$\langle N|\bar{s}s|N\rangle$ via reweighting on (2+1)-flavor DWF lattices

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Introduction

Strange quark content of the nucleon ($\langle N|\bar{s}s|N\rangle$) is one of the quantities which traditionally has needed measurement of disconnected diagrams on the lattice, hence expensive/noisy. Recently, a closely related quantity

$$f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N} = \frac{dm_N}{dm_s} \times \frac{m_s}{m_N}$$

has been getting attention. f_{T_s} can be interpreted as the mass fraction carried by each quark. Related to the cross section of SM Higgs coupling to Nucleon.

Different approaches to calculate $\langle N|\bar{s}s|N\rangle$:

- Direct measurement: measure 3-point funciton

$$\langle N|\bar{s}s|N\rangle = \lim_{0\ll t'\ll t} \frac{\langle O_N(0)\bar{s}s(t')O_N(t)\rangle}{\langle O_N(0)O_N(t)\rangle}$$

  QCDSF, LHPC $\cdots$

- Use Feynman-Hellman theorem

$$\langle N|\bar{s}s|N\rangle = \frac{dm_N}{dm_s}$$

  - First fitting m_N to a ChPT fomula (Thomas & Young)
  - Numerical derivative, via reweighting (JLQCD, This study) or chain rule(MILC).

Here we try to gauge the competitiveness of the "recycling" approach, by using already existing nucleon propagators and strange quark rewighting factors.

(2+1) flavor dynamical DWF+I(D) ensemble

L/a	$m_s a$	$m_l a$	$\tilde{m}_s/\tilde{m}_l$	$m_\pi a$	τ(MD)	Accept.	m_π (Mev)
$\beta = 2.13$, $a^{-1} = 1.73(3)$Gev, $am_{res} = 0.00315(4)$							
$24^3 \times 32 \times 16$	0.04	0.005	5.4	0.192	8980	73%	328
		0.01	3.3	0.242	8540	70%	417
		0.02	1.86	0.323	2800	71%	
		0.03	1.3	0.388	2800	72%	
$\beta = 2.25$, $a^{-1} = 2.28(3)$Gev, $am_{res} = 0.000666(8)$							
$32^3 \times 64 \times 32$	0.03	0.004	7.9	0.127	3428×2	72%	287
		0.006	5.5	0.151	3825×2	76%	344
		0.008	4.2	0.173	2965×2	73%	393
$48^3 \times 64 \times 16$	0.03	0.002	~ 13.8	~ 0.097	~ 500	$\sim 70\%$	
DWF+ID $\beta = 1.75, a \sim 1.4Gev, \epsilon_f/\epsilon_b = 0.02/0.5, am_{res} \sim 0.0019$							
$32^3 \times 64 \times 32$	0.045	0.0042	~ 7.6	~ 0.182	1690	$\sim 70\%$	~ 250
		0.001	~ 16		909	$\sim 70\%$	~ 180

Strange quark resweighting already has been extensively used to tune the strange quark to physical point in the RBC/UKQCD ChPT studies. Reweighting factors already calculated for many configurations.

Reweighting of dynamical strange quark

Motivation: Lattice spacing is a nontrivial function of β and one does not know it until it is measured on thermalized configurations. While typically multiple ensemble of light quark masses are generated to do extrapolations to the chiral limit, it is in principle possible to simulate at the physical strange quark if one guesses the lattice spacing correctly.
In practice, ensembles with different strange quark masses are needed to interpolate (simple linear interpolation or SU(3) ChPT).
It would save a lot of computing resources if this can be avoided.
Reweighting of light quark to approach the chiral limit have been tried by various groups.

