Computing the $B^*B\pi$ coupling with relativistic heavy quarks and domain wall fermions

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FlaviA net

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First of all:

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Warning

No preliminary results available yet!

Content

- Motivation
- Overview of RBC/UKQCD heavy quark project and ...
- ...how our calculation fits into it
- the relevant form factor on the lattice
- computational strategy
- Outlook

Motivation

- phenomenological importance in effective description of hadronic heavy meson decays
- useful tool: *heavy meson chiral perturbation theory* (HMχPT)

$$\mathcal{L}^{\text{int}}_{\text{HM}\chi} = g \cdot \mathrm{Tr}\left[\overline{H}_a H_b \mathcal{A}^{ab}_\mu \gamma^\mu \gamma_5\right]$$

  (heavy mesons + low momentum pions)
- isospin symmetry relates various charge combinations

$$g_{VP\pi} \equiv g_{V^+P^0\pi^+} = -\sqrt{2} g_{V^+P^+\pi^0} = -g_{V^0P^+\pi^-}$$

- first time that Relativistic Heavy Quarks (RHQ) will be used in the computation
- we need it in chiral extrapolation of $B^0 - \overline{B}^0$-mixing calculation

RBC/UKQCD heavy quark project

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- heavy valence quarks with Relativistic Heavy Quark (RHQ) action
- gauge field configurations with *2+1 light dynamical DWF* on $L/a \in \{24, 32\}$ lattices, $T/a = 64$, $a^{-1} \in \{2.28, 1.73\}\mathrm{GeV}$
- ongoing investigations:
  - *non-perturbative tuning of RHQ parameters* (am_0, c_P, ζ) on $L/a = 32$
    $\rightsquigarrow$ talk by Hao Peng [Parallel 46]
  - *B-Meson decay constant &* $B_0 - \overline{B}_0$*-mixing*
    $\rightsquigarrow$ talk by Oliver Witzel [Parallel 30]
  - *neutral B-Meson mixing with static quarks*
    $\rightsquigarrow$ talk by Yasumichi Aoki [Parallel 30]
- we use USQCD software suite CPS, Chroma as well as UKhadron

RBC/UKQCD heavy quark project

heavy valence quarks with Relativistic Heavy Quark (RHQ) action

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we use a variant of the Fermilab action [El-Khadra,Kronfeld,Mackenzie], the ...

RHQ action

$$S = \sum_{n,n'} \overline{\psi}_n \, \mathcal{K}_{n,n'} \, \psi_{n'} + \mathrm{O}\Big[(a\Lambda)^2\Big] \,,$$

$$\mathcal{K} = m_0 + \gamma_0 D_0 - \tfrac{a}{2} D_0^2 + \zeta \Big[\gamma \mathbf{D} - \tfrac{a}{2}\mathbf{D}^2\Big] + a c_P \tfrac{\mathrm{i}}{4} \sigma_{\mu\nu} F_{\mu\nu}$$

claim: accurate to all orders in $(am_{\mathrm{h}})^n$ [Christ,Li,Lin]

- 3 parameters: $\{am_0, \zeta(g_0^2, am_0), c_P(g_0^2, am_0)\}$
- non-perturbative parameter tuning required
- matching to physical spectrum through

$$m_{\mathrm{av}} = \tfrac{1}{4}(m_P + 3m_V), \quad \Delta m = m_V - m_P, \quad E^2 = m^2 + \mathbf{p}^2$$

(this has to be done for each quark flavour separately)

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gauge field configurations with 2+1 light dynamical DWF

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Iwasaki gauge action, $L_s/a = 16, aM_5 = 1.8, T/a = 64$:

$L/a = 24$ — $L \approx 2.75\mathrm{fm},\ a^{-1} \approx 1.732(29)\mathrm{GeV}$

- $am_l \in \{0.005, 0.01, 0.02\}$ or $m_\pi \in \{331, 419, 558\}\mathrm{MeV}$
- strange quark fixed at $am_s = 0.04$
- RHQ parameters fixed [RBC/UKQCD:PoS-Lattice'07,'08]

