NSPT study of the three-loop lattice gluon propagator in Landau gauge

Christian Torrero

Department of Theoretical Physics - University of Regensburg

In collaboration with F. Di Renzo (Parma & INFN), E.-M. Ilgenfritz (Bielefeld), H. Perlt (Leipzig) and A. Schiller (Leipzig)

XXVIII International Symposium on Lattice Field Theories
Villasimius, 15th June 2010

Outline

Thanks to its momentum dependence, the gluon propagator represents a suitable observable to study some properties of a confining theory.
[R. Alkofer, J. Greensite - J. Phys. G34 (2007), S3]

More precisely, non-perturbative aspects are associated to its coupling to local condensates and localized vacuum excitations.
[Ph. Boucaud et al. - Phys. Rev. D79 (2009), 014508; J. Gattnar, K. Langfeld, H. Reinhardt - Phys. Rev. Lett 93 (2004), 061601]

In order to pin down truly non-perturbative features, it is however important to get an understanding of the perturbative behaviour of such an observable.

On the lattice, Perturbation Theory (PT) gets easily too involved to be tackled by means of a diagrammatic approach: Numerical Stochastic Perturbation Theory (NSPT) allows a computer-based procedure to obtain higher-loop results.

Reference: F. Di Renzo, E.-M. Ilgenfritz, H. Perlt, A. Schiller, CT - Nucl. Phys. B831 (2010), 262]

Thanks to its momentum dependence, the gluon propagator represents a suitable observable to study some properties of a confining theory.
[R. Alkofer, J. Greensite - J. Phys. G34 (2007), S3]

More precisely, non-perturbative aspects are associated to its coupling to local condensates and localized vacuum excitations.
[Ph. Boucaud et al. - Phys. Rev. D79 (2009), 014508; J. Gattnar, K. Langfeld, H. Reinhardt - Phys. Rev. Lett 93 (2004), 061601]

In order to pin down truly non-perturbative features, it is however important to get an understanding of the perturbative behaviour of such an observable.

On the lattice, Perturbation Theory (PT) gets easily too involved to be tackled by means of a diagrammatic approach: Numerical Stochastic Perturbation Theory (NSPT) allows a computer-based procedure to obtain higher-loop results.

Reference: F. Di Renzo, E.-M. Ilgenfritz, H. Perlt, A. Schiller, CT - Nucl. Phys. B831 (2010), 262]

Thanks to its momentum dependence, the gluon propagator represents a suitable observable to study some properties of a confining theory.
[R. Alkofer, J. Greensite - J. Phys. G34 (2007), S3]

More precisely, non-perturbative aspects are associated to its coupling to local condensates and localized vacuum excitations.
[Ph. Boucaud et al. - Phys. Rev. D79 (2009), 014508; J. Gattnar, K. Langfeld, H. Reinhardt - Phys. Rev. Lett 93 (2004), 061601]

In order to pin down truly non-perturbative features, it is however important to get an understanding of the perturbative behaviour of such an observable.

On the lattice, Perturbation Theory (PT) gets easily too involved to be tackled by means of a diagrammatic approach: Numerical Stochastic Perturbation Theory (NSPT) allows a computer-based procedure to obtain higher-loop results.

Reference: F. Di Renzo, E.-M. Ilgenfritz, H. Perlt, A. Schiller, CT - Nucl. Phys. B831 (2010), 262]

Thanks to its momentum dependence, the gluon propagator represents a suitable observable to study some properties of a confining theory.
[R. Alkofer, J. Greensite - J. Phys. G34 (2007), S3]

More precisely, non-perturbative aspects are associated to its coupling to local condensates and localized vacuum excitations.
[Ph. Boucaud et al. - Phys. Rev. D79 (2009), 014508; J. Gattnar, K. Langfeld, H. Reinhardt - Phys. Rev. Lett 93 (2004), 061601]

In order to pin down truly non-perturbative features, it is however important to get an understanding of the perturbative behaviour of such an observable.

On the lattice, Perturbation Theory (PT) gets easily too involved to be tackled by means of a diagrammatic approach: Numerical Stochastic Perturbation Theory (NSPT) allows a computer-based procedure to obtain higher-loop results.

