Bottom meson masses from a RHQ action on a fine 2+1 flavor DWF lattice

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Outline

Heavy Quarks

- Current lattices are not fine enough for Charm or Bottom quarks.
  $m_c = 1.25(9) GeV, m_b = 4.20(7) GeV$
  $1/a_{24} = 1.73(2) GeV, 1/a_{32} = 2.32 GeV$
- Solutions:
  - Heavy Quark Effective Theory (HQET)
  - Non-relativistic QCD (NRQCD)
  - Relativistic Heavy Quarks / Fermilab (RHQ)
- RHQ action

$$S = \sum \bar{\psi}(m_0 a + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - \frac{1}{2} r_t (D^0)^2 - \frac{1}{2} r_s (\vec{D})^2 + \sum_{\mu,\nu} \frac{i}{4} C_p \sigma_{\mu\nu} F_{\mu\nu}) \psi$$

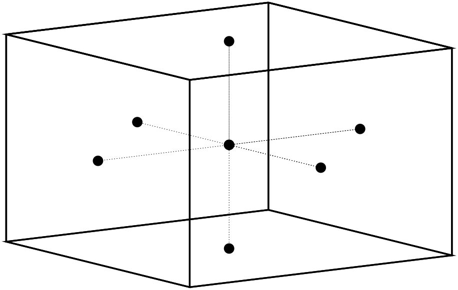
(A.El-Khadra et.al(1997),S.Aoki et al.(2003),N.Chrit et al.(2007))

  - Works for all lattice spacings and allows continuum limit
  - Support non-perturbative methods
  - Only 3 parameters need to be tuned
  - Errors of Order $O((\vec{p}a)^2)$

Lattice 2009 Results of Charm system

- Determine the RHQ parameters for heavy quark systems given the lattice spacing.
  $m_0 a = 0.072(30), C_p = 1.880(78), \zeta = 1.162(22)$
- Predict meson using the determined RHQ parameters
  $m_{\chi_{c0}} = 3.4778(94) GeV$,exp.$3.4148(3) GeV$
  $m_{\chi_{c1}} = 3.6448(112) GeV$,exp.$3.5107(.7) GeV$
  $m_{D_s} = 1.9742(24) GeV$,exp.$1.9682(5) GeV$
  $m_{D_s^*} = 2.1123(19) GeV$,exp.$2.1120(6) GeV$
- Determine the lattice spacing $a_{32}^{-1} = 2.296(32) GeV$

Schemes



- We calculate with 7 different input parameters, $(m_0 a = 3.4 \pm 0.5, C_p = 3 \pm 1, \zeta = 2.5 \pm 0.3)$ on ensembles of $m_{sea} = 0.004, 0.008$ respectively. The source is placed at 0. We took a known lattice spacing $a^{-1} = 2.32GeV$ (see Chris Kelly & Robert Mawhinney's Lattice 2009 talks).
- Step sizes are properly chosen to minimize the nonlinearity between meson masses and RHQ parameters and extrapolation should be avoided.

Meson masses

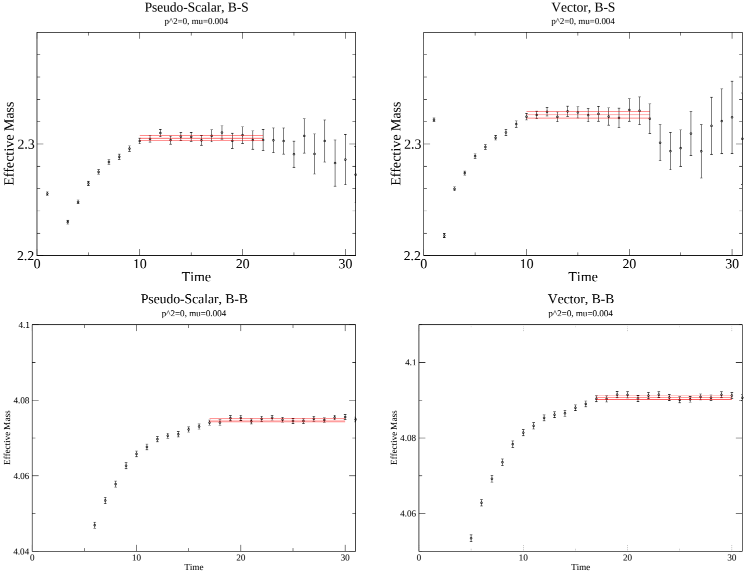
We calculate Bottomium and Bottom-Strange correlators, from which we fit 6 Bottom meson masses, $m_v^{hh}(\Upsilon)$,$m_{ps}^{hh}(\eta_b)$,$m_s^{hh}(\chi_{b0})$,$m_{av}^{hh}(\chi_{b1})$ and $m_v^{hs}(B_s^*)$,$m_{ps}^{hs}(B_s)$.

