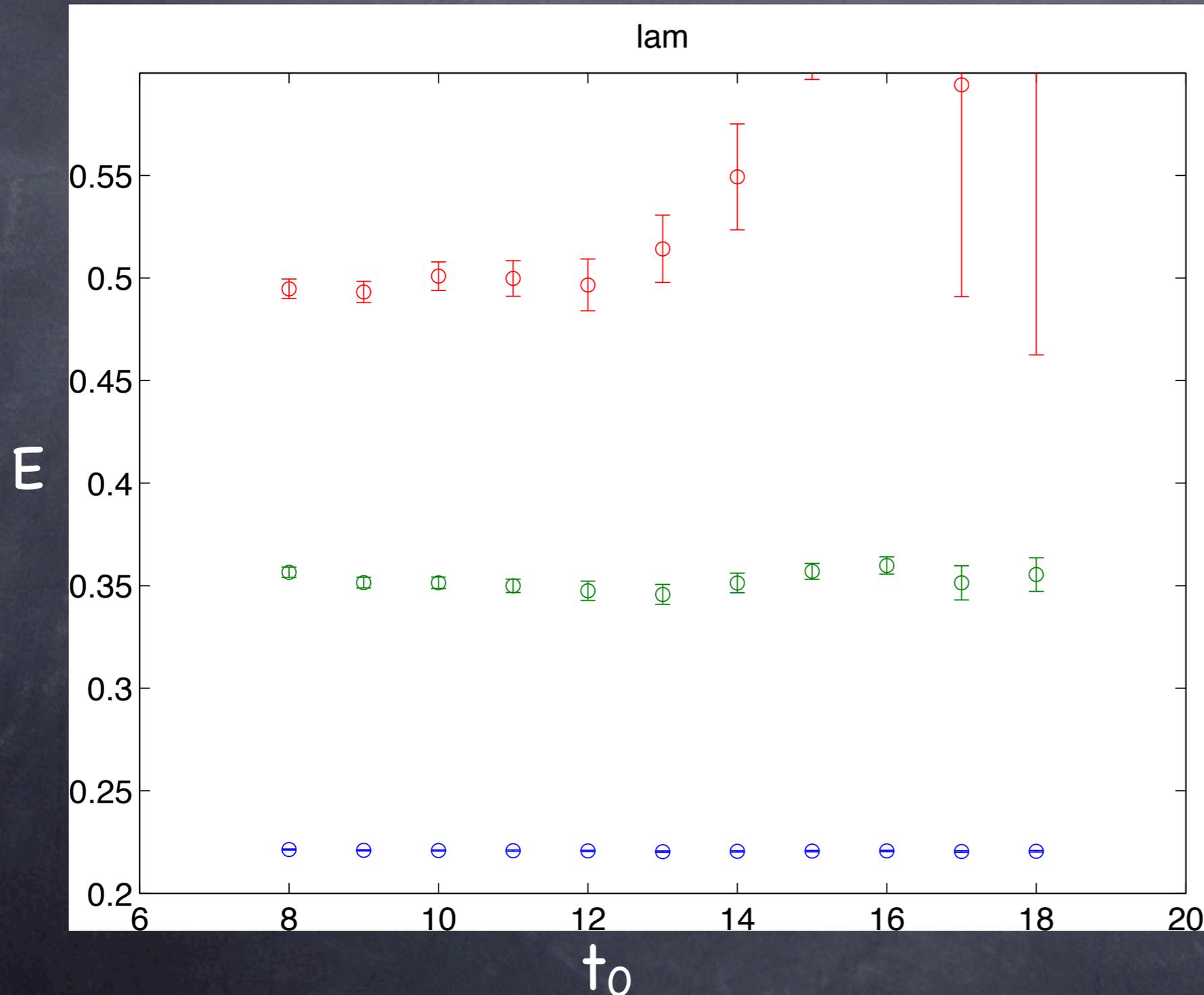
one source smearing

smearing and point sink

VarPro 3 state fits

# Single Baryon

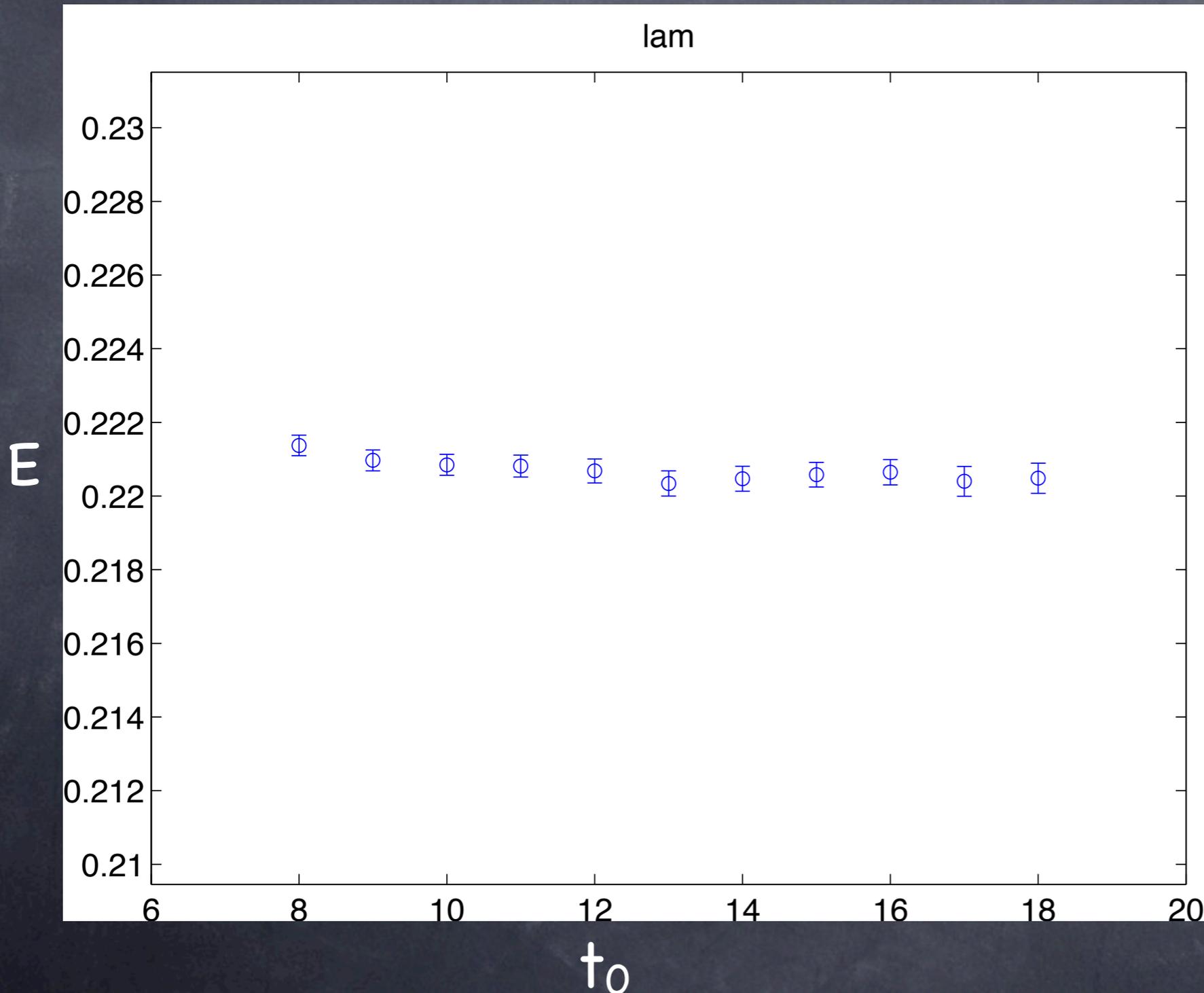
NPLQCD data



single operator  
one source smearing  
smeared and point sink  
VarPro 3 state fits