Reference: F. Di Renzo, E.-M. Ilgenfritz, H. Perlt, A. Schiller, CT - Nucl. Phys. B831 (2010), 262]

Thanks to its momentum dependence, the gluon propagator represents a suitable observable to study some properties of a confining theory.
[R. Alkofer, J. Greensite - J. Phys. G34 (2007), S3]

More precisely, non-perturbative aspects are associated to its coupling to local condensates and localized vacuum excitations.
[Ph. Boucaud et al. - Phys. Rev. D79 (2009), 014508; J. Gattnar, K. Langfeld, H. Reinhardt - Phys. Rev. Lett 93 (2004), 061601]

In order to pin down truly non-perturbative features, it is however important to get an understanding of the perturbative behaviour of such an observable.

On the lattice, Perturbation Theory (PT) gets easily too involved to be tackled by means of a diagrammatic approach: Numerical Stochastic Perturbation Theory (NSPT) allows a computer-based procedure to obtain higher-loop results.

Reference: F. Di Renzo, E.-M. Ilgenfritz, H. Perlt, A. Schiller, CT - Nucl. Phys. B831 (2010), 262]

Outline

The starting point of NSPT is given by ***Stochastic Quantization***.
[G. Parisi, Y. Wu - Sci. Sin. 24 (1981), 483]

Main ingredients

- Introduction of a *stochastic time t* as a new degree of freedom

$$\phi(x) \rightarrow \phi(x, t) \, .$$

- *Langevin equation* with *gaussian noise*

$$\begin{aligned} \frac{\partial \phi(x,t)}{\partial t} &= -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t) \, , \\ \langle \eta(x,t)\eta(x',t') \rangle &= 2\delta(x-x')\delta(t-t') \, . \end{aligned}$$

All this results in

$$\langle O[\phi_1(x_1,t), \phi_2(x_2,t), \ldots] \rangle_\eta \stackrel{t \to +\infty}{\longrightarrow} \frac{1}{Z} \int [D\phi] O[\phi_1(x_1), \phi_2(x_2), \ldots] e^{-S[\phi]} \, .$$

The average on the noise converges to the average on Gibbs measure

The starting point of NSPT is given by ***Stochastic Quantization***.
[G. Parisi, Y. Wu - Sci. Sin. 24 (1981), 483]

Main ingredients

- Introduction of a *stochastic time t* as a new degree of freedom

$$\phi(x) \rightarrow \phi(x, t) \; .$$

- *Langevin equation* with *gaussian noise*

$$\begin{aligned} \frac{\partial \phi(x,t)}{\partial t} &= -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t) \; , \\ \langle \eta(x,t)\eta(x',t')\rangle &= 2\delta(x-x')\delta(t-t') \; . \end{aligned}$$

All this results in

$$\langle O[\phi_1(x_1,t),\phi_2(x_2,t),\ldots]\rangle_\eta \stackrel{t\rightarrow+\infty}{\longrightarrow} \frac{1}{Z}\int [D\phi] O[\phi_1(x_1),\phi_2(x_2),\ldots] e^{-S[\phi]} \; .$$

The average on the noise converges to the average on Gibbs measure

The starting point of NSPT is given by ***Stochastic Quantization***.
[G. Parisi, Y. Wu - Sci. Sin. 24 (1981), 483]

Main ingredients

- Introduction of a *stochastic time t* as a new degree of freedom

$$\phi(x) \longrightarrow \phi(x, t) \; .$$

- *Langevin equation* with *gaussian noise*

$$\begin{aligned} \frac{\partial \phi(x,t)}{\partial t} &= -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t) \; , \\ \langle \eta(x,t)\eta(x',t')\rangle &= 2\delta(x-x')\delta(t-t') \; . \end{aligned}$$

All this results in

$$\langle O[\phi_1(x_1,t), \phi_2(x_2,t), \ldots]\rangle_\eta \stackrel{t\to+\infty}{\longrightarrow} \frac{1}{Z}\int [D\phi] O[\phi_1(x_1), \phi_2(x_2), \ldots] e^{-S[\phi]} \; .$$

The average on the noise converges to the average on Gibbs measure

The starting point of NSPT is given by ***Stochastic Quantization***.

