Lattice QCD with 8 and 12 Degenerate Quark Flavors

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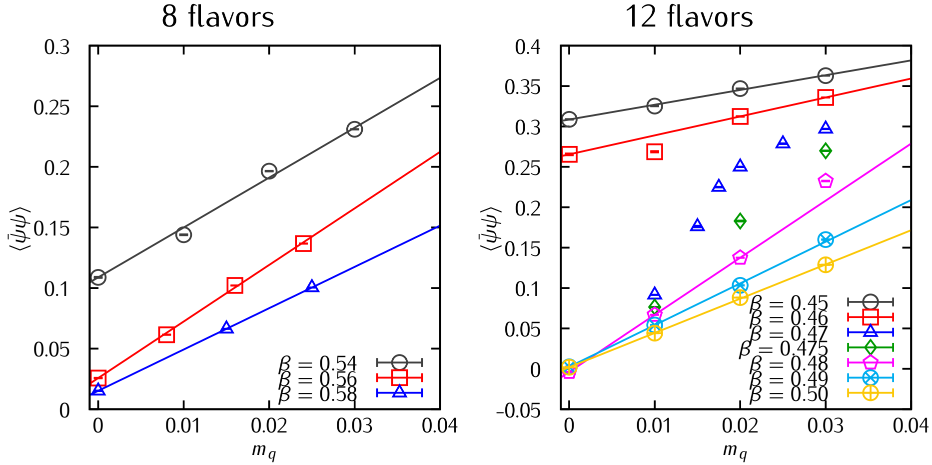
Outline

- Zero temperature, $N_\tau = 32$
  - 8 flavors in LATTICE 2008
  - 12 flavors in LATTICE 2009
  - Detailed report paper soon to be finished, please wait
- Finite temperature, 8 & 12 flavors, $N_\tau = 8$
- First order finite temperature phase transition?
  - 8 flavors: YES! We have CLEAR first order signal in strong coupling
  - 12 flavors: Future work

Our approach to zero temperature dynamics

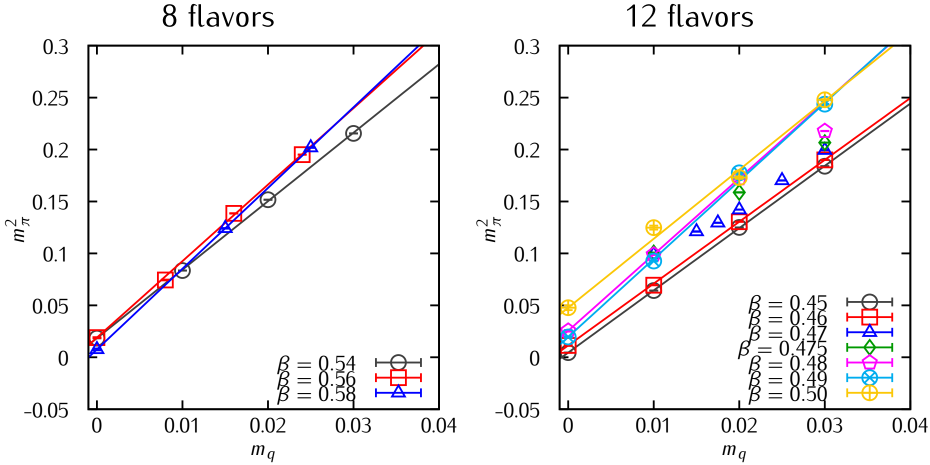
- Goal: hadronic observables from zero temperature simulations
  - Meson propagators and masses
  - π decay constant, f_π
  - Quark potential and string tension
- DBW2 gauge with naïve staggered fermion action
- RHMC algorithm
- Each measurements are separated by 10 trajectories
- Each trajectory has a length of 0.5 MD unit
- Simulation parameters of 8 flavors, LATTICE 2008
  - 3 major β values: 0.54, 0.56, 0.58
  - m_f ranges from 0.008 to 0.03
  - Lattice size of $16^3\times 32$, $24^3\times 32$ and $32^3\times 32$
- Simulation parameters of 12 flavors, LATTICE 2009
  - β ranges from 0.45 to 0.50
  - m_f ranges from 0.01 to 0.03
  - Lattice size of $16^3\times 32$, $24^3\times 32$ and $32^3\times 32$

