Propagators in lattice Coulomb gauge and confinement mechanisms

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Together with:

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- H. Reinhardt
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Outline

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Static (equal-time) propagators in Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$

- $D(\vec{p}) = \delta^{ab}\delta_{ij}\langle \widetilde{A}_i^a(\vec{p},t)\widetilde{A}_j^b(-\vec{p},t)\rangle$
- $G(\vec{p}) = |\vec{p}|^{-2}d(\vec{p}) = \delta^{ab}\langle \widetilde{\bar{c}}^a(\vec{p},t)\widetilde{c}^b(-\vec{p},t)\rangle = \langle(-\vec{D}\cdot\vec{\nabla})^{-1}\rangle$
- $S(\vec{p}) = \delta^{AB}\langle \widetilde{\bar{\psi}}^A(\vec{p},t)\widetilde{\psi}^B(-\vec{p},t)\rangle$
- $D_0(\vec{p}) = \delta^{ab}\langle \widetilde{A}_0^a(\vec{p},t)\widetilde{A}_0^b(-\vec{p},t)\rangle$

Motivations

Static (equal-time) propagators in Coulomb gauge:

- Test for:
  - Gribov-Zwanziger confinement scenario
  - Hamiltonian variational approach
- Comparison with Landau gauge
- Renormalization issues need to be addressed

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GZ mechanism Gribov NPB 1978; Zwanziger NPB 1997

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to:
    - Gribov region Ω, local maxima $(-\vec{D}\cdot\vec{\nabla} > 0)$
    - or Fundamental modular region Λ, absolute maxima
- Radius of Λ (or Ω) introduces an IR scale $\simeq O(\sigma)$

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Continuum Coulomb gauge

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  $\sigma \leq \sigma_c$ Zwanziger PRL 2003
- DSE, gap equations, variational approach:
- IR $D(\vec{p})^{-1} \propto |\vec{p}|^{\kappa_{gl}}$; $d(\vec{p})^{-1} \propto |\vec{p}|^{\kappa_{gh}}$;
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Residual gauge on the lattice

- Weyl gauge not viable on the lattice
- Temporal gauge in theory unnecessary for static quantities
- In practice?
  - Finite volume effects?
  - Discretization effects?
- Non-static propagators as intermediate step
- Are they renormalizable?
- Different choices for global gauge at fixed time
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Gluon propagator G. B., Reinhardt. Quandt, PRL 2009

- p_0 dependence makes $D(\vec{p}, p_0)$ not renormalizable
- temporal UV cut-off $a_t \longrightarrow$ scaling violations in $\sum_{p_0} D(\vec{p}, p_0)$
  - Either go to lattice Hamiltonian limit or
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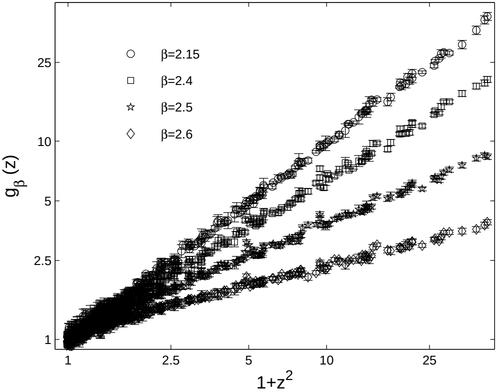
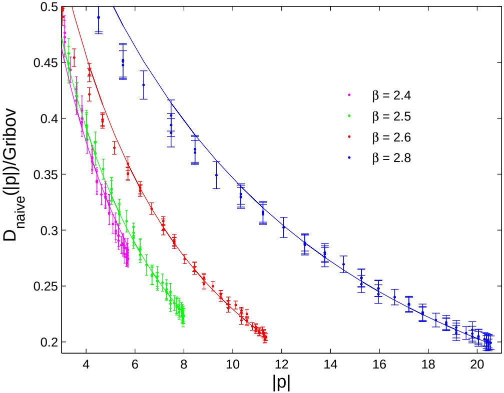
p_0 dependence hinders renormalizability