[G. Parisi, Y. Wu - Sci. Sin. 24 (1981), 483]

Main ingredients

- Introduction of a *stochastic time t* as a new degree of freedom

$$\phi(x) \longrightarrow \phi(x, t) \; .$$

- *Langevin equation* with *gaussian noise*

$$\begin{aligned} \frac{\partial \phi(x,t)}{\partial t} &= -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t) \; , \\ \langle \eta(x,t)\eta(x',t') \rangle &= 2\delta(x-x')\delta(t-t') \; . \end{aligned}$$

All this results in

$$\langle O[\phi_1(x_1,t), \phi_2(x_2,t), \ldots] \rangle_\eta \xrightarrow{t \to +\infty} \frac{1}{Z} \int [D\phi] O[\phi_1(x_1), \phi_2(x_2), \ldots] e^{-S[\phi]} \; .$$

The average on the noise converges to the average on Gibbs measure

The starting point of NSPT is given by ***Stochastic Quantization***.
[G. Parisi, Y. Wu - Sci. Sin. 24 (1981), 483]

Main ingredients

- Introduction of a *stochastic time t* as a new degree of freedom

$$\phi(x) \longrightarrow \phi(x, t) \ .$$

- *Langevin equation* with *gaussian noise*

$$\begin{aligned} \frac{\partial \phi(x,t)}{\partial t} &= -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t) \ , \\ \langle \eta(x,t)\eta(x',t')\rangle &= 2\delta(x-x')\delta(t-t') \ . \end{aligned}$$

All this results in

$$\langle O[\phi_1(x_1,t), \phi_2(x_2,t), \ldots]\rangle_\eta \stackrel{t\rightarrow+\infty}{\longrightarrow} \frac{1}{Z}\int [D\phi] O[\phi_1(x_1), \phi_2(x_2), \ldots] e^{-S[\phi]} \ .$$

The average on the noise converges to the average on Gibbs measure

For lattice gauge variables, the Langevin equation is modified as

$$\frac{\partial}{\partial t} U_\mu(x,t) = -i \sum_A T^A \left[\nabla_{x,\mu,A} S_G[U] + \eta^A_\mu(x,t) \right] U_\mu(x,t) \; ,$$

where the group derivative is defined as

$$\mathcal{F}\left[e^{i\alpha^A T^A} U_\mu(x), U' \right] = \mathcal{F}\left[U_\mu(x), U' \right] + \alpha^A \nabla_{x,\mu,A} \mathcal{F}[U_\mu(x), U'] + \ldots$$

Perturbation Theory is introduced by means of a *formal* expansion like

$$U_\mu(x,t) = 1 + \sum_{k>0} \beta^{-\frac{k}{2}} U^{(k)}_\mu(x,t) \qquad (\beta^{-1/2} = g_0/\sqrt{2N_c}),$$

which, plugged into Langevin equation, gives a *hierarchical system of differential equations*.

The stochastic time can now be discretized as $t = n\varepsilon$ and the system numerically integrated: this is the core of **NSPT**.

[F. Di Renzo, E. Onofri, G. Marchesini, P. Marenzoni - Nucl. Phys. B426 (1994) 675]

For lattice gauge variables, the Langevin equation is modified as

$$\frac{\partial}{\partial t} U_\mu(x,t) = -i \sum_A T^A \left[\nabla_{x,\mu,A} S_G[U] + \eta^A_\mu(x,t) \right] U_\mu(x,t) \; ,$$

where the group derivative is defined as

$$\mathcal{F}\left[e^{i\alpha^A T^A} U_\mu(x), U'\right] = \mathcal{F}\left[U_\mu(x), U'\right] + \alpha^A \nabla_{x,\mu,A} \mathcal{F}[U_\mu(x), U'] + \dots$$

Perturbation Theory is introduced by means of a *formal* expansion like

$$U_\mu(x,t) = 1 + \sum_{k>0} \beta^{-\frac{k}{2}} U^{(k)}_\mu(x,t) \qquad (\beta^{-1/2} = g_0/\sqrt{2N_c}),$$

which, plugged into Langevin equation, gives a *hierarchical system of differential equations*.

The stochastic time can now be discretized as $t = n\varepsilon$ and the system numerically integrated: this is the core of **NSPT**.

[F. Di Renzo, E. Onofri, G. Marchesini, P. Marenzoni - Nucl. Phys. B426 (1994) 675]

For lattice gauge variables, the Langevin equation is modified as

$$\frac{\partial}{\partial t} U_\mu(x,t) = -i \sum_A T^A \left[\nabla_{x,\mu,A} S_G[U] + \eta^A_\mu(x,t) \right] U_\mu(x,t) \ ,$$

where the group derivative is defined as

$$\mathcal{F}\left[e^{i\alpha^A T^A} U_\mu(x), U' \right] = \mathcal{F}\left[U_\mu(x), U' \right] + \alpha^A \nabla_{x,\mu,A} \mathcal{F}[U_\mu(x), U'] + \dots$$

Perturbation Theory is introduced by means of a *formal* expansion like

$$U_\mu(x,t) = 1 + \sum_{k>0} \beta^{-\frac{k}{2}} U^{(k)}_\mu(x,t) \qquad (\beta^{-1/2} = g_0/\sqrt{2N_c}),$$

which, plugged into Langevin equation, gives a *hierarchical system of differential equations*.

