Results for Light Pseudoscalar Mesons

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Lattice 2010
Sardinia, Italy, June 14-19

MILC Collaboration

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MILC Ensembles

- Since 1999, MILC Collaboration has been generating asqtad staggered configurations with 2+1 sea flavors.
- Use fourth root procedure to reduce unwanted 4 taste degrees of freedom to 1.
  - good analytic and numerical evidence that this works:
    - Shamir (2005,2007); CB, Golterman, & Shamir (2006,2008); CB (2006); CB, Golterman, Shamir & Sharpe (2007,2008).
    - Dürr & Hoelbling (2004,2005); Follana, Hart & Davies (2004); MILC (2005).
- Lattice spacings from a=0.18 fm to 0.045 fm
  - but only a=0.09, 0.06, and 0.045 fm used here.

MILC Asqtad Ensembles

- Simulation strange sea quark mass (m'_s) usually $\approx m_s$.
- Simulation light sea mass ($\hat{m}'$) usually $0.05 m_s \leq \hat{m}' \leq 0.4 m_s$
- 3 ensembles have $m'_s \approx 0.6 m_s$
- 1 ensemble has $m'_s = \hat{m}' \approx 0.1 m_s$
  - "lighter-than-physical strange" ensembles are useful in fixing SU(3) LECs
- Lowest Goldstone pion:
  - about 175 MeV at a=0.09 fm.
  - about 220 MeV at a=0.06 fm.
  - about 320 MeV at a=0.045 fm.
- Volumes from (2.4 fm)^3 to (5.4 fm)^3; $m_\pi L > 4$ always.

MILC Asqtad Ensembles

a (fm)	$\hat{m}' \;/\; m'_s$	$m_\pi L$	# lattices
0.09	0.0124 / 0.031	5.78	531
0.09	0.0093 / 0.031	5.04	1124
0.09	0.0062 / 0.031	4.14	591
0.09	0.00465 / 0.031	4.11	984
0.09	0.0031 / 0.031	4.21	945
0.09	0.00155 / 0.031	4.80	751
0.09	0.0062 / 0.0186	4.09	985
0.09	0.0031 / 0.0186	4.22	781
0.09	0.0031 / 0.0031	4.20	555
0.06	0.0072 / 0.018	6.33	594
0.06	0.0054 / 0.018	5.48	465
0.06	0.0036 / 0.018	4.49	751
0.06	0.0025 / 0.018	4.39	768
0.06	0.0018/ 0.018	4.27	826
0.06	0.0036 / 0.0108	5.96	601
0.045	0.0028 / 0.014	4.56	801

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lighter than physical m_s'

Chiral fitting

- Partially quenched data for pseudoscalar meson masses and decay constants.
- Systematic SU(3) chiral fits through NNLO.
  - NLO includes complete (rooted) staggered chiral logs.
  - at NNLO only continuum version exists (Bijnens *et al*., 2004, 2005, 2006).*
  - input the taste RMS meson mass to continuum formulas.
  - this is systematic only if taste splittings are significantly smaller than the meson masses.
  - starts to be true at a=0.09 fm (if we avoid lightest meson mass); better obeyed at a=0.06 and 0.045 fm).

*Thanks to Hans Bijnens for computer code for PQ NNLO logarithms

Chiral fitting

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With our stat errors, have never been able to get good fits at NLO: SU(3) *or* SU(2)

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Pion Masses & Splittings

a(fm)	Goldstone	RMS	Singlet
0.15	236	532	660
0.12	268	455	550
0.09 ($\hat{m}' = 0.05 m_s'$)	174	275	340
0.09 (other)	240	320	377
0.06	219	253	274
0.045	318	327	337

Low Mass Chiral Fits

- systematic NNLO, using only ensembles with:

$$m'_s \leq 0.6 m_s$$

$$m_x + m_y \leq 0.6 m_s$$

  - (m_x, m_y are the valence masses.)

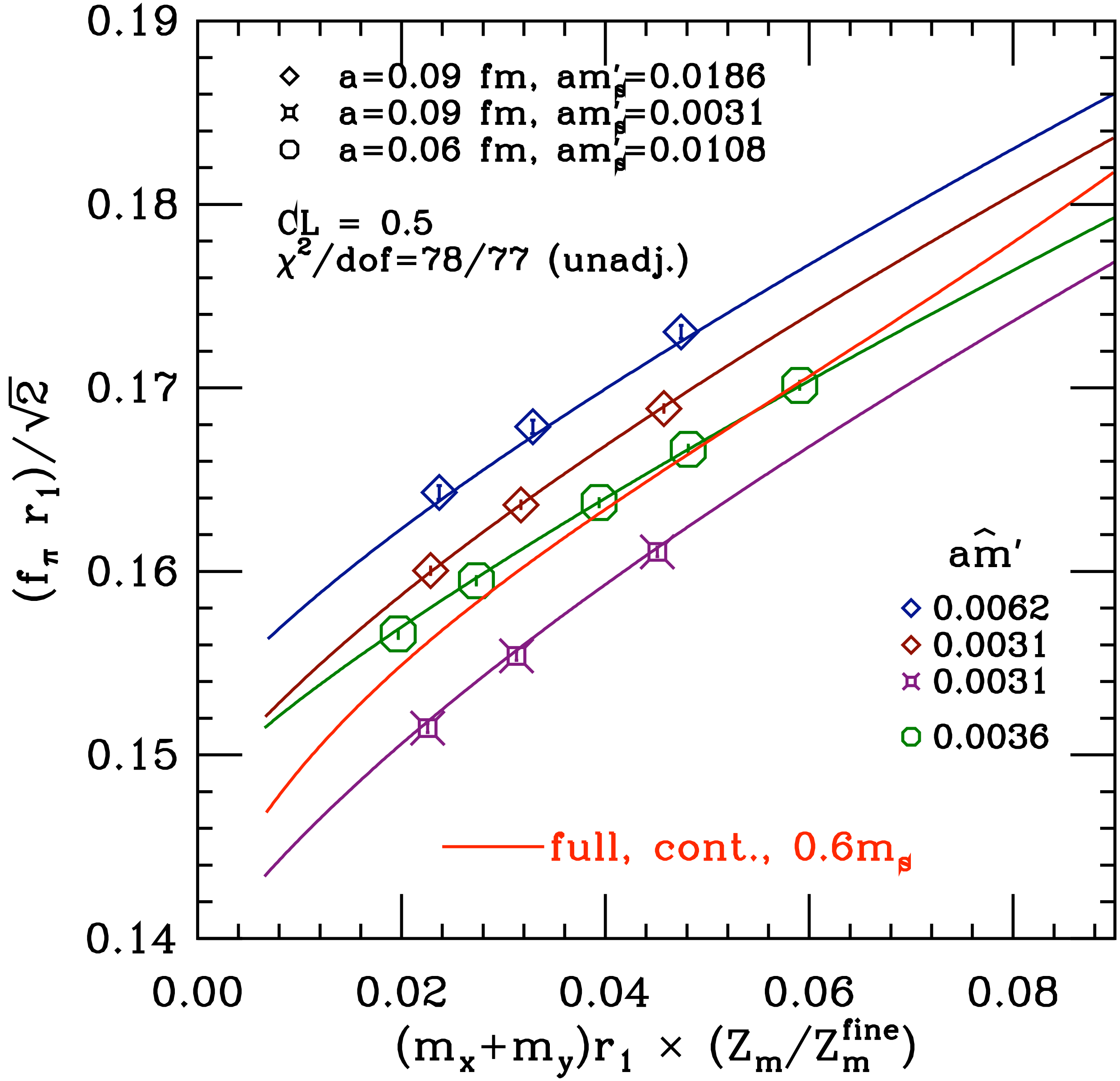
- 4 of the 5 p^4 LECs that first appear at NNLO (L_1, L_2, L_3, L_7) are constrained by priors from continuum info (Bijnens, 2007).

- All other LO, NLO, and NNLO LECs are unconstrained:
  - 19 unconstrained params.
  - 4 constrained p^4 LECs + up to 8 constrained a^2 variations of physical params.
  - gives 31 params total; ~110 points

- Need to use a "renormalized chiral coupling" $\sim f_\pi$ to get acceptable fits, not 3-flavor chiral limit f_3 .

Low Mass Chiral Fits

- Fit to PQ data for decay constants & masses simultaneously.
- Full covariance matrix.
- Here, show f_π for $m_x = m_y$.
- These fits used to determine LECs.



