Flavour Symmetry and Flavour Symmetry Breaking in $2+1$ flavour lattice simulations

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2+1 project - results arriving - talks and posters at LAT10.

Introduction

The QCD interaction is flavour-blind. Neglecting electromagnetic and weak interactions, the only difference between flavours comes from the mass matrix. We investigate how flavour-blindness constrains hadron masses after flavour SU(3) is broken by the mass difference between the strange and light quarks, to help us extrapolate 2+1 flavour lattice data to the physical point.

We have our best theoretical understanding when all 3 quark flavours have the same masses (because we can use the full power of flavour $SU(3)$); nature presents us with just one instance of the theory, with $m_s/m_l \approx 25.$ We are interested in interpolating between these two cases.

Introduction

We consider possible behaviours near the symmetric point, and find that flavour blindness is particularly helpful if we approach the physical point along a path with $m_u + m_d + m_s$ held constant. We also show that on this trajectory the errors of the partially quenched approximation are much smaller than on other trajectories.

Strategy

Start from a point with all 3 sea quark masses equal,

$$m_u = m_d = m_s \equiv m_0$$

and extrapolate towards the physical point, keeping the average sea quark mass

$$\overline{m} \equiv \frac{1}{3}(m_u + m_d + m_s)$$

constant.
Starting point has

$$m_0 \approx \frac{1}{3} m_s^{phys}$$

As we approach the physical point, the u and d become lighter, but the s becomes heavier. Pions are decreasing in mass, but K and η increase in mass as we approach the physical point.

Quark Masses

Notation

$$\overline{m} \equiv \frac{1}{3}(m_u + m_d + m_s)$$
$$\hat{m}_u \equiv m_u - m_0$$
$$\hat{m}_d \equiv m_d - m_0$$
$$\hat{m}_s \equiv m_s - m_0$$

Quark Masses

The quark mass matrix is

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = \overline{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \tfrac{1}{2}(\hat{m}_u - \hat{m}_d) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \tfrac{1}{2}\hat{m}_s \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$\mathcal{M}$ has a singlet part (proportional to I) and an octet part, proportional to λ_3, λ_8.
In clover case, the singlet and non-singlet parts of the mass matrix renormalise differently.

Quark Masses

The quark mass matrix is

$$\begin{aligned}\mathcal{M} &= \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \\ &= \overline{m}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \tfrac{1}{2}(\hat{m}_u - \hat{m}_d)\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \tfrac{1}{2}\hat{m}_s\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\end{aligned}$$

We argue that the theoretically cleanest way to approach the physical point is to keep the singlet part of $\mathcal{M}$ constant, and vary only the non-singlet parts.

Singlet Quantities

Consider a flavour singlet quantity (eg r_0, P) at the symmetric point (m_0, m_0, m_0).

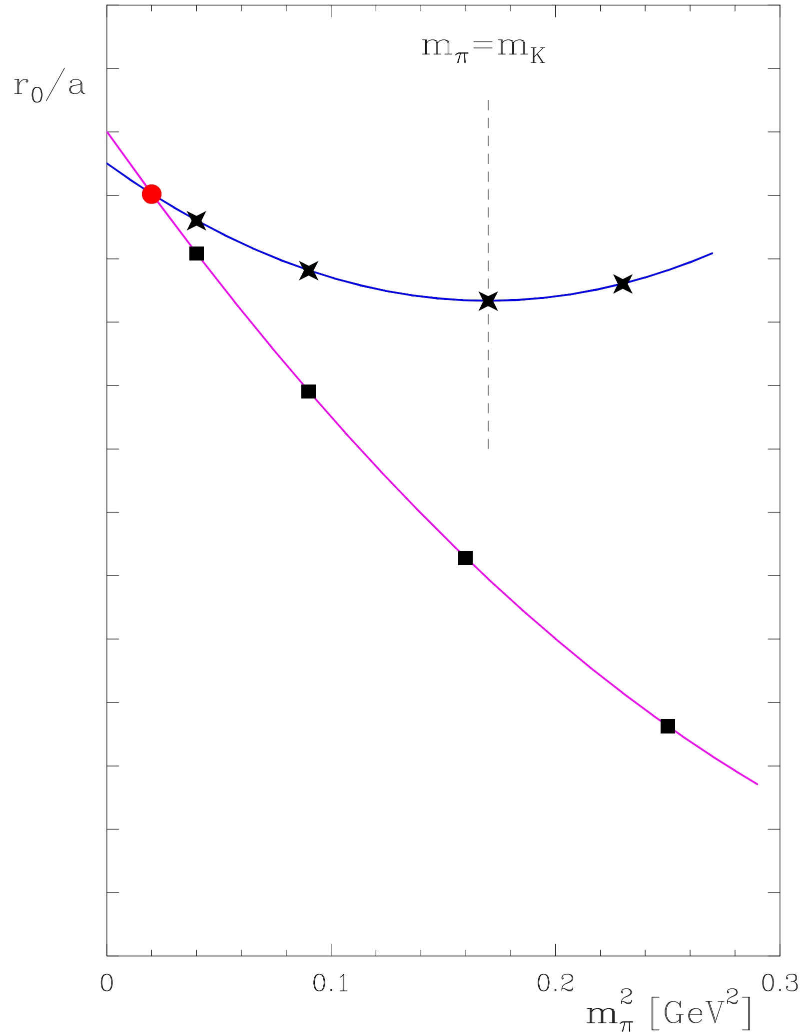
$$\frac{\partial r_0}{\partial m_u} = \frac{\partial r_0}{\partial m_d} = \frac{\partial r_0}{\partial m_s} \, .$$