The stochastic time can now be discretized as $t = n\varepsilon$ and the system numerically integrated: this is the core of **NSPT**.

[F. Di Renzo, E. Onofri, G. Marchesini, P. Marenzoni - Nucl. Phys. B426 (1994) 675]

Outline

Including its gauge-fixing part, the lattice QCD action reads

$$S_{LQCD} = S_W + S_{FP} + S_{gf} \;,$$

with

$$S_W = \beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \left(1 - \frac{Tr}{2N_c} \left[U_{\mu\nu}(x) + U^{\dagger}_{\mu\nu}(x) \right] \right) \;,$$

$$S_{FP} = \sum_{x,y,a,b} \bar{c}^a_x M^{ab}_{xy}[U] c^b_y \;,$$

$$S_{gf} = \frac{1}{2\xi} \sum_x Tr\left(\left[\partial_\mu A_\mu(x) \right]^2 \right) \qquad \left(A_\mu(x) = log\, U_\mu(x) \right) \;,$$

where the second term can be rewritten - by means of functional integration - as

$$S_{FP} = Tr(log\, M[U]) \;.$$

Including its gauge-fixing part, the lattice QCD action reads

$$S_{LQCD} = S_W + S_{FP} + S_{gf} \ ,$$

with

$$S_W = \beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \left(1 - \frac{Tr}{2N_c} \left[U_{\mu\nu}(x) + U^\dagger_{\mu\nu}(x) \right] \right) \ ,$$

$$S_{FP} = \sum_{x,y,a,b} \bar{c}^a_x M^{ab}_{xy}[U] c^b_y \ ,$$

$$S_{gf} = \frac{1}{2\xi} \sum_x Tr\left(\left[\partial_\mu A_\mu(x) \right]^2 \right) \qquad (A_\mu(x) = log\, U_\mu(x)) \ ,$$

where the second term can be rewritten - by means of functional integration - as

$$S_{FP} = Tr(log\, M[U]) \ .$$

Including its gauge-fixing part, the lattice QCD action reads

$$S_{LQCD} = S_W + S_{FP} + S_{gf} \ ,$$

with

$$S_W = \beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \left(1 - \frac{Tr}{2N_c} \left[U_{\mu\nu}(x) + U^{\dagger}_{\mu\nu}(x) \right] \right) \ ,$$

$$S_{FP} = \sum_{x,y,a,b} \bar{c}^a_x M^{ab}_{xy}[U] c^b_y \ ,$$

$$S_{gf} = \frac{1}{2\xi} \sum_x Tr\left(\left[\partial_\mu A_\mu(x) \right]^2 \right) \qquad \left(A_\mu(x) = \log U_\mu(x) \right) \ ,$$

where the second term can be rewritten - by means of functional integration - as

$$S_{FP} = Tr(\log M[U]) \ .$$

Including its gauge-fixing part, the lattice QCD action reads

$$S_{LQCD} = S_W + S_{FP} + S_{gf} \ ,$$

with

$$S_W = \beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \left(1 - \frac{Tr}{2N_c} \left[U_{\mu\nu}(x) + U^\dagger_{\mu\nu}(x) \right] \right) \ ,$$

$$S_{FP} = \sum_{x,y,a,b} \bar{c}^a_x M^{ab}_{xy}[U] c^b_y \ ,$$

$$S_{gf} = \frac{1}{2\xi} \sum_x Tr\left(\left[\partial_\mu A_\mu(x) \right]^2 \right) \qquad (A_\mu(x) = log\, U_\mu(x)) \ ,$$

where the second term can be rewritten - by means of functional integration - as

$$S_{FP} = Tr(log\, M[U]) \ .$$

Including its gauge-fixing part, the lattice QCD action reads

$$S_{LQCD} = S_W + S_{FP} + S_{gf} \ ,$$

with

$$S_W = \beta \sum_{\substack{x,\mu,\nu \\ \mu>\nu}} \left(1 - \frac{Tr}{2N_c} \left[U_{\mu\nu}(x) + U^\dagger_{\mu\nu}(x) \right] \right) \ ,$$

$$S_{FP} = \sum_{x,y,a,b} \bar{c}^a_x M^{ab}_{xy}[U] c^b_y \ ,$$

$$S_{gf} = \frac{1}{2\xi} \sum_x Tr\left(\left[\partial_\mu A_\mu(x) \right]^2 \right) \qquad (A_\mu(x) = log\, U_\mu(x)) \ ,$$

where the second term can be rewritten - by means of functional integration - as

$$S_{FP} = Tr(log\, M[U]) \ .$$

However, since the gauge one wants to work within is Landau's (where $\xi = 0$), the previous expression is actually useless.