# Single Baryon

NPLQCD data



single operator

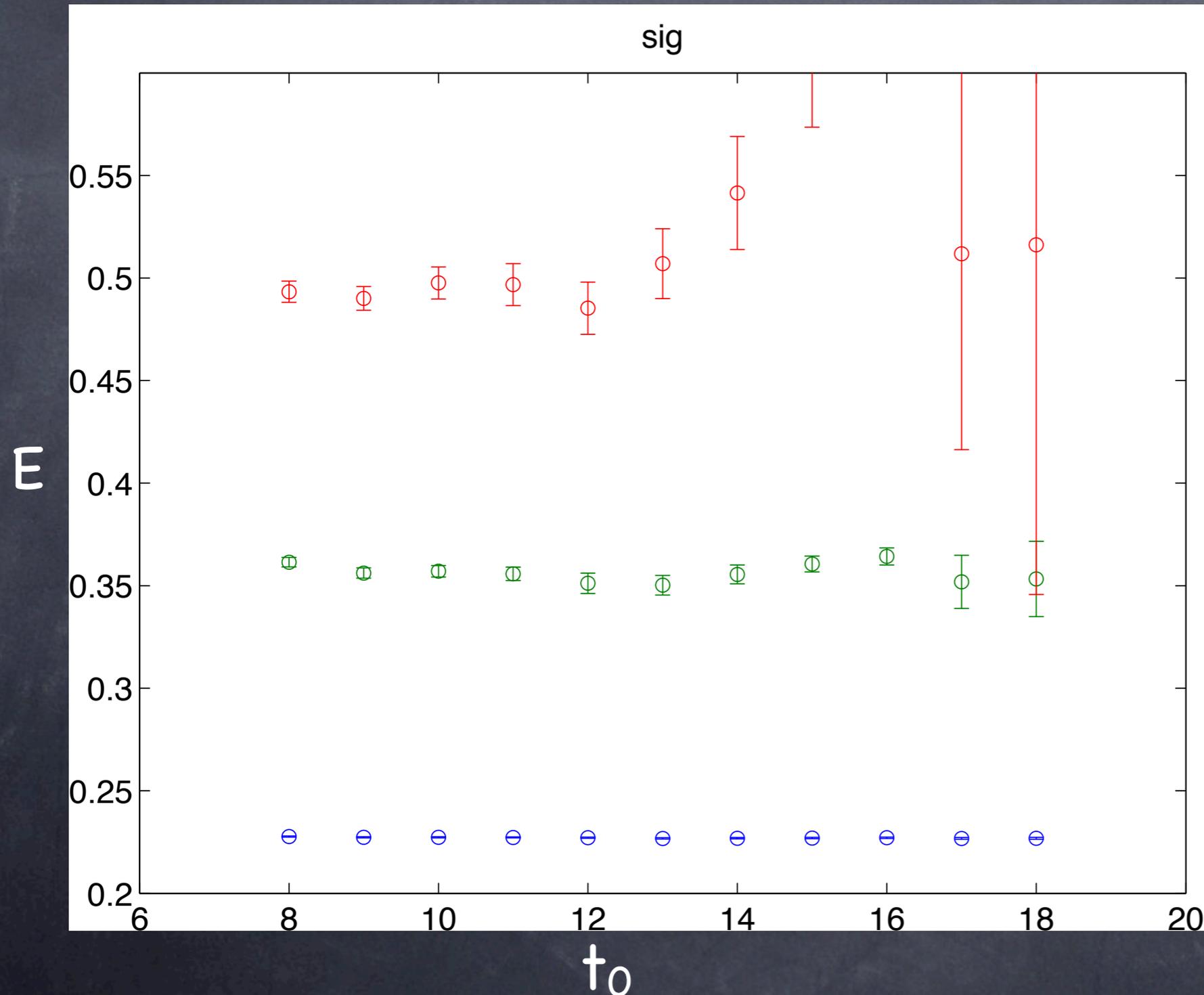
one source smearing

smearing and point sink

VarPro 3 state fits

# Single Baryon

NPLQCD data



single operator

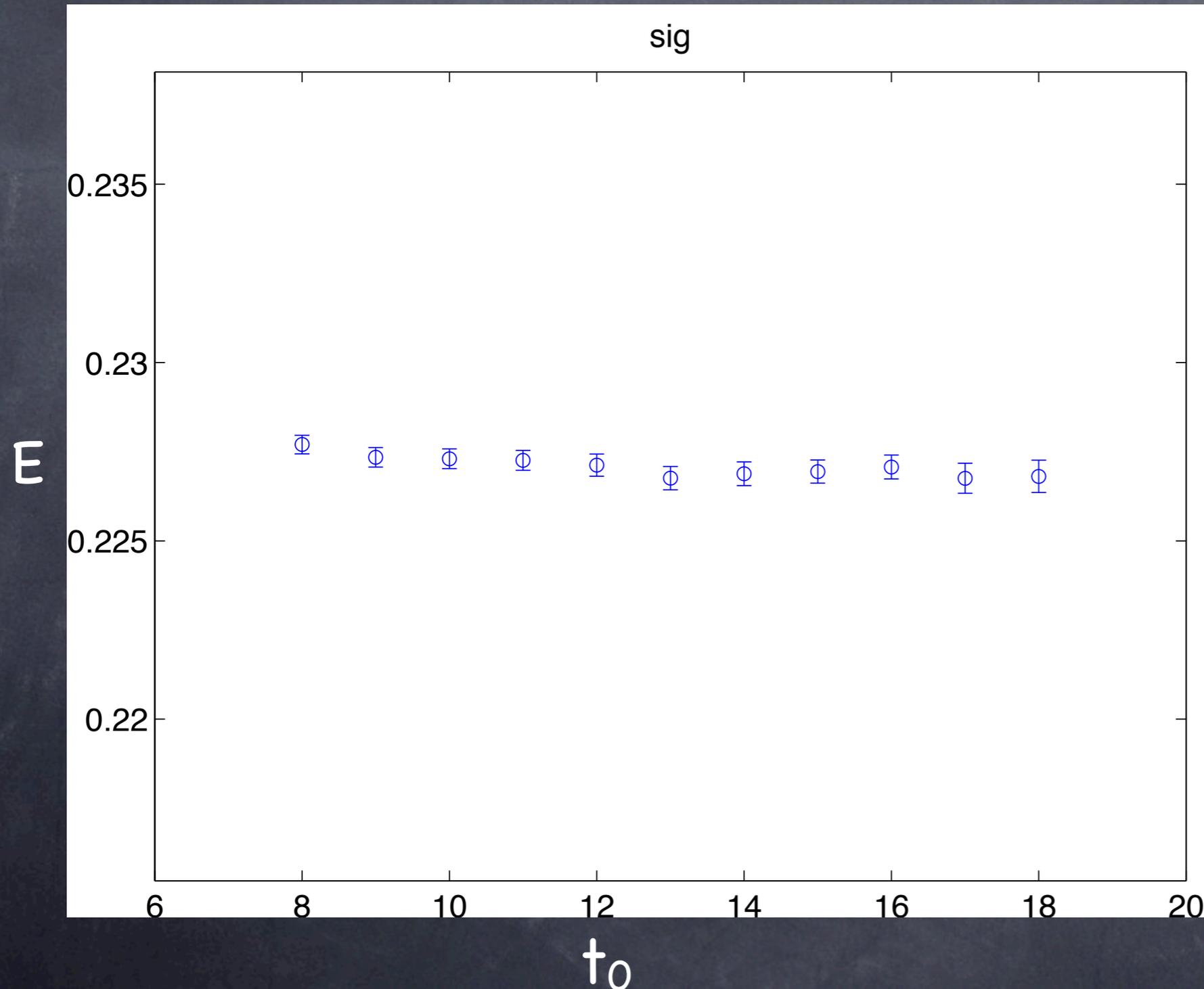
one source smearing

smearing and point sink

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NPLQCD data



single operator

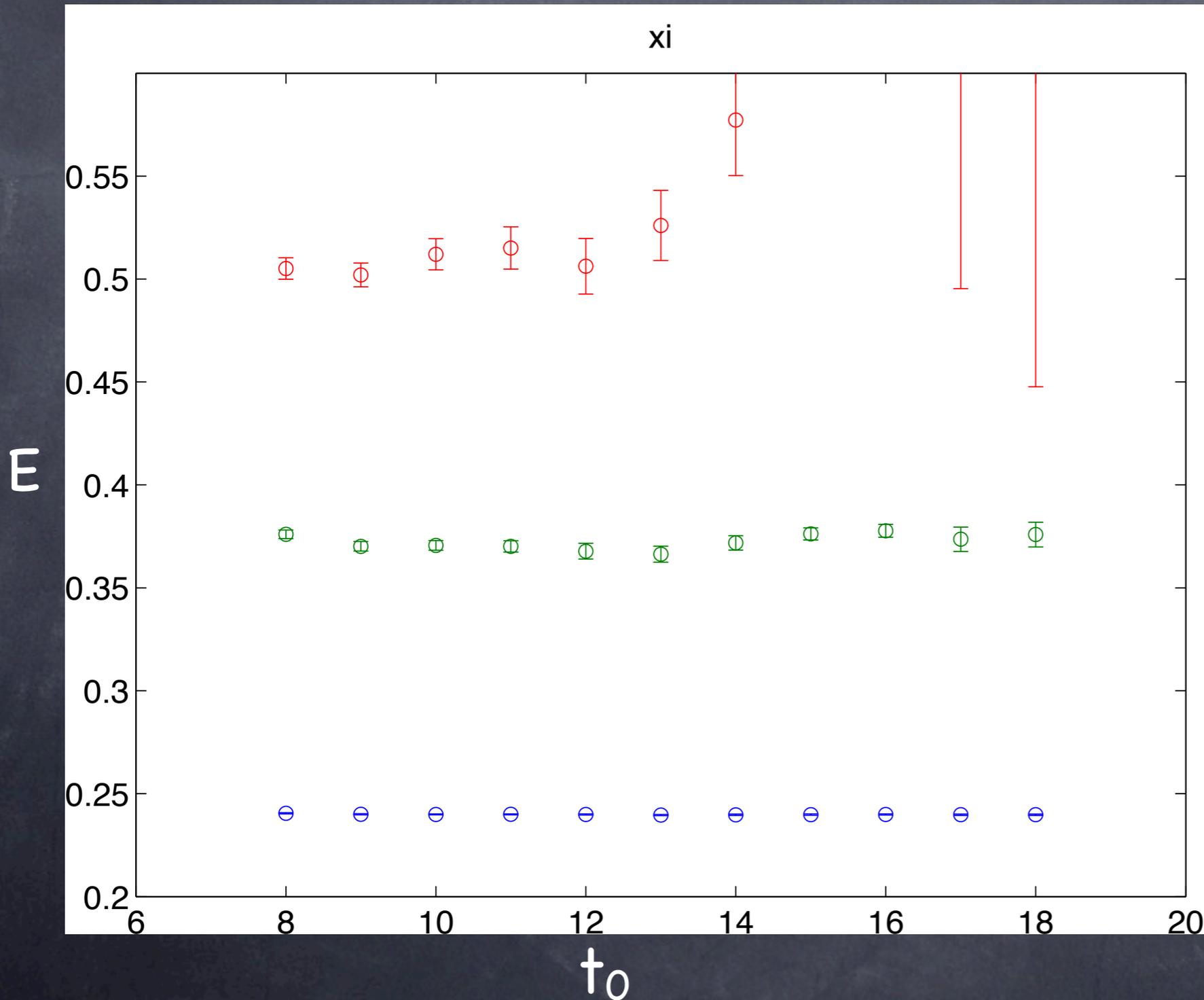
one source smearing

smearred and point sink

VarPro 3 state fits

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NPLQCD data



single operator