$L/a = 32$ — $L \approx 2.72\mathrm{fm},\ a^{-1} \approx 2.284(25)\mathrm{GeV}$

- $am_l \in \{0.004, 0.006, 0.008\}$ or $m_\pi \in \{307, 366, 418\}\mathrm{MeV}$
- strange quark fixed at $am_s = 0.03$
- RHQ parameters fixed [RBC/UKQCD:PoS-Lattice'09] for charm only yet

$\rightsquigarrow$ talk by C.Kelly, today 12:30 [Parallel 18]

RBC/UKQCD heavy quark project

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Defining an effective coupling

. . . is a matter of convention

our conventions follow [DeDivitiisETAL:JHEP10(1998)010]:

effective coupling

$$\langle P(p)\pi(q)|V(\lambda,p')\rangle = g_{VP\pi}(q^2)\cdot q_\mu \epsilon^\mu(\lambda,p')\cdot(2\pi)^4\delta(p'-p-q)$$

$$\langle P(p)|P(p')\rangle = 2p^0(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')$$

- fully relativistic treatment
- heavy-light pseudo-scalar P and vector meson V (in either c-quark or b-quark channel)
- $\epsilon^\mu(\lambda,p')$: polarisation vector of V

computational strategy is mainly the standard one, see [AbadaETAL:PhysRevD66:074504:2002] for instance

The relevant matrix element

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LSZ reduction formula & PCAC relation

$$\langle B^0(p)\pi^+(q)|B^{\star+}(p')\rangle = \mathrm{i}(m_\pi^2 - q^2)\int_x \mathrm{e}^{\mathrm{i}qx}\langle\overline{B}(p)|\pi(x)|B^\star(p+q)\rangle$$
$$= q^\mu \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\int_x \mathrm{e}^{\mathrm{i}qx}\langle\overline{B}(p)|A_\mu(x)|B^\star(p+q)\rangle$$

$\langle P(p)|A_\mu|V(\lambda,p')\rangle =$

$$A_0(q^2)2m_V\frac{\epsilon^\lambda\cdot q}{q^2}q_\mu + A_1(q^2)(m_V+m_P)\left[\epsilon^\lambda{}_\mu - \frac{\epsilon^\lambda\cdot q}{q^2}q_\mu\right]$$
$$+ A_2(q^2)\frac{\epsilon^\lambda\cdot q}{m_V+m_P}\left[(p'+p)_\mu - \frac{m_V^2-m_P^2}{q^2}q_\mu\right]$$

- parametrized by **three form factors** $A_{0,1,2}(q^2)$
- we will set $\mathbf{p}=0 \Rightarrow q_0 = E_V(\mathbf{p}') - m_P$, with $\mathbf{p}'=\mathbf{q}$

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The relevant matrix element

$$g_{B^\star B\pi} \equiv \lim_{q^2\to 0} g_{B^\star B\pi}(q^2)\,,$$

$$g_{B^\star B\pi}(q^2)\cdot\epsilon_\mu(\lambda,p') = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\left\langle \overline{B}(p)\middle| A_\mu(q) \middle| B^\star(\lambda,p')\right\rangle$$

at zero recoil $q^2 = 0$ only A_0 survives: $g_{B^\star B\pi} = \frac{2m_V}{f_\pi} A_0(0)$

A_0 dominated by pion; strong dependence on q^2 & mass

no massless state can couple to the axial current $\Rightarrow$

$$2m_V A_0(0) = (m_V + m_P)A_1(0) + (m_V - m_P)A_2(0)$$

$$g_{B^\star B\pi} = \frac{m_V + m_P}{f_\pi} A_1(0)\left[1 + \frac{m_V - m_P}{m_V + m_P}\frac{A_2(0)}{A_1(0)}\right]$$

The lattice 3pt function $\langle P A_\mu V_\nu^\dagger \rangle$

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inserting full set of physical states (twice) and performing polarisation sum:

$\langle P A_\mu V_\nu^\dagger \rangle \propto$

$$A_0(q^2) 2m_\mathrm{V} \left\{ \frac{q_\nu q_\mu}{q^2} - p_\nu q_\mu \frac{pq}{p^2 q^2} \right\} +$$

$$A_1(q^2)(m_\mathrm{V} + m_\mathrm{P}) \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - \frac{q_\mu q_\nu}{q^2} + p_\nu q_\mu \frac{pq}{p^2 q^2} \right] +$$

$$A_2(q^2) \left[\frac{(p+p')_\mu}{m_\mathrm{V} + m_\mathrm{P}} \left\{ q_\nu - p_\nu \frac{pq}{p^2} \right\} - (m_\mathrm{V} - m_\mathrm{P}) \left\{ \frac{q_\mu q_\nu}{q^2} - p_\nu q_\mu \frac{pq}{p^2 q^2} \right\} \right]$$

with a factor like

$$\frac{\mathrm{e}^{-E_V t}}{2E_V} \frac{\mathrm{e}^{-E_P(t_s - t)}}{2E_P} \sqrt{\mathcal{Z}_V \mathcal{Z}_P}$$

with *overlap factors*: $\mathcal{Z}_P = |\langle 0|P|B\rangle|^2$, $\mathcal{Z}_V = |\langle 0|V|B^\star\rangle|^2$

Building appropriate ratios

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$$C_{\mu\nu} = \Big\langle \sum_{\mathbf{x},\mathbf{y}} \mathrm{e}^{-\mathrm{i}\mathbf{qx}} P(\mathbf{y}, t_s) A_\mu(\mathbf{x}, t) V_\nu^\dagger(0) \Big\rangle \,, \qquad 0 < t < t_s \leq \frac{T}{2} \,,$$

$$C_{V_\mu V_\nu} = \Big\langle \sum_{\mathbf{x}} \mathrm{e}^{\mathrm{i}\mathbf{px}} V_\mu(\mathbf{x}, t) V_\nu^\dagger(0) \Big\rangle \simeq \mathcal{Z}_V \frac{\mathrm{e}^{-E_V t}}{2E_V} \Big(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \Big) \,,$$

$$C_{PP} = \Big\langle \sum_{\mathbf{x}} \mathrm{e}^{\mathrm{i}\mathbf{px}} P(\mathbf{x}, t) P^\dagger(0) \Big\rangle \simeq \mathcal{Z}_P \frac{\mathrm{e}^{-E_P t}}{2E_P}$$

$$R_1(t) = \frac{C_{ii}(t, \mathbf{0}) \sqrt{\mathcal{Z}_V \mathcal{Z}_P}}{C_{V_i V_i}(t, \mathbf{0}) C_{PP}(t_s - t, \mathbf{0})} \Rightarrow A_1(0)$$

$$R_{2,3,4}(t) = \frac{C_{10,11,22}(t, \mathbf{q}) \sqrt{\mathcal{Z}_V \mathcal{Z}_P}}{C_{V_2 V_2}(t, \mathbf{q}) C_{PP}(t_s - t, \mathbf{q})} \Rightarrow A_1(q), \frac{A_2(q)}{A_1(q)}$$

Ongoing lattice computations

. . . use

- sequential source technique for 3pt function
- point sources at origin of randomly translated gauge field . . .
- corresponding DWF light quark propagators available (on disk)
- producing heavy quark propagators is cheap and negligible w.r.t. . . .
- 2×DWF inversion with seq. source at $t_s = T/2$
- only A_μ needs to be renormalized, Z_A known!
- 1st: $q^2 \to 0$, 2nd: $m_\pi \to 0$

Outlook

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- we are ready to go the way described here
- within next 1-2 months we expect to have all point source results of $\mathcal{O}(400-800)$ on $L/a = 24$ analysed (c+b) $\rightsquigarrow$ PoS:Lattice'10
- **next steps:**
  1. repeat the procedure using stochastic (Z_2) noise
  2. start same computation on $L/a = 32$ as soon as RHQ parameters for bottom system are reliably known

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Thank you for your patience!

Let's go to
Micheal Donnellan's talk, next session at 10:30 [Parallel 18],
to see some new results.