- Use 2 meson masses and the mass ratio m_1/m_2, from dispersion relation $E^2 = m_1^2 + \frac{m_1}{m_2}p^2$, to fix the RHQ parameters m_0a,C_p,ζ.
- Explore 3 approaches
  - m_{η_b}, m_Υ, $(m_1/m_2)_\Upsilon$
    small statistical error, large intrinsic error
    $O(p^2a^2) \sim (\alpha_s m_b)^2 \sim (1GeV \cdot a_{32})^2 \sim 20\%$
  - m_{B_s}, $m_{B_s^*}$, $(m_1/m_2)_{B_s}$
    large statistical error, small intrinsic error
    $O(p^2a^2) \sim (\Lambda_{QCD})^2 \sim (300MeV \cdot a_{32})^2 \sim 2\%$
  - m_{B_s}, $m_{B_s^*}$, $(m_1/m_2)_\Upsilon$
    Compromised statistical and intrinsic errors.

Linear Approach

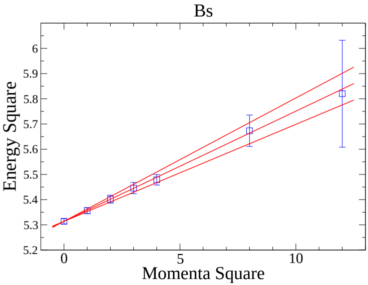
- Linear approximation in an appropriate region
  $Y = A + J \cdot X$
  with $Y^T = (m_{sa}^{hh}a, m_{hs}^{hh}a, m_1/m_2), X^T = (m_0 a, C_p, \zeta)$
  or $Y^T = (m_{sa}^{hh}a, m_{hs}^{hh}a, m_{sa}^{hs}a, m_1/m_2), X^T = (m_0 a, C_p, \zeta, a)$
- J and A are determined from the finite difference approximation to Y derivatives w.r.t. X
- Obtain RHQ parameters X by minimizing the χ^2 defined as
  $\chi^2 = \sum (A + J \cdot X - Y_{exp})^T \cdot W^{-1} \cdot (J \cdot X + A - Y_{exp})$
  where W is the covariance matrix among the meson mass combinations and the solution can be written explicitly as
  $X = (J^T \cdot W^{-1} \cdot J)^{-1} \cdot J^T \cdot W^{-1} \cdot (Y_{exp} - A)$
- Prediction of other meson masses using known RHQ parameters
  $Y_{new} = A_{new} + J_{new} \cdot X$

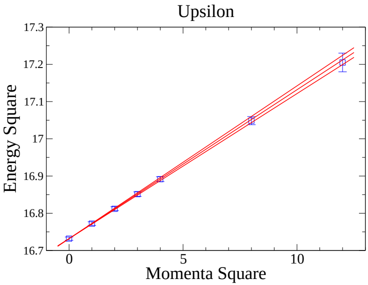
Sample Effective Mass Spectra Plots



Dispersion Relation

- We use the 4 smallest momenta, $p^2 = (\frac{2\pi}{L})^2(0, 1, 2, 3)$, to calculate the mass ratio. The dispersion relation of Υ is preferred other than that of B_s for the reason that the former calculate reduce errors in m_1/m_2 and in final results as well.





