$\Delta I = 3/2$, $K \to \pi\pi$ Decays with a Nearly Physical Pion Mass

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Introduction

- Calculations of $K \to \pi\pi$ decays could help to solve the puzzle of the $\Delta I = 1/2$ rule.
- Can put constraints on CKM matrix elements, including the CP violating phase and ϵ'/ϵ, when lattice calculations are compared with experimental results.

Introduction

- Chiral extrapolations from unphysical masses are problematic (Christ and Li, Lattice 2008), so we attempt a calculation with very close to physical masses and kinematics.
- Use RBC/UKQCD $32^3 \times 64$, $L_s = 32$ lattices with 2+1 flavors of domain wall fermions (DWF), and strong coupling ($a^{-1} = 1.4$ GeV). Iwasaki + DSDR gauge action.
- Valence pion mass $m_\pi = 145.6(5)$ MeV (unitary pion mass $m_\pi \approx 180$ MeV). Physics mostly determined by valence mass.

Effective Hamiltonian

- The weak interactions are included in an effective Hamiltonian

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i$$

  where $z_i(\mu)$, $y_i(\mu)$ are Wilson coefficients, $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$, and Q_i are four quark operators. (Buchalla et. al. Rev. Mod. Phys. 68, (1996), 1125)

- Operators can be classified by their $SU(3)_L \times SU(3)_R$ transformation properties, and by isospin.

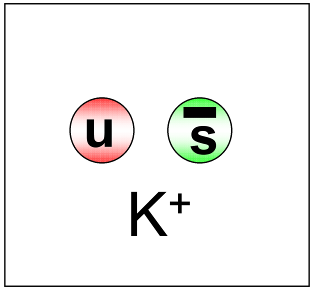
- More difficult $\Delta I = 1/2$ channel discussed in talk by Qi Liu. This talk: $\Delta I = 3/2$ channel.

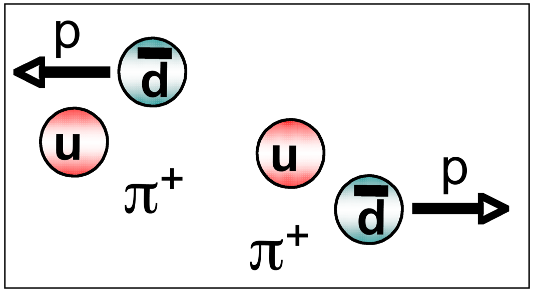
- $Q_1, Q_2, Q_9, Q_{10} \rightarrow$ (27,1) operator

- $Q_7 \rightarrow$ (8,8) operator, $Q_8 \rightarrow$ (8,8) mixed operator

Twisted Boundary Conditions

- To give the pions momentum without having to fit excited states, we use *twisted boundary conditions* (Kim and Christ, Lattice 2002, hep-lat/0210003; Sachrajda and Villadoro hep-lat/0411033).
- In particular, antiperiodic boundary conditions in two of the spatial directions so that pions have equal and opposite momentum of magnitude $p_\pi = \frac{\sqrt{2}\pi}{L}$.
- Twist the d quark only so that pions have momentum, but kaon doesn't (can calculate $K^+ \rightarrow \pi^+\pi^+$ and relate to physical decays using Wigner-Eckhart theorem).



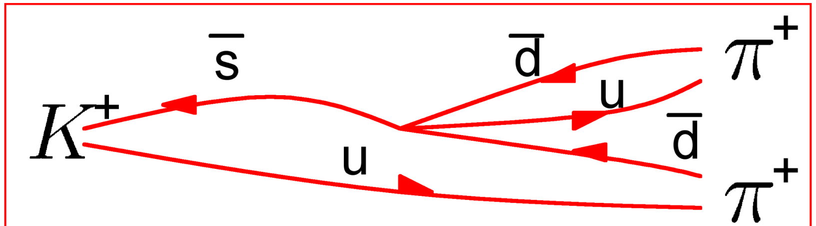


Details of the Calculation

- A similar calculation was performed by Changhoan Kim on $16^3 \times 32$ quenched lattices (Changhoan Kim, Doctoral Thesis).
- On the lattice, must calculate matrix elements

$$\mathcal{M} = \langle \pi\pi | Q | K \rangle$$

where Q is a four quark operator.



$\Delta I = 3/2$ quark flow diagram

Details of the Calculation

- 47 configurations of RBC/UKQCD $32^3 \times 64$, $L_s = 32$ lattices, with Domain Wall Fermions and 2+1 dynamical quark flavors, using the DSDR action, generated on BG/P at ANL (talk by Bob Mawhinney).
- Inverse lattice spacing $a^{-1} = 1.4$ GeV, box of side length $L = 4.51$ fm.
- Set quark masses $m_s^{val} = 0.049$, $m_s^{sea} = 0.045$, $m_l^{val} = 0.0001$, $m_l^{sea} = 0.001$ (some partial quenching). Partially quenched and unitary pion less different than indicated by m_l because $m_{res} = 0.0018$. $\rightarrow$ Valence pion mass $m_\pi = 145.6(5)$ MeV.