$\langle\bar{\psi}\psi\rangle$ in linear extrapolation



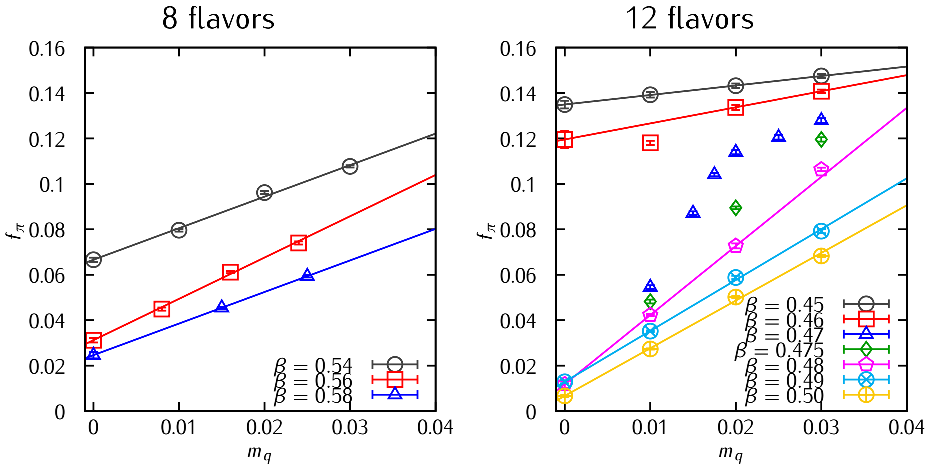
- Nonzero in the chiral limit with linear extrapolation
- Large lattice scale change with small change in bare coupling

Goldstone Boson behavior $m_\pi^2 = 2Bm_q$



- Dimensionful parameter $B = \frac{\langle\bar\psi\psi\rangle}{F^2}$
- Dynamical chiral symmetry breaking
- In weak couplings, B obtained by linear extrapolations almost a constant in lattice unit for both 8 and 12 flavors

f_π in linear extrapolation



- Falling rapidly with the quark mass, steeper in 12 flavors than in 8 flavors
- Lattice scale change consistent with $\langle\bar{\psi}\psi\rangle$

Linear extrapolation & chiral perturbation

- Explicit and implicit (in $B, F \ldots$) N_f dependence alter the convergence of ChPT

$$M_\pi^2 = 2Bm_q\Big\{1 + \xi m_q \frac{1}{N_f}\log\Big(\frac{\xi m_q}{\Lambda_M}\Big)\Big\}$$

$$F_\pi = F\Big\{1 - \xi m_q \frac{N_f}{2}\log\Big(\frac{\xi m_q}{\Lambda_F}\Big)\Big\} \qquad \text{where} \quad \xi = \frac{2B}{(4\pi F)^2}$$

$$\langle\bar\psi\psi\rangle = F^2 B\Big\{1 - \xi m_q \frac{N_f^2 - 1}{N_f}\log\Big(\frac{\xi m_q}{\Lambda_C}\Big)\Big\}$$

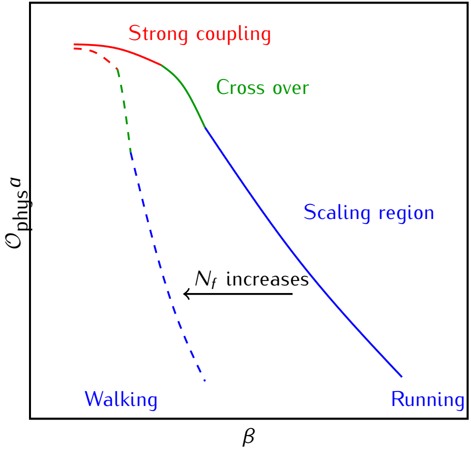
- In weak couplings, our data show

  $\xi m_q \sim 1$ for 8 flavors, $\xi m_q \sim 10$ for 12 flavors

  $\therefore$ we need $m_q \lesssim 0.001$ for 8 flavors, $m_q \lesssim 0.0001$ for 12 flavors
- Linear extrapolations give good approximations in $2+1$ flavors at physical quark mass
- Linear extrapolation fit well in the mass range of our simulations
- m_π^2 clearly Goldstone behavior in linear extrapolation

