Tuning the strange quark mass and the hadron mass spectrum for $2 + 1$ quark flavours

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[Lattice 2010, Villasimius, Sardinia, Italy] with

W. Bietenholz, V. Bornyakov, N. Cundy, M. Göckeler, T. Hemmert, A. D. Kennedy, W. G. Lockhart, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, A. Schäfer, G. Schierholz, A. Schiller, T. Streuer, H. Stüben, F. Winter, J. M. Zanotti . . .

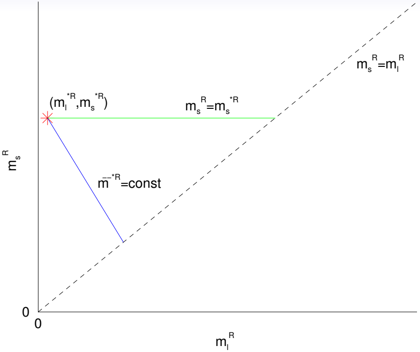
related talks/poster:

P. Rakow, F. Winter, J. Zanotti, Y. Nakamura, H. Stüben

Introduction

- The m_l^R–m_s^R plane and our choice of path to physical point
- This path choice with clover fermions
- Tuning results
- Spectrum results
  - f_K/f_π
  - Baryon octet: $N(lll)$, $\Lambda(lls)$, $\Sigma(lls)$, $\Xi(lss)$
  - Baryon decuplet: $\Delta(lll)$, $\Sigma^*(lls)$, $\Xi^*(lss)$, $\Omega(sss)$
- Partially Quenched results
- Conclusions

Many paths to approach the physical point



Choice here:
Extrapolate from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_l^{R\,(0)}, m_s^{R\,(0)}) \longrightarrow (m_l^{R\,*}, m_s^{R\,*})$$

keeping the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}\,(2m_l^R + m_s^R)$$

Expanding a flavour singlet quantity about a point on the $SU(3)_F$-flavour line:

Let $X_S(m_u^R, m_d^R, m_s^R)$ be a flavour singlet object

[X_S invariant under the quark permutation symmetry between u, d and s]

$$
\begin{aligned}
& X_S(\overline{m}^{R(0)} + \delta m_l^R, \overline{m}^{R(0)} + \delta m_l^R, \overline{m}^{R(0)} + \delta m_s^R) \\
&= X_{S\,sym}^{(0)} + \left.\frac{\partial X_S}{\partial m_u^R}\right|_{sym}^{(0)} \delta m_l^R + \left.\frac{\partial X_S}{\partial m_d^R}\right|_{sym}^{(0)} \delta m_l^R + \left.\frac{\partial X_S}{\partial m_s^R}\right|_{sym}^{(0)} \delta m_s^R + O((\delta m_q^R)^2)
\end{aligned}
$$

On the symmetric line:

$$
\left.\frac{\partial X_S}{\partial m_u^R}\right|_{sym} = \left.\frac{\partial X_S}{\partial m_d^R}\right|_{sym} = \left.\frac{\partial X_S}{\partial m_s^R}\right|_{sym} ,
$$

On the chosen trajectory $\overline{m}^R = \frac{1}{3}(2m_l^R + m_s^R) = \text{constant}$

$$
2\delta m_l^R + \delta m_s^R = 0
$$

Together these imply that

$$
X_S(\overline{m}^{R(0)} + \delta m_l^R, \overline{m}^{R(0)} + \delta m_l^R, \overline{m}^{R(0)} + \delta m_s^R) = X_{S\,sym}^{(0)} + O((\delta m_q^R)^2)
$$

Potential Advantages:

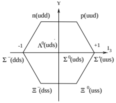
- X_S flat at symmetric point, so this value close to value at physical point
- $\overline{m} =$ const. means that as we extrapolate $m_l^R \searrow m_l^{R*}$ then $m_s^R \nearrow m_s^{R*}$
- No knowledge of χ-PT necessary (eg r_0)
- Simulation cost change should be moderate along this path

Examples of singlet quantities

- Octet baryons: (centre of mass)

  $X_N = \frac{1}{3}(m_N + m_\Sigma + m_\Xi) = 1.150\,\text{GeV}$

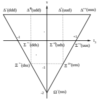
stable under strong ints.



- Decuplet baryons: (centre of mass)

  $X_\Delta = \frac{1}{3}(2m_\Delta + m_\Omega) = 1.379\,\text{GeV}$

decay under strong ints.



- Gluonic:

$$X_r = 1/r_0$$

$r_0 = 0.5$ fm ?