Details of the Calculation

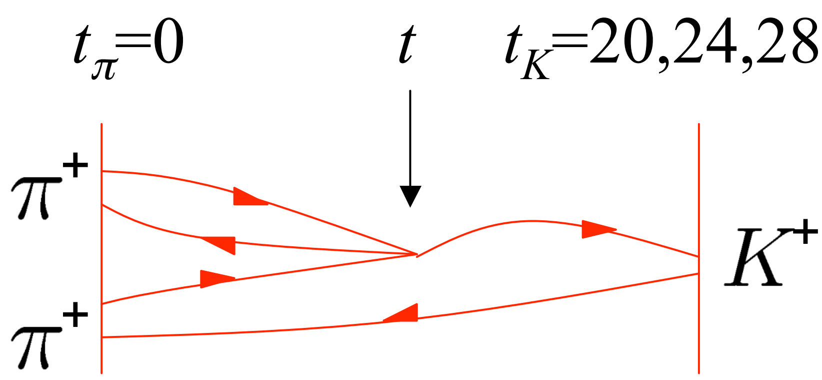
- Coulomb gauge fixed wall sources for untwisted quark propagators.
- Coulomb gauge fixed "cosine" source for twisted d quark propagators.

$$s_{\mathbf{p},cosine}(\mathbf{x}) = \cos(p_x x)\cos(p_y y)\cos(p_z z)$$

- Requires fewer inversions than pure momentum sources $e^{i\mathbf{p}\cdot\mathbf{x}}$ and $e^{-i\mathbf{p}\cdot\mathbf{x}}$.
- Non-zero total momentum terms don't contribute due to zero total momentum sink.

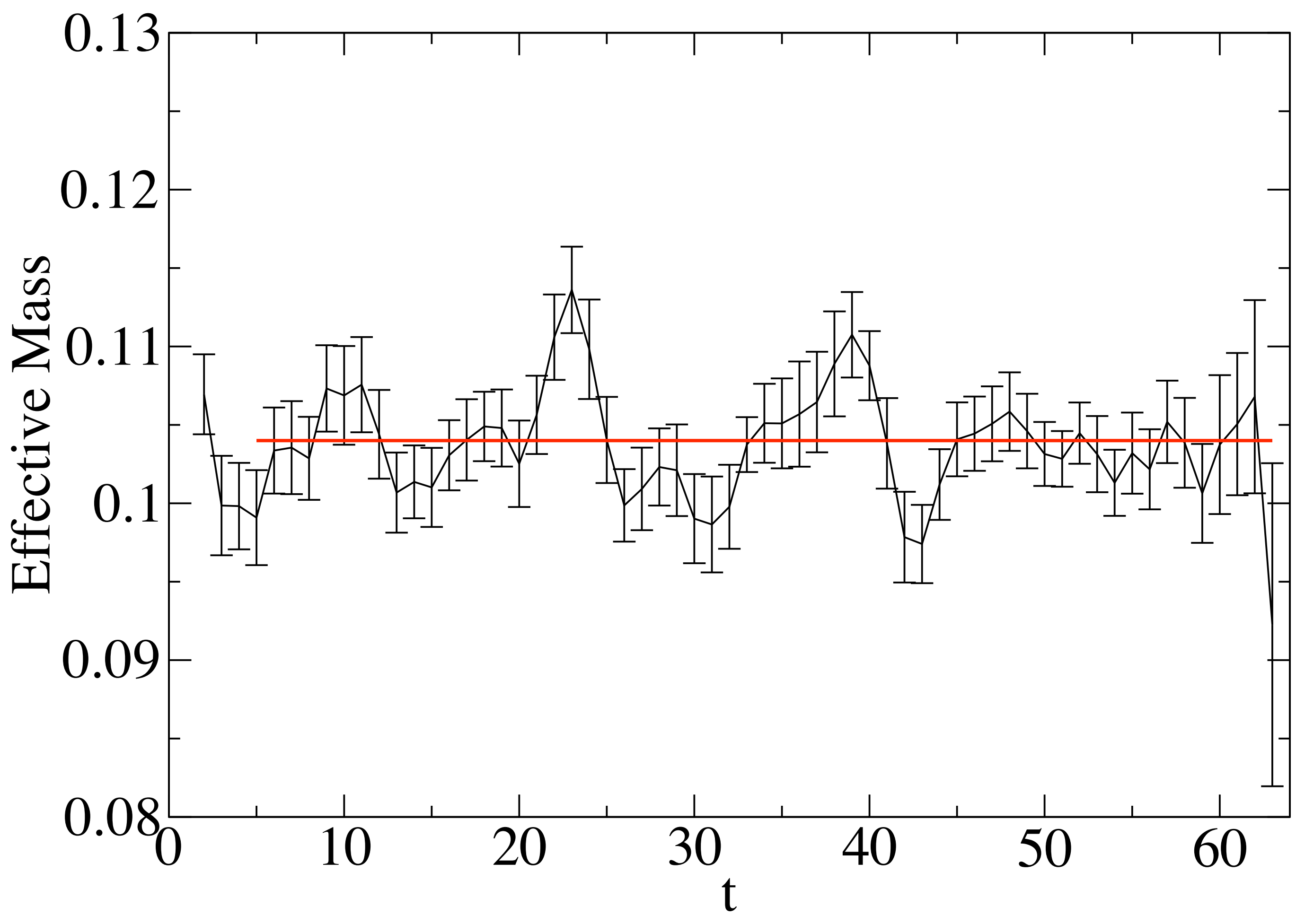
Details of the Calculation

- We add and subtract quark propagators with periodic and antiperiodic boundary conditions in the *time* direction from each other to double the effective time length.
- Two pion source at $t_\pi = 0$, kaon source at $t_K = 20, 24, 28, 32$. Time location t of four quark operator is varied.



