Non-perturbative W mass?

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Electroweak gauge action

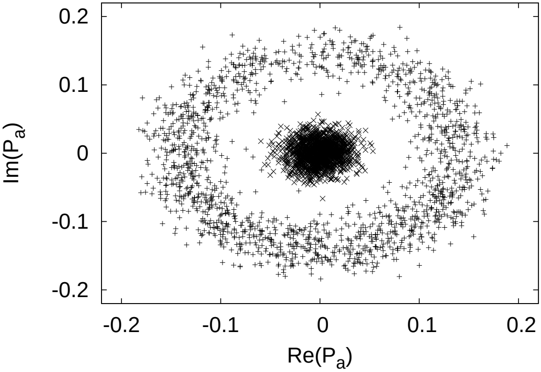
In Euclidean field theory notation the action of the electroweak gauge part of the standard model reads

$$
\begin{aligned}
S &= \int d^4x\, L^{\mathrm{ew}}\,, \\
L^{\mathrm{ew}} &= -\frac{1}{4}\, F^{\mathrm{em}}_{\mu\nu} F^{\mathrm{em}}_{\mu\nu} - \frac{1}{2}\, \mathrm{Tr}\, F^{b}_{\mu\nu} F^{b}_{\mu\nu}\,, && (1) \\
F^{\mathrm{em}}_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu\,, && (2) \\
F^{b}_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + i g_b \left[B_\mu, B_\nu\right]\,, && (3)
\end{aligned}
$$

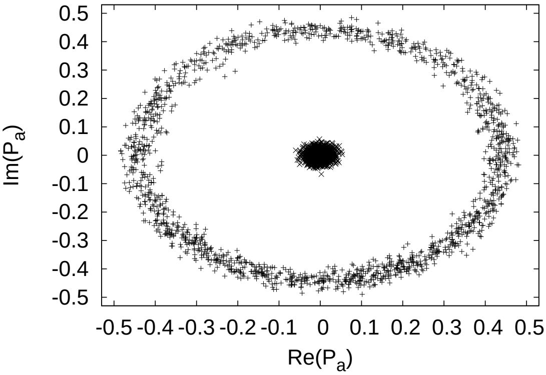
where a'_μ are U(1) and B_μ are SU(2) gauge fields.

Higgs mechanism, confinement-Higgs transition. Explicit W mass in perturbation theory. Non-perturbative mechanisms? Motivation:

U(1) deconfining phase transition



Scatter plot for U(1) Polyakov loops with the Wilson action on a 12^4 lattice at $\beta_e = 0.9$ (center) versus $\beta_e = 1.1$ (ring), $\beta_e = 1/g_e^2$.



Scatter plot for U(1) Polyakov loops with the Wilson action on a 12^4 lattice at $\beta_e = 0.9$ (center) versus $\beta_e = 2$ (ring), $\beta_e = 1/g_e^2$.

With the notation $U_l = e^{i\phi_l}$ for U(1) link matrices phases ϕ_l on parallel, nearest-neighbor links become aligned for large β_e.

SU(2) alignment

Add to the U(1) and SU(2) Wilson actions a term

$$S^{\rm int} = \sum_p S_p^{\rm int}\,, \quad S_p^{\rm int} =$$

$$\frac{\lambda}{2}\,{\rm Re\,Tr}\,\left[U_\mu(x)\,V_\nu(x+\hat{\mu}a)\,U_\mu^*(x+\hat{\nu}a)\,V_\nu^*(x)\right]$$

with $U \in U(1)$ taken as diagonal 2×2 matrices and $V \in SU(2)$. For aligned U(1) matrices the SU(2) matrices become aligned too and one may expect a SU(2) deconfining phase transition for large enough λ (breaking of the Z_2 center symmetry).

