Probing the Yang-Mills vacuum with adjoint zero-modes

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Introduction

- The identification of the topological structures contained in Monte Carlo configurations is fundamental to understand the QCD vacuum.
- Smearing and Cooling algorithms have been used in this way. But the updating process could modify the original structures.
- Differential operators have been proposed to filter the high frequency noise of Monte Carlo configurations without modifying the initial configuration.
- The Dirac operator in the fundamental representation is the most common one.
- In this talk an alternative operator based in the adjoint representation of the Dirac operator is studied.

The Supersymmetric Mode. I

- In the adjoint representation the Index Theorem is given by

$$n_+ - n_- = 2NQ.$$

- If $A_\mu(x)$ is a solution of the equation of motion,

$$D_\mu F_{\mu\nu}(x) = 0.$$

- One of the zero-modes can be constructed as

$$\psi^a_{ss}(x) = \frac{1}{8} F^a_{\mu\nu}(x)[\gamma_\mu, \gamma_\nu]V; \quad \not{D}\psi_{ss}(x) = 0.$$

The Supersymmetric Mode. II

- This mode can be decomposed in two Weyl fermions

$$\psi^a_{ss} = \begin{pmatrix} \psi^a_{ss+} \\ \psi^a_{ss-} \end{pmatrix}$$

- The Weyl modes satisfy the equations:

$$\bar{\sigma}_\mu D_\mu \psi_{ss+} = 0; \;\; \sigma_\mu D_\mu \psi_{ss-} = 0$$

- The density of these modes corresponds to the (anti)self-dual part of the gauge action density:

$$|\psi_{ss+}(x)|^2 = (F^{SF}_{\mu\nu}(x))^2; \;\; |\psi_{ss-}(x)|^2 = (F^{ASF}_{\mu\nu}(x))^2.$$

The Method

- We propose to use this eigenvector as a filtering method of Monte Carlo configurations.
- This mode is obtained as the lowest eigenvector of a projection of $(\gamma_5 \not{D})^2$ operator.

How to select the Supersymmetric mode?

- A method is necessary to select $\psi_{ss\pm}$ within the space of zero-modes.
- If we take $V^\dagger = (1,0,0,0)$, $\psi^a_{ss}(x)$ is given by

$$\psi^a_{ss}(x) = \imath \begin{pmatrix} \frac{E^3_a(x)+B^3_a(x)}{2} \\ \frac{E^1_a(x)+B^1_a(x)}{2} - \imath \frac{E^2_a(x)+B^2_a(x)}{2} \\ 0 \\ 0 \end{pmatrix}$$

- The real part of the first spinorial component is zero in every space-time point, and all components in Colour-space.
- This condition defines a subspace of dimension one in the space of zero-modes.

The Supersymmetric operators, $O^{\pm}$

- We define two operators, $O^{\pm}$, given by $-\hat{D}\bar{D}$ $(-\bar{D}\hat{D})$ operators projected onto the space of states satisfying the reality condition:

$$O^{\pm}_{ij} = -\delta_{ij} D^2_{\mu} \pm i\epsilon_{ijk}(E_k \pm B_k)$$

- The $O^{\pm}$ operators are real.
- $\psi_{ss\pm}(x)$ correspond with the zero-modes of these operators.

Lattice Implementation

- Our lattice version of the $O^{\pm}$ is implemented in terms of the adjoint Neuberger-Dirac operator.

$$D_{ov} = \frac{1}{2}\left(1 + \gamma_5 \epsilon(\gamma_5 D_{WD}^{Q})\right).$$

- D_{WD}^{Q} is the quaternionic version of the adjoint Wilson-Dirac operator.
- The lattice version of the operator $O^{\pm}$ is defined by,

$$O^{L}_{\pm} = P_{\sigma_0} H^2_{\pm} P_{\sigma_0},$$

where,

$$H^2_{\pm} = P_{\pm}(\gamma_5 D_{ov})^2 P_{\pm} = P_{\pm}(1 \pm \epsilon) P_{\pm},$$

and $P_{\sigma_0}\Psi = \Psi - \sigma_0 \langle \sigma_0, \Psi \rangle$.

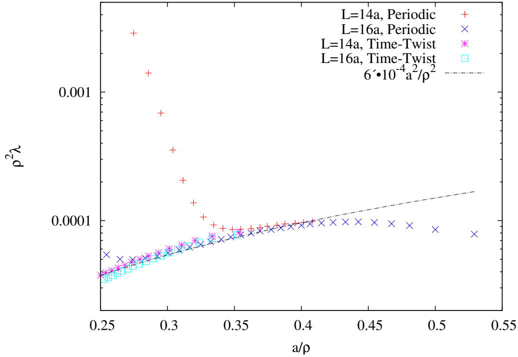
Testing the Method for SU(2).