- Some other possibilities

$$X_S = \begin{cases} \frac{1}{2}(m_\Sigma + m_\Lambda) \\ m_{\Sigma^*},\ \frac{1}{2}(m_\Delta + m_{\Xi^*}) \\ \frac{1}{3}(2m_K^2 + m_\pi^2) \\ \frac{1}{3}(2m_{K^*} + m_\rho) \end{cases}$$

Check using χ-PT

Choose your favourite χ-PT result

Expand about a $SU(3)_F$ flavour symmetric point:

$$X_S = X_S^{(0)}|_{sym} + O((\delta m_q^R)^2)$$

Clover fermions

- Above discussion holds for all fermions
- For clover fermions, path $\overline{m}^R = \text{const.}$ solves another problem – singlet and non-singlet quarks renormalise differently

$q = l, s$

$$m_q^R = Z_m^{NS}(m_q - \overline{m}) + Z_m^S \overline{m} \equiv Z_m^{NS}(m_q + \alpha_Z \overline{m})$$

$\alpha_Z = (Z_m^S - Z_m^{NS})/Z_m^{NS} \sim O(1)$

$$am_q = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{sym;c}}\right)$$

Vanishing of the quark mass along the symmetric line (i.e. for 3 mass degenerate flavours) $\Longrightarrow \kappa_{sym;c}$

LO χPT:

$$\tfrac{1}{3}(2(am_K)^2 + (am_\pi)^2) \propto \tfrac{2}{9}(1 + \alpha_Z)a\overline{m} = \text{const.}$$

so (equivalently) for path $a\overline{m} = \text{const.}$ then

$$\kappa_s = \frac{1}{\frac{3}{\kappa_{sym}^{(0)}} - \frac{2}{\kappa_l}}$$

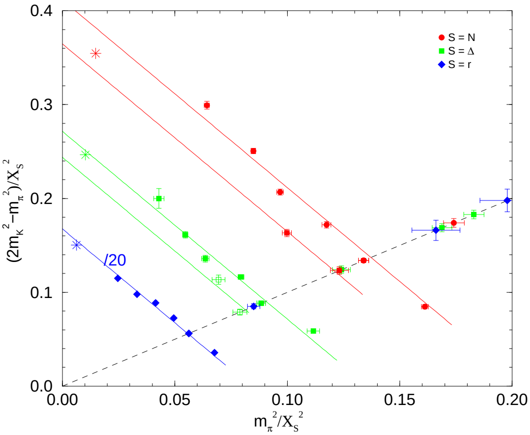
where $\kappa_{sym}^{(0)}$ is a point on the $SU(3)$ flavour symmetric line

Present runs

$N_S^2 \times N_T$	(κ_l, κ_s)		(κ_l, κ_s)
	$\kappa_{sym} = 0.120900$		$\kappa_{sym} = 0.120920$
$24^3 \times 48$	(0.120830,0.121040)	$m_l > m_s$	
$24^3 \times 48$	(0.120900,0.120900)	$m_l = m_s$	(0.120920,0.120920)
$24^3 \times 48$	(0.120950,0.120800)	$m_l < m_s$	(0.120980,0.120800)
$32^3 \times 64$	(0.121000,0.120700)	$m_l < m_s$	
$32^3 \times 64$	(0.121040,0.120620)	$m_l < m_s$	

Possible to choose $(\kappa_l,\ \kappa_s)$ values (here $(0.12083, 0.12104)$) such that $m_l > m_s$. In this 'strange' world, would expect to see an *inversion* of the particle spectrum, with (eg) the nucleon being the heaviest octet particle.

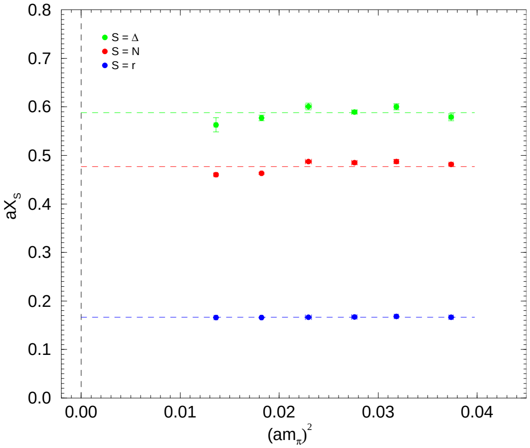
m_l^R–m_s^R plane



Also shown: fit to constant $2m_K^2 + m_\pi^2$ by:

$$\frac{2m_K^2 - m_\pi^2}{X_S^2} = c_S - 2\,\frac{m_\pi^2}{X_S^2} \qquad [m_\pi^{(0)}|_{sym} \sim 410\,\text{MeV}]$$