(A. Hasenfratz, et. al., PRD78 014515(2008), Lüscher et. al., arXiv:0810.0946)

Reweighting: Basics

$$w_i(m_s', m_s) = \det\left(\frac{D_2^\dagger D_2}{D_1^\dagger D_1}\right)^{1/2} = \det(\Omega)^{1/2}, \Omega = D_2^{-1} D_1 D_1^\dagger (D_2^\dagger)^{-1}$$

$$D_1 = D([U_i], m_l, m_s), D_2 = ([U_i], m_l, m_s')$$

$$w = \frac{\int e^{-\xi^\dagger \Omega^{1/2} \xi}}{\int e^{-\xi^\dagger \xi}} = \left\langle e^{-\xi^\dagger (\Omega^{1/2} - 1)\xi} \right\rangle$$

Now observables for reweighted ensemble is calculated by

$$\langle O \rangle (m_s') = \frac{\Sigma_i O[U_i] w_i}{\Sigma_i w_i} \quad (1)$$

Propagators & reweighting factors

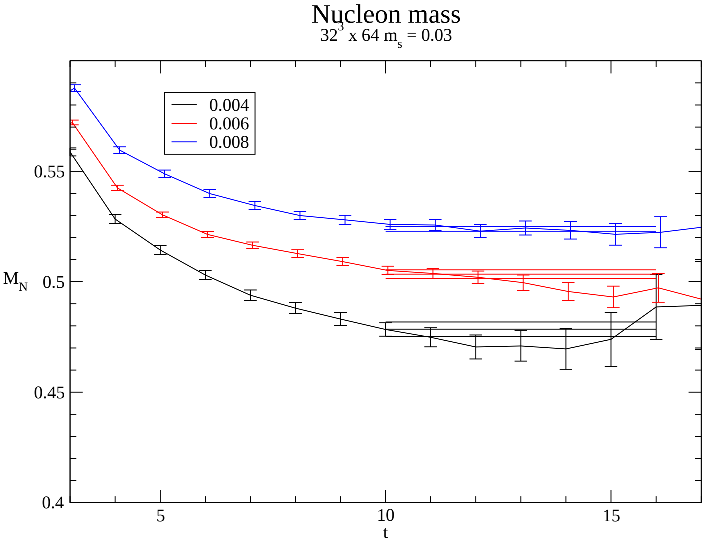
- 32^3 :Gaussian, generated by LHPC, $\langle r^2 \rangle^{1/2} \sim 6.0$
  $am'_s = 0.025, 0.0255 \cdots 0.03$

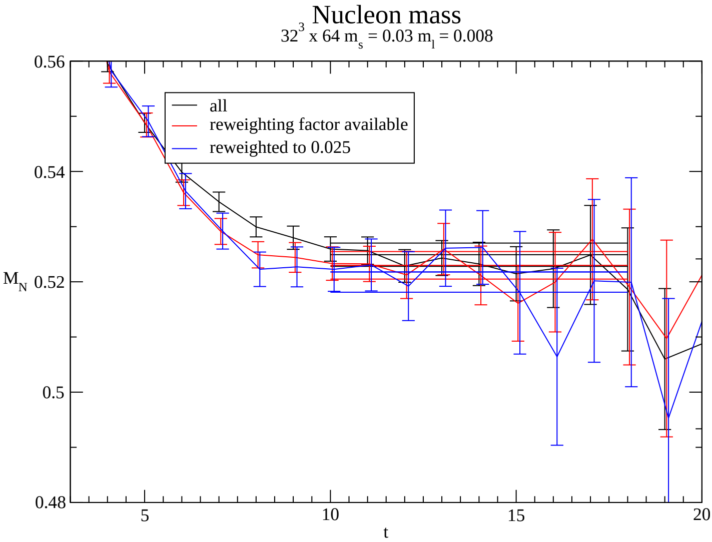
am_l	Reweighting factor	Total	With r.f.
0.004	260-3350(310)	1723	688
0.006	500-3610(312)	3232	604
0.008	260-2800(255)	1687	892