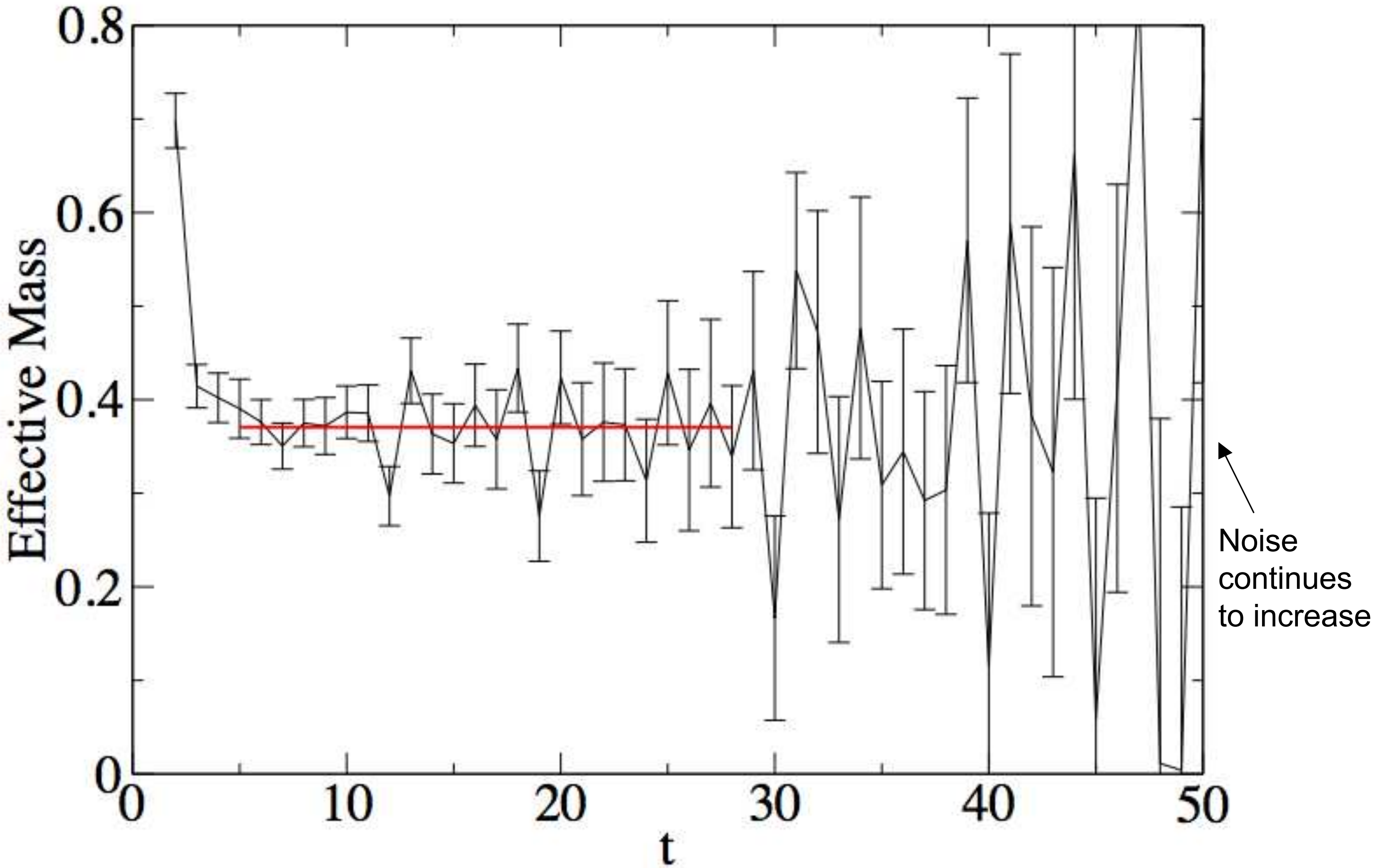
Pion Effective Mass

Fit: m_π=0.10400(37) $\rightarrow$ 145.6(5) MeV



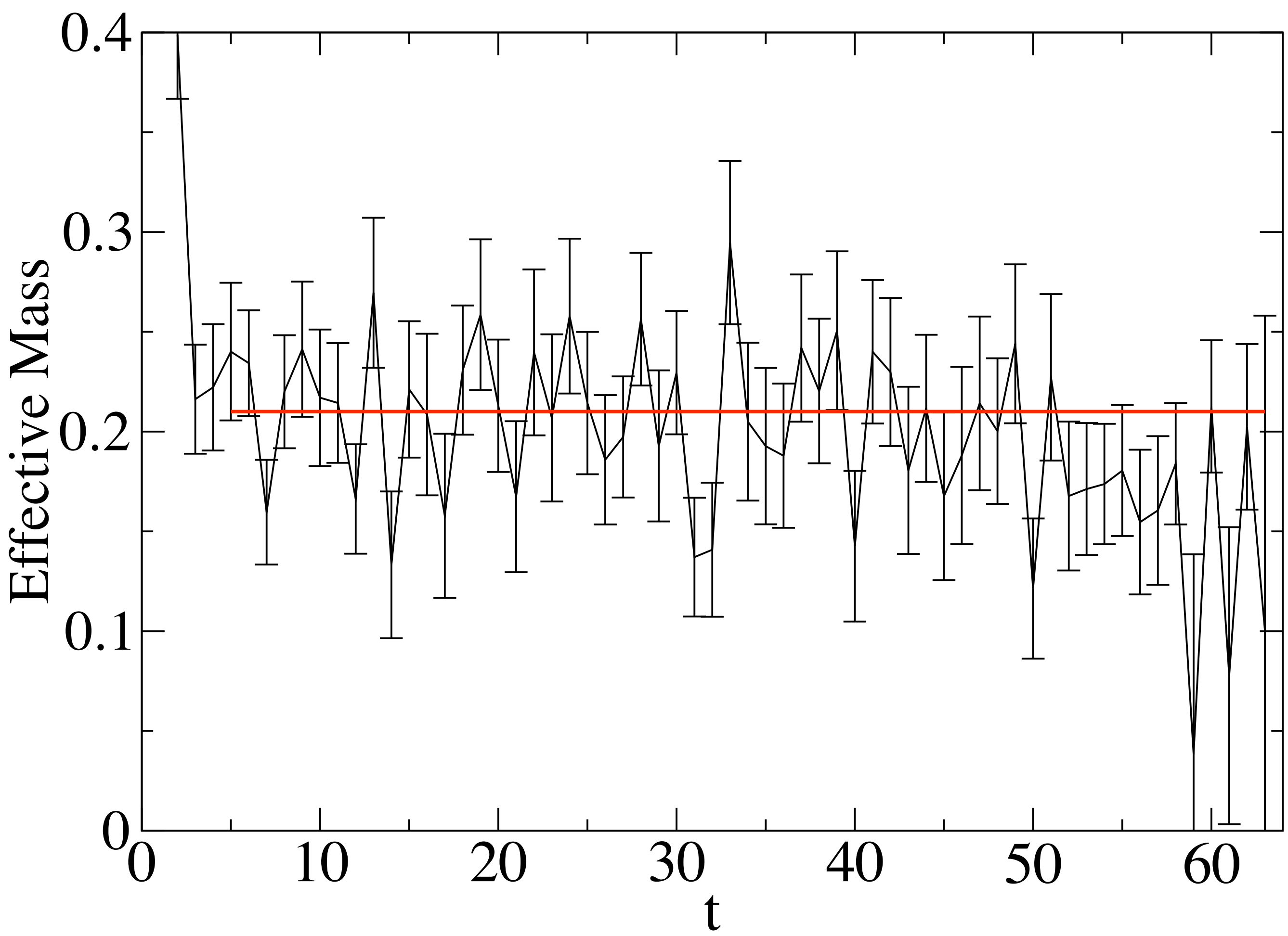