Meaning of rapid scale change — Walking

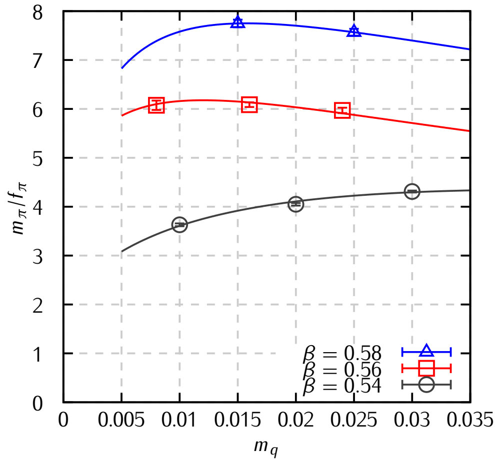
Scaling on the lattice



- Cross over from strong coupling (with higher order operators irrelevant to continuum) to the scaling region (continuously connected to the continuum limit)
- When N_f increases, small change in β results in increasingly larger change in observables in lattice unit
- Hard to tell from cross over to scaling region, no direct evidence that current simulations are in the scaling region

8 flavors m_π/f_π

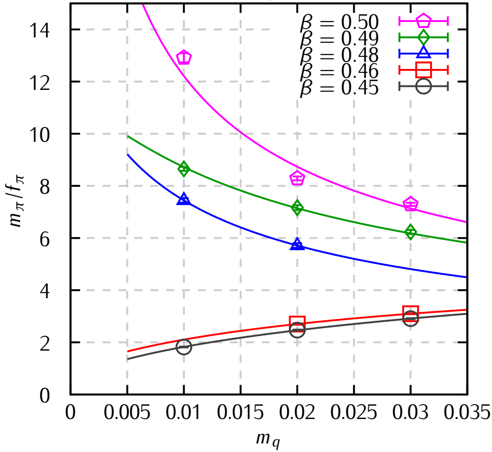
m_π/f_π in the simulated quark mass range



- Increasing function at $\beta = 0.54$
- Decreasing function for $m \gtrsim 0.015$ at $\beta = 0.56$ and 0.58,
- Values in 3 couplings differ a lot in the mass range we have simulated
- No way to follow constant m_π/f_π in the mass range we have simulated
- Mainly caused by lattice scale change
- Need smaller mass in weaker coupling