RHQ parameters

- Simulation Specifications

volume	L_s	(m_{sea}, m_s)	Trajectory(step)	# of configs
$32^3 \times 64$	16	(0.004, 0.03)	260-3000(10)	257
$32^3 \times 64$	16	(0.008, 0.03)	2000-2880(10)	88

Table: Fitting Range for B_s, B_s^* is 10-22, for η_b, Υ is 17-30, for χ_{b0}, χ_{b1} is 10-18

- RHQ parameters are determined using experimental results m_{B_s},$m_{B_s^*}$and m_1/m_2=1, assuming $1/a = 2.32GeV$.

m_{sea}	Inputs	m_0a	C_p	ζ
0.004	m_{B_s},$m_{B_s^*}$,$(m_1/m_2)_\Upsilon$	3.526(82)	3.167(352)	2.440(46)
0.008	m_{B_s},$m_{B_s^*}$,$(m_1/m_2))_\Upsilon$	3.512(138)	3.036(594)	2.395(59)
0.004	m_{B_s},$m_{B_s^*}$,$(m_1/m_2)_{B_s}$	3.694(232)	3.441(483)	2.232(301)
0.008	m_{B_s},$m_{B_s^*}$,$(m_1/m_2)_{B_s}$	3.936(493)	3.683(1043)	1.845(628)

Predictions

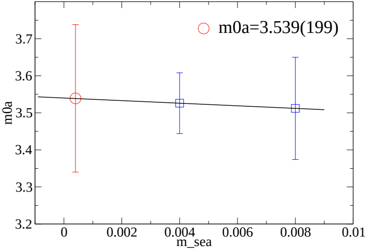
We've also predicted other meson masses m_{η_b},m_{Υ},$m_{\chi_{b0}}$ and $m_{\chi_{b1}}$ with these RHQ parameters for each ensemble.

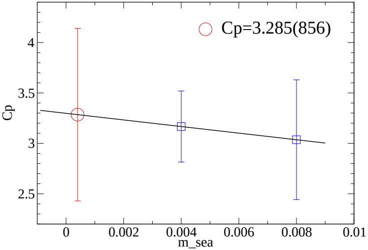
$meson/m_{sea}$	0.004	0.008	Experiment
$m_{\chi_{b0}}(GeV)$	9910(16)	9928(21)	9859(3)
$m_{\chi_{b1}}(GeV)$	9942(17)	9961(24)	9893
$m_{\chi_{b1}} - m_{\chi_{b0}}(GeV)$	32(5)	33(10)	33.34
$m_{\eta_b}(GeV)$	9479(13)	9465(18)	9389(3)(3)
$m_{\Upsilon}(GeV)$	9519(15)	9503(23)	9460
$m_{\Upsilon} - m_{\eta_b}(GeV)$	40(5)	39(8)	71(3)(3)

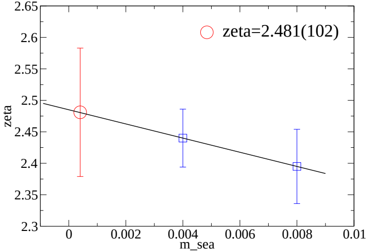
Table: All quantities are in unit of MeV

RHQ parameters at physical light quark mass limit

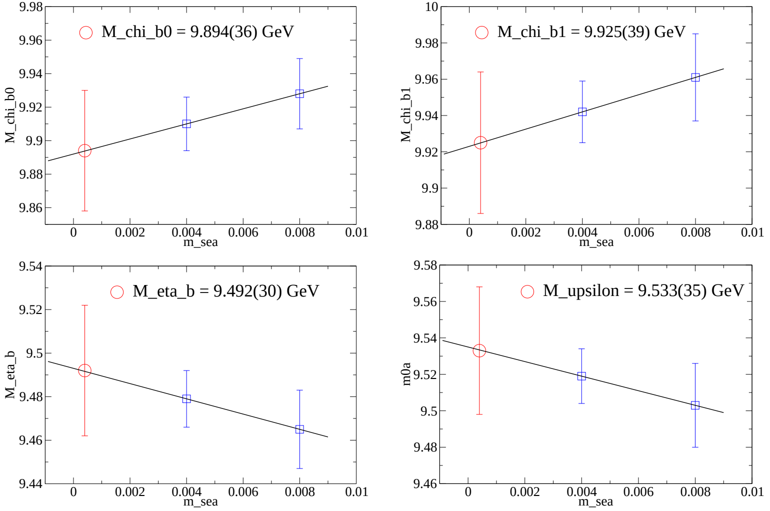
Physical light quark mass limit
$m_{sea} = 0.000399$
Extrapolated RHQ parameters
$m_0 a = 3.539(199)$
$C_p = 3.285(856)$
$\zeta = 2.481(102)$