Kaon Effective Mass

Fit: m_K=0.3706(13) $\rightarrow$ 519(2) MeV



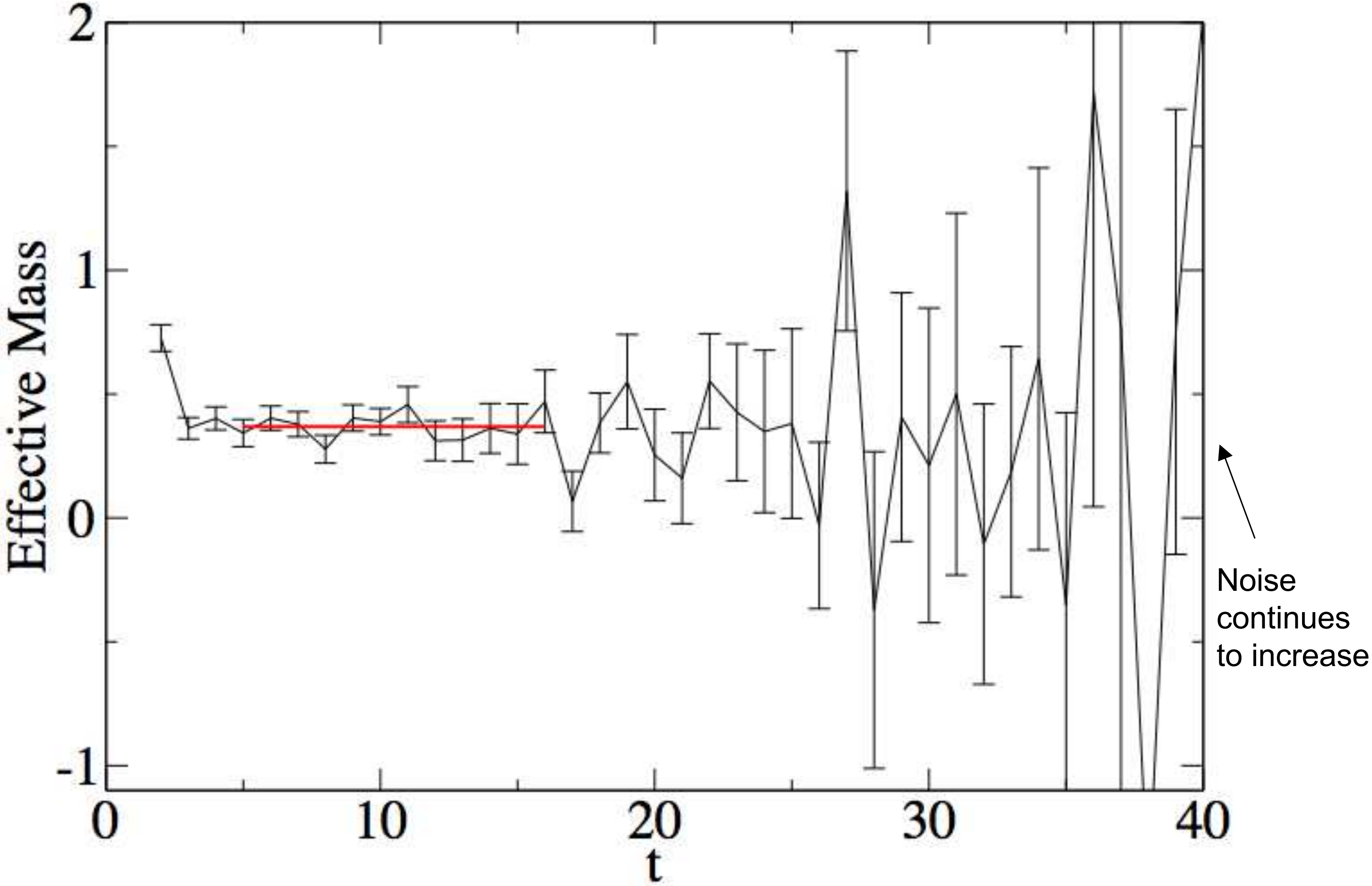
Two Pion Effective Mass ($p_\pi \approx 0$)