12 flavors m_π/f_π

m_π/f_π in the simulated quark mass range

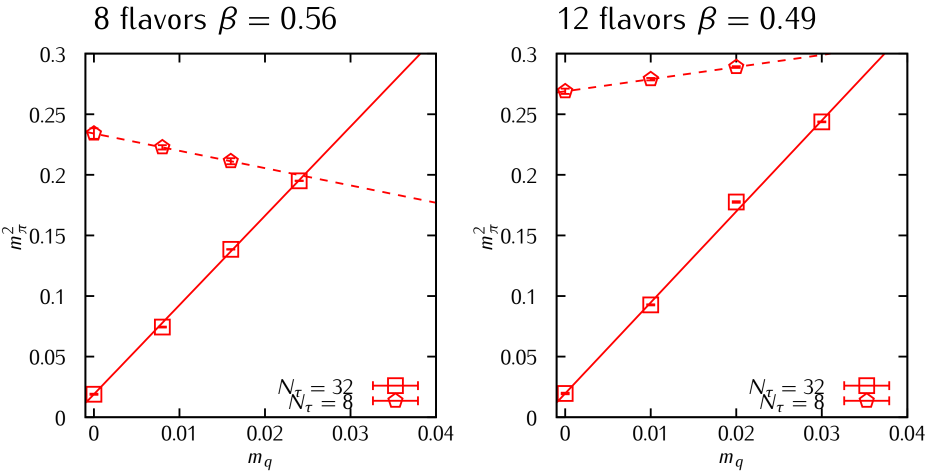


- Much profound change in 12 flavors from strong to weak coupling
- Warning: backward flow of running coupling in weaker couplings — If we keep m_π/f_π fixed, m_q increases with β
- Need much smaller quark mass to see m_π/f_π going to zero with decreasing quark mass
- Consequences of a much larger change in lattice scale

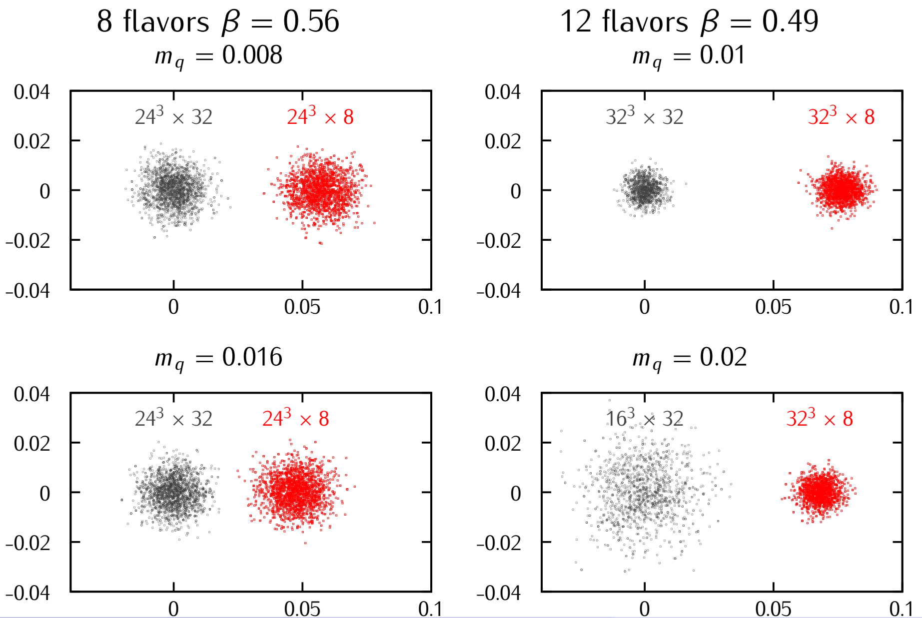
Volume and T_c

- Finite volume effect exists in 12 flavors at weak coupling and small mass ($\beta = 0.50$, $m_q = 0.01$)
- In weak couplings, m_ρ, f_π in lattice unit much smaller in 12 flavors than in 8 flavors
- T_c expected to be lower in 12 flavors, thus larger N_τ in 12 flavors
- Proceed to heat up the system, and see what happens