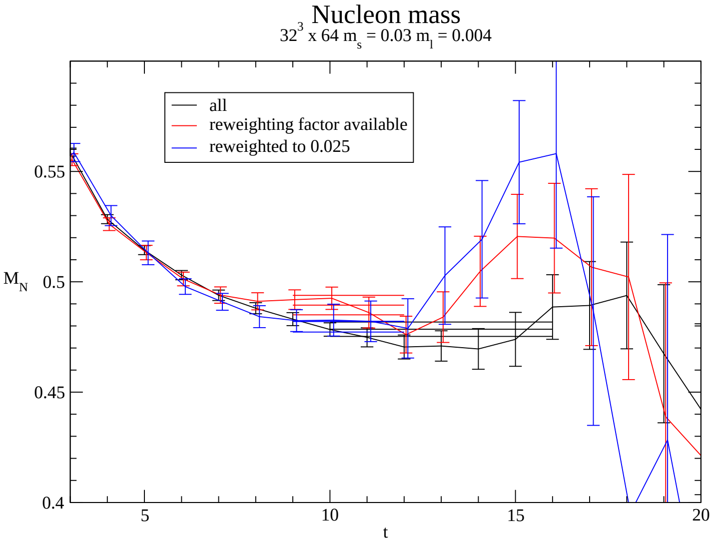
- 24^3: Box, size=16
  $am'_s = 0.03, 0.03025, 0.0305, \cdots 0.04$

am_l	Reweighting factor	Total	with r.f.
0.005	900-8980 (203)	182	182
0.010	1460-8540(178)	182	182

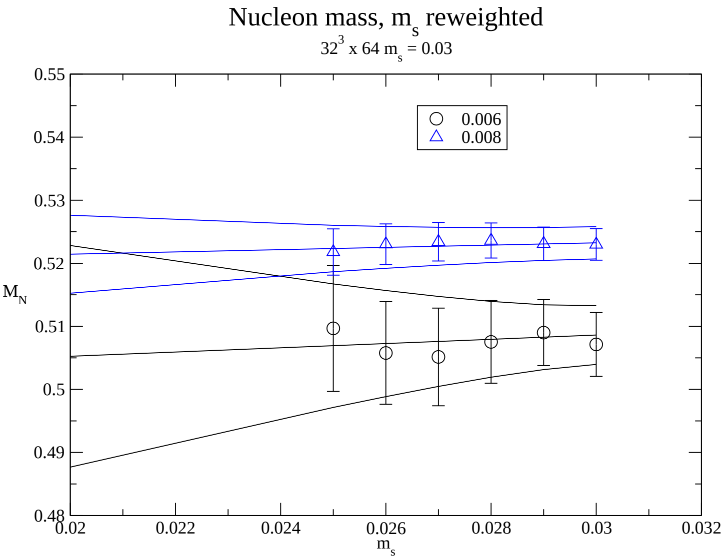
32^3 DWF nucleon effective mass



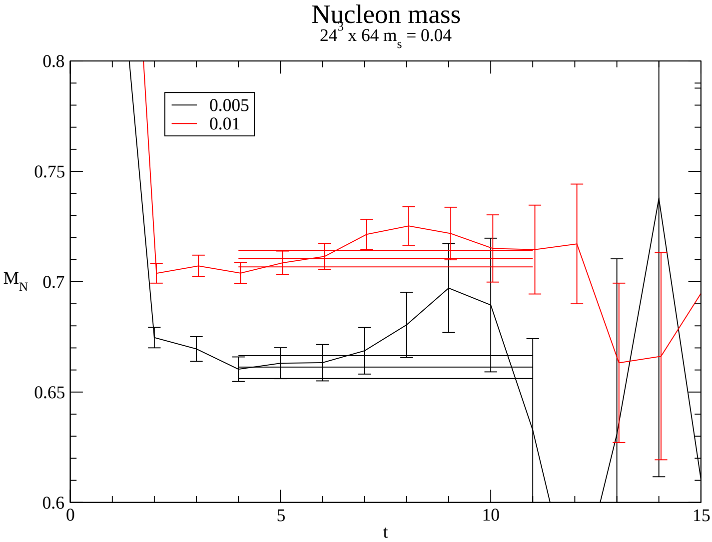

Nucleon mass
32^3 x 64 m_s = 0.03 m_l = 0.008
all
reweighting factor available
reweighted to 0.025
M_N
t
0.56
0.54
0.52
0.5
0.48
5
10
15
20