Fit: $E_{\pi\pi}$=0.2100(10) $\rightarrow$ 294(1) MeV



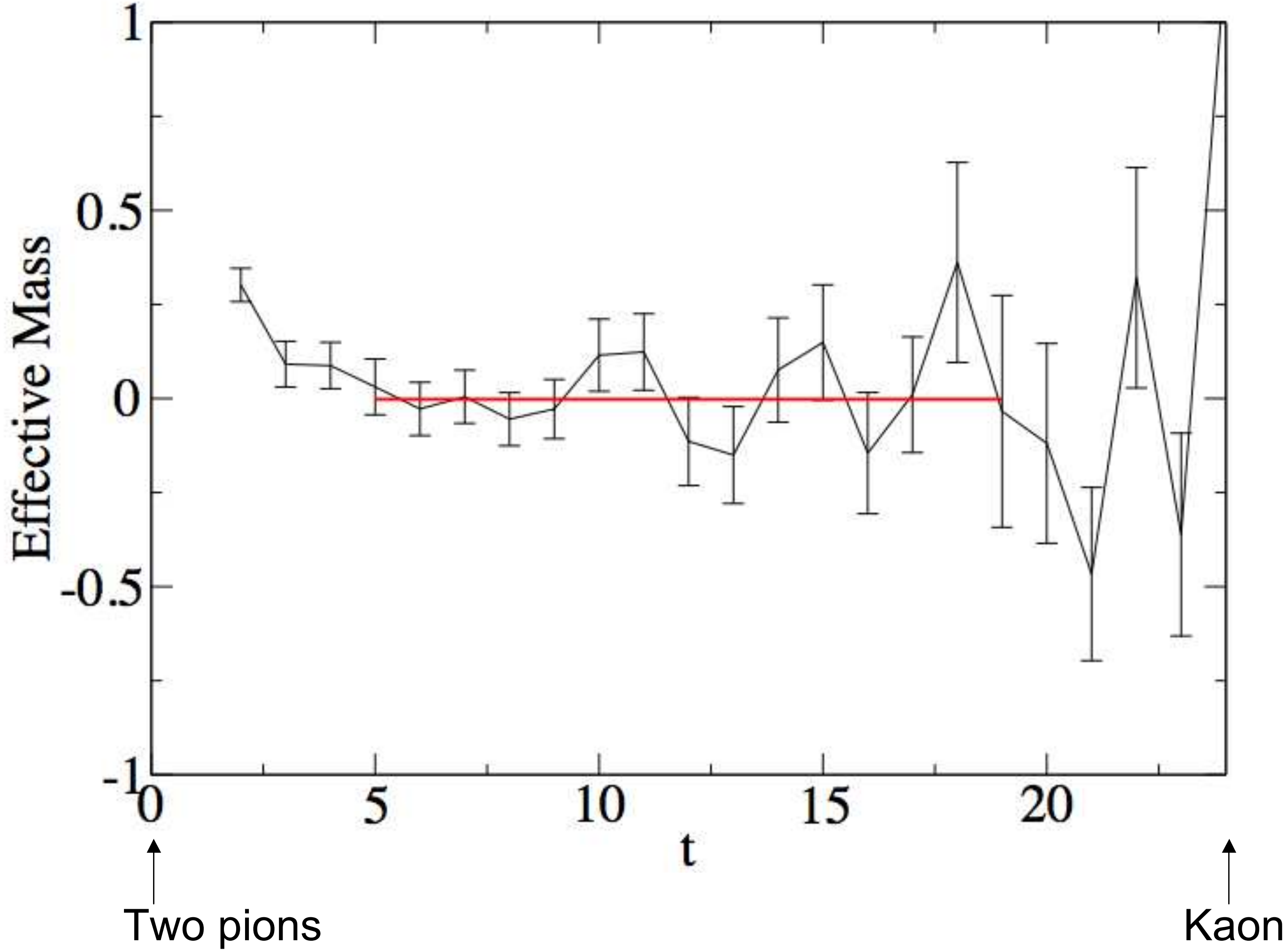
Two Pion Effective Mass ($p_\pi \approx \frac{\sqrt{2}\pi}{L}$)