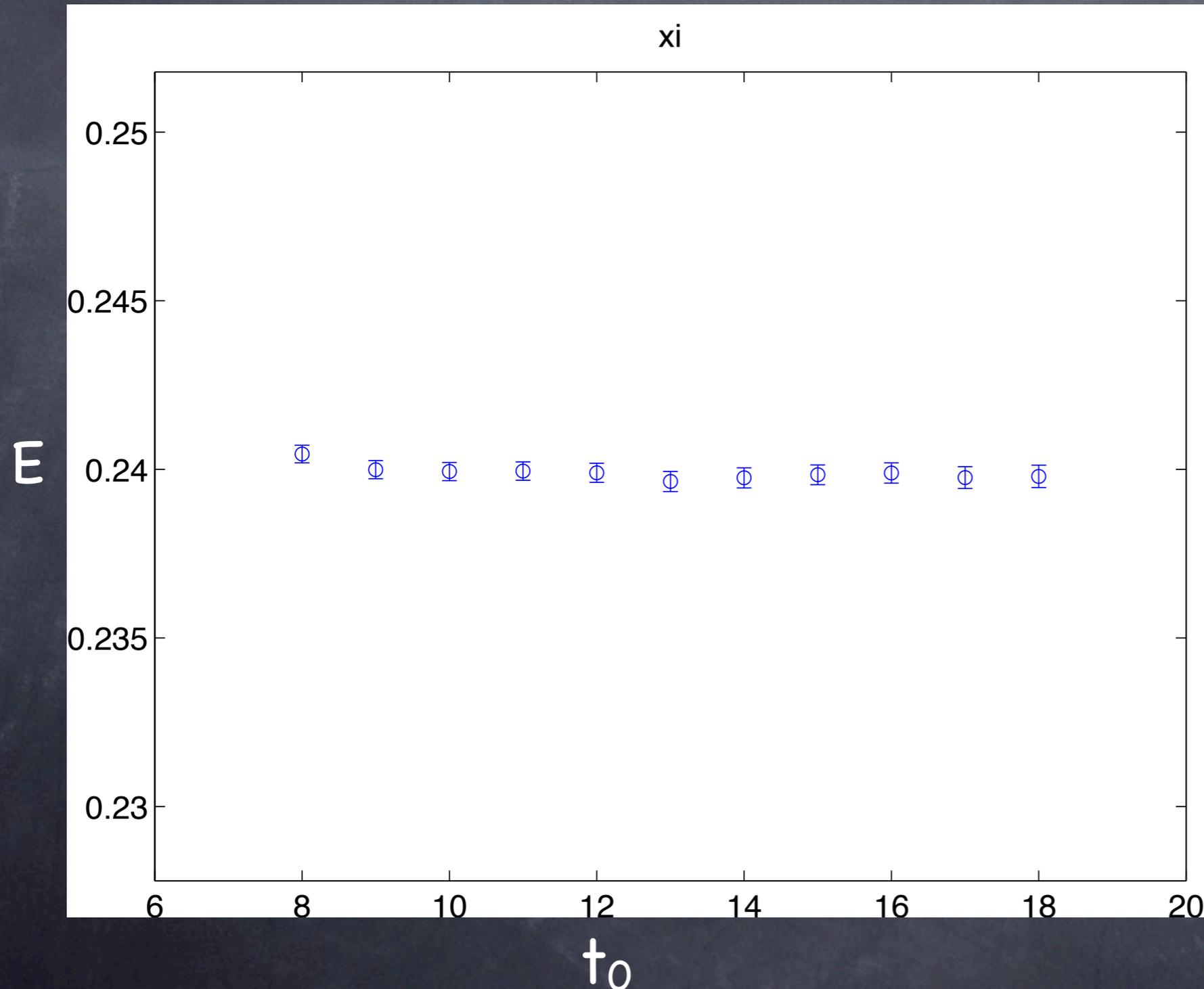
one source smearing

smearing and point sink

VarPro 3 state fits

# Single Baryon

NPLQCD data



single operator

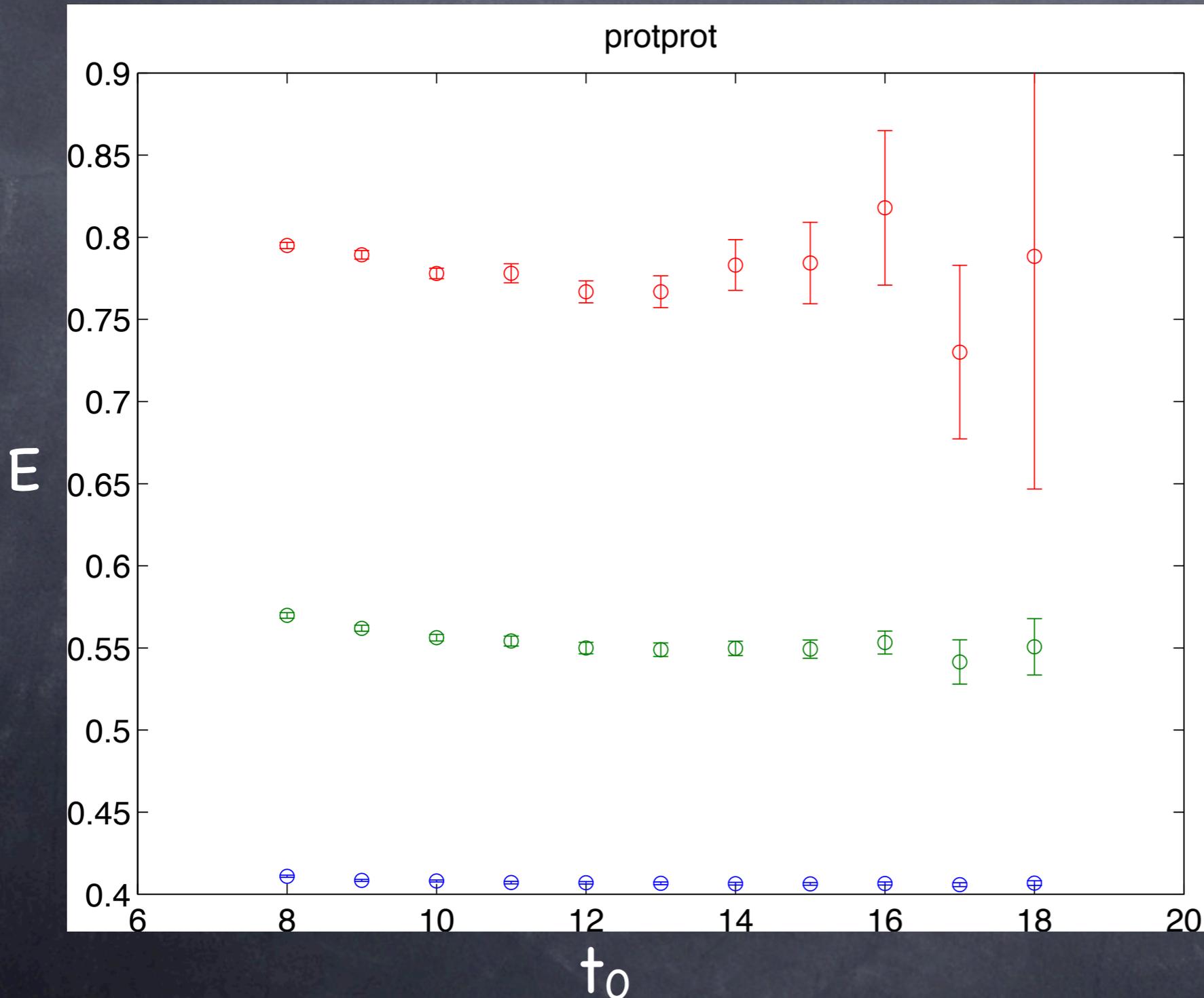
one source smearing

smearing and point sink

VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

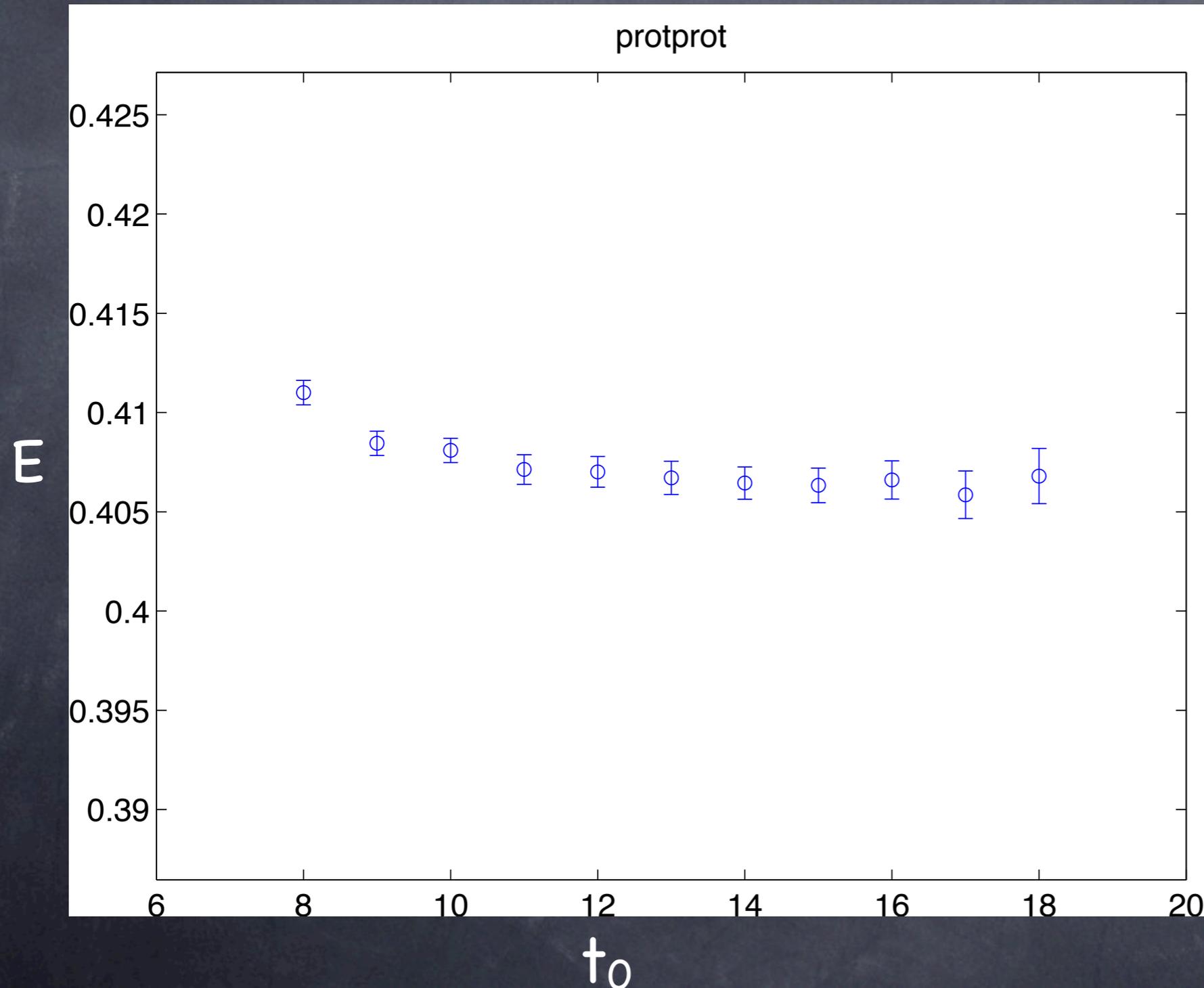
one source smearing

smearing, point and  
smearing-point sink

VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

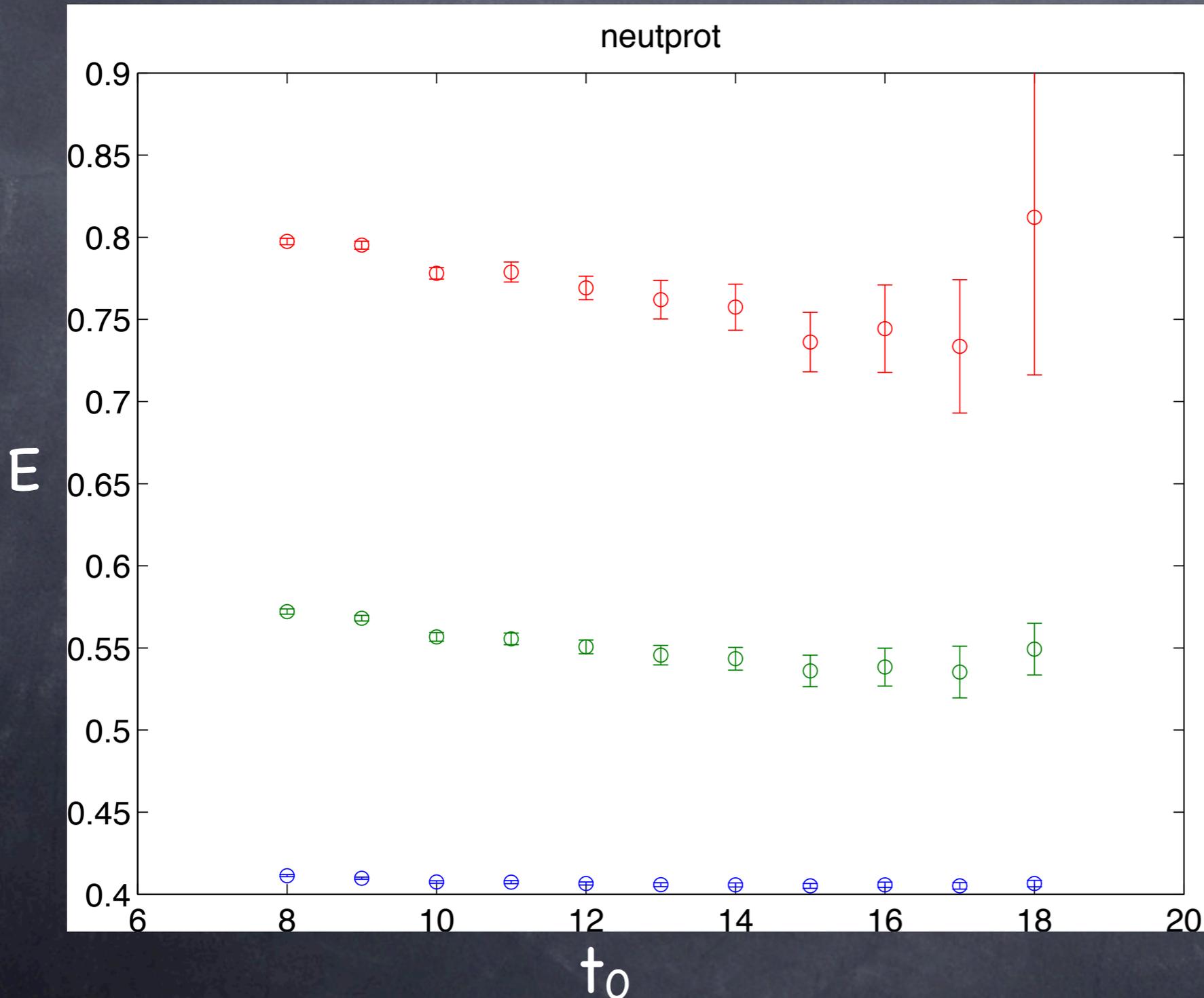
one source smearing

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VarPro 3 state fits