- The zero-mode condition can be violated by several reasons:
  - Numerical implementation: lattice artefacts, finite-volume effects.
  - Non-classical solutions: instanton-antiinstanon pairs, configurations with ultraviolet noise.
- We check whether the lowest eigenvector of the $O^{\pm}$ operator reproduces the gauge action density in these cases.

Smooth Q=1 configuration. Spectrum

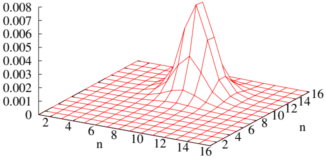
- We analyse the lowest eigenvalue of the O^+ operator for a smooth instanton configuration.
- Volume effects are important for $L/\rho < 4$ in the periodic case. [1]
- For Time-Twist boundary conditions the volume effects disappear.
- The lowest eigenvalues can be fitted as

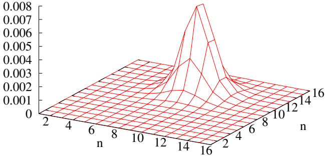
$$\lambda_1 = \frac{6 \cdot 10^{-4}}{\rho^2}\frac{a^2}{\rho^2} + O(\frac{a^2}{\rho^2})$$

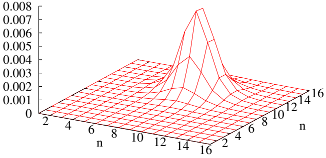


[1] Q=1 solutions do not exist on a periodic torus. [Braam-Van Baal,89]

Smooth Q=1 configuration. Density





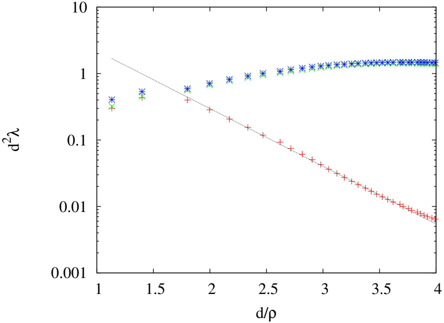


Smooth Q=+1-1 configuration. Spectrum

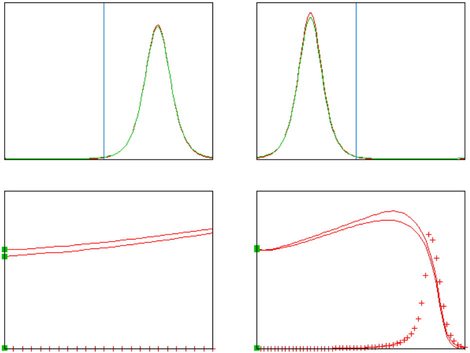
- In this case the SSM is not in correspondence with a zero-mode
- We compute the spectrum in terms of the IA distance
- The lowest eigenvalue decreases exponentially with the distance as

$$\lambda_1 = \frac{16}{d^2} \exp^{-\frac{2d}{\rho}} .$$

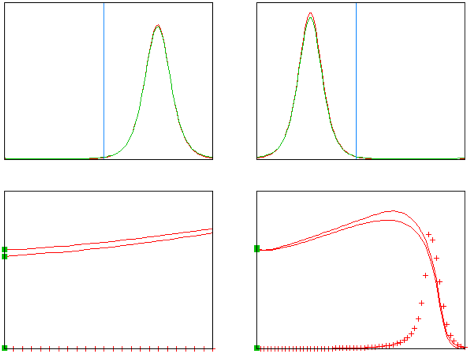
- When $d/\rho < 2$, the SSM is not in correspondence with the lowest eigenvalue.



Intanton-Antiinstanton annihilation

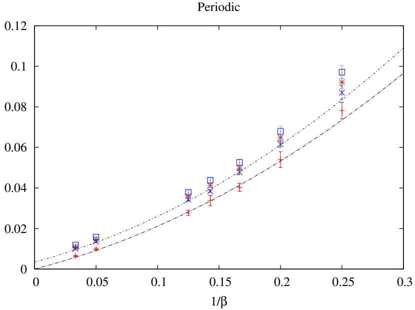


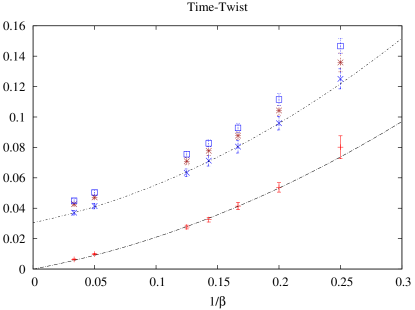
Intanton-Antiinstanton annihilation



Heated Q=1 configurations. Spectrum

- Finally we test the filtering properties of the O^+ operator.
- We apply 10 heat-bath sweeps to a smooth Q=1 configuration, for different values of β and analyse the spectrum.