Nevertheless, Landau gauge can be reached by repeatedly implementing the gauge transformation reading

$$U_\mu(x,\varepsilon) \longrightarrow e^{i\omega(x,\varepsilon)} U_\mu(x,\varepsilon) e^{-i\omega(x+\hat{\mu},\varepsilon)} ,$$

where

$$\omega(x,\varepsilon) = \alpha \sum_\mu \partial^L_\mu A_\mu(x,\varepsilon) \quad (0 < \alpha < 1) ,$$

Indeed, it is well-known that iterating the transformation above leads to an extremum of the norm of the field

$$\sum_\mu \partial_\mu A^{(l)}_\mu(x) = 0 ,$$

i.e. Landau gauge.

However, since the gauge one wants to work within is Landau's (where $\xi = 0$), the previous expression is actually useless.

Nevertheless, Landau gauge can be reached by repeatedly implementing the gauge transformation reading

$$U_\mu(x,\varepsilon) \longrightarrow e^{i\omega(x,\varepsilon)} U_\mu(x,\varepsilon) e^{-i\omega(x+\hat{\mu},\varepsilon)} ,$$

where

$$\omega(x,\varepsilon) = \alpha \sum_\mu \partial^L_\mu A_\mu(x,\varepsilon) \quad (0 < \alpha < 1) ,$$

Indeed, it is well-known that iterating the transformation above leads to an extremum of the norm of the field

$$\sum_\mu \partial_\mu A^{(l)}_\mu(x) = 0 ,$$

i.e. Landau gauge.

However, since the gauge one wants to work within is Landau's (where $\xi = 0$), the previous expression is actually useless.

Nevertheless, Landau gauge can be reached by repeatedly implementing the gauge transformation reading

$$U_\mu(x,\varepsilon) \longrightarrow e^{i\omega(x,\varepsilon)} U_\mu(x,\varepsilon) e^{-i\omega(x+\hat{\mu},\varepsilon)} \ ,$$

where

$$\omega(x,\varepsilon) = \alpha \sum_\mu \partial^L_\mu A_\mu(x,\varepsilon) \quad (0 < \alpha < 1) \ ,$$

Indeed, it is well-known that iterating the transformation above leads to an extremum of the norm of the field

$$\sum_\mu \partial_\mu A^{(l)}_\mu(x) = 0 \ ,$$

i.e. Landau gauge.

Outline

By definition, the gluon propagator reads

$$D^{ab}_{\mu\nu}(x-y) = \langle A^a_\mu(x) A^b_\nu(y) \rangle = \frac{1}{Z} \int DU \, A^a_\mu(x) A^b_\nu(y) \, e^{-S_W} \,,$$

whose momentum-space representation $D^{ab}_{\mu\nu}(p)$ can be decomposed as

$$D^{ab}_{\mu\nu}(p) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{F(p^2)}{p^2} \,.$$

The quantity that will be eventually studied is ($F(p^2) = 0$ in Landau gauge)

$$D(p^2) = \frac{1}{3} \frac{1}{N_c^2 - 1} \sum_\mu \sum_a D^{aa}_{\mu\mu}(p) \,.$$

For future convenience, let's define the quantities $\left(k_\mu \in (-L_\mu/2a, \; L_\mu/2a] \right)$,

$$p_\mu(k_\mu) = \frac{2\pi k_\mu}{L_\mu} \,, \qquad \hat{p}_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{N_\mu} \right) = \frac{2}{a} \sin\left(\frac{a p_\mu}{2} \right) \,.$$

By definition, the gluon propagator reads

$$D^{ab}_{\mu\nu}(x-y) = \langle A^a_\mu(x) A^b_\nu(y) \rangle = \frac{1}{Z} \int DU \, A^a_\mu(x) A^b_\nu(y) \, e^{-S_W} \ ,$$

whose momentum-space representation $D^{ab}_{\mu\nu}(p)$ can be decomposed as

$$D^{ab}_{\mu\nu}(p) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) + \frac{p_\mu p_\nu}{p^2} \frac{F(p^2)}{p^2} \ .$$