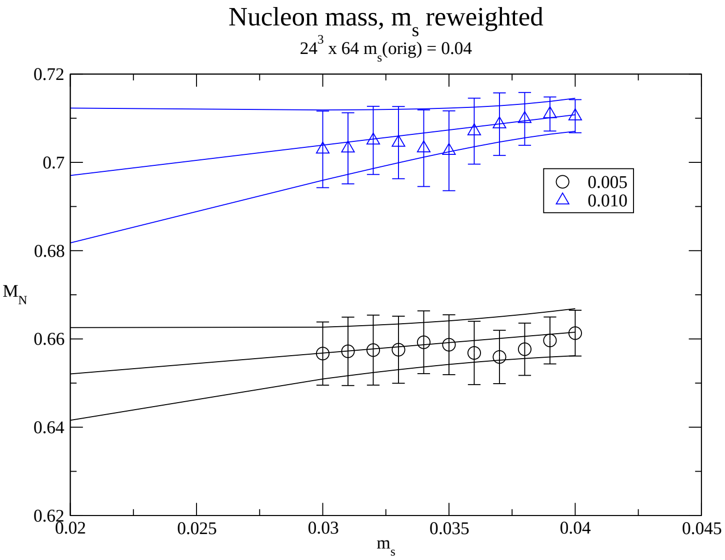


Nucleon mass
32^3 x 64 m_s = 0.03 m_l = 0.004
all
reweighting factor available
reweighted to 0.025
M_N
0.55
0.5
0.45
0.4
5
10
15
20
t

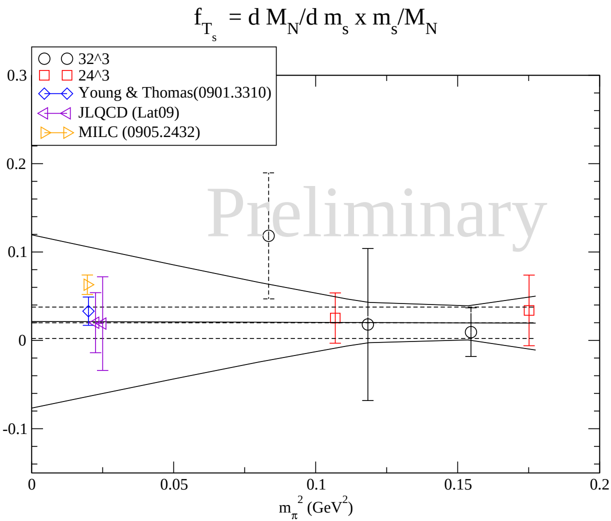

Nucleon mass, m_s reweighted
32^3 x 64 m_s = 0.03
0.006
0.008
M_N
m_s
0.55
0.54
0.53
0.52
0.51
0.5
0.49
0.48
0.02
0.022
0.024
0.026
0.028
0.03
0.032

24^3 DWF nucleon effective mass





Nucleon mass, m_s reweighted
24^3 x 64 m_s(orig) = 0.04
M_N
m_s
0.72
0.7
0.68
0.66
0.64
0.62
0.02
0.025
0.03
0.035
0.04
0.045
0.005
0.010



$f_{T_s} \sim 0.019(17)$ (constant fit)

$\sim 0.021(85)$ (linear fit) at physical m_π.

Discussions & Conclusions

- Shifting of dynamical nucleon mass by up to $\sim$ 20% from the dynamical strange mass is stable, allowing the extraction of dm_N/dm_s.
- Increase in the error between different 32^3 ensembles is likely from slower convergence of gaussian source/point sink combination to the nucleon ground state, Different sink could improve the signal. Factor of > 2 improvement in statistics is possible without more nucleon measurements by adding more reweighting factor measurements.
- Combination of "good" nucleon source and strange quark reweighting can give a competitive measurement of f_{T_s} without measurement of disconnected diagrams.
- Measurements at lighter pion mass would help greatly in controlling the systematic error in chiral extrapolation. It may be possible to combine these with DSDR lattices ($m_\pi \sim$ 180,250Mev).