This can be obtained in the London limit ($\kappa \to \infty$) from the following gauge invariant expression:

$$
\begin{aligned}
S^{int} &= \frac{\lambda}{2} \mathrm{Re}\,\mathrm{Tr}\,\{U_\mu(x) U^*_\mu(x+\hat{\nu}a) \\
&\quad [\Phi^+(x+\hat{\mu}a) V_\nu(x+\hat{\mu}a)\Phi(x+\hat{\mu}a+\hat{\nu}a)] \\
&\quad [\Phi^+(x) V_\nu(x)\Phi(x+\hat{\nu}a)]^+\} + \kappa\,\mathrm{Tr}\,[(\Phi^+\Phi - \mathbf{1})^2]\,\}
\end{aligned}
$$

where Φ is a 2×2 matrix scalar field that is charged with respect to $U(1)$ and $SU(2)$. The gauge transformations are: $\Phi \to e^{-i\alpha} g \Phi$, where $g \in SU(2)$, $e^{i\alpha} \in U(1)$. The vacuum value of Φ is a pure gauge value $\Phi = e^{-i\alpha} g$, where $g \in SU(2)$ and $e^{i\alpha} \in U(1)$.

We choose the potential in the London limit and fix the gauge at $\Phi = \mathbf{1}$. Similar results can be expected from simulations at sufficiently large finite κ values.

Classical continuum limit

In the limit $a \to 0$ the a^4 contributions of $S_p^{\rm int}$ (after gauge fixing) give

$$L^{\rm int} = -\lambda\,{\rm Tr}\left(F^{\rm int}_{\mu\nu}F^{\rm int}_{\mu\nu}\right)\,,$$
$$F^{\rm int}_{\mu\nu} = g_a\partial_\mu A_\nu - g_b\partial_\nu B_\mu$$

where A_μ are the photon and B_μ the gluon fields. There is no explicit mass term $\sim B_\mu B_\mu$ which would be obtained by applying the London limit to conventional Higgs coupling.

Monte Carlo updating

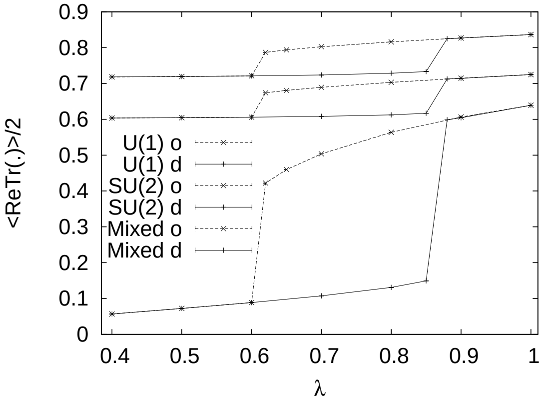
Our MC procedure proposes the usual U(1) and SU(2) changes. For the update of a SU(2) matrix $V_\mu(n)$ we need the contribution to the action, which comes from the six staples containing this matrix

$$
\begin{aligned}
V_{\sqcup,\mu}(n) &= \frac{\beta_b}{2} \sum_{\nu\neq\mu} \left[V_\nu(n+\hat{\mu})\, V^*_\mu(n+\hat{\nu})\, V^*_\nu(n) \right. \\
&+ \left. V^*_\nu(n+\hat{\mu}-\hat{\nu})\, V^*_\mu(n-\hat{\nu})\, V_\nu(n-\hat{\nu}) \right] \\
&+ \frac{\lambda}{2} \sum_{\nu\neq\mu} \left[U_\nu(n+\hat{\mu})\, V^*_\mu(n+\hat{\nu})\, U^*_\nu(n) \right. \\
&+ \left. U^*_\nu(n+\hat{\mu}-\hat{\nu})\, V^*_\mu(n-\hat{\nu})\, U_\nu(n-\hat{\nu}) \right] .
\end{aligned}
$$

and correspondingly for the U(1) matrices $U_\mu(n)$. This is well suited for updates with a biased Metropolis-heatbath algorithm with an acceptance rates for U(1) as well as for SU(2) updates larger than 95% in the range of parameters considered.