Fit: $E_{\pi\pi}$=0.3687(61) $\rightarrow$ 516(9) MeV



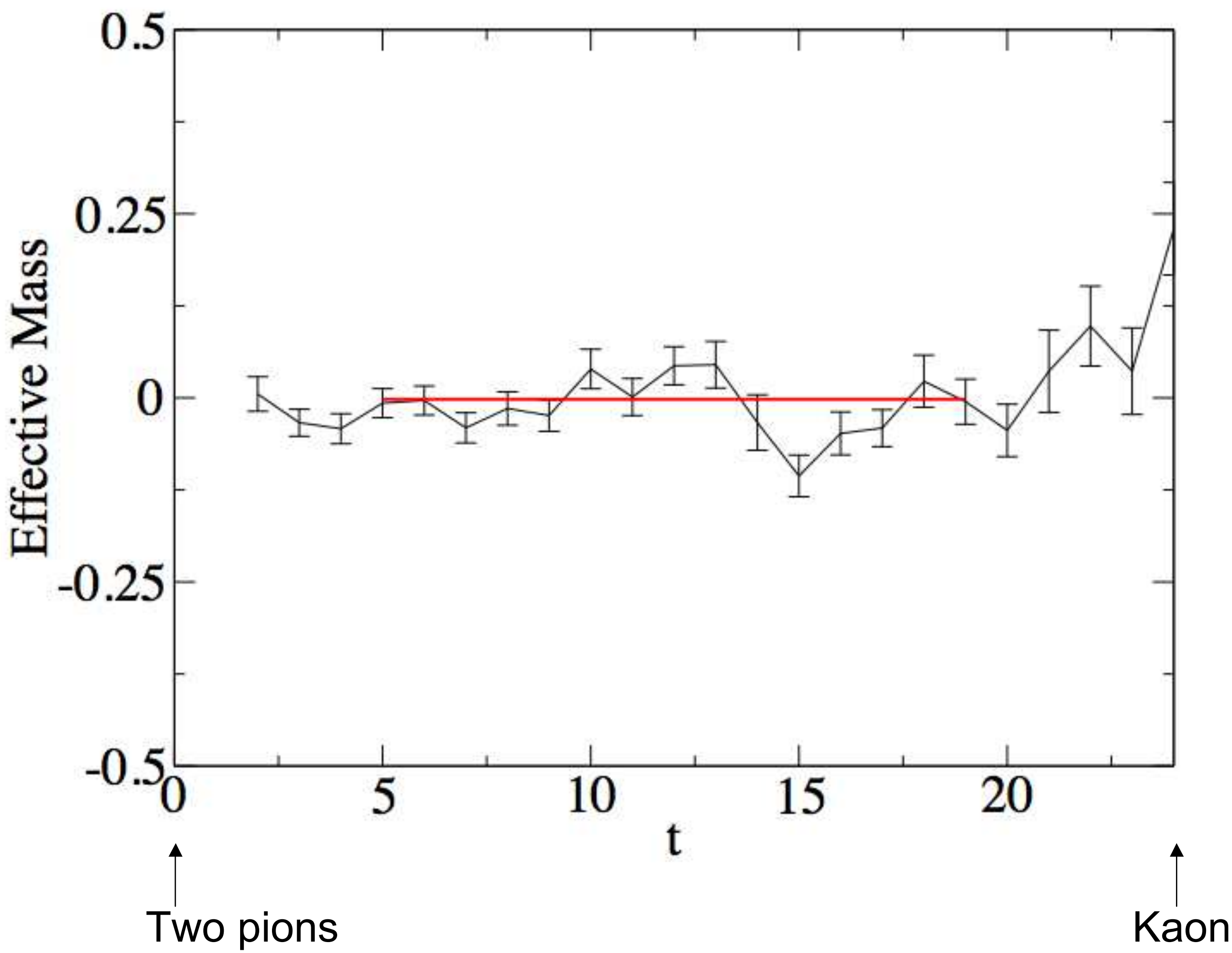
(27,1) Operator Effective Mass

From previous fits: $m_{eff} = E_{\pi\pi} - m_K$=-0.0019(59) $\rightarrow$ -2.7(8.3) MeV



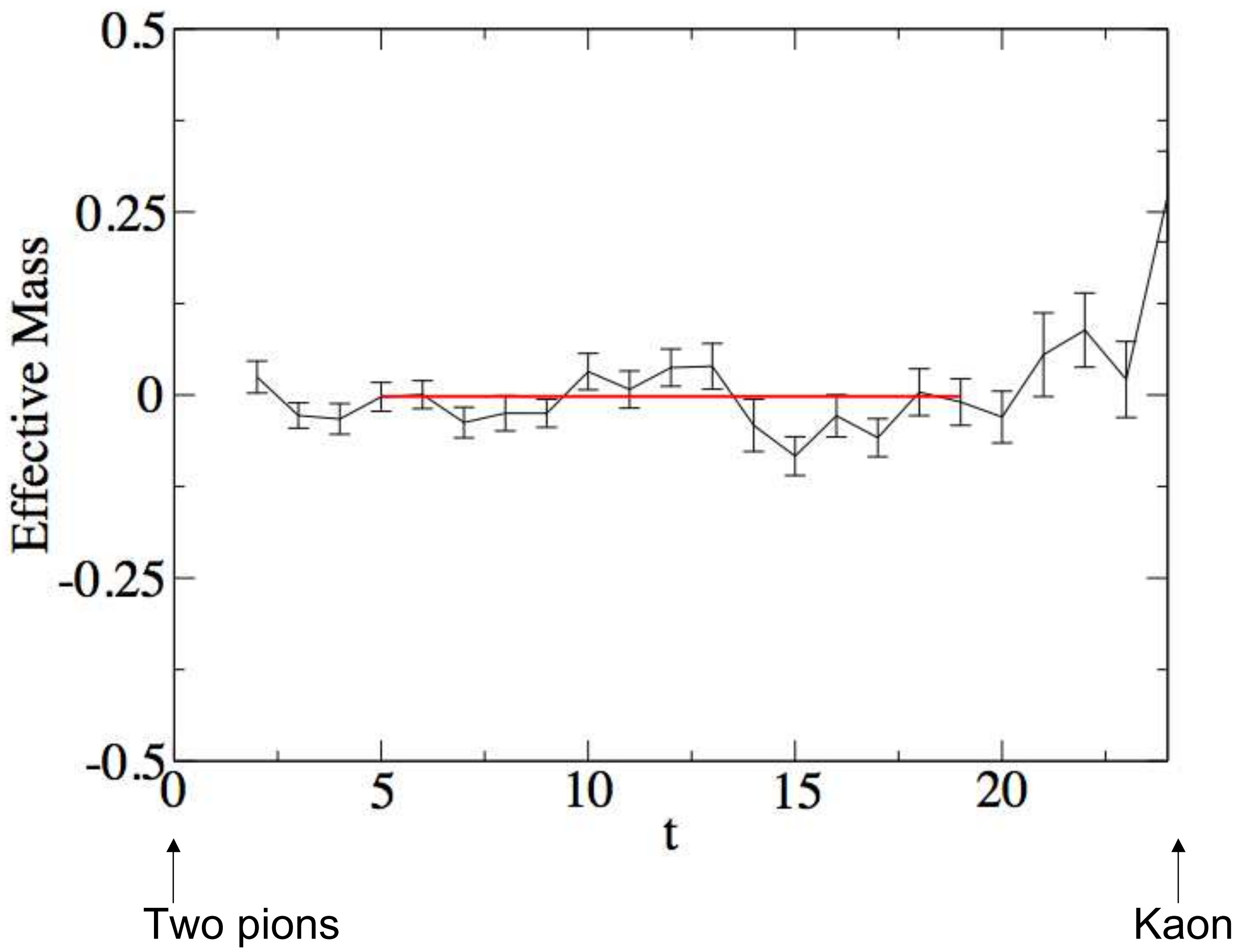
(8,8) Operator Effective Mass

From previous fits: $m_{eff} = E_{\pi\pi} - m_K$=-0.0019(59) $\rightarrow$ -2.7(8.3) MeV