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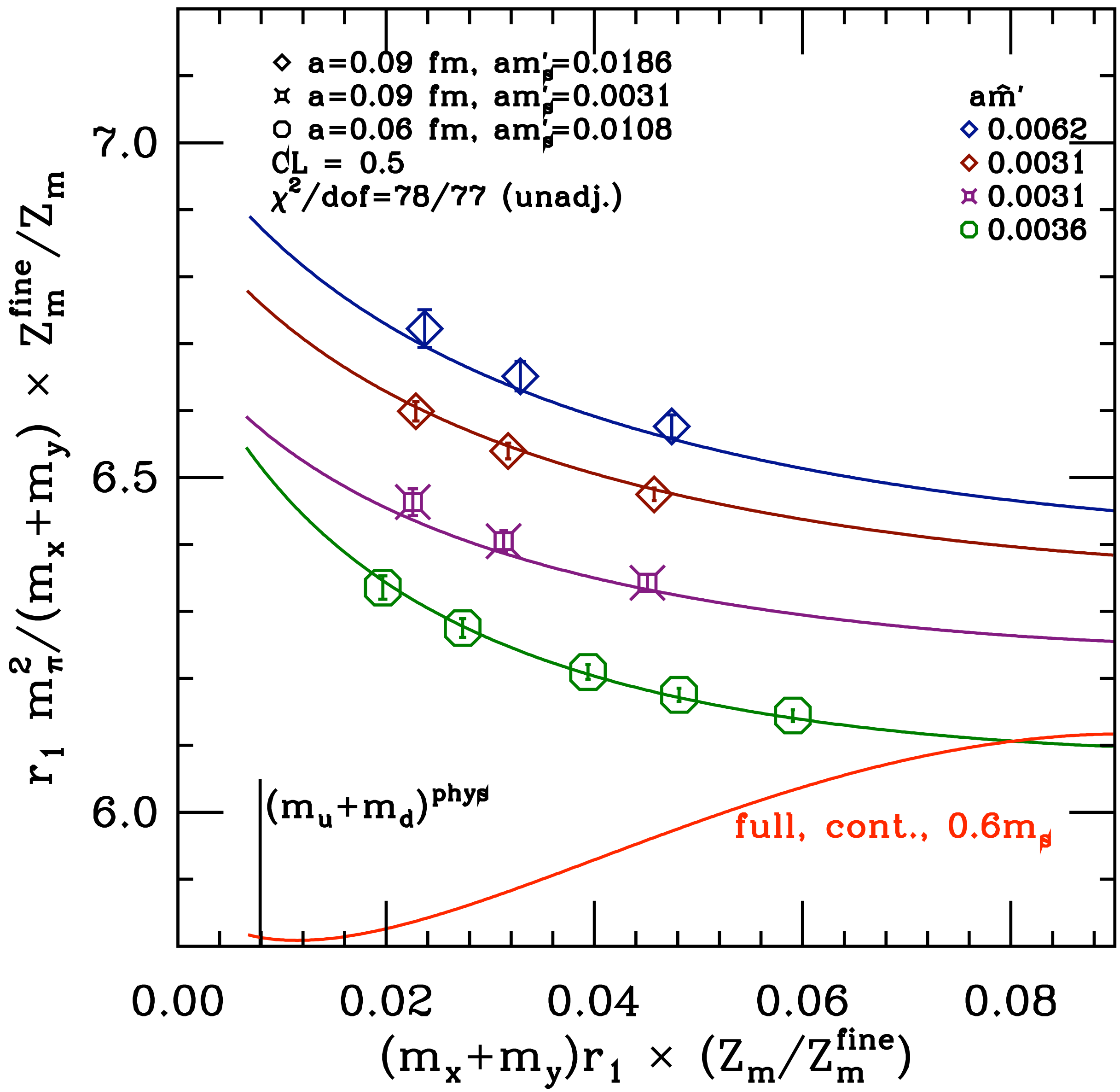
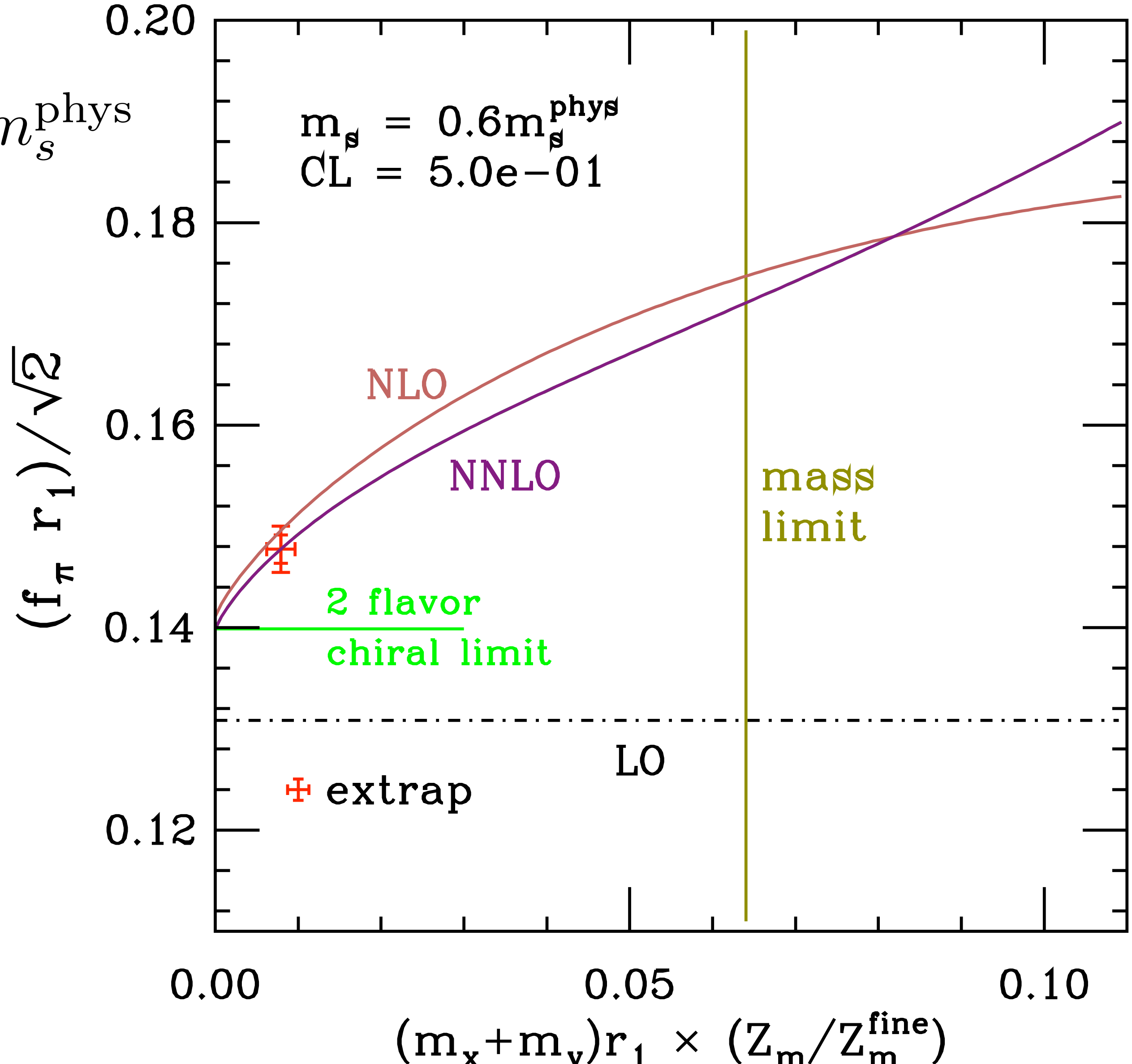