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NPLQCD data



single operator

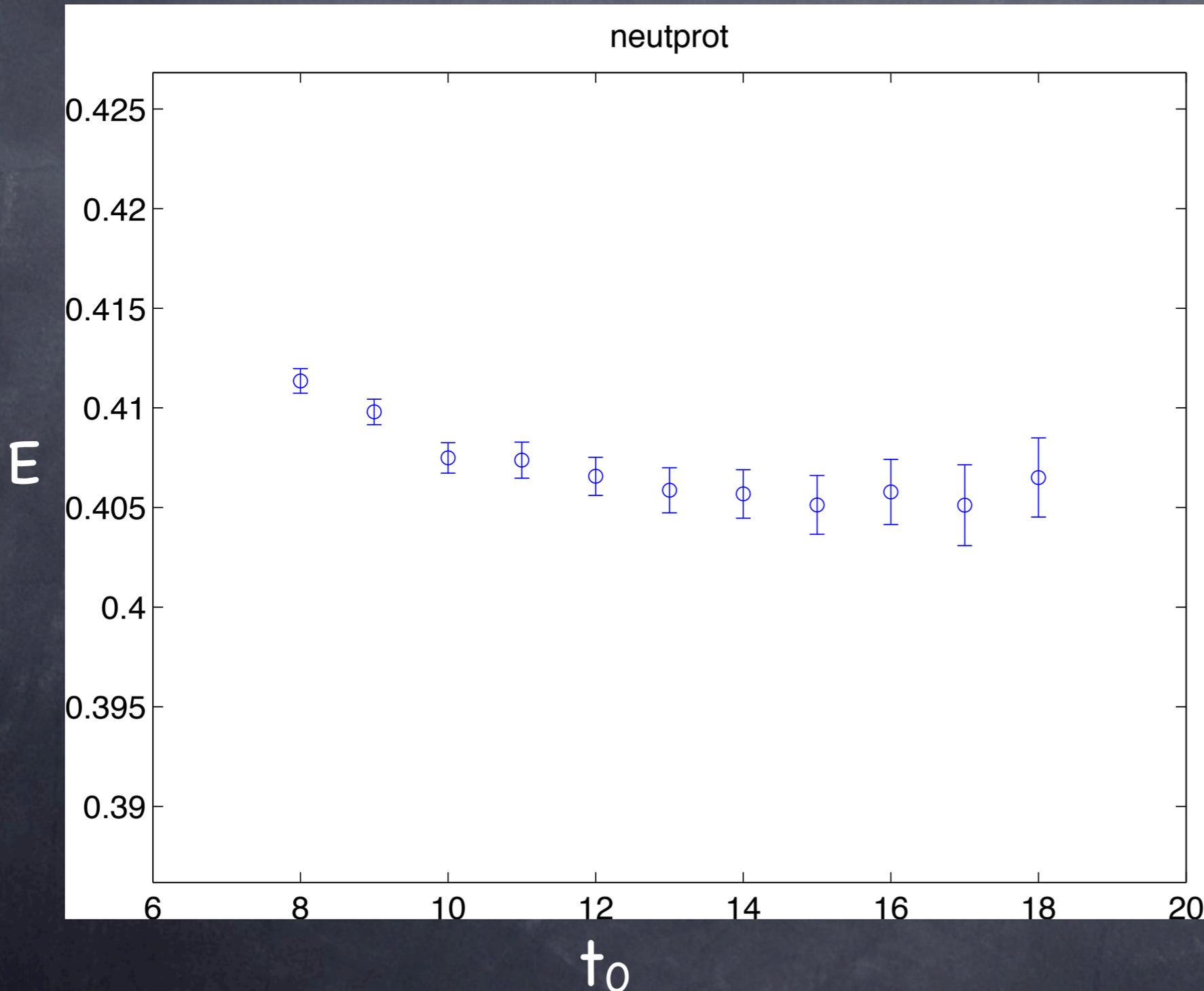
one source smearing

smearing, point and  
smearing-point sink

VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

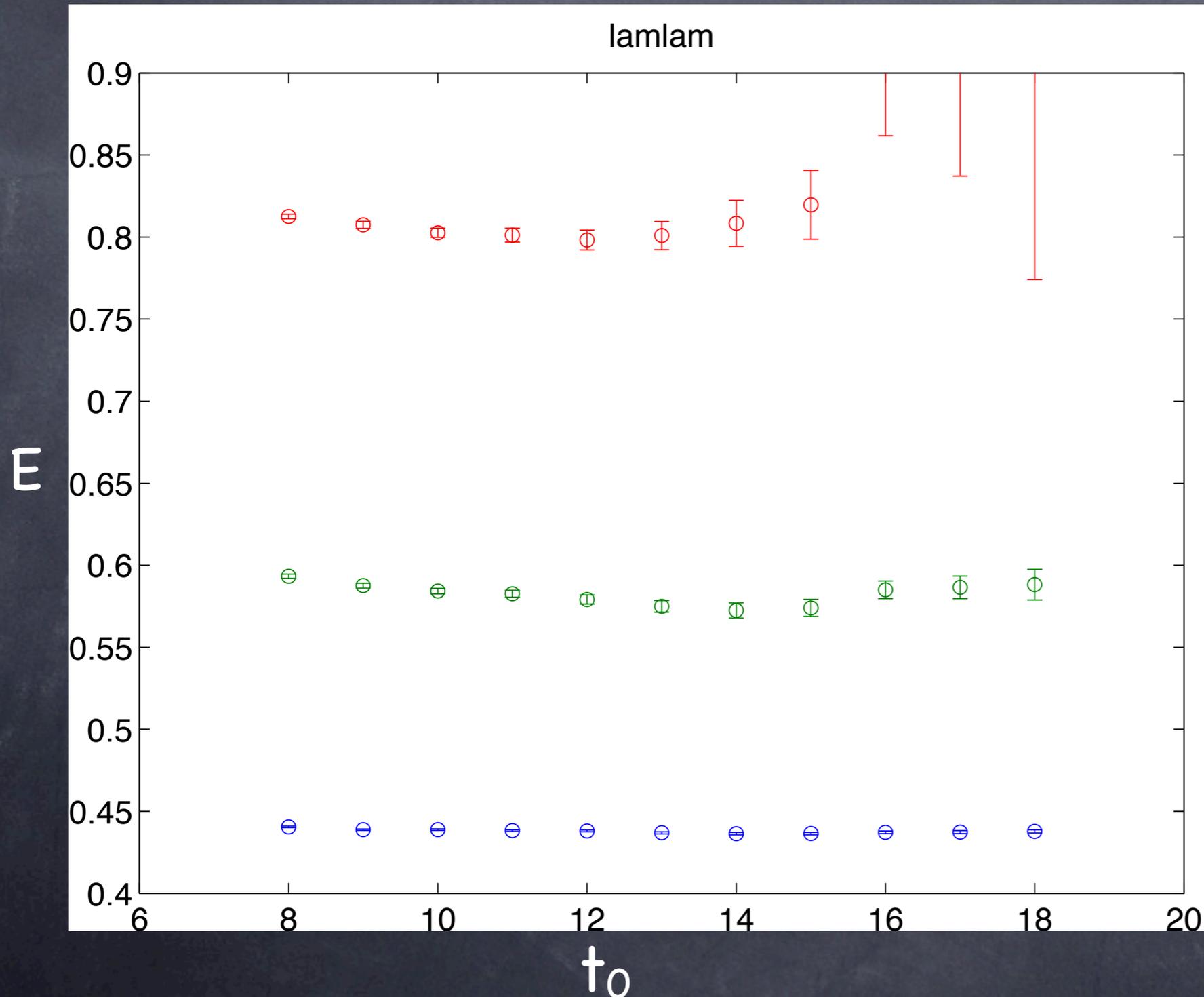
one source smearing

smearing, point and  
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VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

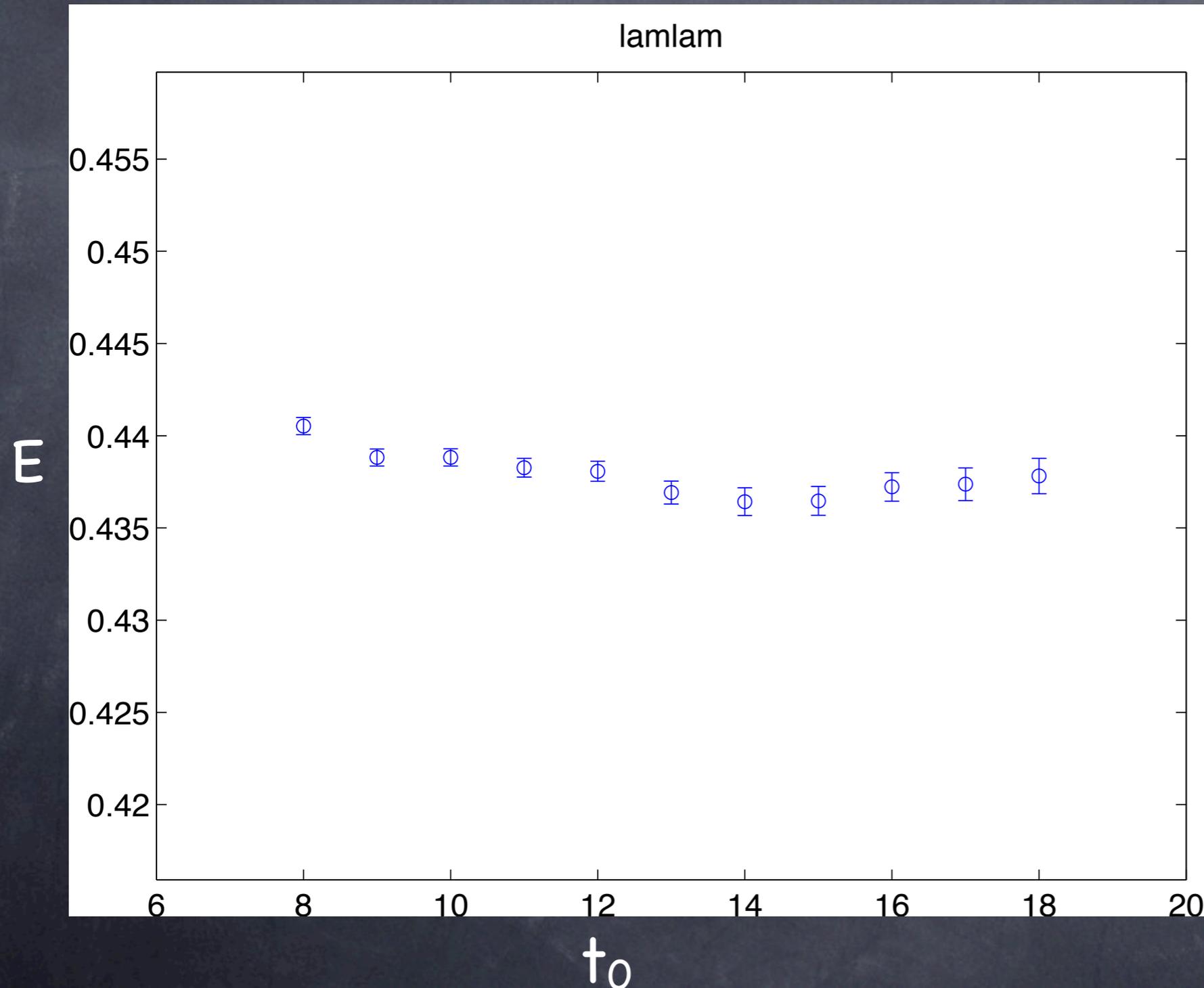
one source smearing

smearing, point and  
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VarPro 3 state fits