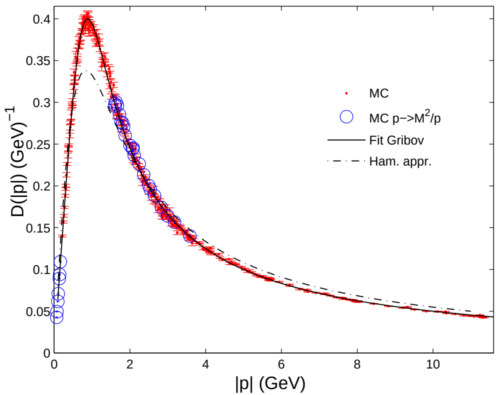
$$\frac{D_\beta(\vec{p},p_0)}{D_\beta(\vec{p},0)} = g_\beta\left(\frac{|\vec{p}|}{p_0}\right) \propto \left(1+\frac{\vec{p}^2}{p_0^2}\right)^{\delta-1}$$

scaling violations in $\sum_{p_0} D(\vec{p}, p_0)$



$$\propto B\left(\frac{4\xi^2}{4\xi^2+\hat{p}^2}, \frac{1}{2}, -\delta + \frac{1}{2}\right) \quad \left[\frac{a_s}{a_t} = \xi \geq 1,\ \hat{p} = a_s|\vec{p}|\right]$$

$D(\vec{p})$ agrees with Gribov's formula



$$D(|\vec{p}|) \propto (|\vec{p}|^2 + \frac{M^4}{|\vec{p}|^2})^{-1/2},\ M = 0.856(8)GeV,\ \chi^2/\text{d.o.f.} = 1.6$$

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Comparison with Landau gauge G. B., Reinhardt. Quandt, PRD 2010

- Coulomb and Landau gluon are surprisingly similar
- Coincide rescaling momentum. No new scale needed!
- Simple description for Landau gluon for all momenta
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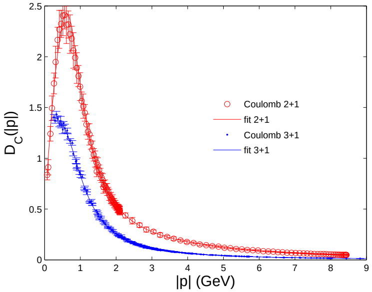
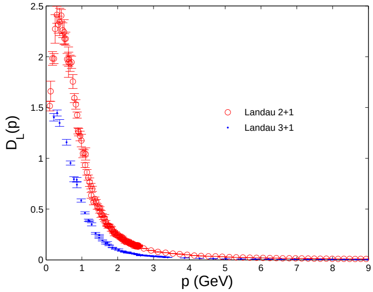
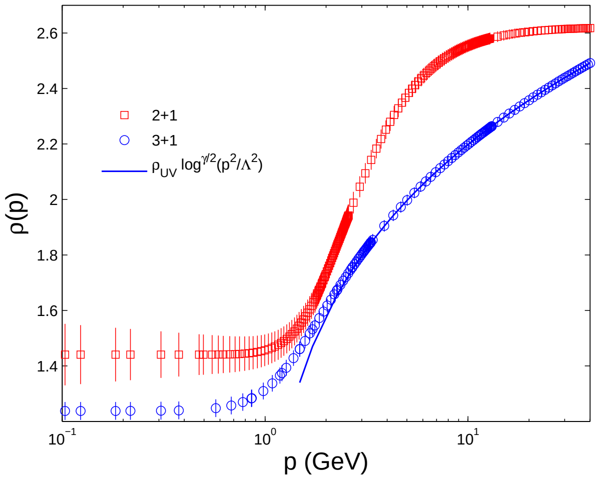
Coulomb and Landau gluon are surprisingly similar