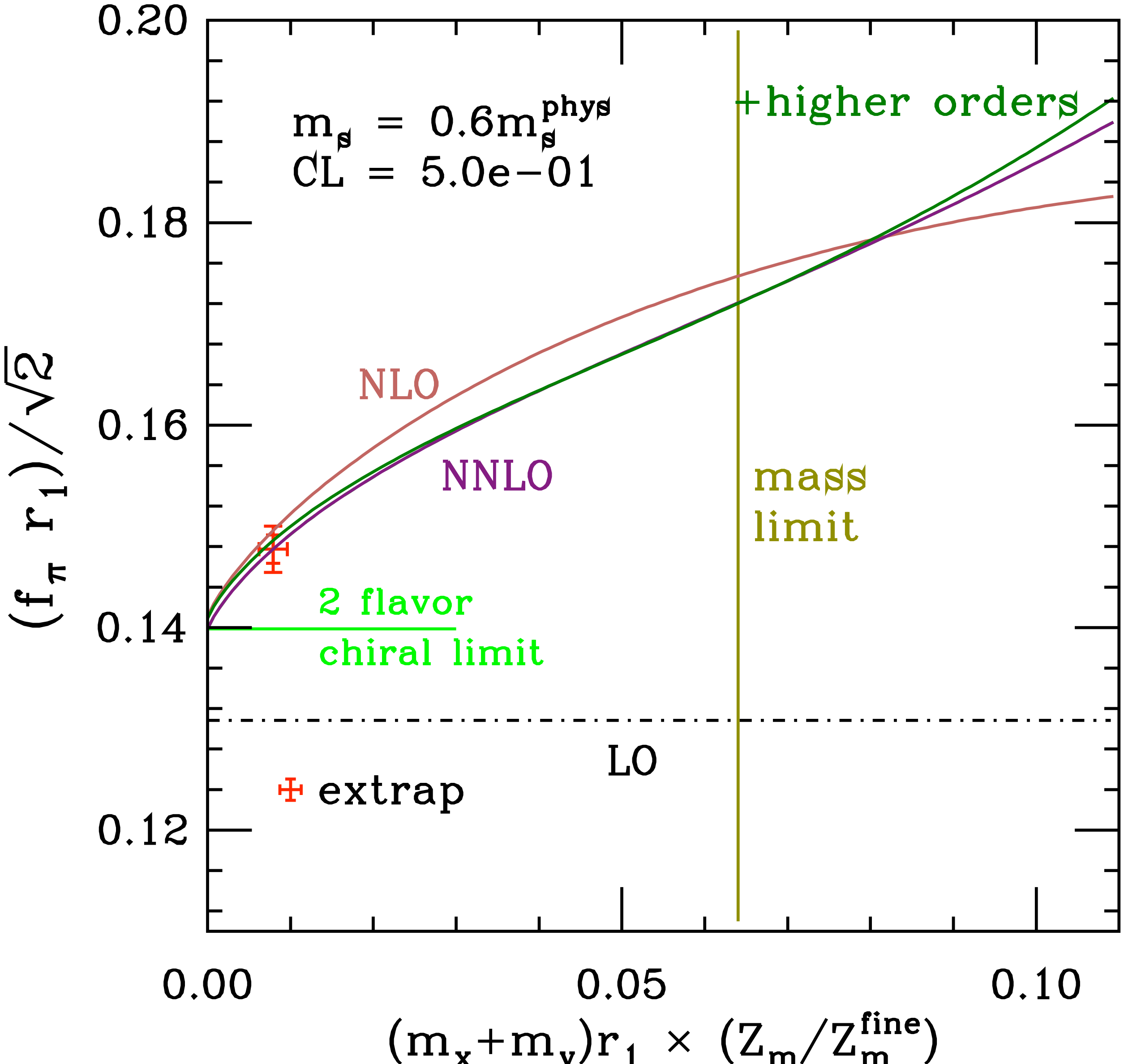
ChPT Convergence: Low Mass

- strange mass held fixed at $0.6\ m_s^{\text{phys}}$
- plotted for $m_x = m_y = \hat{m}'$



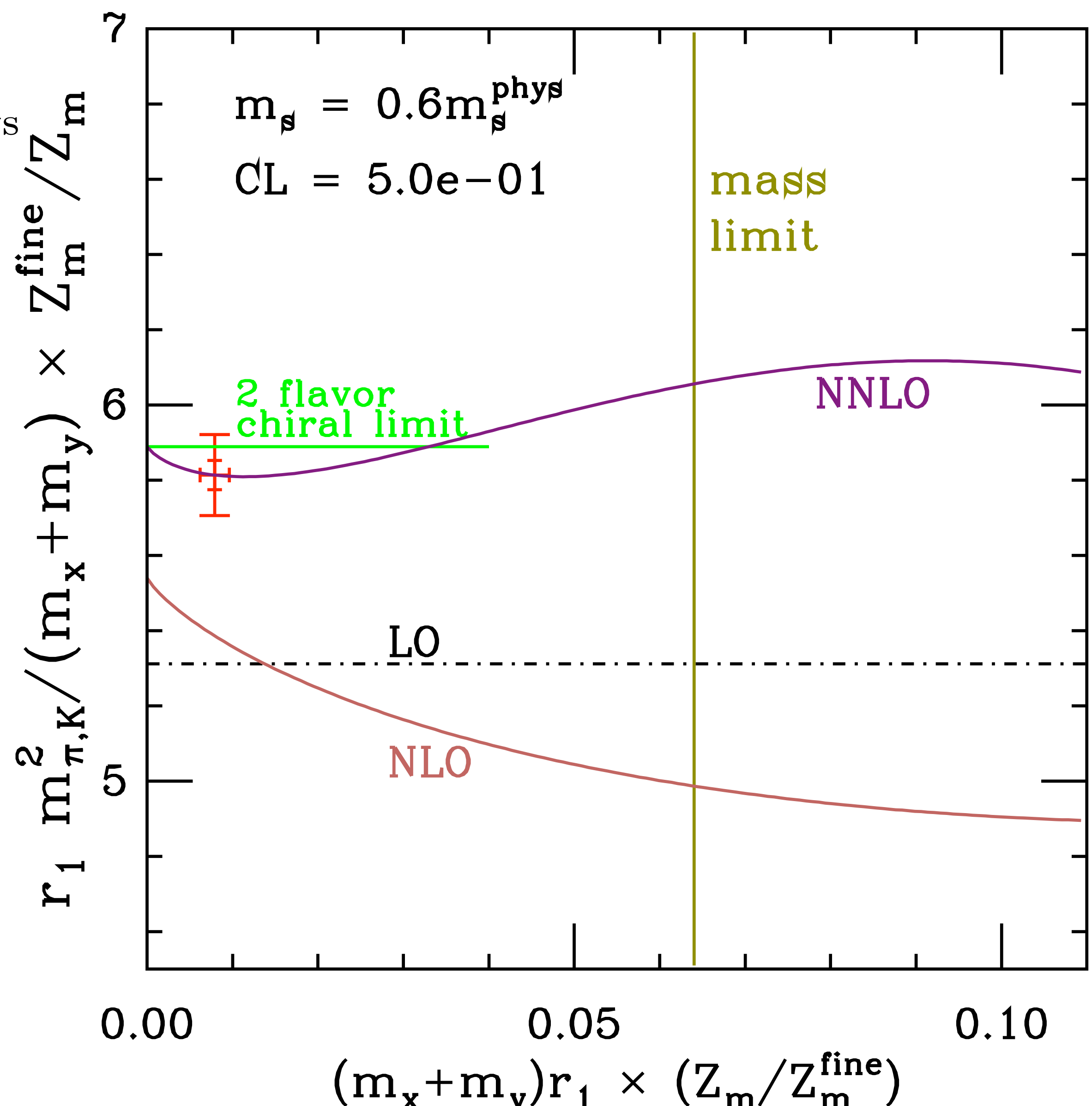
ChPT Convergence: Low Mass

- Test of convergence:
- Add in all N^{3}LO & N^{4}LO analytic terms.
  - keep LO, NLO, & NNLO fixed.



ChPT Convergence: Low Mass

- strange mass held fixed at $0.6\ m_s^{\text{phys}}$
- plotted for $m_x = m_y = \hat{m}'$
- NLO term is anomalously small for mass, so NNLO is a relatively big change.
- But full correction to LO is only ~11%.