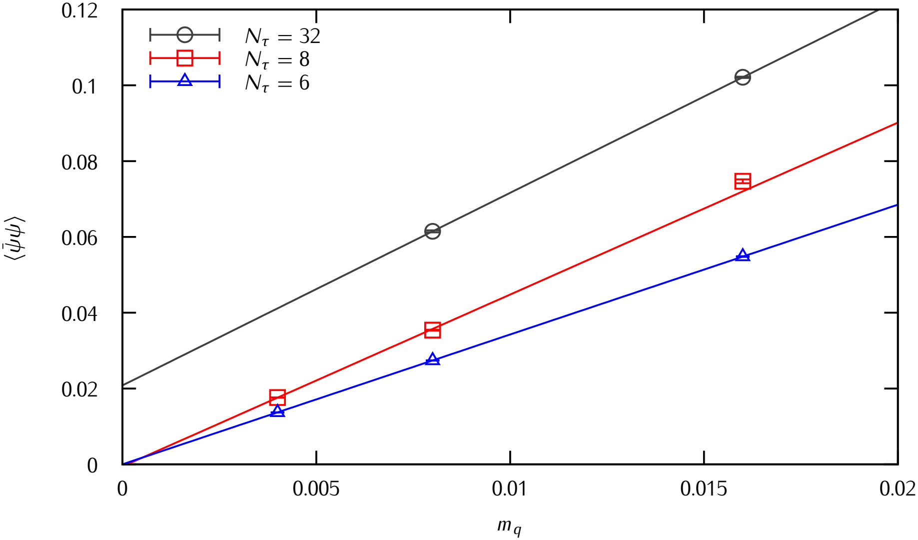
m_π^2 at $N_\tau = 8$ and $N_\tau = 32$



- Goldstone behavior at zero temperature ($N_\tau = 32$)
- No Goldstone behavior at finite temperature ($N_\tau = 8$)
- Finite temperature phase transition between $N_\tau = 32$ and $N_\tau = 8$


8 flavors $\beta = 0.56$
12 flavors $\beta = 0.49$
$m_q = 0.008$
$m_q = 0.01$
$m_q = 0.016$
$m_q = 0.02$
$24^3 \times 32$
$24^3 \times 8$
$32^3 \times 32$
$32^3 \times 8$
$16^3 \times 32$
0.04
0.02
0
-0.02
-0.04
0
0.05
0.1

8 flavors $\langle\bar{\psi}\psi\rangle$ $\beta = 0.56$

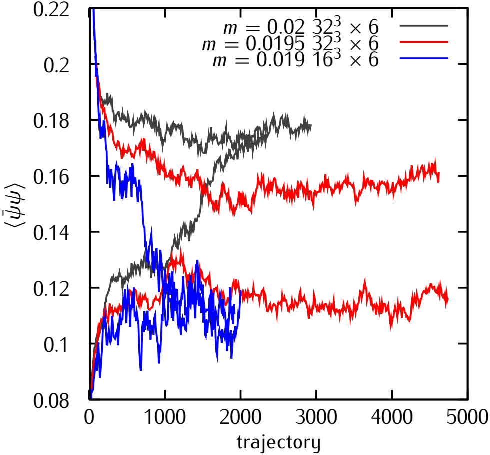


- Clearly in high temperature phase at $N_\tau = 8$ and 6

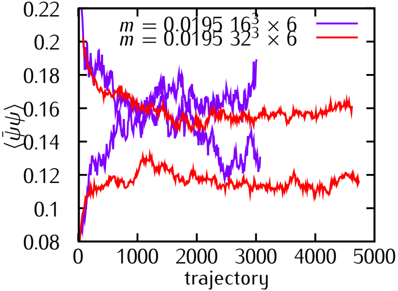
Order of the finite T phase transition

- For $N_f > 2$, finite T phase transition expected to be of first order (R. Pisarski, F. Wilczek, 1984)
- Earlier studies: M. Fukugita *et. al.* (1988); E. Pallante *et. al.* (2008)
- Can we see first order signal clearly? (Metastable lattice evolution)
- Our approach
  - Fixed β, because of rapid scale change in changing β
  - Work on discrete N_τ: 6, 8, 10, 12...
  - Tune quark masses
  - Start with strong coupling ($\beta = 0.54$), where it should be easier to see

Evolution of $\langle\bar{\psi}\psi\rangle$

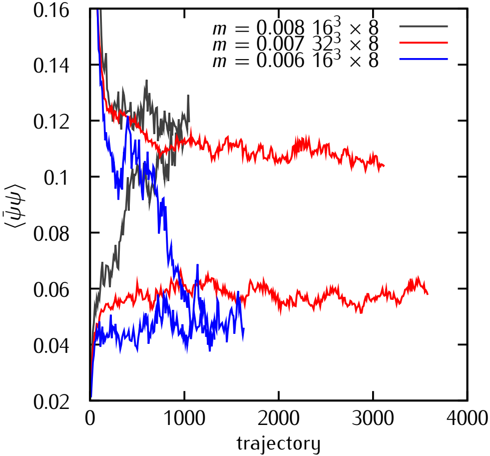


- CLEAR metastability observed
- Quark mass has to be tuned within 3%
- Need large volume, 32^3 rather than 16^3