Predictions at physical light quark mass limit

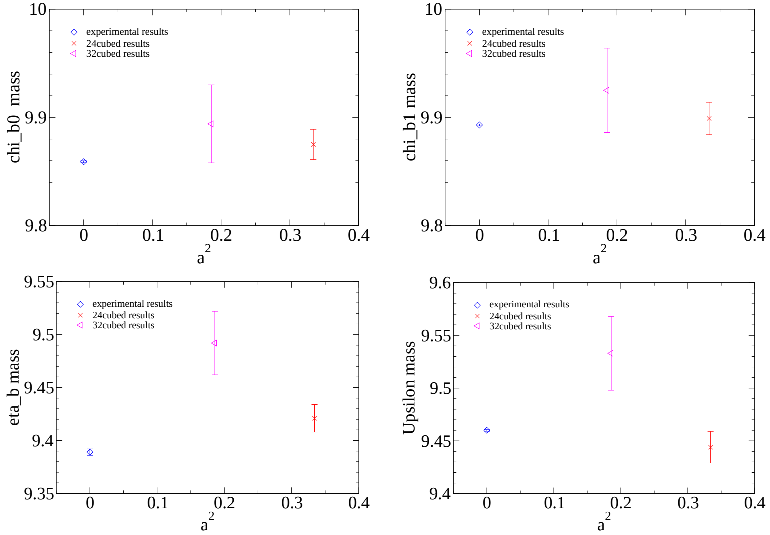


Comparison to the experimental results

meson	32^3RHQ(GeV)	24^3RHQ(GeV)	Exp.(GeV)
$m_{\chi_{b0}}$	9.894(36)	9.875(14)	9.859
$m_{\chi_{b1}}$	9.925(39)	9.899(15)	9.893
$m_{\chi_{b1}} - m_{\chi_{b0}}$	0.0311(131)	0.0236(32)	0.0333
m_{η_b}	9.492(30)	9.421(13)	9.389(3)
m_{Υ}	9.533(35)	9.444(15)	9.460
$m_{\Upsilon} - m_{\eta_b}$	0.0409(119)	0.0235(34)	0.071(3)

- $m_{\chi_{b0}}$ and $m_{\chi_{b1}}$ agree with the experimental results within one standard deviation.
- The RHQ predicted $m_{\Upsilon} - m_{\eta_b}$ is just half of the experimental value, which indicates a limitation of RHQ. m_{η_b} and m_{Υ} are larger than the experimental results and require more statistics to study.

Scaling Limit



Conclusion

- These are very preliminary results for Bottom system on $32^3 \times 64$ lattice
- RHQ parameters are determined by matching to experimental results and extrapolated to physical light quark mass limit. $(m_0 a, C_p, \zeta) = (3.539(199), 3.285(856), 2.481(102))$
- The meson masses $m_{\chi_{b0}}$,$m_{\chi_{b1}}$, m_{η_b} and m_{Υ} are predicted.

Outlook

- We will finish the ensemble of $m_{sea} = 0.006$ and run more configurations to get more accurate answer.
- The results of RHQ parameters will be applied to $B - \bar{B}$,$B_s - \bar{B_s}$ mixing, f_B calculations, etc, inside RBC-UKQCD collaborations.

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WSpect p-label	$\vec{p}$ with all permutations	p^2
0	(0,0,0)	0
1	(0,0,1)	1
2	(0,0,2)	4
3	(0,1,1)	2
4	(0,2,2)	8
5	(1,1,1)	3
6	(2,2,2)	12

Throughout our calculation the meson masses are recombined in the following way since they are sensitive to the change of input RHQ parameters.