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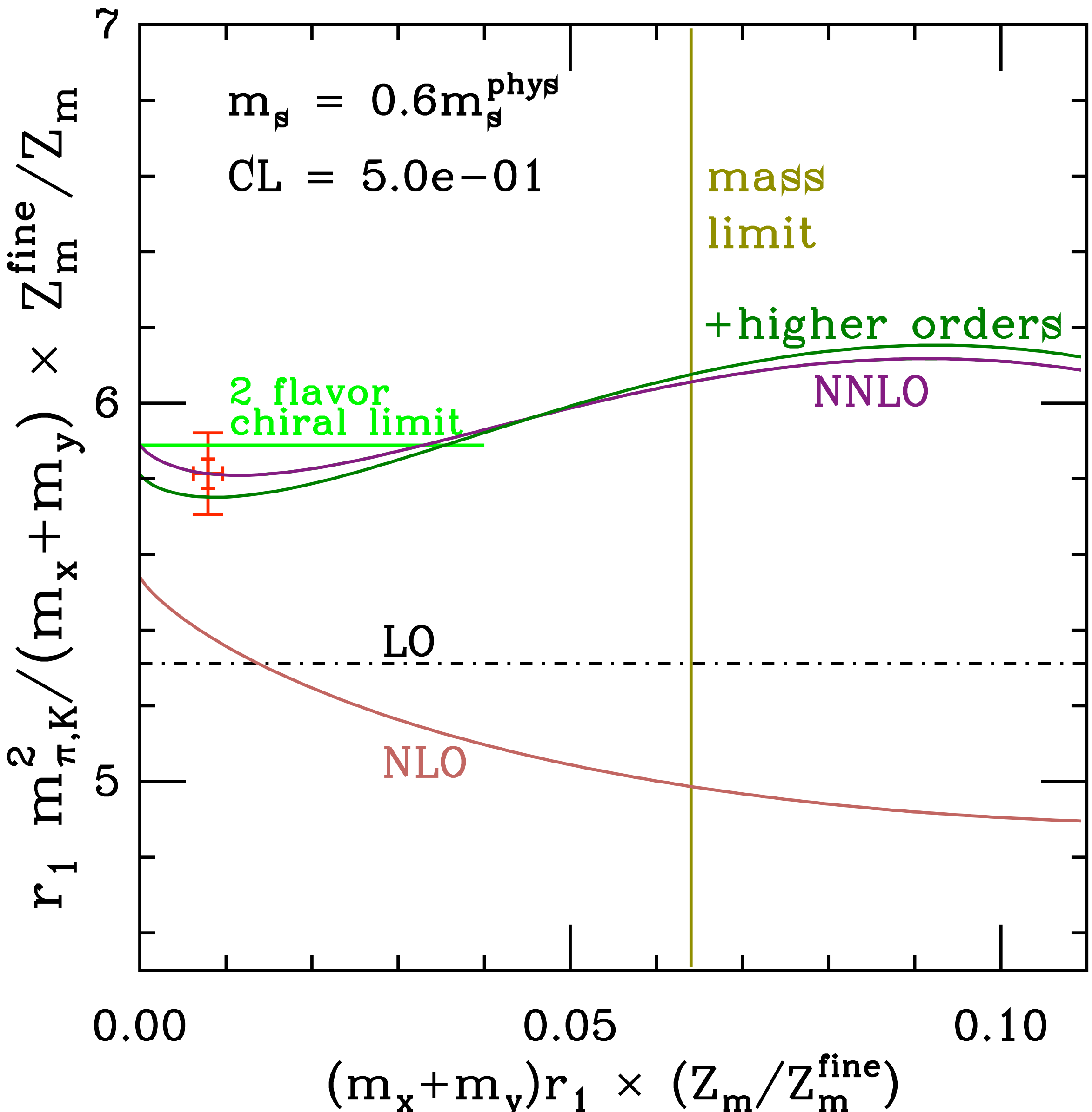
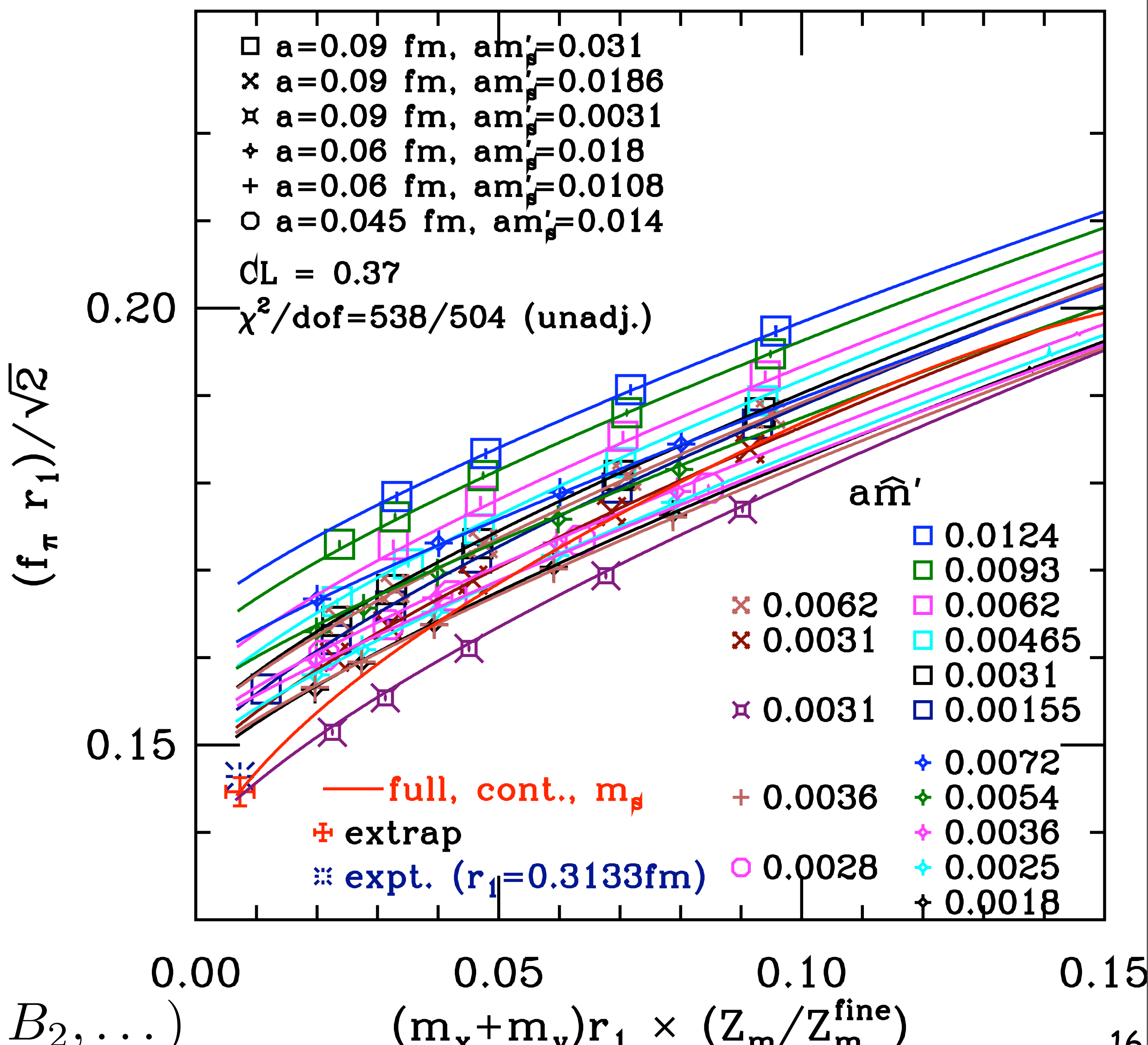


High Mass Chiral Fits

- Need higher masses for quantities involving strange valence or sea quark.
- Now fit to all ensembles with a=0.09, 0.06, & 0.045 fm.
- Valence masses limited only by $m_x + m_y \leq 1.2 m_s$
- Fix LO, NLO, & NNLO LECs from low mass fits.
  - (sometimes also allow variations with width determined by statistical errors.)
- Add in N^3LO (18) & N^4LO (32) analytic terms + constrained a^2 variations of NNLO & N^3LO terms (33) = 83 params total.
  - necessary for a good CL.
  - ~polynomial interpolation around strange mass.
- Not systematic (no higher chiral logs), but still controlled:
  - LO, NLO (&NNLO) terms dominate slope of extrapolation to physical light quark masses.

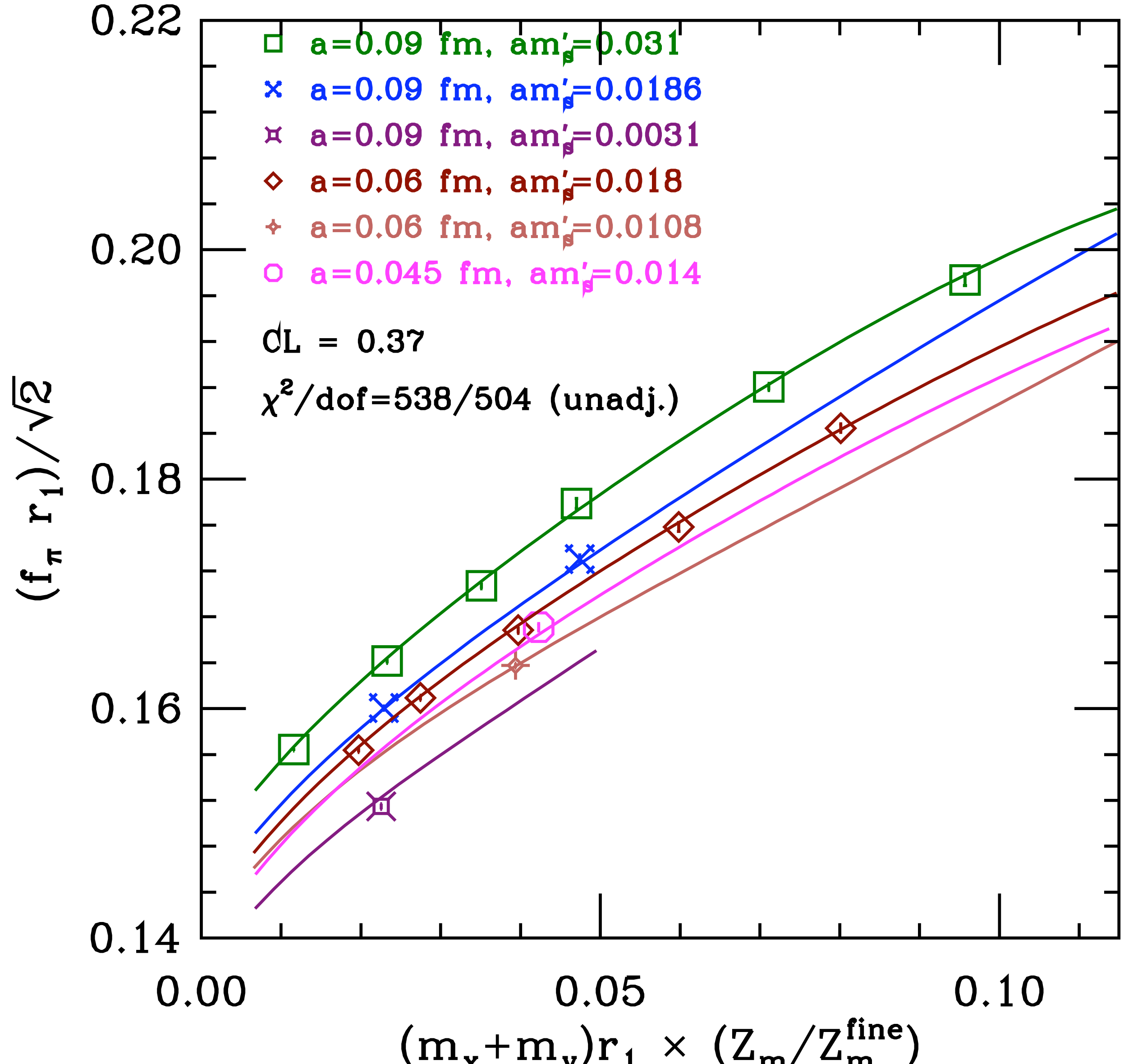
High Mass Chiral Fits

- Fit to PQ data for decay constants & masses simultaneously.
- Full covariance matrix.
- Here, show f_π for $m_x = m_y$.
- These fits used to determine decay constants, quark masses, & 2-flavor chiral limit quantities $(f_2, B_2, \ldots)$