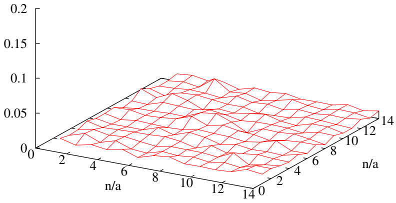


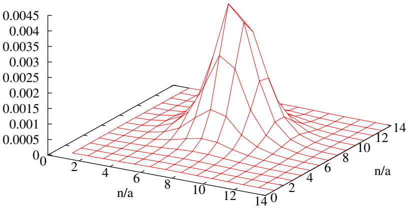


$$\lambda_1^P = 3.2\cdot 10^{-5} + 0.15\beta^{-1} + 0.55\beta^{-2}, \lambda_1^T = 7.2\cdot 10^{-6} + 0.15\beta^{-1} + 0.56\beta^{-2}$$
$$\lambda_2^P = 3.5\cdot 10^{-3} + 0.16\beta^{-1} + 0.63\beta^{-2}, \lambda_2^T = 3.0\cdot 10^{-2} + 0.17\beta^{-1} + 0.77\beta^{-2}$$

Heated Q=1 configurations. SSM density

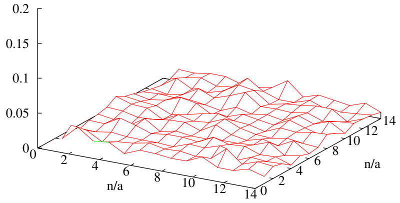
$\beta = 30$

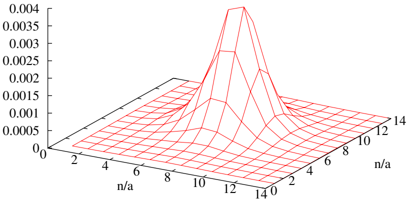




Heated Q=1 configurations. SSM density

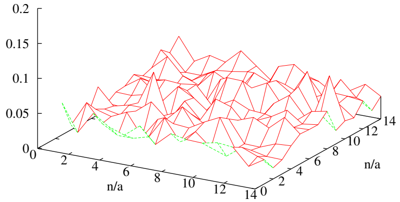
$$\beta = 20$$

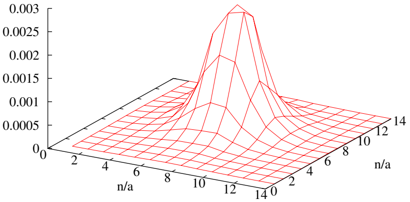




Heated Q=1 configurations. SSM density

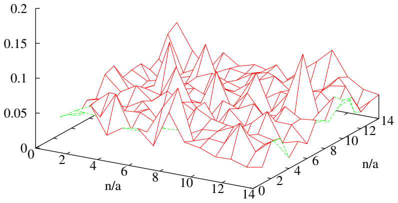
$$\beta = 8$$

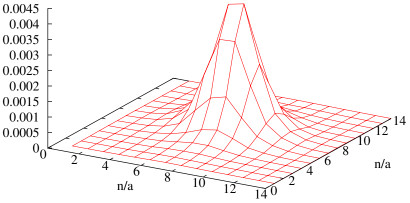




Heated Q=1 configurations. SSM density

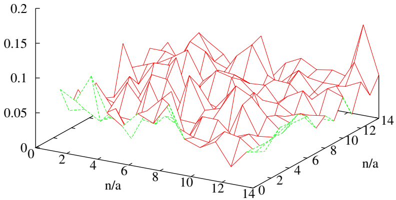
$\beta = 7$

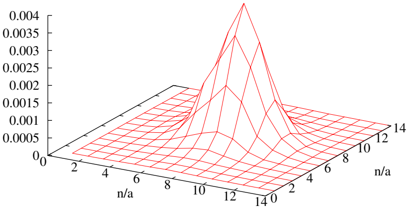




Heated Q=1 configurations. SSM density

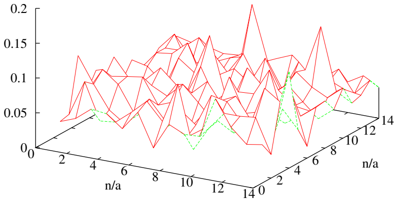
$\beta = 6$

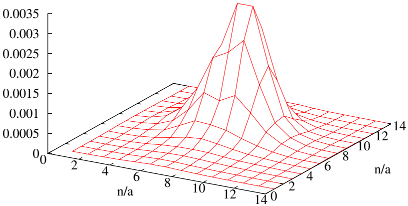




Heated Q=1 configurations. SSM density

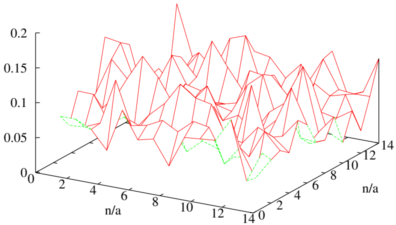