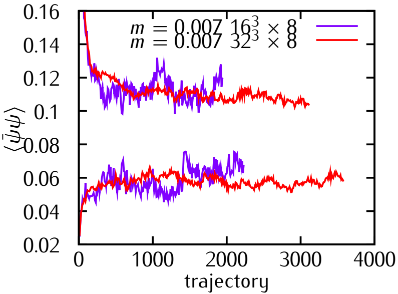


8 flavors, $\beta = 0.54$ Strong coupling, $N_\tau = 8$

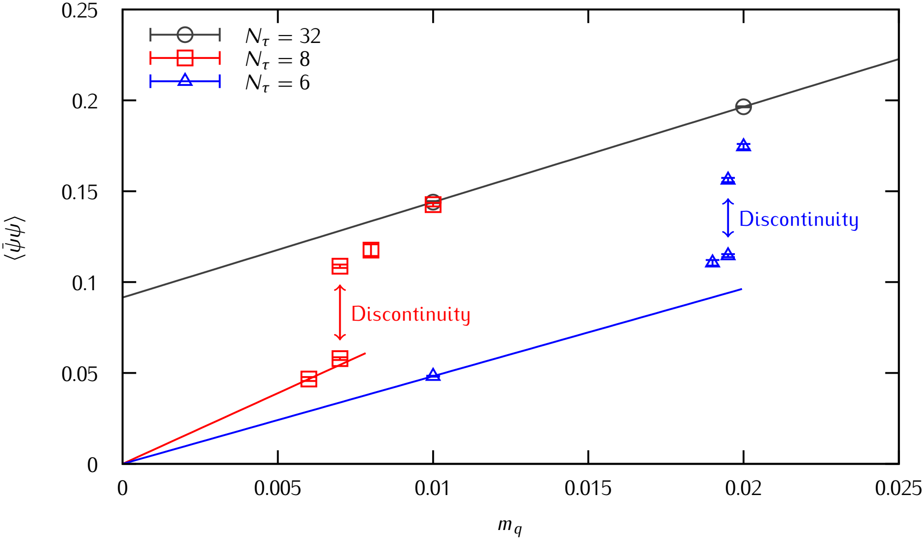
Evolution of $\langle\bar{\psi}\psi\rangle$



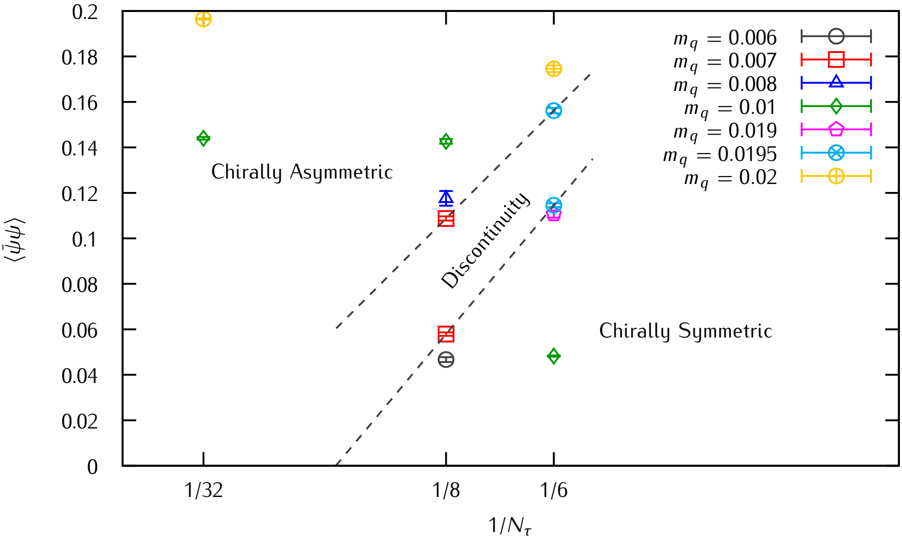
- CLEAR metastability observed
- Quark mass has to be tuned within 10%
- Metastability observed in both spatial volumes (16^3 and 32^3), but 16^3 much noisier