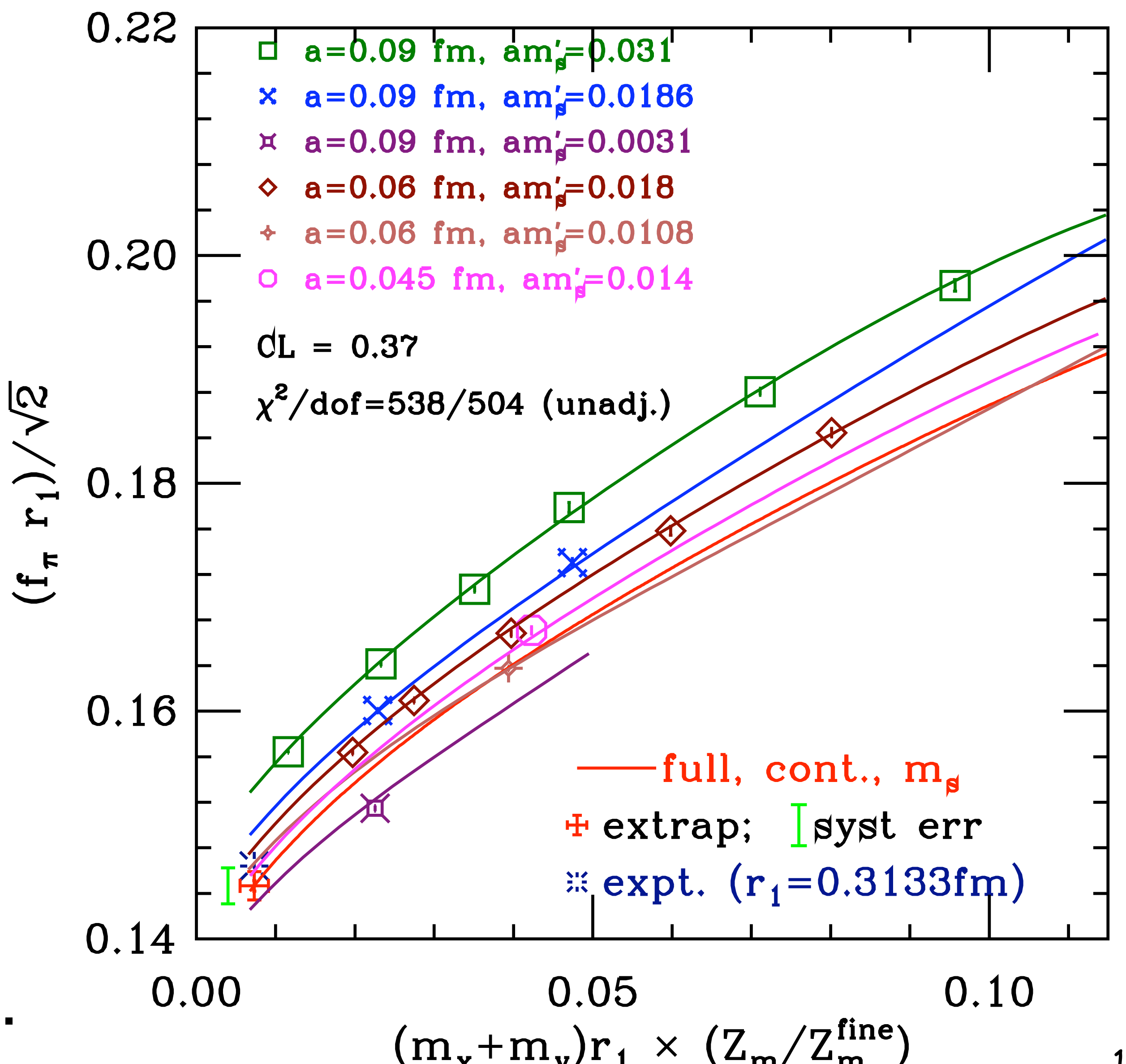
High Mass Chiral Fits

- Same fit, but show full QCD points only, for clarity.



High Mass Chiral Fits

- Add in continuum extrapolated line, with $m_s' = m_s$
- Show extrapolated point & comparison with experiment.
- This uses the scale from splitting by HPQCD group: $r_1 = 0.3133(23)$ fm.



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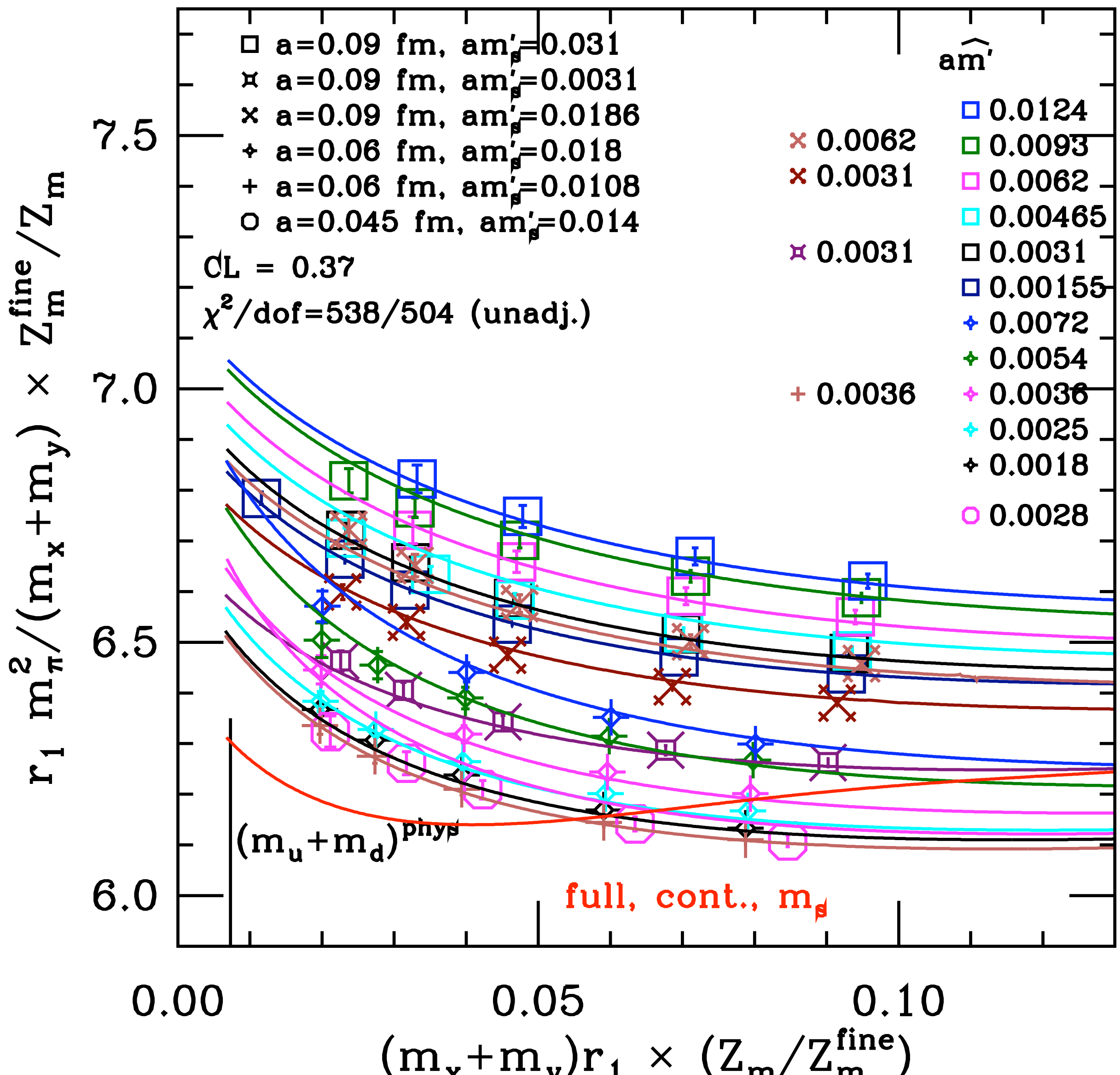
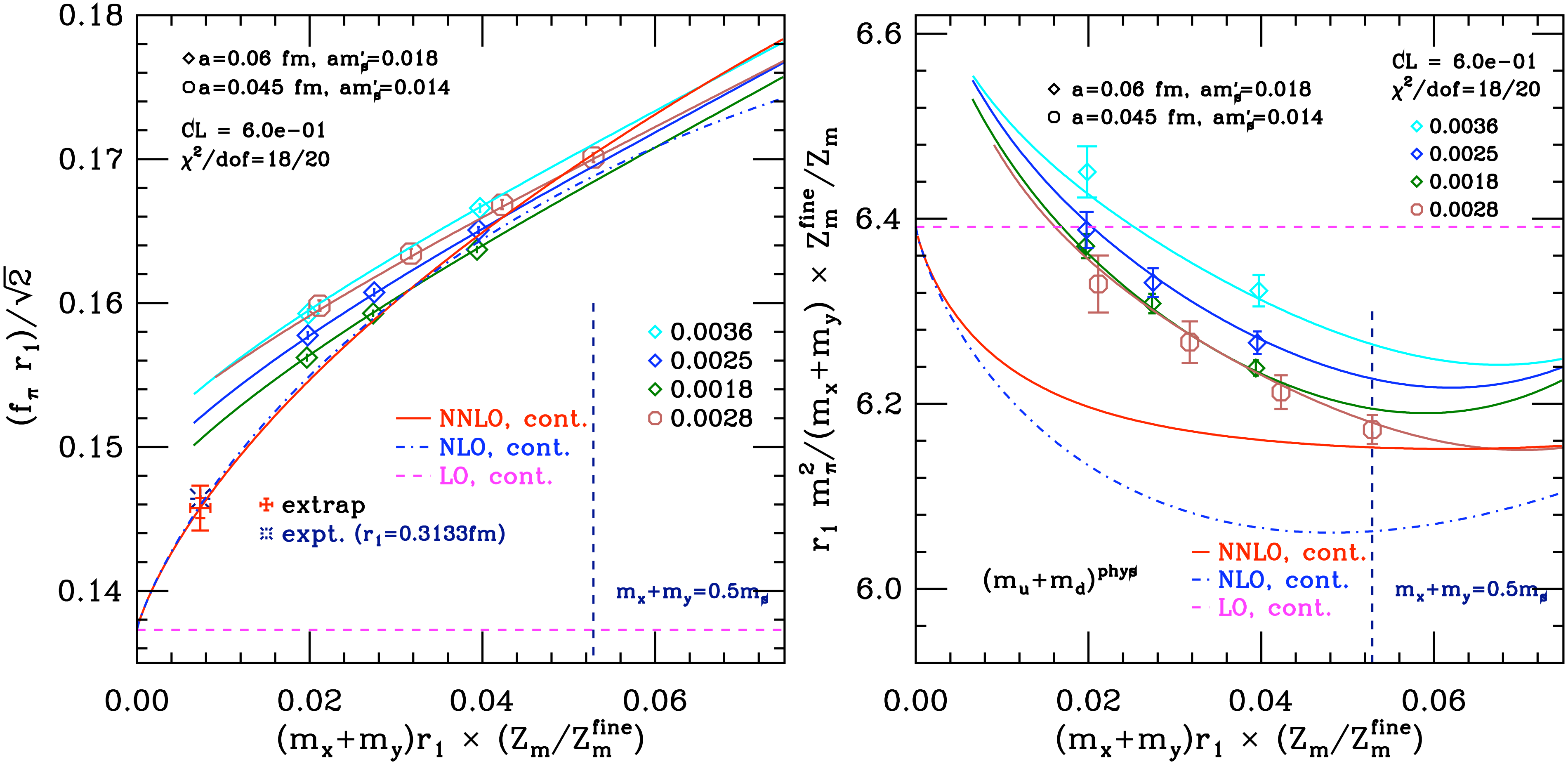