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NPLQCD data



single operator

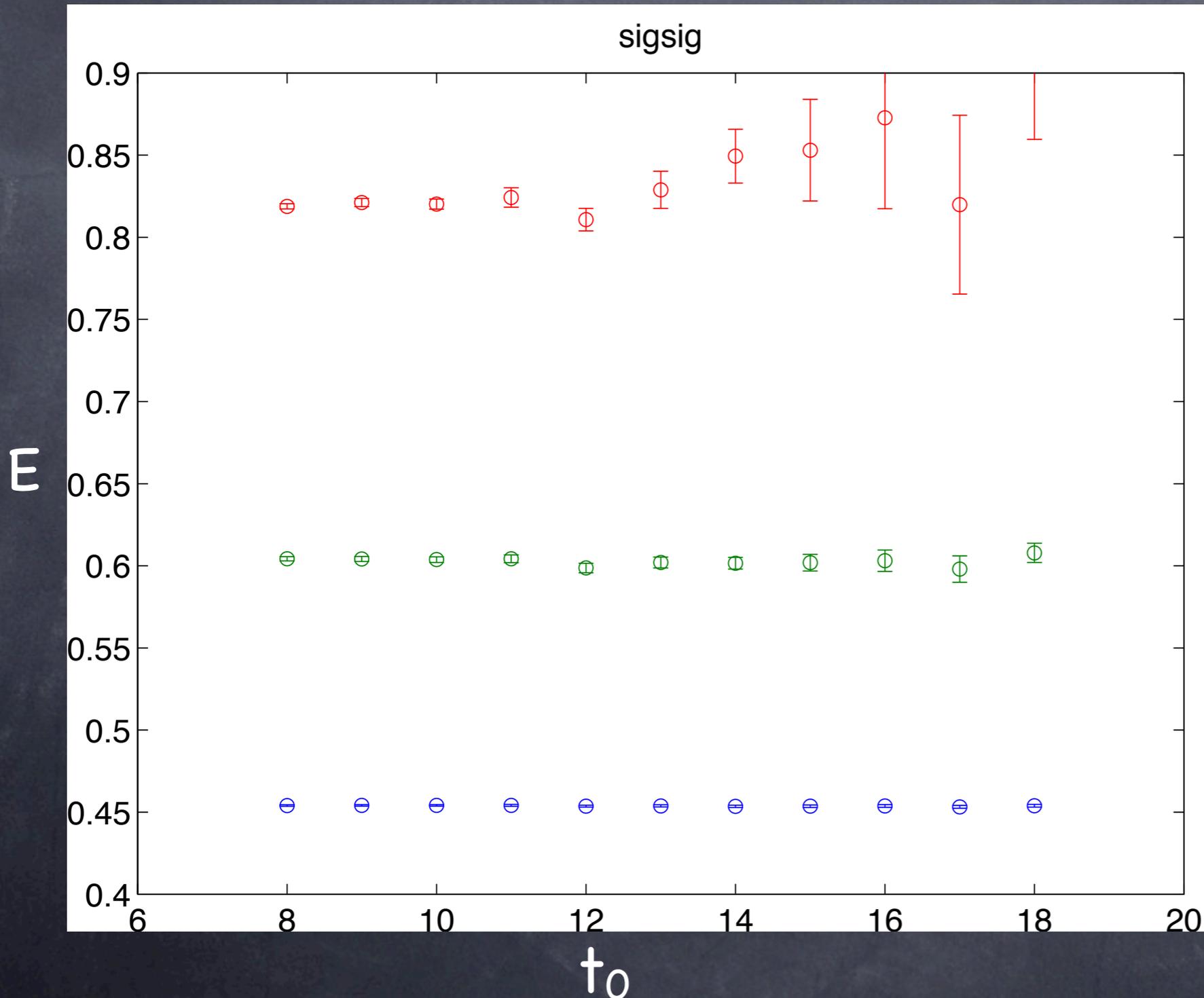
one source smearing

smearing, point and  
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VarPro 3 state fits