The quantity that will be eventually studied is $(F(p^2) = 0$ in Landau gauge)

$$D(p^2) = \frac{1}{3} \frac{1}{N_c^2 - 1} \sum_\mu \sum_a D^{aa}_{\mu\mu}(p) \ .$$

For future convenience, let's define the quantities $\left(k_\mu \in (-L_\mu/2a, \ L_\mu/2a] \ \right)$,

$$p_\mu(k_\mu) = \frac{2\pi k_\mu}{L_\mu} \ , \qquad \hat{p}_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{N_\mu} \right) = \frac{2}{a} \sin\left(\frac{a p_\mu}{2} \right) \ .$$

The gluon dressing function $J^{(n)}(p)$ to a given loop n reads

$$J^{(n)}(a,p,\beta) \quad = \quad p^2 D^{(n)}(p(k)) = p^2 \langle D(p(k)) \rangle^{(l=2n)}$$

but also (rewriting β in terms of $\alpha_{\mathrm{RI}'} = g^2_{\mathrm{RI}'}/16\pi^2$)

$$J(a,p,\alpha_{\mathrm{RI}'}) = Z(a,\mu,\alpha_{\mathrm{RI}'}) J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'}) \; ,$$

with the renormalization condition

$$J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'})|_{p^2=\mu^2} = 1 \; .$$

Recalling the perturbative decomposition of $J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'})$, $Z(a,\mu,\alpha_{\mathrm{RI}'})$ ([J. Gracey - Nucl. Phys. B662 (2003), 247]) and the relation between $\alpha_{\mathrm{RI}'}$ and α_0 ([M. Lüscher, P. Weisz - Nucl. Phys. B452 (1995), 234]) ***at infinite volume***

$$J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'}) = 1 + \sum_{i>0} \alpha_{\mathrm{RI}'}^i \sum_{k=1}^{i} z_{i,k}^{\mathrm{RI}'} \left(\frac{1}{2} \log \frac{p^2}{\mu^2} \right)^k$$

$$Z(a,\mu,\alpha_{\mathrm{RI}'}) = 1 + \sum_{i>0} \alpha_{\mathrm{RI}'}^i \sum_{k=0}^{i} z_{i,k}^{\mathrm{RI}'} \left(\log(a\mu)\right)^k$$

$$\alpha_{\mathrm{RI}'} = \alpha_0 + d_1\,\alpha_0^2 + d_2\,\alpha_0^3 + \dots ,$$

one gets

$$J^{3-\mathrm{loop}}(a,p,\beta) = 1 + \sum_{i=1}^{3} \frac{1}{\beta^i} \sum_{k=0}^{i} J_{i,k} \left(\log(pa)^2\right)^k ,$$

where, in particular,

$$J_{1,0} = 0.0379954\, z_{1,0}^{\mathrm{RI}'} ,$$

$$J_{2,0} = 0.106737\, z_{1,0}^{\mathrm{RI}'} + 0.00144365\, z_{2,0}^{\mathrm{RI}'} ,$$

$$J_{3,0} = 0.37599\, z_{1,0}^{\mathrm{RI}'} + 0.00811106\, z_{2,0}^{\mathrm{RI}'} + 0.0000548523\, z_{3,0}^{\mathrm{RI}'}$$

Recalling the perturbative decomposition of $J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'})$, $Z(a,\mu,\alpha_{\mathrm{RI}'})$ ([J. Gracey - Nucl. Phys. B662 (2003), 247]) and the relation between $\alpha_{\mathrm{RI}'}$ and α_0 ([M. Lüscher, P. Weisz - Nucl. Phys. B452 (1995), 234]) ***at infinite volume***

$$\begin{aligned} J^{\mathrm{RI}'}(p,\mu,\alpha_{\mathrm{RI}'}) &= 1+\sum_{i>0}\alpha_{\mathrm{RI}'}^{i}\sum_{k=1}^{i} z_{i,k}^{\mathrm{RI}'}\left(\frac{1}{2}\log\frac{p^2}{\mu^2}\right)^k \\ Z(a,\mu,\alpha_{\mathrm{RI}'}) &= 1+\sum_{i>0}\alpha_{\mathrm{RI}'}^{i}\sum_{k=0}^{i} z_{i,k}^{\mathrm{RI}'}\,(\log(a\mu))^k \\ \alpha_{\mathrm{RI}'} &= \alpha_0+d_1\,\alpha_0^2+d_2\,\alpha_0^3+\dots\,, \end{aligned}$$