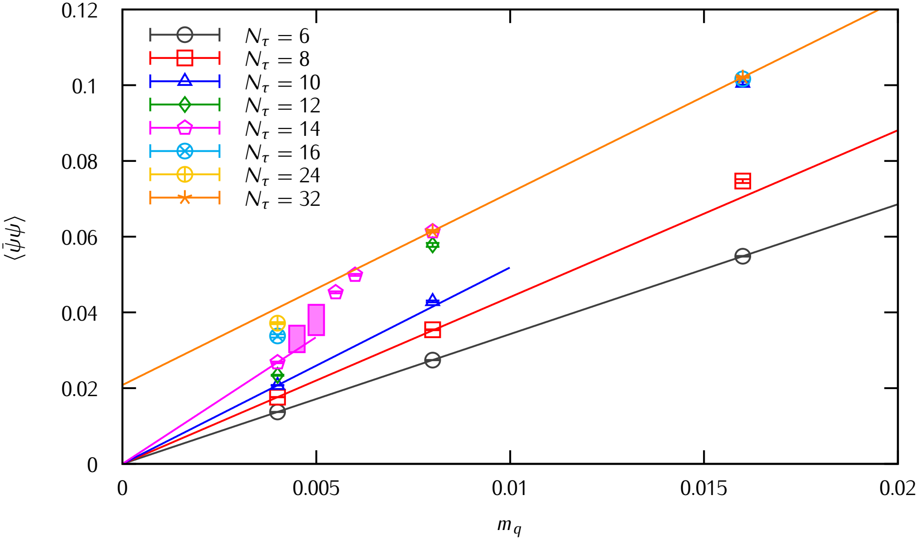
$\langle\bar{\psi}\psi\rangle$ vs. m_q, 8 flavors, $\beta = 0.54$



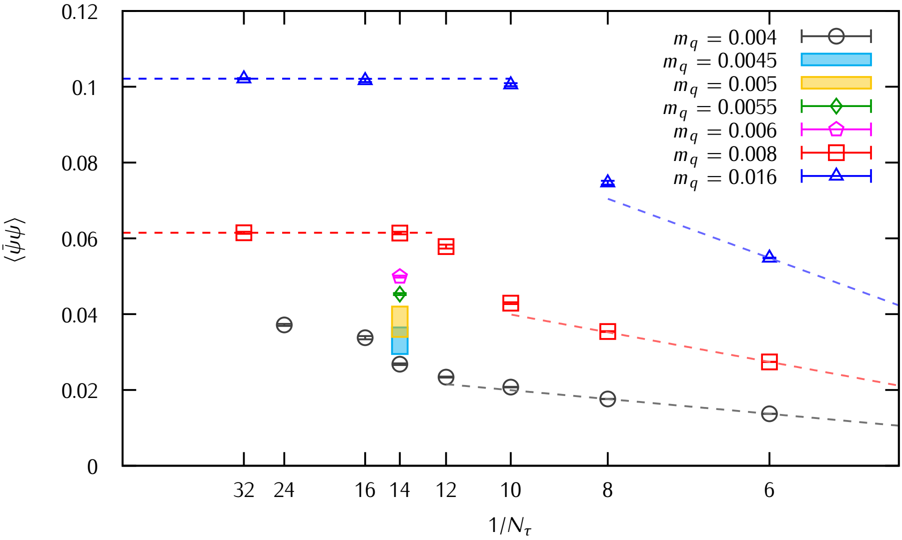
$\langle\bar{\psi}\psi\rangle$ vs. T, 8 flavors, $\beta = 0.54$