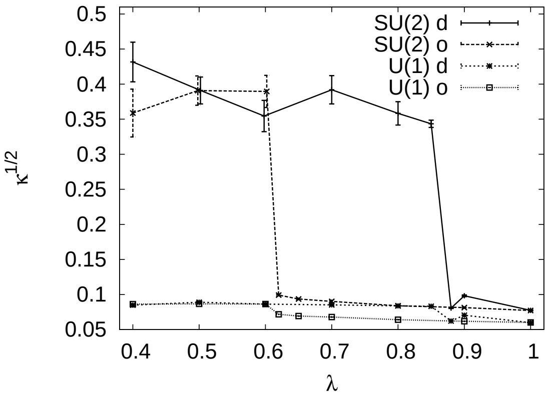
Numerical Results

Simulations reported here are at $\beta_e = 1.1$ in the Coulomb phase and $\beta_b = 2.3$ in the SU(2) scaling region.



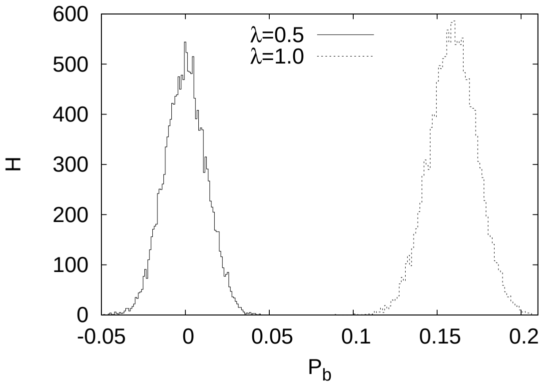
Plaquette expectation values on a 12^4 lattice as function of λ (ordered o and disordered d starts): Strong first-order transition.

String tensions



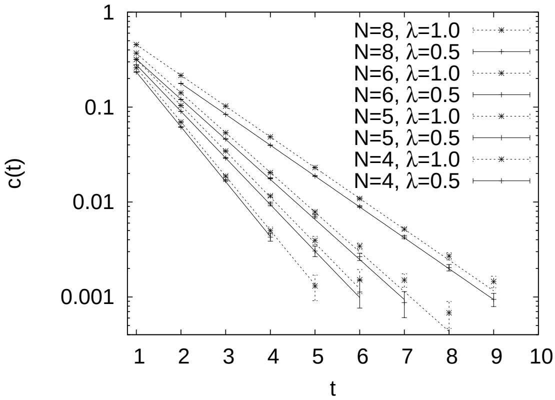
SU(2) and U(1) string tensions from Creutz ratios on a 12^4 lattice as function of λ (disordered d and ordered o starts).

Polyakov loop histograms



SU(2) Polyakov loop histogram H at $\lambda = 0.5$ (left) and $\lambda = 1$ (right).

Photon



Correlation functions data and fits for photon energy E_{k_1}, $k_1 = 2\pi/N$ estimates on $N^3 N_t$, $N_t \gg N$ lattices. The up-down order of the curves agrees with that of the labeling. Relying on the the dispersion relation:

$$m^2_{\rm photon} = E^2_{k_1} - 4\,\sin^2(k_1/2) \to 0$$

with increasing N on both sides of the transition.

Mass spectrum

Fits from zero-momentum correlations functions.

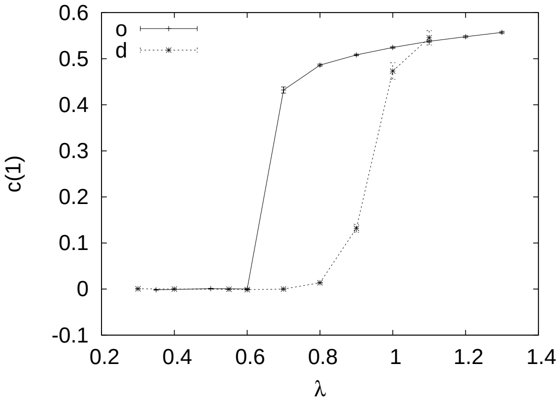
Glueballs: Correlations are very noisy, best for 0^+. Signals up to distance 2 in the disordered phase and even worse (higher masses) in the ordered phase.