one gets

$$J^{3-\mathrm{loop}}(a,p,\beta)=1+\sum_{i=1}^{3}\frac{1}{\beta^i}\sum_{k=0}^{i}J_{i,k}\left(\log(pa)^2\right)^k\ ,$$

where, in particular,

$$\begin{aligned} J_{1,0} &= 0.0379954\,z_{1,0}^{\mathrm{RI}'}\,, \\ J_{2,0} &= 0.106737\,z_{1,0}^{\mathrm{RI}'}+0.00144365\,z_{2,0}^{\mathrm{RI}'}\,, \\ J_{3,0} &= 0.37599\,z_{1,0}^{\mathrm{RI}'}+0.00811106\,z_{2,0}^{\mathrm{RI}'}+0.0000548523\,z_{3,0}^{\mathrm{RI}'}\,. \end{aligned}$$

Outline

To give a taste of how the analysis in performed, let's consider the first loop and its decomposition as

$$J^{(1)}(pa) = J_{1,1} \log(pa)^2 + J_{1,0} \ ,$$

which, taking into account lattice artifacts, can be rewritten as

$$J^{(1)}(pa) = J_{1,1} \log(pa)^2 + J_{1,0}(pa) \ ,$$

with

$$\begin{aligned} J_{1,0}(pa) &= J_{1,0} + c_{1,1}\,(pa)^2 + c_{1,2}\,\frac{(pa)^4}{(pa)^2} + c_{1,3}\,(pa)^4 + c_{1,4}\left((pa)^2\right)^2 + \\ &+ c_{1,5}\,\frac{(pa)^6}{(pa)^2} + \cdots \end{aligned}$$

being $(pa)^n$ the hypercubic invariant

$$(pa)^n = \sum_{\mu} (ap_{\mu})^n \ .$$

To give a taste of how the analysis in performed, let's consider the first loop and its decomposition as

$$J^{(1)}(pa) = J_{1,1}\log(pa)^2 + J_{1,0}\ ,$$

which, taking into account lattice artifacts, can be rewritten as

$$J^{(1)}(pa) = J_{1,1}\log(pa)^2 + J_{1,0}(pa)\ ,$$

with

$$\begin{aligned} J_{1,0}(pa) &= J_{1,0} + c_{1,1}\,(pa)^2 + c_{1,2}\,\frac{(pa)^4}{(pa)^2} + c_{1,3}\,(pa)^4 + c_{1,4}\,\left((pa)^2\right)^2 + \\ &+ c_{1,5}\,\frac{(pa)^6}{(pa)^2} + \cdots \end{aligned}$$

being $(pa)^n$ the hypercubic invariant

$$(pa)^n = \sum_\mu (ap_\mu)^n\ .$$

However, measurements are actually taken at finite volume so that a more correct decomposition would read

$$\begin{aligned} J^{(1)}(ap, pL) &= J_{1,1}\log(pa)^2 + J_{1,0;L}(pa, pL) = \\ &= J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \left[J_{1,0;L}(pa, pL) - J_{1,0}(pa)\right] = \\ &= J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \delta J_{1,0}(pa, pL) \rightarrow \\ &\rightarrow J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \delta J_{1,0}(pL) \qquad (\textit{ansatz})\ , \end{aligned}$$

with $\delta J_{1,0}(pL) \equiv \delta J_{1,0}(0, pL)$: in other words, ***corrections on corrections are assumed to be negligible***.

Moreover, a given 4-tuple (k_1, k_2, k_3, k_4) has the same pL-effects on different lattice extents because of the trivial identity

$$p_\mu L = p_\mu aN = 2\pi k_\mu\ .$$

However, measurements are actually taken at finite volume so that a more correct decomposition would read

$$\begin{aligned} J^{(1)}(ap, pL) &= J_{1,1}\log(pa)^2 + J_{1,0;L}(pa, pL) = \\ &= J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \left[J_{1,0;L}(pa, pL) - J_{1,0}(pa)\right] = \\ &= J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \delta J_{1,0}(pa, pL) \rightarrow \\ &\rightarrow J_{1,1}\log(pa)^2 + J_{1,0}(pa) + \delta J_{1,0}(pL) \qquad (\textit{ansatz})\ , \end{aligned}$$

with $\delta J_{1,0}(pL) \equiv \delta J_{1,0}(0, pL)$: in other words, ***corrections on corrections are assumed to be negligible***.