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NPLQCD data



single operator

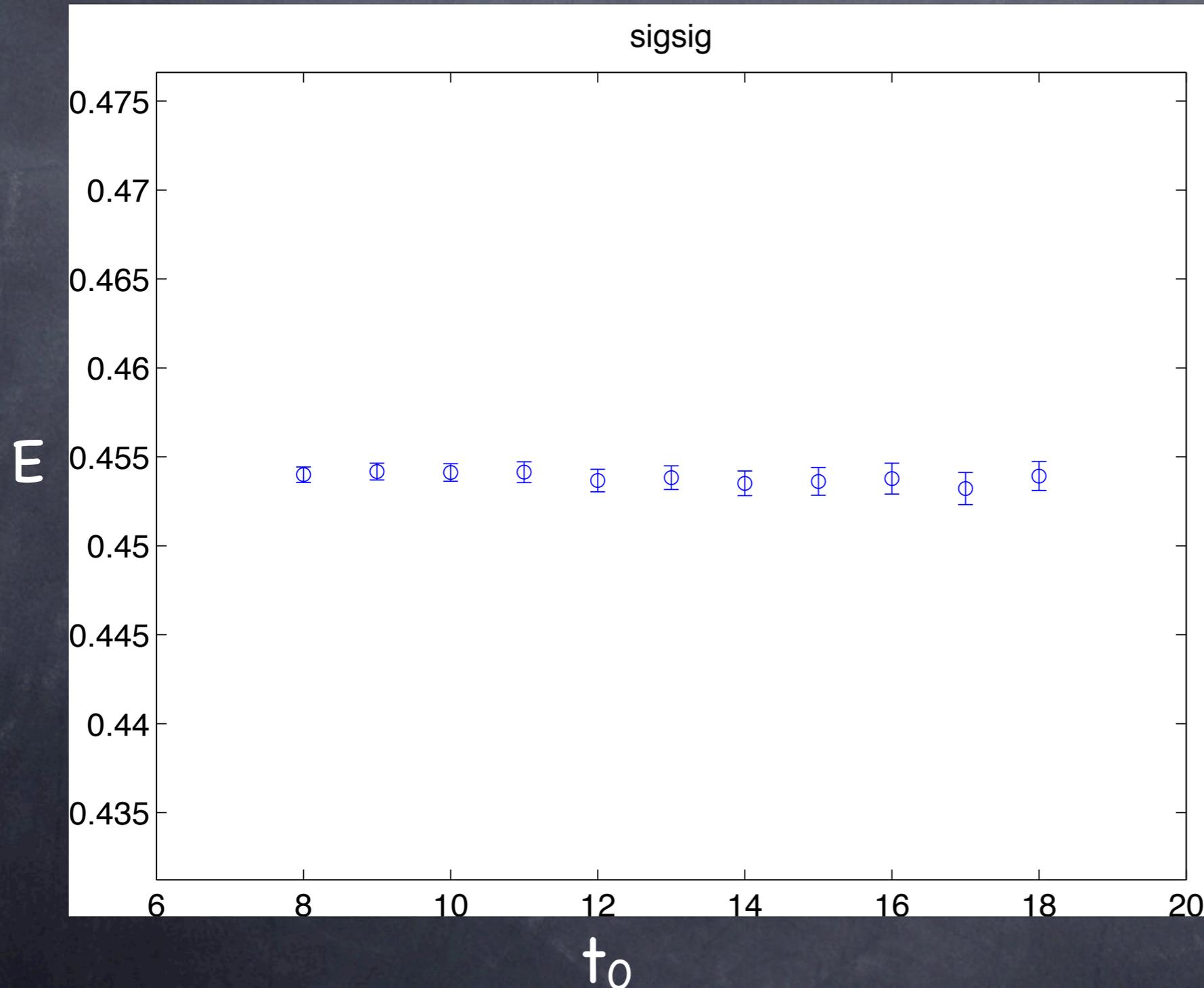
one source smearing

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NPLQCD data



single operator

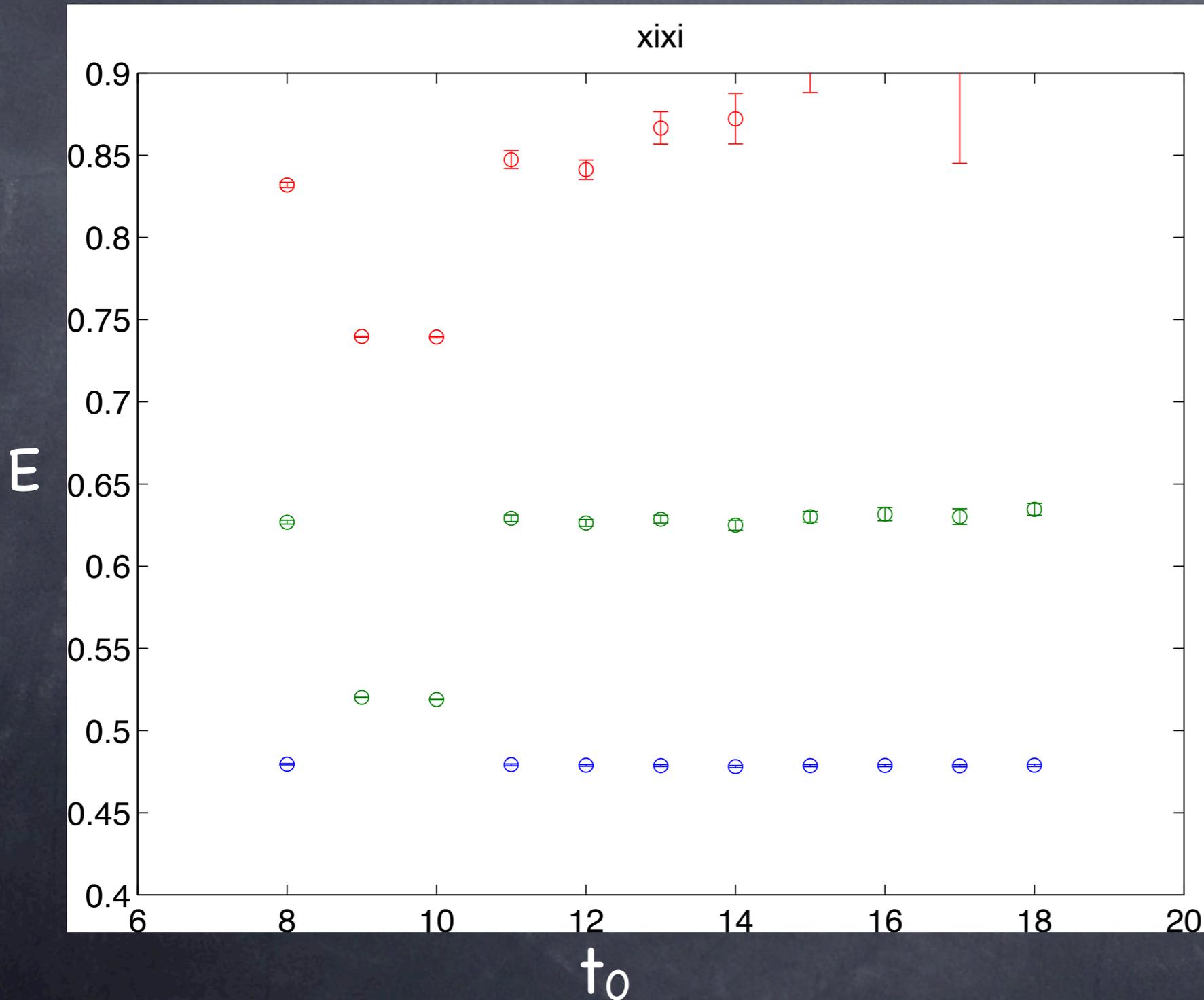
one source smearing

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VarPro 3 state fits

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NPLQCD data



single operator

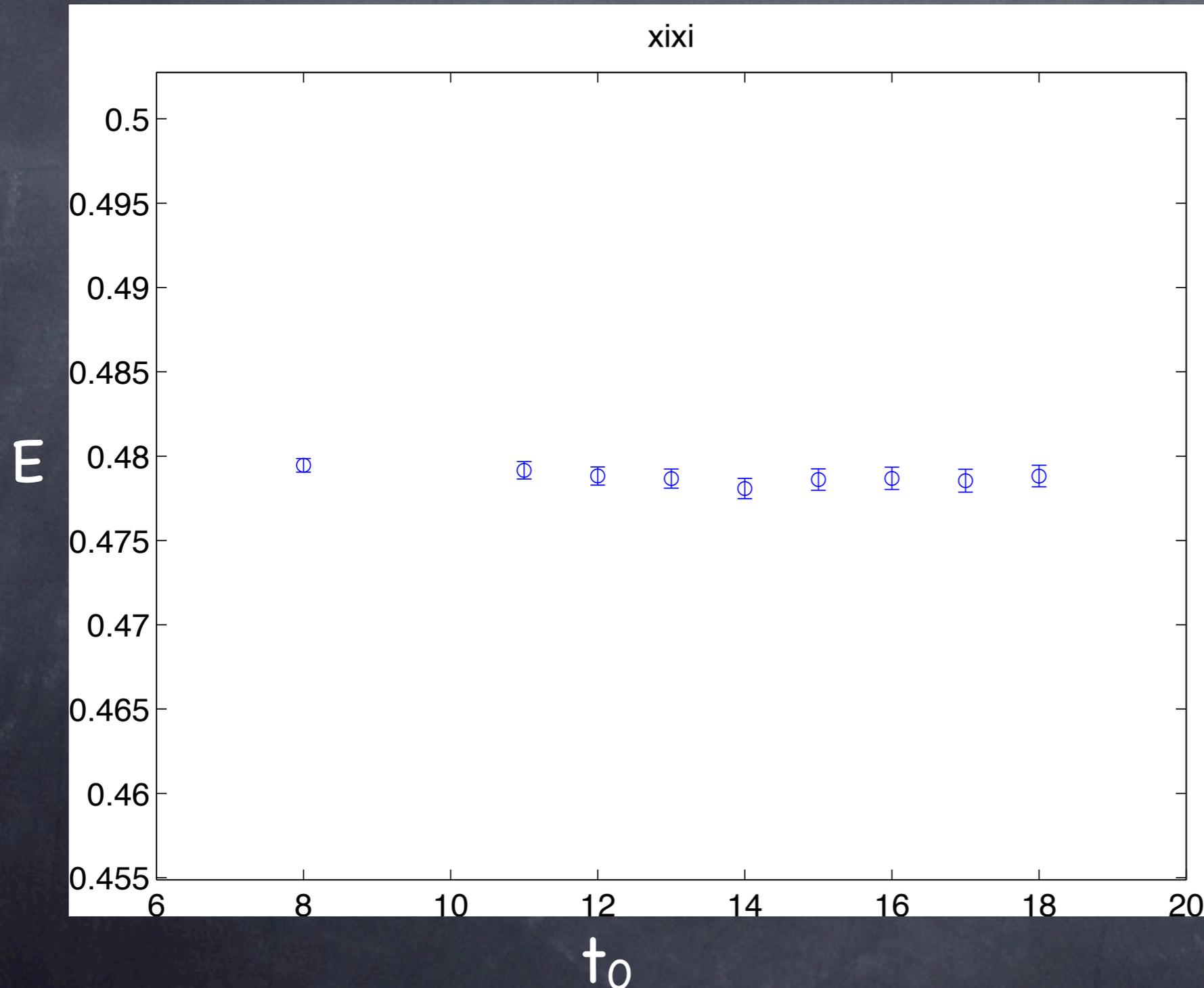
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smeared, point and  
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VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

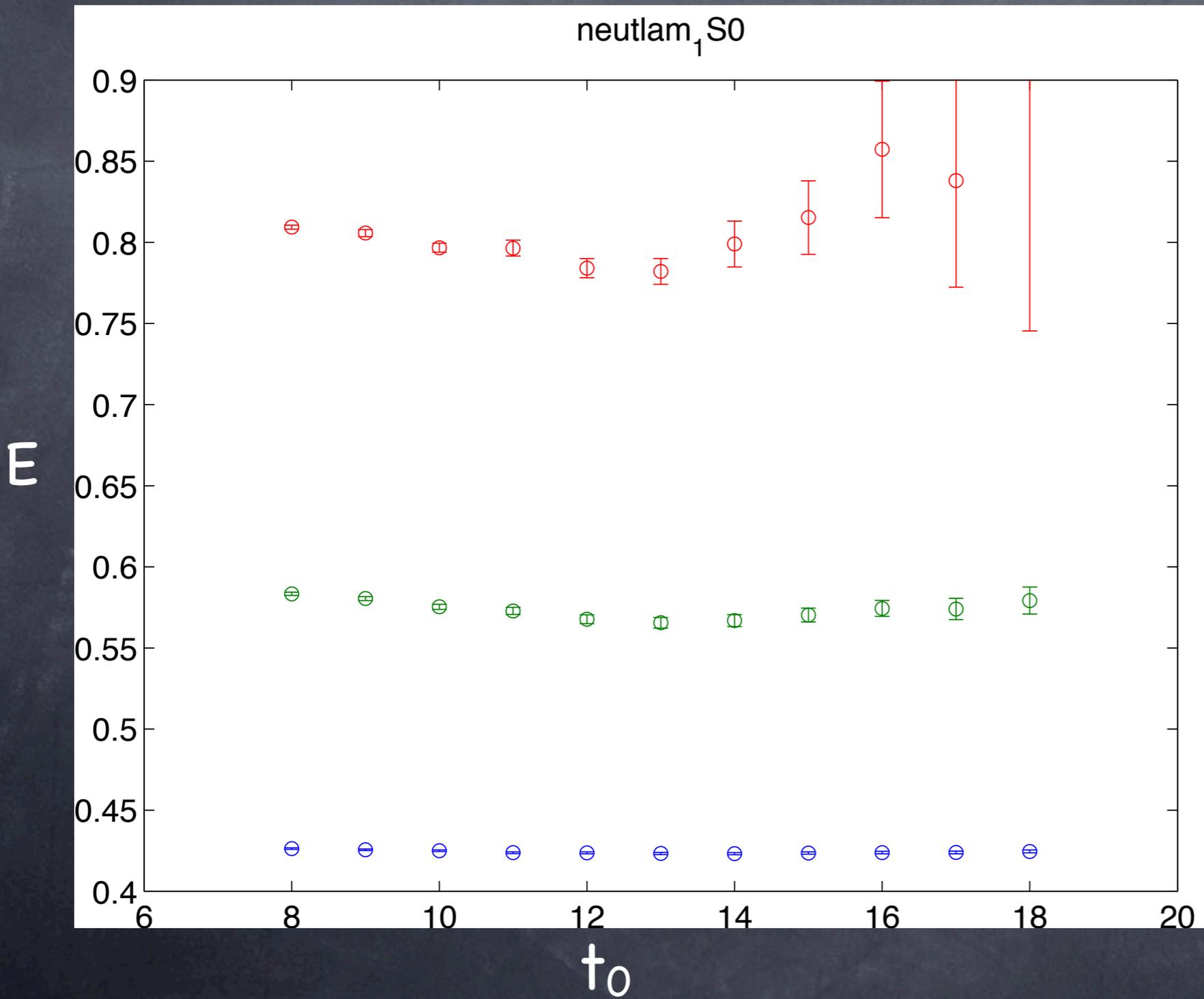
one source smearing

smearing, point and  
smearing-point sink

VarPro 3 state fits

# Two Baryons

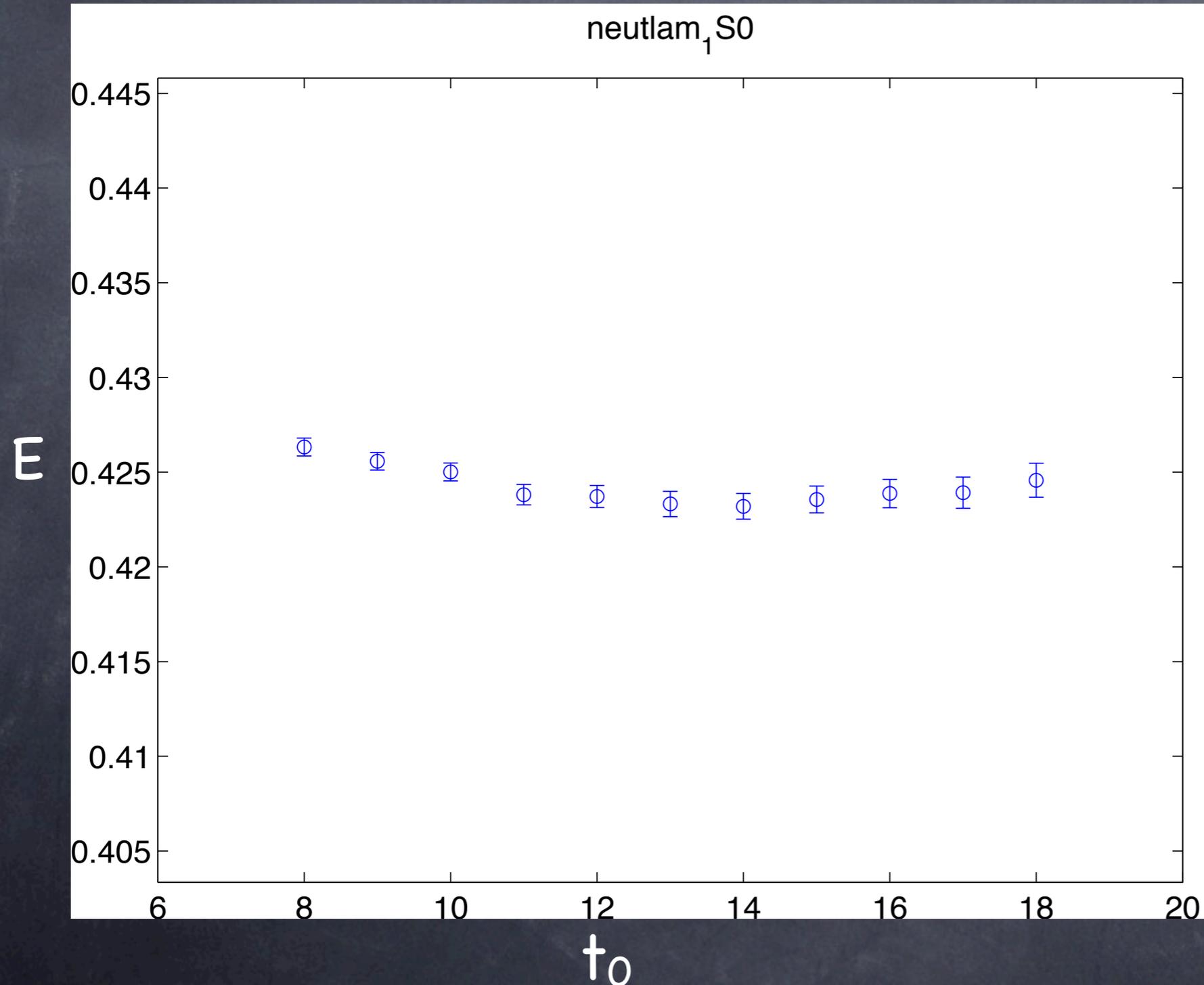
NPLQCD data



single operator  
one source smearing  
smearing, point and  
smearing-point sink  
VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