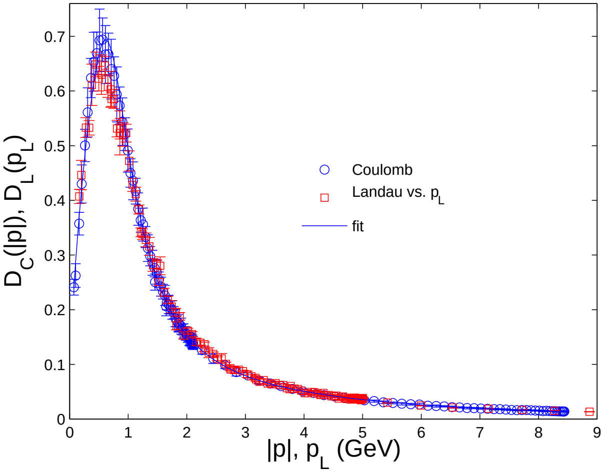


$$D_c(\vec{p}) = |\vec{p}|^{-1} D(\vec{p})$$

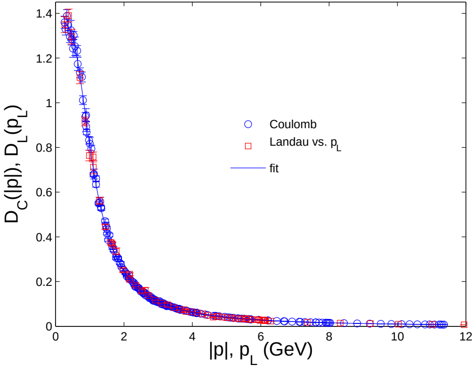
$p \longrightarrow p_L = p\,\rho(\frac{p}{\Lambda})$



2+1 dimensions



3+1 dimension



$$D(p)^{-1} \propto \sqrt{M^4 + \rho_{IR}^4 p^4 \log^{2\gamma}(e + ap^2/\Lambda^2 + p^4/\Lambda^4)},$$
$$M = 0.856(8)GeV, \gamma = 0.52(3)\ \Lambda = 1.05(15), \chi^2/\text{d.o.f.} = 1.6$$

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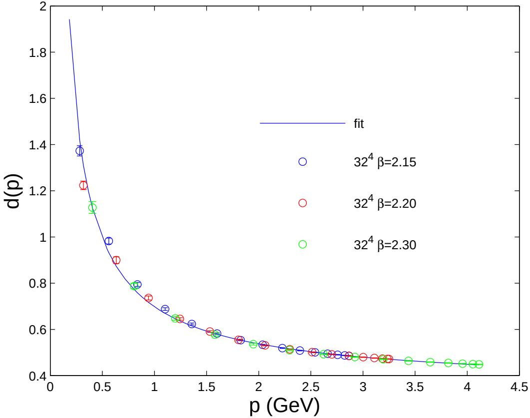
Coulomb Ghost form factor



$$d(p) \propto \sqrt{\frac{m^2}{\vec{p}^2} + \log^{-1}(e + \frac{\vec{p}^2}{m^2})},\ m = 0.33(3)\mathrm{GeV},\ \chi^2/\mathrm{d.o.f.} = 0.6$$

Outline

Quark propagator

- $S^{-1}(\vec{p}, p_0) = i\not{p}_0 A_t(\vec{p}, p_0) + i\vec{\not{p}} A_s(\vec{p}, p_0) + B_m(\vec{p}, p_0)$
  $S(\vec{p}, p_0) = -i\not{p}_0 \mathcal{A}_t(\vec{p}, p_0) - i\vec{\not{p}} \mathcal{A}_s(\vec{p}, p_0) + \mathcal{B}_m(\vec{p}, p_0)$
- If renormalizable
  - $S^{-1}(\vec{p}, p_0) = Z^{-1}(\vec{p}, p_0)\left(i\vec{\not{p}} + i\not{p}_0 \alpha(\vec{p}, p_0) + M(\vec{p}, p_0)\right)$
  - $\alpha(\vec{p}, p_0)$ and $M(\vec{p}, p_0)$ should be cut-off independent
- Static propagator $S^{(-1)}(\vec{p}) = \int d\,p_0 S^{(-1)}(\vec{p}, p_0)$
  $\rightarrow \alpha$ cancels for parity
- $S^{-1}(\vec{p}) = Z^{-1}(\vec{p})\left(i\vec{\not{p}} + M(\vec{p})\right)$
- $Z(\vec{p}) = \left(\vec{p}^2 + M^2(\vec{p})\right) \int dp_0\, \mathcal{A}_s(p) = \left(\int dp_0\, A_s(p)\right)^{-1}$
  $M(\vec{p}) = \frac{\int dp_0\, \mathcal{B}_m(p)}{\int dp_0\, \mathcal{A}_s(p)} = \frac{\int dp_0\, B_m(p)}{\int dp_0\, A_s(p)}$
- $Z(\vec{p})$ renormalizable, $M(\vec{p})$ invariants