Moreover, a given 4-tuple (k_1, k_2, k_3, k_4) has the same pL-effects on different lattice extents because of the trivial identity

$$p_\mu L = p_\mu a N = 2\pi k_\mu \ .$$

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\mathrm{min}}, (pa)^2_{\mathrm{max}}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\min}, (pa)^2_{\max}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\rm min}, (pa)^2_{\rm max}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\rm min}, (pa)^2_{\rm max}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\rm min}, (pa)^2_{\rm max}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Thus, data can be handled by means of the following strategy:

- a sufficiently large amount of 4-tuples is measured on different lattice sizes;
- an interval $[(pa)^2_{\mathrm{min}}, (pa)^2_{\mathrm{max}}]$ is determined where a hypercubic expansion of $J_{i,0}$ with a manageable number of terms can be performed;
- the resulting number of data points should clearly be such that it is sufficiently large with respect to the number of fit parameters;
- it is assumed that the point in the above interval with maximum $(pa)^2$ on the largest lattice is free from finite-size corrections, i.e. it is taken as a reference point;
- the final fit is stable enough.

Outline

A first look at the collected statistics:

ε	$L = 6$	$L = 8$	$L = 10$	$L = 12$	$L = 16$
0.01	1500	750	3000	1000	3000
0.02	1000	750	2000	1000	2000
0.03	1000	750	2000	1000	2000
0.05	1000	750	2000	1000	2000
0.07	1000	750	2000	1000	2000

Table 1: Number of gluon propagator measurements - up to the third loop - at different lattices sizes done with the Leipzig NSPT code at Leipzig PC clusters.

ε	$N = 16$	$N = 20$	$N = 32$
0.010	7436	5965	810
0.015	3053	3896	-
0.020	4725	3015	715
0.040	2827	2633	835

Table 2: Number of gluon propagator measurements - up to the second loop - at different lattices sizes using the Parma code.

The *pa*−extrapolation.

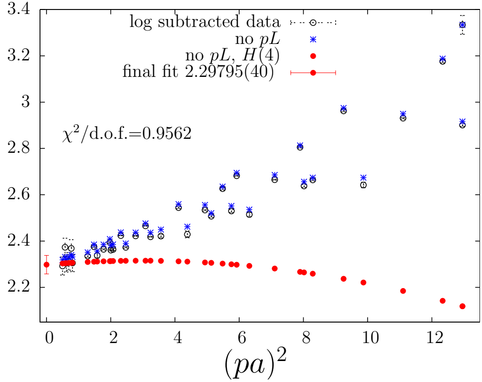

Figure 1: Fitting of the one-loop coefficient $J_{1,0}(pa)$ according to the outlined procedure. Black open circles are raw data (on different lattice sizes $N = L/a$); blue stars are data after correction for finite-volume effects; red circles are data after correcting finite-volume and (some) hypercubic effects.

A pictorial comparison among outcomes from fitting procedures with different parameters...

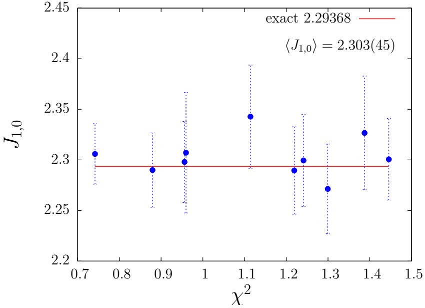

Figure 2: One-loop results compared to the analytical value for $J_{1,0}$ (straight red line).

...and some numbers.

Δk_{max}	1^{st} *loop*	2^{nd} *loop*	3^{rd} *loop*
1	2.318(57)	7.981(41)	31.6(4)
2	2.303(45)	7.962(33)	31.3(6)
3	2.292(54)	7.976(120)	31.0(8)
Analytical	2.293680	-	-

Table 3: Numerical values for different choices of the k-tuple set ($\Delta k_{max} \equiv max(|k_i - k_j|) \;\; \forall i, j \in [1, 4]$).

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.

Summary

- The gluon propagator has been computed up to the third loop and non-logarithmic contributions to its dressing function have been extracted;
- the treatement of lattice artifacts and finite-size corrections seems to be accurate (the one-loop result is satisfactory when compared to its already-known value).

Outlook

- Comparison with MC data with the same definition for $A_\mu(x)$;
- Improve results by employing tadpole improvement.