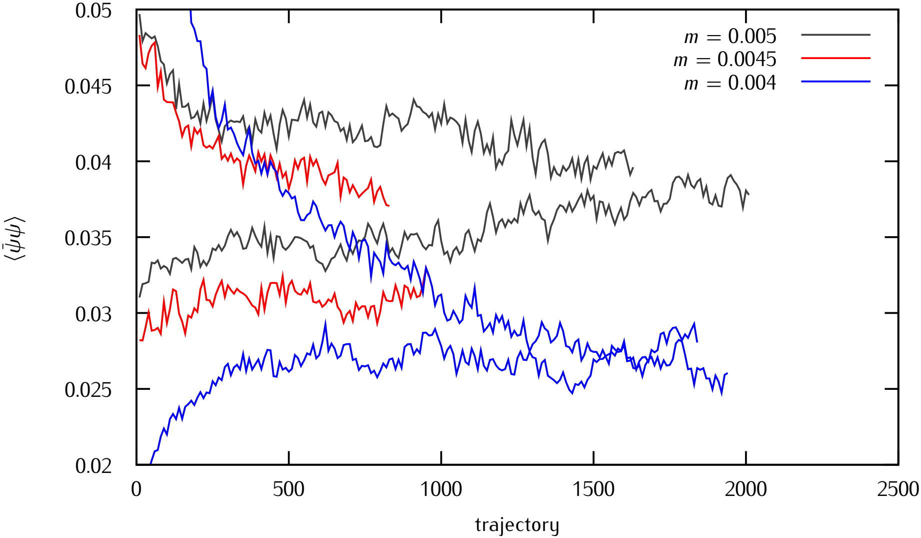
$\langle\bar{\psi}\psi\rangle$ vs. m_q, 8 flavors, $\beta = 0.56$ Weaker coupling



$\langle\bar{\psi}\psi\rangle$ vs. T, 8 flavors, $\beta = 0.56$

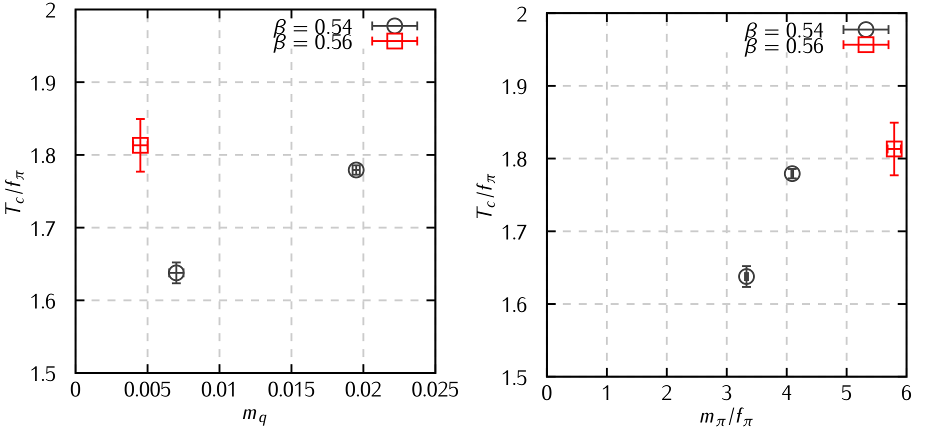


8 flavors, $\beta = 0.56$, $N_\tau = 14$



- Very long thermalization time
- Need finer tuning of quark mass or larger volume to confirm

8 flavors, T_c/f_π vs. m_q



- $\beta = 0.56$ point hasn't confirmed

Summary

- 8 flavors is clearly in a chiral symmetry breaking phase with rapid crossover, which changes scale by $\sim 2\times$
- 12 flavors clearly has a marked change in system, which changes scale by $\sim 10\times$; plausible arguments that it is chiral symmetry breaking phase with some finite volume effects
- Significant differences between $N_\tau = 32$ and $N_\tau = 8$ suggest finite temperature phase transition occurs for both 8 and 12 flavors
- First order finite T phase transition observed with 8 flavors
  - At strong coupling $\beta = 0.54$, clear first order signal
  - At weaker coupling $\beta = 0.56$, needs more work

Acknowledgment

- Simulations are done using NYBlue BG/L and QCDOC at Columbia University and BNL.
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