SU(2) Fits & Convergence

- SU(2) fits for pure light quantities only, so far (no "heavy strange" yet).



- Good SU(2) ChPT convergence in both cases.
- Physical results agree well with SU(3) fits (and expt.).

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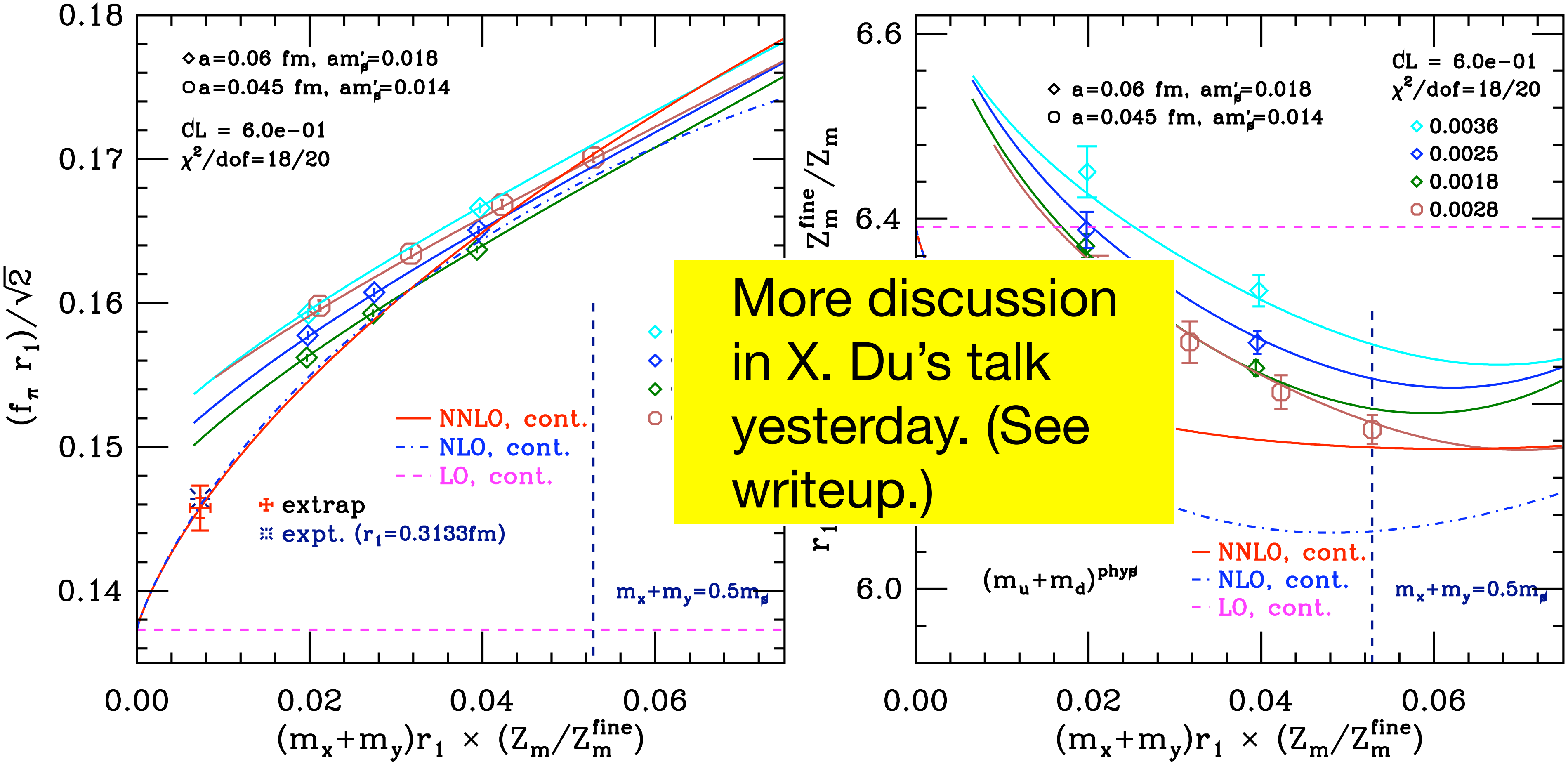

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(Selected) Preliminary Results

- With HPQCD r_1 = 0.3133(23):

  SU(3): $f_\pi = 129.2 \pm 0.4 \pm 1.4 \text{ MeV}$

  SU(2): $f_\pi = 130.2 \pm 1.4 \left(^{+2.0}_{-1.6}\right) \text{ MeV}$

- Using f_π to set the scale, find:

  $$r_1 = 0.3106(8)(18)(4) \text{ fm}$$

  (last error from experimental uncertainty: 130.4(2) MeV)

- From now on use f_π to set scale for all physical quantities.
- All results use SU(3) ChPT unless noted otherwise.

(Selected) Preliminary Results

- Decay constants:

$$f_K = 156.1 \pm 0.4 \left({}^{+0.6}_{-0.9}\right) \text{ MeV}$$

$$f_K/f_\pi = 1.197(2) \left({}^{+3}_{-7}\right)$$

$$V_{us} = 0.2247 \left({}^{+14}_{-9}\right)$$

- Also get decay constants in the chiral limits (2-flavor, f_2, and 3-flavor, f_3).

$$f_2 = 123.0 \pm 0.5 \pm 0.7 \text{ MeV}$$

$$f_2 = 123.8 \pm 1.4 \left({}^{+1.0}_{-3.7}\right) \text{ MeV}$$ ← SU(2)

$$f_3 = 118.0 \pm 3.6 \pm 4.6 \text{ MeV}$$

- Useful for scale setting:

$$f_{ss.4} \equiv f(m_x = m_y = 0.4 m_s, \hat{m}, m_s) = 154.0 \pm 0.4 \pm 0.6 \text{ MeV}$$

(Selected) Preliminary Results

- Masses (at 2 GeV scale):

$$m_s^{\overline{\mathrm{MS}}} = 87.0(0.2)(1.5)(4.4)(0.1)\ \mathrm{MeV}$$

$$\hat{m}^{\overline{\mathrm{MS}}} = 3.17(1)(7)(16)(0)\ \mathrm{MeV}$$

$$\hat{m}^{\overline{\mathrm{MS}}} = 3.19(4)\left({}^{+5}_{-3}\right)(16)(0)\ \mathrm{MeV}$$ ← SU(2)

$$m_s/\hat{m} = 27.46(4)(16)(0)(4)$$

$$m_u^{\overline{\mathrm{MS}}} = 1.91(1)(6)(10)(12);\mathrm{MeV}$$

$$m_d^{\overline{\mathrm{MS}}} = 4.43(1)(8)(22)(12);\mathrm{MeV}$$

$$m_u/m_d = 0.432(1)(7)(0)(39)$$

  - errors: statistical, lattice systematics, perturbation theory, EM effects
  - perturbation theory (2 loop): Q. Mason et al., Phys. Rev. D73 (2006) 114501 [hep-lat/0511160].
  - perturbative error assumed: $2\alpha^3$

(Selected) Preliminary Results

- EM effects are by far largest systematic in m_u/m_d.
- At present use continuum phenomenology for EM effects of K^+ mass.
- To improve situation, we are calculating EM effects on the lattice: see talks by E. Freeland (this session); and A. Turok (parallel 45, Thursday.)