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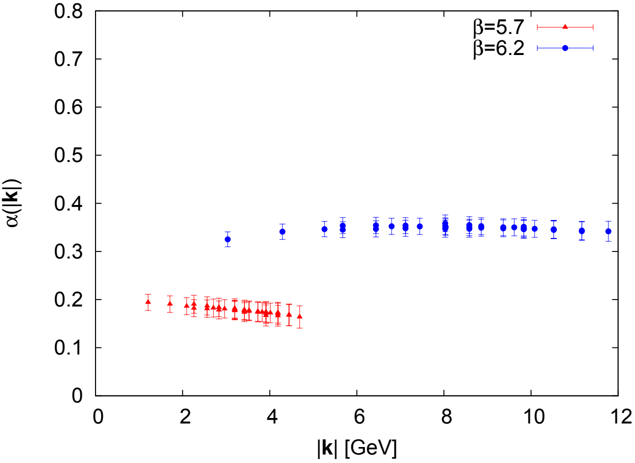
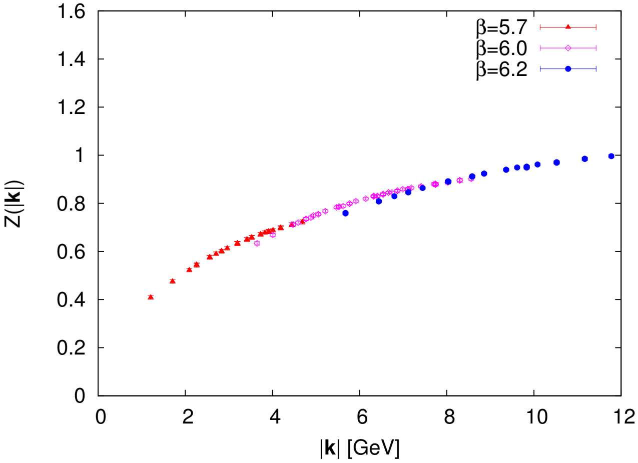
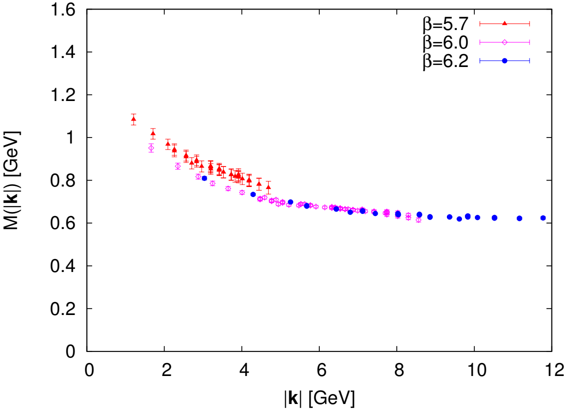
$S(\vec{p}, p_0)$ not renormalizable



$S(\vec{p})$ renormalizable



Running mass



Quenched 12^4x24, asqtad staggered, $m_b \simeq 212$ MeV

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- Landau propagators at $\beta = 0$ to check reliability of IR lattice results
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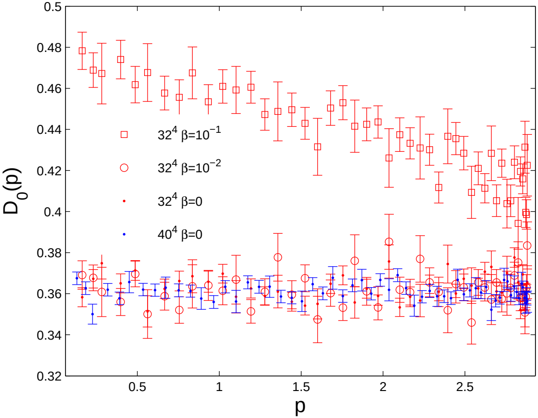
$D_0(\vec{p})$ at strong coupling



Consistent only if $a \to \infty$

Summary

- Static propagators in Coulomb gauge renormalizable
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