(8,8) Mixed Op. Effective Mass

From previous fits: $m_{eff} = E_{\pi\pi} - m_K$=-0.0019(59) $\rightarrow$ -2.7(8.3) MeV



Preliminary Results - Summary

Quantity	This Calculation	Physical
m_π	145.6(5) MeV	139.6 MeV
m_K	519(2) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx 0)$	294(1) MeV	-
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L)$	516(9) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L) - m_K$	-2.7(8.3) MeV	0 MeV

Two Pion Phase Shift

- Two pion S-wave phase shift can be found from energy via Luscher relation (Luscher, M., Nucl. Phys. B, 354, p. 531-578)

$$n\pi - \delta(p_\pi) = \phi(q_\pi), \qquad q_\pi = \frac{p_\pi L}{2\pi} \tag{1}$$

where

$$\tan\phi(q) = -\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1, q^2)} \tag{2}$$

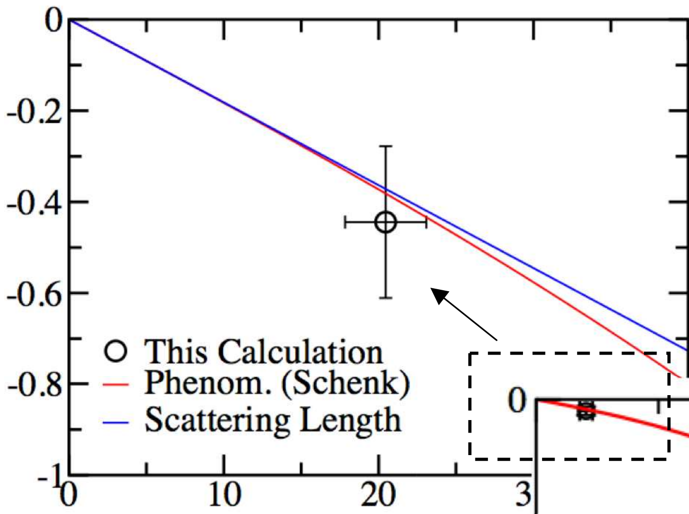
$$\mathcal{Z}_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n}} (\mathbf{n}^2 - q^2)^{-s} \tag{3}$$

(the components of $\mathbf{n}$ are integers or half integers depending on whether or not that direction is twisted)

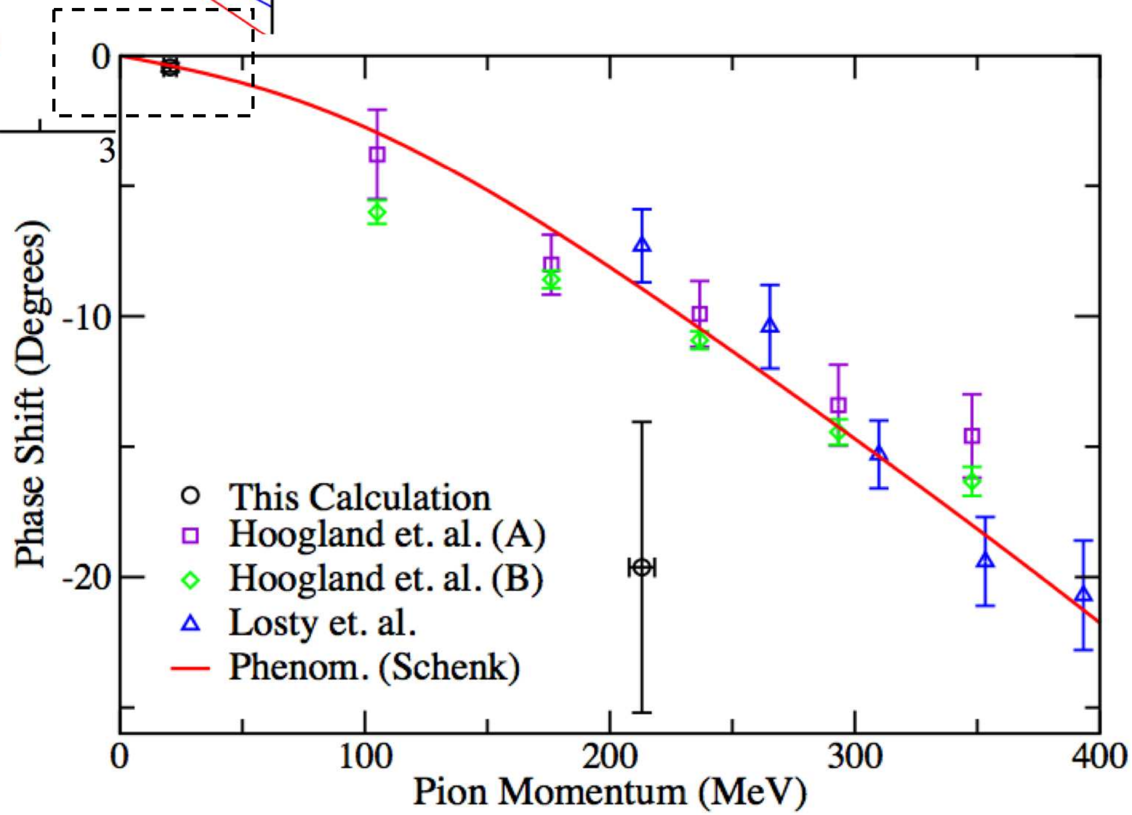
- p_π determined exactly from two pion energy via dispersion relation

$$E_{\pi\pi} = 2\sqrt{m_\pi^2 + p_\pi^2} \tag{4}$$

Two Pion Phase Shift



p_π (Exact)	δ
20(3) MeV	-0.44(17) degrees
213(5) MeV	-19.6(5.6) degrees



Derivative of Phase Shift

- Will need $\frac{\partial\delta}{\partial q}$ evaluated at pion momentum for normalization of matrix element.
- Use phenomenological curve (A. Schenk, Nucl. Phys. B 363 (1991) 97) shown on previous slide

$$\tan\delta_{l=0}^{I=2} = \sqrt{1-\frac{4m_\pi^2}{s}}\left(A + B\frac{p^2}{m_\pi^2} + C\frac{p^4}{m_\pi^4} + D\frac{p^6}{m_\pi^6}\right)\left(\frac{4m_\pi^2 - s_{l=0}^{I=2}}{s - s_{l=0}^{I=2}}\right) \tag{5}$$

with values of constants fit from experiment.