(Selected) Preliminary Results

- NLO Low Energy Constants for SU(3) (chiral scale m_η):

$$L_5 = 1.79(16)(33) \times 10^{-3}$$

$$L_4 = 0.19(21)(14) \times 10^{-3}$$

$$2L_6 - L_4 = 0.09(23)(27) \times 10^{-3}$$

$$2L_8 - L_5 = -0.51(11)(35) \times 10^{-3}$$

$$L_6 = 0.14(19)(15) \times 10^{-3}$$

$$L_8 = 0.64(7)(6) \times 10^{-3}$$

- NLO Low Energy Constants for SU(2):

$$\bar{l}_3 = 3.7(1.2)(1.4)$$ ⟵ from SU(3) @ NLO

$$\bar{l}_3 = 2.85(81)\left({}^{+37}_{-92}\right)$$ ⟵ direct SU(2)

$$\bar{l}_4 = 3.96(26)(27)$$ ⟵ from SU(3) @ NLO

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- NLO Low Energy Constants for SU(2):

Big errors because of NNLO terms in fit

$$\bar{l}_3 = 3.7(1.2)(1.4) \quad \longleftarrow \text{from SU(3) @ NLO}$$

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(Selected) Preliminary Results

- NNLO Low Energy Constants for SU(3) [definitions of Bijnens, Colangelo, Ecker (1999)]
  - PQ SU(3) LECs:

$$K_{21} = 6.7(2.2)(3.4) \times 10^{-6}$$

$$K_{27} = 0.4(2)(3) \times 10^{-6}$$

$$K_{39} - K_{17} = 3.9(1.1)(1.6) \times 10^{-6}$$

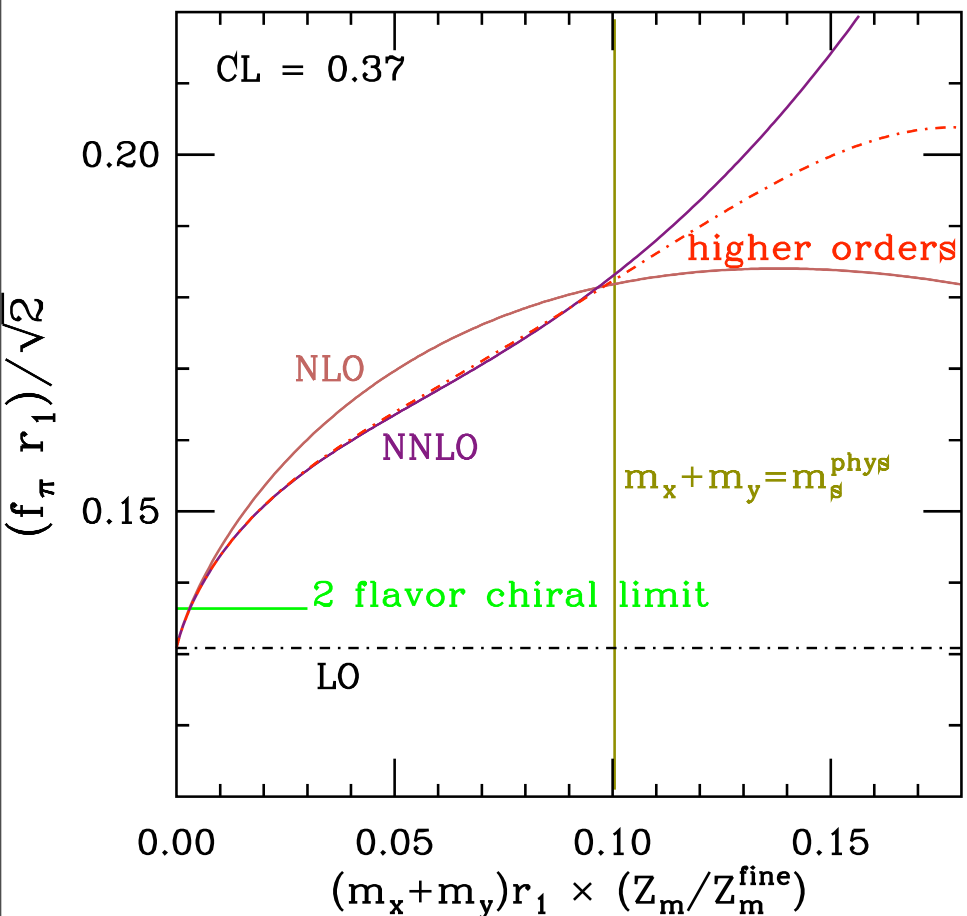
  - full SU(3) LECs:

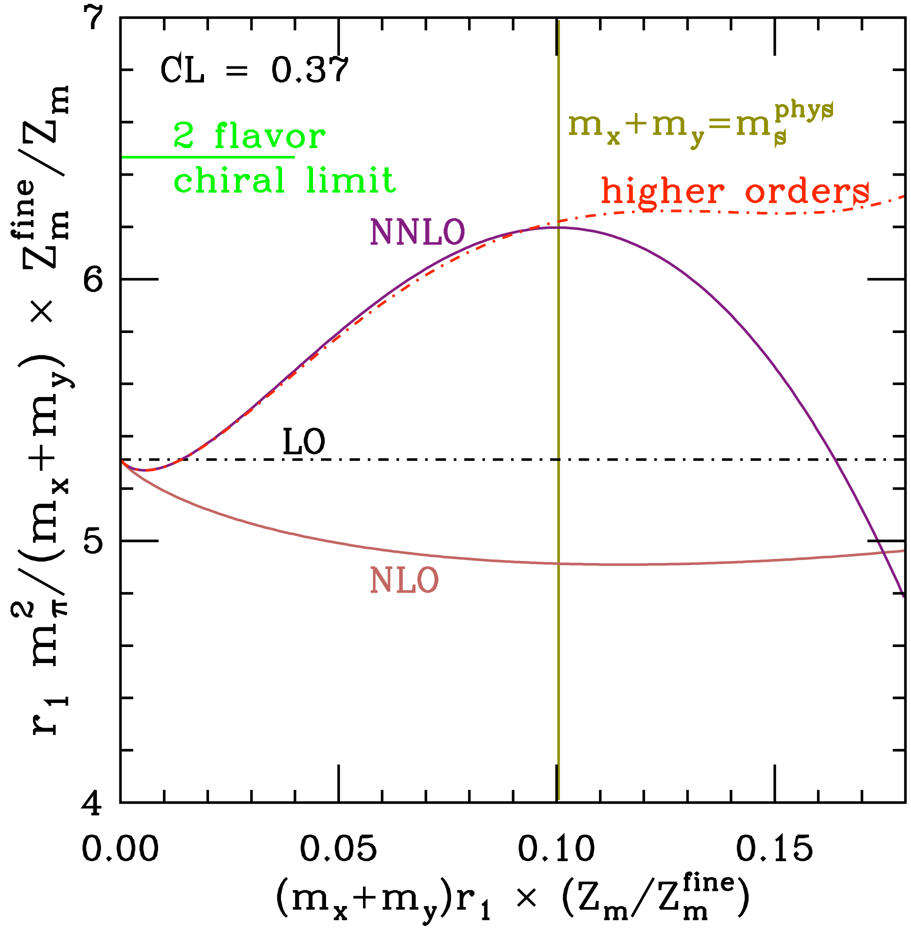
$$C_{16} = 7.1(2.3)(4.0) \times 10^{-6}$$

- Other NNLO LECs are also ~10^{-6}, but statistical or systematic errors (or both) are more than 100%.