- Spin-average:
  $m_{sa}^{hh} = \frac{1}{4}(m_{ps}^{hh} + 3m_{v}^{hh}), m_{sa}^{hs} = \frac{1}{4}(m_{ps}^{hs} + 3m_{v}^{hs})$
- Hyperfine splitting:
  $m_{hs}^{hh} = m_{v}^{hh} - m_{ps}^{hh}, m_{hs}^{hs} = m_{v}^{hs} - m_{ps}^{hs}$
- Spin-orbit average and splitting:
  $m_{soa}^{hh} = \frac{1}{4}(m_{s}^{hh} + 3m_{av}^{hh})$, $m_{sos}^{hh} = m_{av}^{hh} - m_{s}^{hh}$
- Mass ratio (from dispersion relation):
  m_1/m_2, where $E^2 = m_1^2 + \frac{m_1}{m_2}p^2$, with m_1 the rest mass and m_2 the kinetic mass

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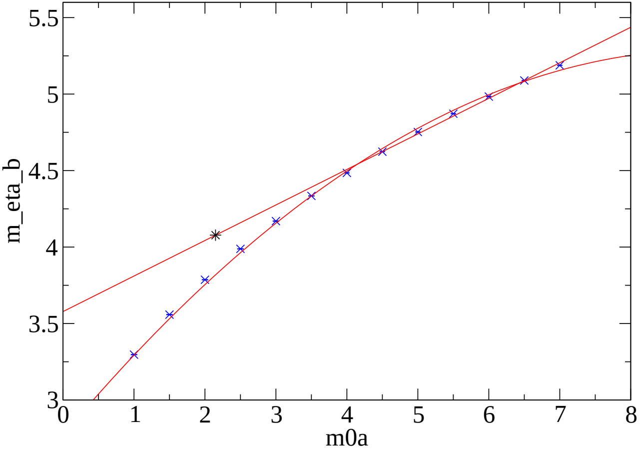

The meson mass has some nonlinear dependence on the input RHQ parameter $m_0 a$ in a long range.

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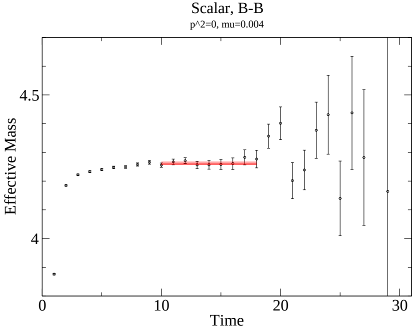
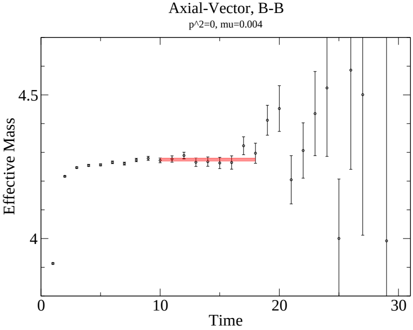


Scalar, B-B
p^2=0, mu=0.004
Effective Mass
Time
4.5
4
0
10
20
30



Axial-Vector, B-B
p^2=0, mu=0.004
Effective Mass
Time
4.5
4
0
10
20
30

backup

Calculate RHQ parameters using m_{η_b},m_{Υ},$(m_1/m_2)_{\Upsilon}$

m_{sea}	Input m_{η_b}	m_0a	C_p	ζ
0.004	9.3889GeV	3.871(68)	5.258(88)	2.380(85)
0.008	9.3889GeV	3.901(65)	5.315(94)	2.345(87)
0.004	(9.3889+0.03)GeV	3.480(31)	3.214(40)	2.417(43)
0.008	(9.3889+0.03)GeV	3.511(40)	3.246(51)	2.376(56)

We could have a consistent results with those from Bottom-Strange correlations, (3.539(199),3.285(856),2.481(102)), if we increase m_{η_b} by 30MeV.

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Lattice spacing is determined with
$Y^T_{exp} = (\frac{1}{4}(m_{B_s} + 3m_{B_s^*})a, (m_{B_s^*} - m_{B_s})a, \frac{1}{4}(m_{\eta_b} + 3m_{\Upsilon})a, m_1/m_2)$

m_{sea}	0.004	0.008	0.000399
$1/a$	2.459(33)	2.426(48)	2.489(76)