- With p_π=213 MeV, get

$$\frac{\partial\delta}{\partial q} = -0.305 \tag{6}$$

Decay Amplitude A_2

- Look at quantity A_2 which can be compared to experiment.

$$\langle \pi\pi(I=2)|\mathcal{L}_W(0)|K\rangle = A_2 e^{i\delta_2} \tag{7}$$

- Related to lattice matrix element by

$$A_2 = \frac{\sqrt{3}}{2\sqrt{2}} \frac{1}{\pi q_\pi} \sqrt{\frac{\partial\phi}{\partial q_\pi} + \frac{\partial\delta}{\partial q_\pi}} L^{3/2} a^{-3} G_F V_{ud} V_{us} \sqrt{m_K} E_{\pi\pi} \times \sum_{i,j} C_i(\mu) Z_{ij}(\mu) \langle \pi\pi|Q_j|K\rangle \tag{8}$$

- Z_{ij} is operator renormalization (NPR)
- All terms purely real except for Wilson coefficients $C_i(\mu) = z_i(\mu) + \tau y_i(\mu)$.

Lellouch-Luscher Factor

- Lellouch-Luscher factor (hep-lat/0003023v1) is a finite volume correction to the matrix element that takes $\pi\pi$ interactions into account.
- Proportional to

$$\text{LL factor} \propto \sqrt{\frac{\partial \phi}{\partial q_\pi} + \frac{\partial \delta}{\partial q_\pi}} \qquad (9)$$

- $\frac{\partial \delta}{\partial q_\pi}$ from phenomenological curve as before.

$\frac{\partial \phi}{\partial q_\pi}$	5.141(64)
$\frac{\partial \delta}{\partial q_\pi}$	-0.305

- For systematic error, compare to LL factor with $\frac{\partial \delta}{\partial q_\pi}$ obtained by drawing a straight line between the two phase shift data points from this calculation. Get a 2% difference.

Real Part of A_2 (Preliminary)

- Final result is an error-weighted average over separations between the kaon and two pions $t_K = 20, 24, 28, 32$
- Note: NPR performed on $L_s = 16$ rather than $L_s = 32$. (See previous talk by Nicholas Garron).

	Re(A_2) (10^{-8} GeV)
$t_K = 20$	1.52(12)
$t_K = 24$	1.52(10)
$t_K = 28$	1.71(13)
$t_K = 32$	1.35(22)
Error Weighted Average	1.555(73)
Experimental	1.5

Imaginary Part of A_2 (Preliminary)

- NOTE: NPR note yet done, approximate $Z_{ij} = 0.9 Z_q^2 \delta_{ij}$ for (8,8) and (8,8) mixed operator. NPR to be done soon!

	Im(A_2) (10^{-13} GeV)
$t_K = 20$	-9.20(50)
$t_K = 24$	-10.03(70)
$t_K = 28$	-9.51(73)
$t_K = 32$	-10.10(84)
Error Weighted Average	-9.58(44)
RBC Quenched 16^3 χPT	-12.64(72)
Dynamical 24^3 χPT (Shu Li)	-7.9(16)(39)

- Im(A_2)/Re$(A_2) = -6.16(29) \times 10^{-5}$ from error weighted averages.

Systematic Error Budget

Finite lattice spacing (Scaling from Iwasaki to DSDR action, 32^3)	5% $\times$ 3=15% (just 5% for Im(A_2)/Re(A_2))
Finite volume (Note $m_\pi L$=3.3) (Comparison of f_π, f_K on 16^3 and 24^3 lattices)	4%
$m_\pi^{val} \neq m_\pi^{sea}$ (Effect of partial quenching on 32^3 Iwasaki, Lightman Lattice 2008)	2%
Derivative of Phase Shift	2%

Systematic Error Budget

Masses not exactly physical (Study of mass dependence on 24^3 quenched)	T.B.D.
Operator Renormalization	T.B.D. (20% for Im(A_2) due to guess for Z_{ij})
Wilson Coefficients	T.B.D.
Total (Added in quadrature)	16% Re(A_2) 25% Im(A_2) 21% Im(A_2)/Re(A_2)

Conclusion

- Preliminary results for $\Delta I = 3/2\ K \to \pi\pi$ decay amplitude on 32^3 lattices with 2+1 flavors of dynamical domain wall fermions and the Iwasaki + DSDR gauge action.
- $m_\pi =$145.6(5) MeV, $m_K =$519(2) MeV, $E_{\pi\pi} =$516(9) MeV.
- Re(A_2)=$1.56(07)_{stat}(25)_{sys} \times 10^{-8}$ GeV
- Im(A_2)=$-9.6(0.4)_{stat}(2.4)_{sys} \times 10^{-13}$ GeV
- Im(A_2)/Re(A_2)=$-6.2(0.3)_{stat}(1.3)_{sys} \times 10^{-5}$ GeV
- NPR for (8,8) operators will be done soon, reducing systematic error in Im(A_2) and Im(A_2)/Re(A_2) to same as for Re(A_2).