Scale



Scale determination – given aX_S

- $X_N = \frac{1}{3}(m_N + m_\Sigma + m_\Xi) = 1.150\,\mathrm{GeV}$

$$a = 0.082\,\mathrm{fm}$$

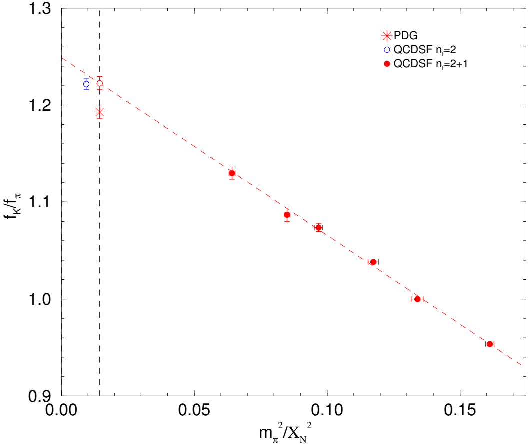
- $X_\Delta = \frac{1}{3}(2m_\Delta + m_\Omega) = 1.379\,\mathrm{GeV}$

$$a = 0.084\,\mathrm{fm}$$

- $r_0 =??$, so invert: from $aX_r = a/r_0$

$$a = 0.083\,\mathrm{fm} \quad \Longrightarrow \quad r_0 = 0.50\,\mathrm{fm}$$

f_K/f_π


PDG
QCDSF n_f=2
QCDSF n_f=2+1
1.3
1.2
1.1
1.0
0.9
0.00
0.05
0.10
0.15
f_K/f_π
m_π^2/X_N^2

Baryon Spectrum

Gell-Mann–Okubo or LO χ-PT gives constrained fits for baryon octet/decuplet:

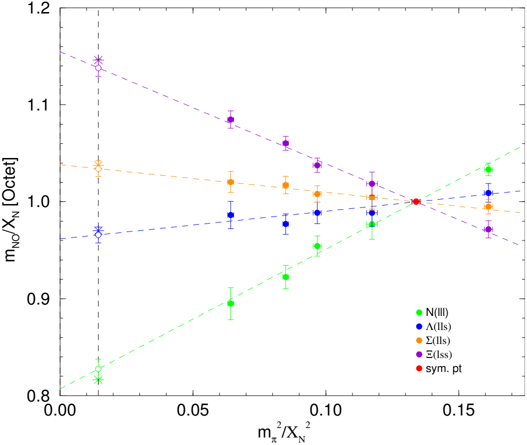
$$\begin{aligned} m_N &= m_0 + 3A_1 \delta m_l \\ m_\Lambda &= m_0 + 3A_2 \delta m_l \\ m_\Sigma &= m_0 - 3A_2 \delta m_l \\ m_\Xi &= m_0 - 3(A_1 - A_2)\delta m_l \end{aligned}$$

$$\begin{aligned} m_\Delta &= m_0 + 3A \delta m_l \\ m_{\Sigma^*} &= m_0 \\ m_{\Xi^*} &= m_0 - 3A \delta m_l \\ m_\Omega &= m_0 - 6A \delta m_l \end{aligned}$$

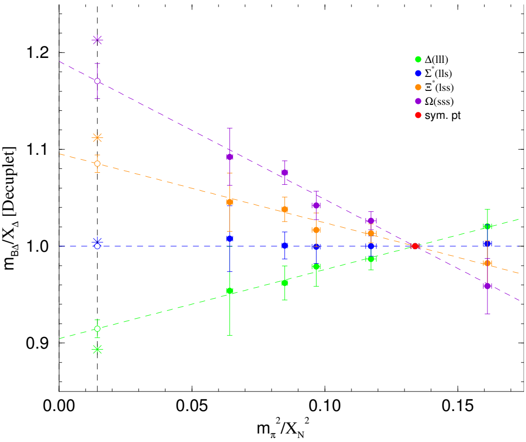
$[m_0 = \alpha + \beta \overline{m} \qquad \text{but} \quad \overline{m} = \frac{1}{3}(2m_l + m_s) = \text{const.} \Longrightarrow m_0 = \text{const.}]$

Octet: 3 coefficients; Decuplet: 2 coefficients

Baryon Octet 'fan plot'



Baryon Decuplet ‘fan plot’



Quadratic mass terms – work in progress:

For example for the baryon decuplet:

$$
\begin{aligned}
m_\Delta &= m_0 + 3A\delta m_l + (B_0 + 3B_1)(\delta m_l)^2 \\
m_{\Sigma^*} &= m_0 + (B_0 + 6B_1 + 9B_2)(\delta m_l)^2 \\
m_{\Xi^*} &= m_0 - 3A\delta m_l + (B_0 + 9B_1 + 9B_2)(\delta m_l)^2 \\
m_\Omega &= m_0 - 6A\delta m_l + (B_0 + 12B_1)(\delta m_l)^2
\end{aligned}
$$

Octet: $3 + 4$ coefficients; Decuplet: $2 + 3$ coefficients

To help determine these coefficients generalise to non-unitary valence quarks - partially quenched or *pq*

Quadratic mass terms and partially quenched Baryon Spectrum

- m_l, m_s sea quarks, constrained by $\frac{1}{3}(2m_l + m_s) = \overline{m} = \text{const.}$; $\delta m = m - \overline{m}$
- μ_l, μ_s valence quarks, unconstrained $\delta\mu = \mu - \overline{m}$

On unitary line, $\mu \to m$; together with $2\delta m_l + \delta m_s = 0$ results collapse to previous results

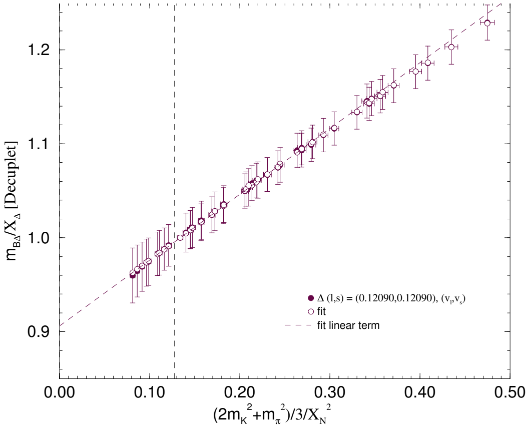
$$\begin{aligned}
m_\Delta &= m_0 + 3A\delta\mu_l + B_0\delta m_l^2 + 3B_1\delta\mu_l^2 \\
m_{\Sigma^*} &= m_0 + A(2\delta\mu_l + \delta\mu_s) + B_0\delta m_l^2 + B_1(2\delta\mu_l^2 + \delta\mu_s^2) + B_2(\delta\mu_s - \delta\mu_l)^2 \\
m_{\Xi^*} &= m_0 + A(\delta\mu_l + 2\delta\mu_s) + B_0\delta m_l^2 + B_1(\delta\mu_l^2 + 2\delta\mu_s^2) + B_2(\delta\mu_s - \delta\mu_l)^2 \\
m_\Omega &= m_0 + 3A\delta\mu_s + B_0\delta m_s^2 + 3B_1\delta\mu_s^2
\end{aligned}$$

No additional constants required

Compact (master) formula: [eg m_Δ set $\mu_s \to \mu_l$]

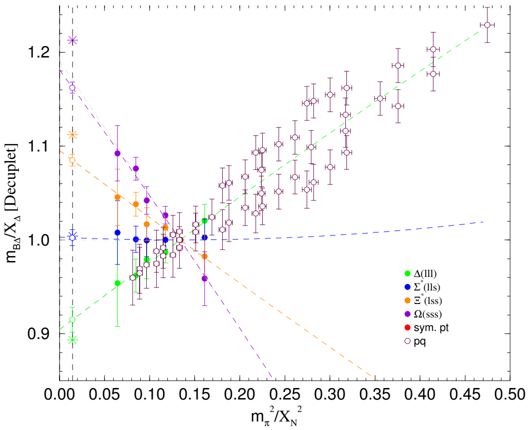
$$m = m_0 + A\left(2\delta\mu_l + \delta\mu_s\right) + B_0\delta m_l^2 + B_1\left(2\delta\mu_l^2 + 2\delta\mu_s^2\right) + B_2\left(\delta\mu_l - \delta\mu_s\right)^2$$

pq data



Little sign of curvature

Using *pq* fit results



- Fit only to *pq* data
- Non-linearity small
- Extrapolations of *pq* data give very similar results to previous results

Conclusions

- Programme:
  Tune strange and light quark masses to their physical values simultaneously by keeping

$$\overline{m}^R = \frac{1}{3}\left(2m_l^R + m_s^R\right) = \text{const.}$$

- $m_\pi \searrow$; $m_K \nearrow$
- Singlet quantities remain constant
  [helps to determine scale]
- Can have situation with a heavy l-quark and a light s-quark
- Baryon Octet and Decuplet mass spectrum – fan plots
- pq results help to give a handle on constraining $SU(3)_F$-flavour expansion coefficients, in particular quadratic coefficients