Vector boson: Correlations from

$$V_{i,\mu}(x) = -i \, \mathrm{Tr} \, [\tau_i \, V_\mu(x)]$$

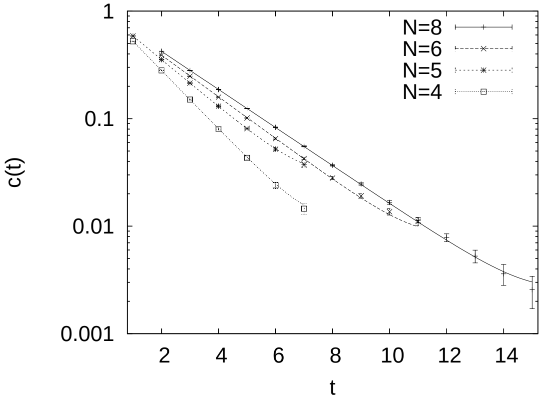
are zero in the disordered phase and strong in the ordered phase.

Vector boson order parameter



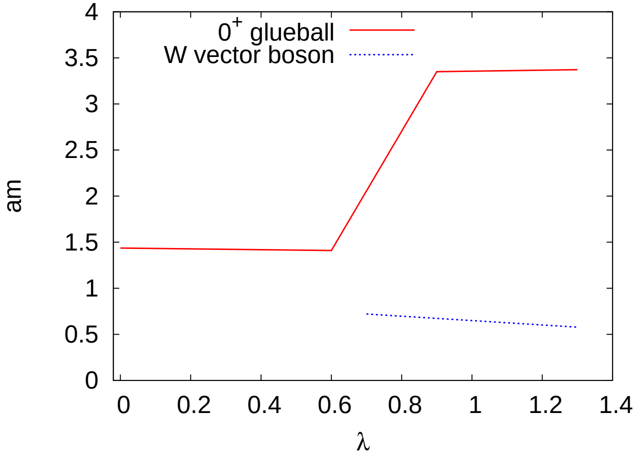
Vector boson correlation function at $t = 1$ (o ordered and d disordered SU(2) starts).

Vector boson mass estimates



Correlation functions and fits for m_W mass estimates.

Mass spectrum sketch



Extended gauge transformations

Can we get entirely rid of the scalar boson field?

In the London limit the U(1)⊗SU(2) scalar matrix field Φ has become unphysical, because it takes its vacuum value and does not fluctuate, while its gauge transformations have survived. In the present model they can be absorbed by extending the gauge transformations of the U(1) and SU(2) vector fields to U(1)⊗SU(2):

$$\begin{aligned} U_\mu(x) &\to e^{-i\alpha(x)}\, g(x)\, U_\mu(x)\, g^{-1}(x+\hat{\mu}a)\, e^{i\alpha(x+\hat{\mu}a)}\,, \\ V_\mu(x) &\to e^{-i\alpha(x)}\, g(x)\, V_\mu(x)\, g^{-1}(x+\hat{\mu}a)\, e^{i\alpha(x+\hat{\mu}a)}\,. \end{aligned}$$

In this way all remnants of the scalar field disappear without destroying invariance of the action, so that a U(1)⊗SU(2) local invariance of matter fields can be kept. A similar construction is not possible in the London limit of the Higgs mechanism.

Summary and conclusions

To the extend that similar results hold also for finite (sufficiently large) values of κ, we have constructed a gauge-invariant theory with a U(1)$\otimes$SU(2) scalar boson, which generates a deconfining transition and W boson mass.

In the London limit (for which the simulations were done) the scalar field is frozen in its vacuum state and its gauge transformations can be absorbed by extending the gauge transformations of the the photon and gluon fields to U(1)$\otimes$SU(2).

Does this allow for a quantum continuum limit?