ChPT Convergence: High Mass

- Behavior for 3 degenerate flavors:

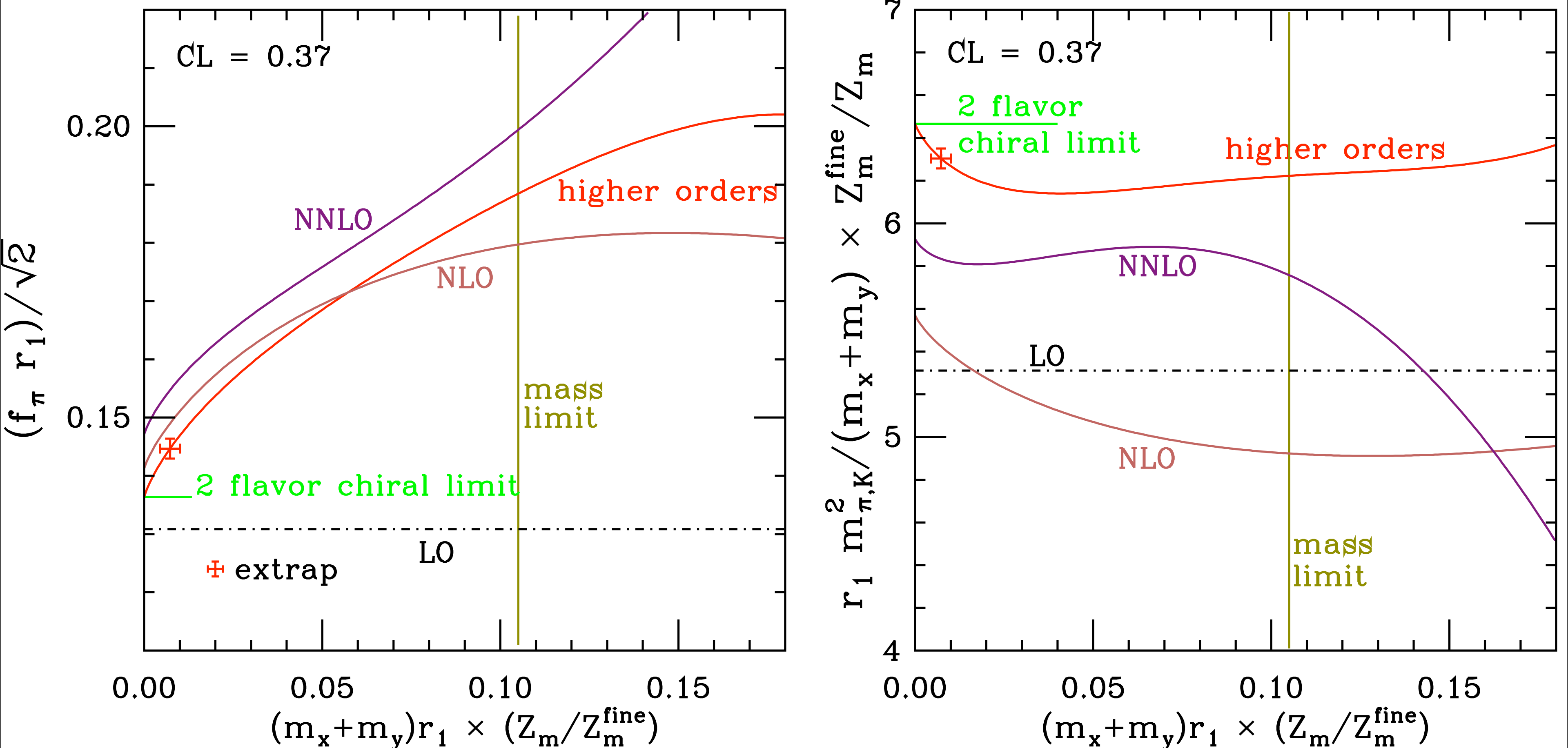




- Good convergence up to $m_x + m_y \sim 1.2 m_s$, well beyond m_K.

ChPT Convergence: High Mass

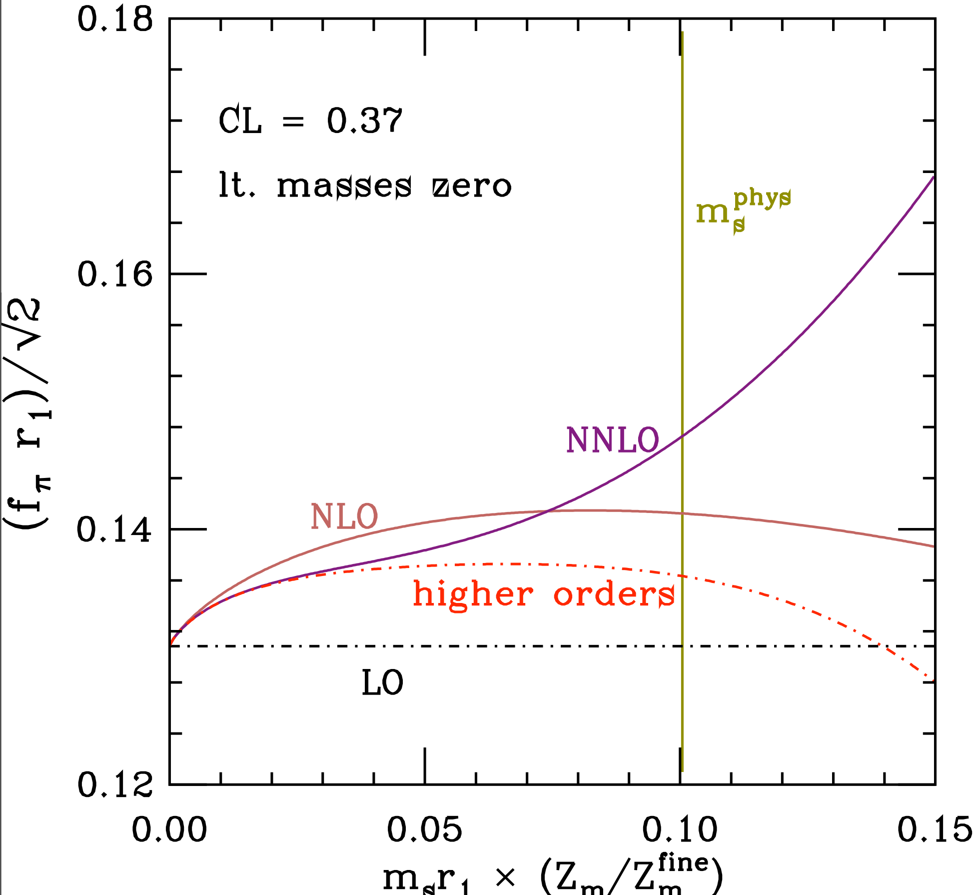
- Behavior for strange sea quark fixed at m_s.

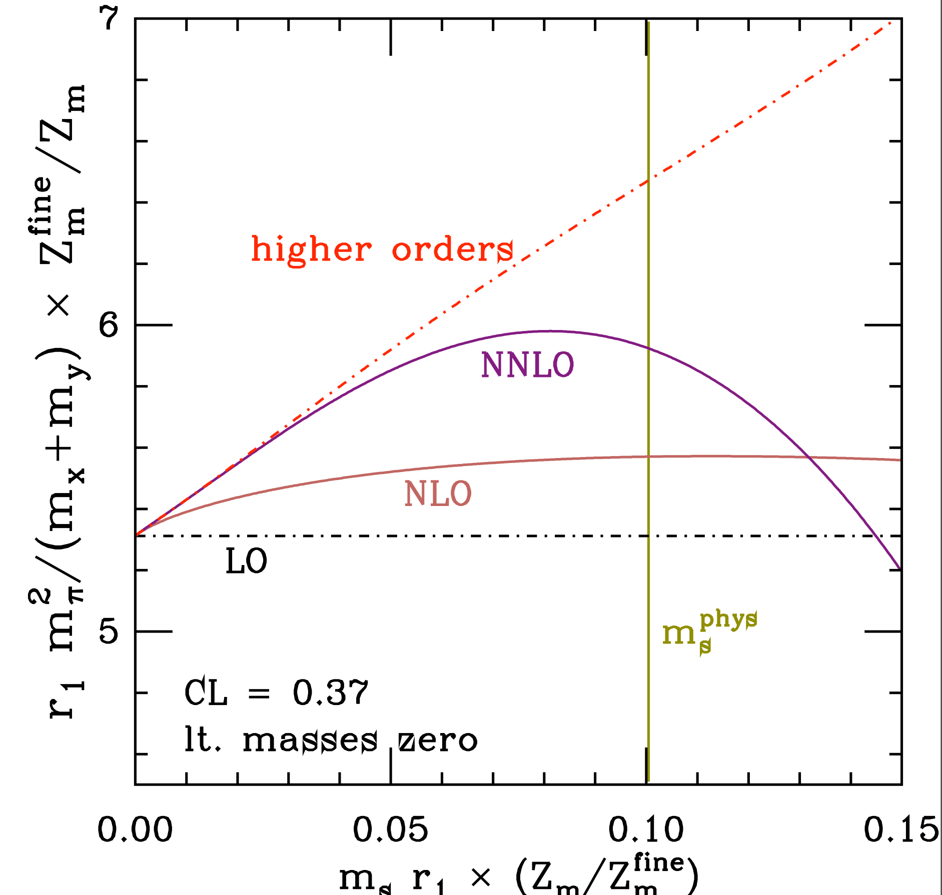


- no convergence at NNLO for f_π; fair for $m_\pi^2/(m_x+m_y)$
  - not completely surprising: ChPT is an asymptotic expansion.
  - NLO does quite well though, for f_π

ChPT Convergence: High Mass

- To study how breakdown occurs at m_s, consider m_s' dependence at vanishing light sea mass & valence masses.





- Breakdown occurs at $m_s' \approx 0.6 m_s$ to $0.8 m_s$

Conclusions

- Nearing completion of asqtad staggered analysis of pseudoscalar-meson quantities.
- Precise results (<1%) for several quantities in continuum limit & for physical quark masses.
  - e.g. $f_K\,,\ \ f_K/f_\pi$
  - SU(3) and SU(2) fits give good agreement.
- ChPT through NNLO gives very good representation of our data up through 0.6 m_s.
- At m_s (with other quarks light), asymptotic nature of chiral expansion evident.
  - NNLO terms can start to show divergent behavior.
  - adding effective, higher order analytic terms necessary to describe data.

Outlook

- HISQ program of 2+1+1 simulations is well under way.
- Current method should work well starting at 0.12 fm.
- Expect significantly smaller systematic errors.
- Need to extend staggered ChPT to include heavy staggered charm quark.