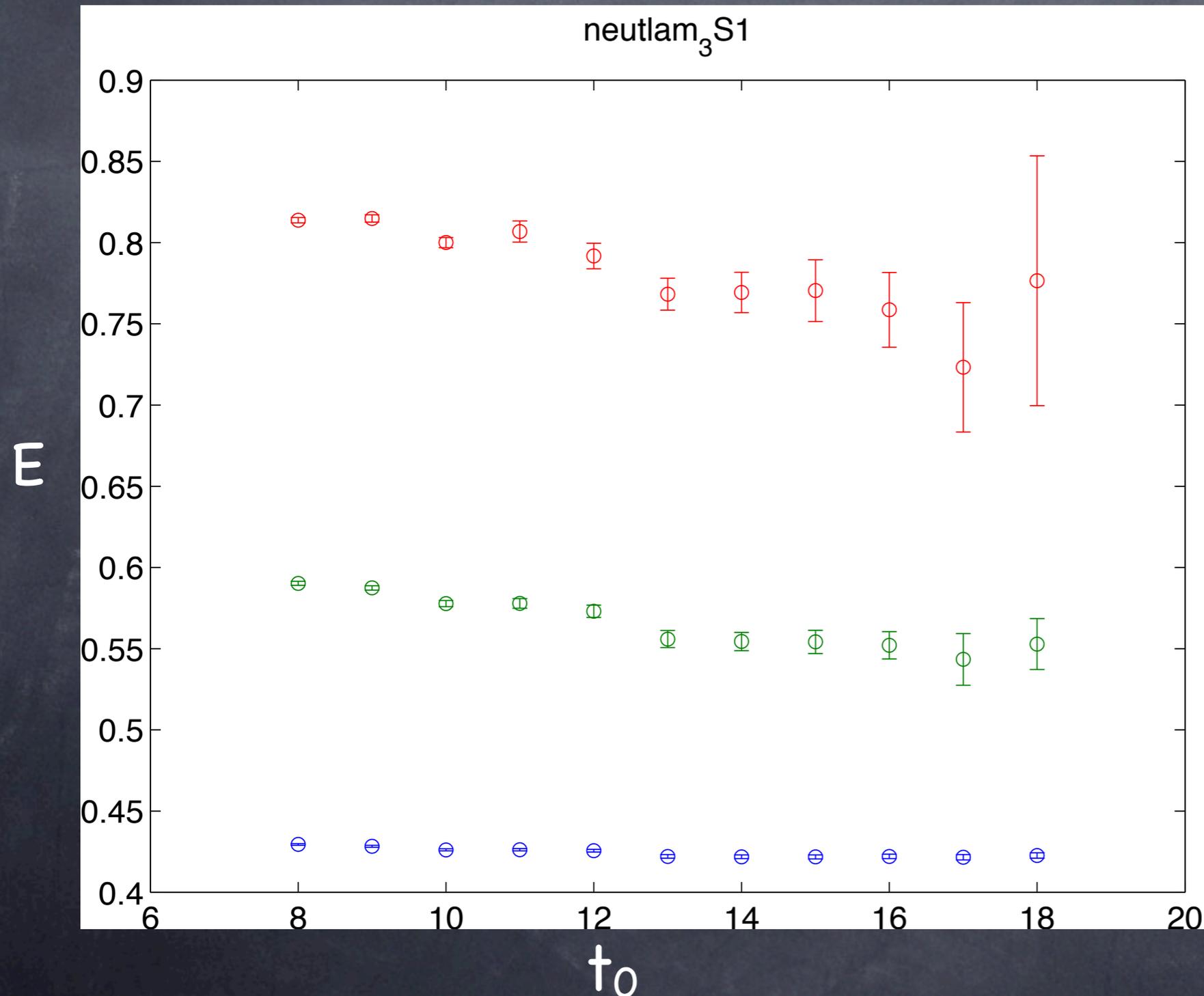
one source smearing

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# Two Baryons

NPLQCD data



single operator

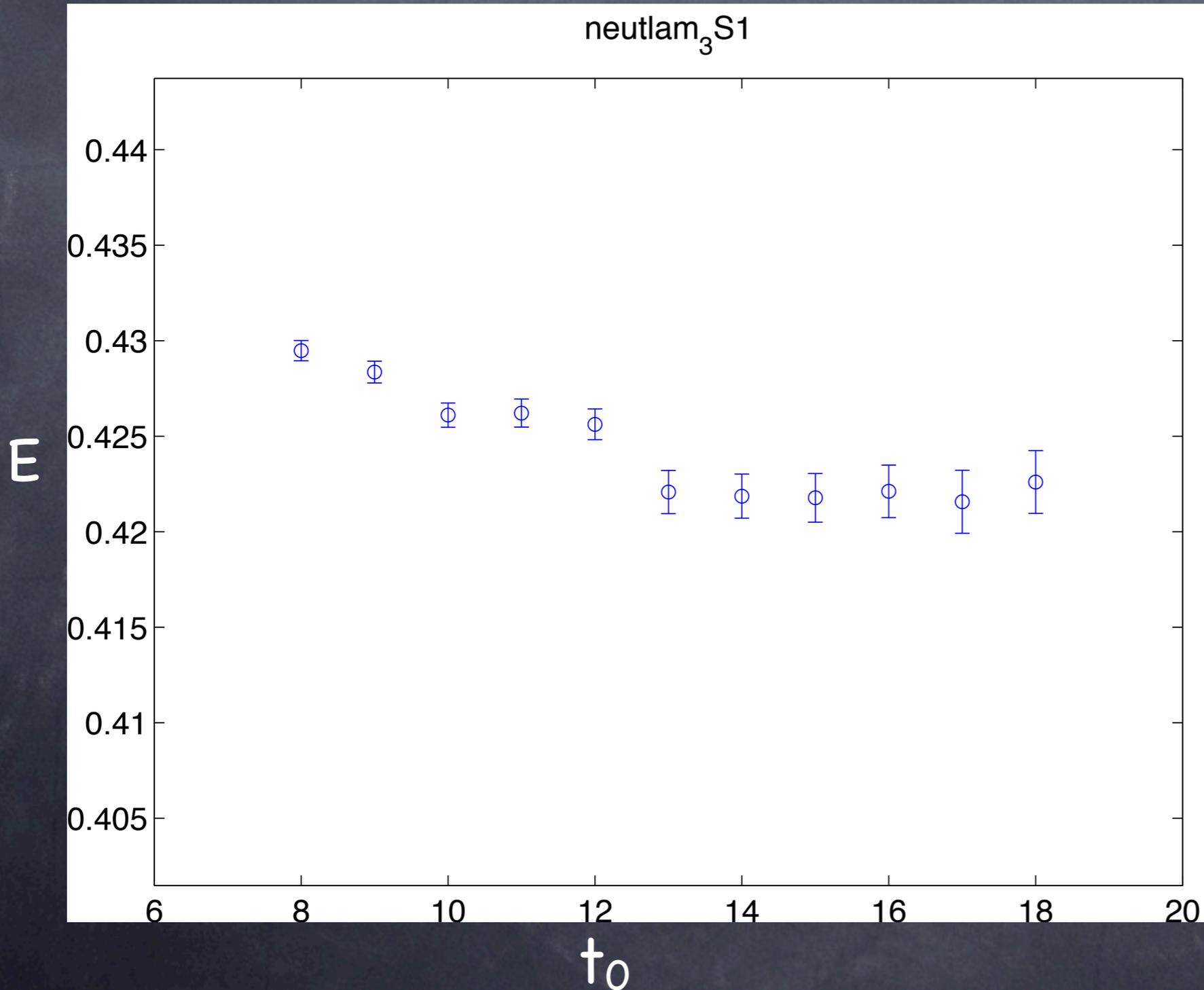
one source smearing

smearing, point and  
smearing-point sink

VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

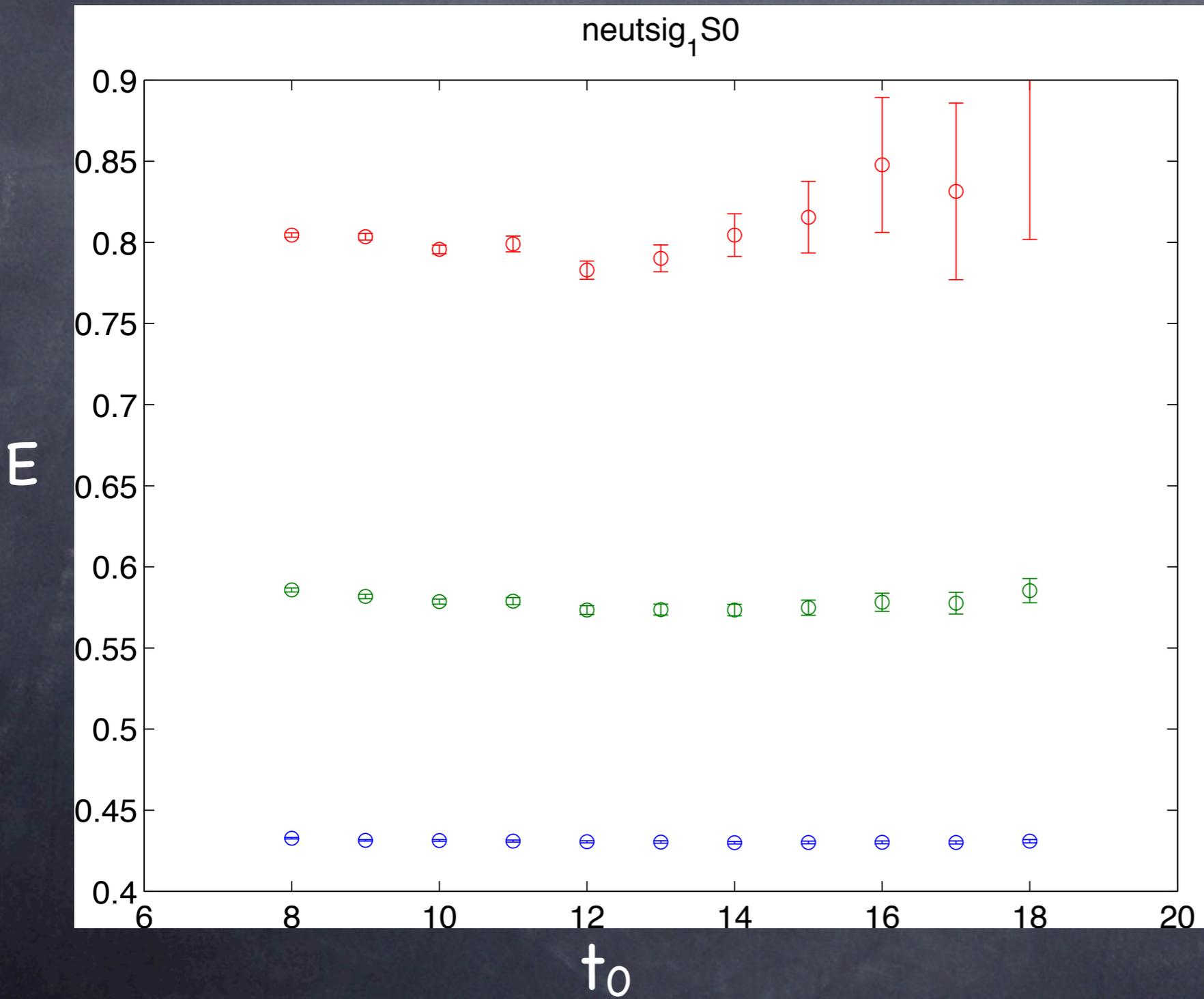
one source smearing

smearing, point and  
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NPLQCD data