$\beta = 5$

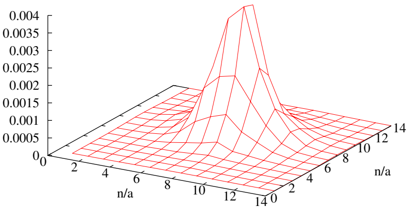




Heated Q=1 configurations. SSM density

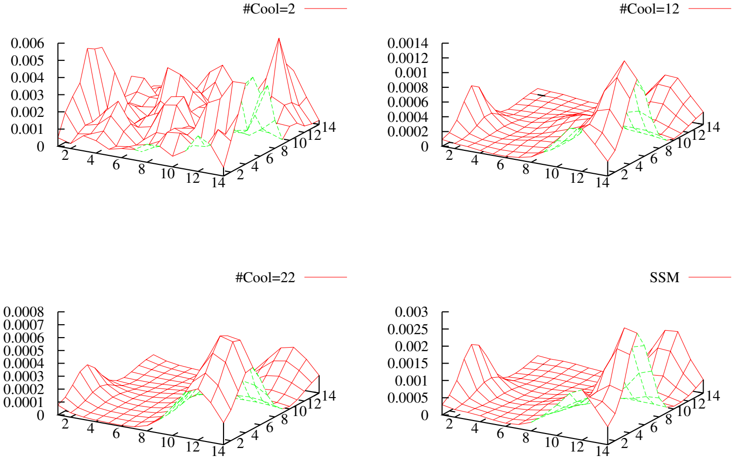
$\beta = 4$





Monte Carlo Configurations: preliminary analysis

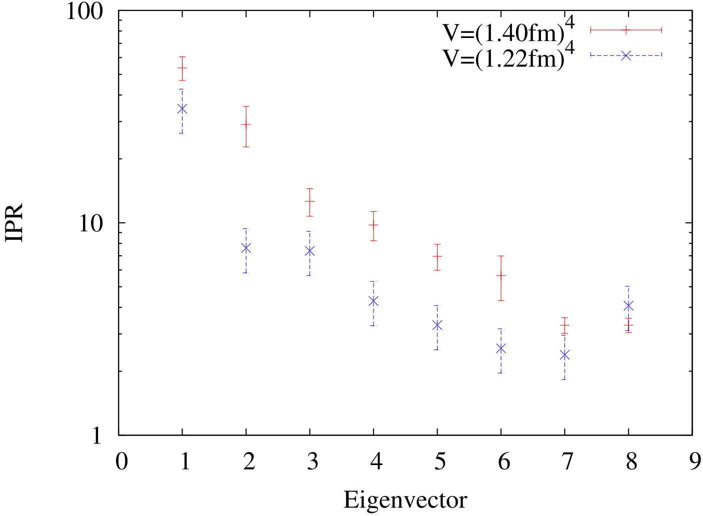
- Wilson Action. $\beta = 2.518$



Conclusions

- For smooth classical solutions the lowest eigenvalue is a function of the lattice spacing.
- Topological structures contained in each quiral sector can be separated.
- The high frequency noise is filtered by the $O^{\pm}$ operators.
- No level crossing is observed by adding high frequency noise.
- The lowest eigenvector of $O^{\pm}$ reproduces the gauge action density in all the analysed cases.
- Smooth densities with a rich structure are obtained from Monte Carlo configurations.

Montecarlo Configurations: Preliminary results



Smooth Q=2 configuration. SSM Spectrum

- We compute the spectrum of two-instanton configurations in terms of the distance.
- When the instantons distance is $\frac{L}{\rho} > 2$ the second eigenvalues becomes small.

- This eigenvalue decrease exponentially as

$$\lambda_2 = \frac{\pi^2}{L^2} e^{-\frac{3d}{2\rho}}$$