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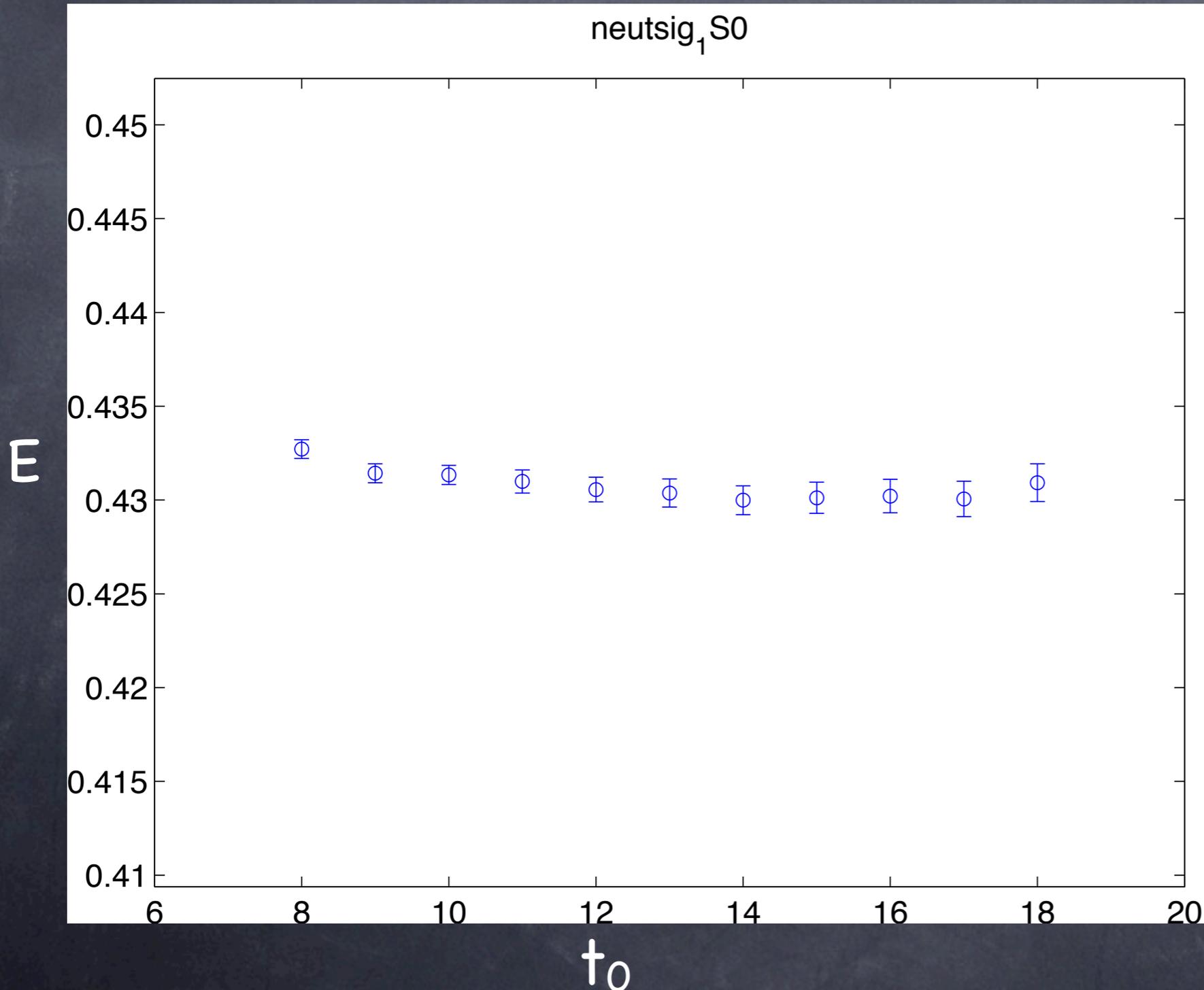
one source smearing

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VarPro 3 state fits

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NPLQCD data



single operator

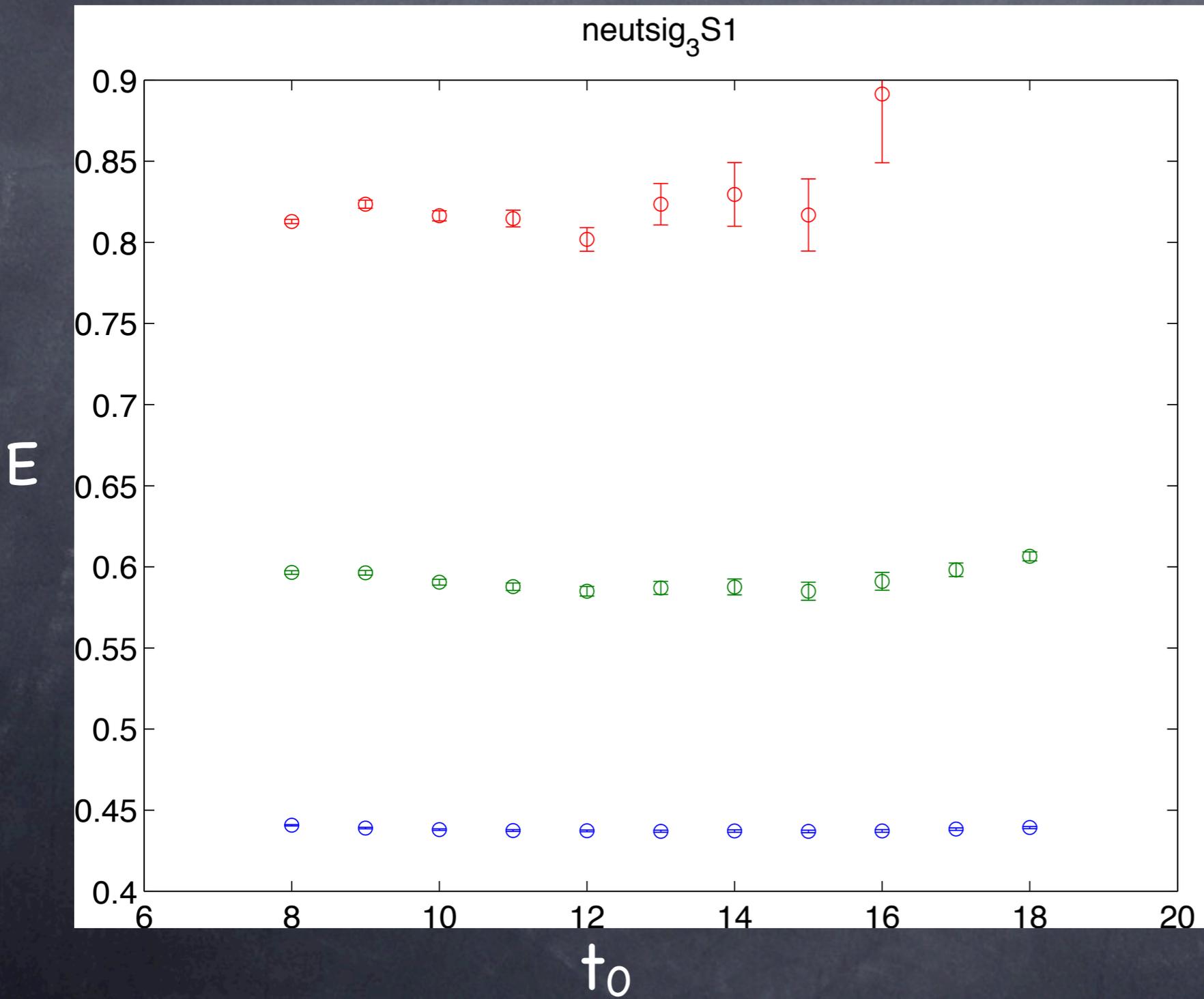
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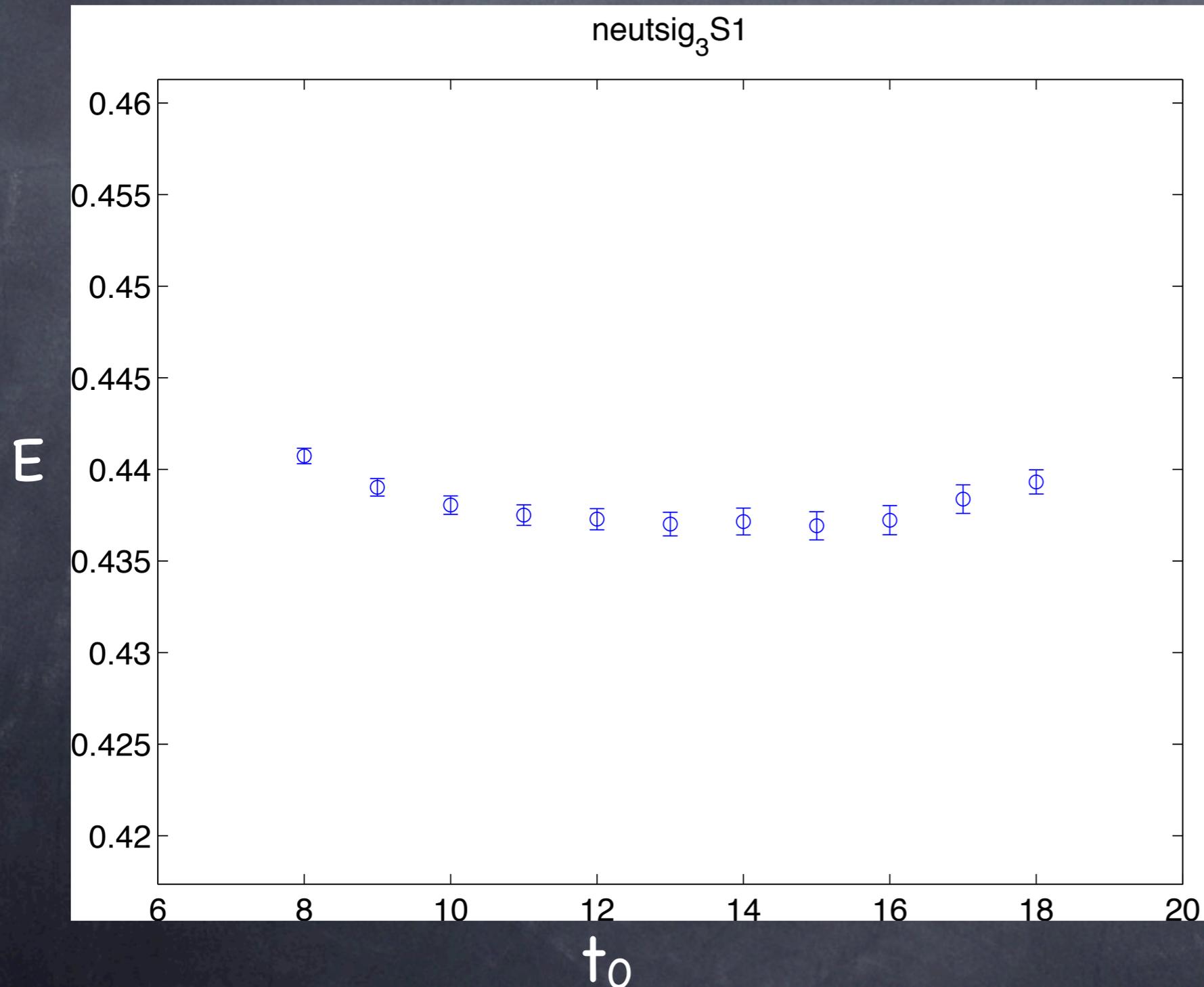
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VarPro 3 state fits

# Two Baryons

NPLQCD data



single operator

one source smearing

smearing, point and  
smearing-point sink

VarPro 3 state fits

# Conclusions

- The Generalized Pencil of Matrix provides an alternative “diagonalization” method for correlator matrices
- “Shifting” is not a substitute for a good set of interpolating fields
- Non-symmetric correlator matrices can be treated
- Symmetric positive definite correlator matrices are preferable
- Could be useful for variational approach to Form Factor calculations