If we keep $m_u + m_d + m_s$ constant, $dm_s = -dm_u - dm_d = -2dm_l$ so

$$dr_0 = dm_u \frac{\partial r_0}{\partial m_u} + dm_d \frac{\partial r_0}{\partial m_d} + dm_s \frac{\partial r_0}{\partial m_s} = 0$$

The effect of making the strange quark heavier exactly cancels the effect of making the light quarks lighter, so we know that r_0 must have a stationary point at the symmetrical point.

Singlet Quantities



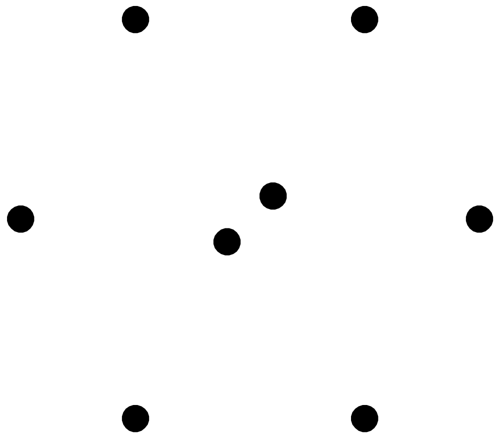
Singlet Quantities

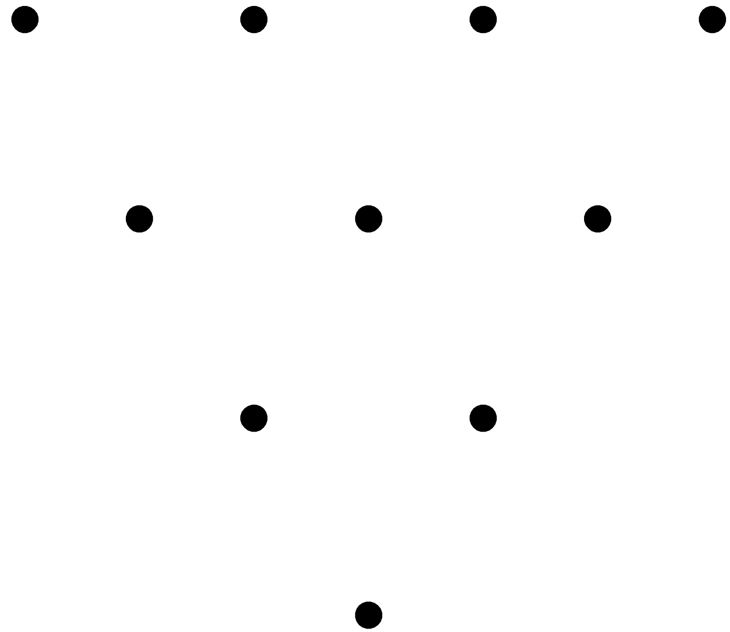
Any permutation of the quarks

$$u \leftrightarrow s, \qquad u \to d \to s \to u$$

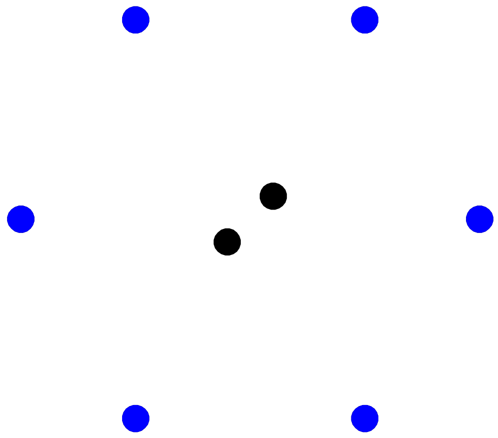
doesn't really change physics, it just renames the quarks. Any quantity unchanged by all permutations will also be flat at the symmetric point.

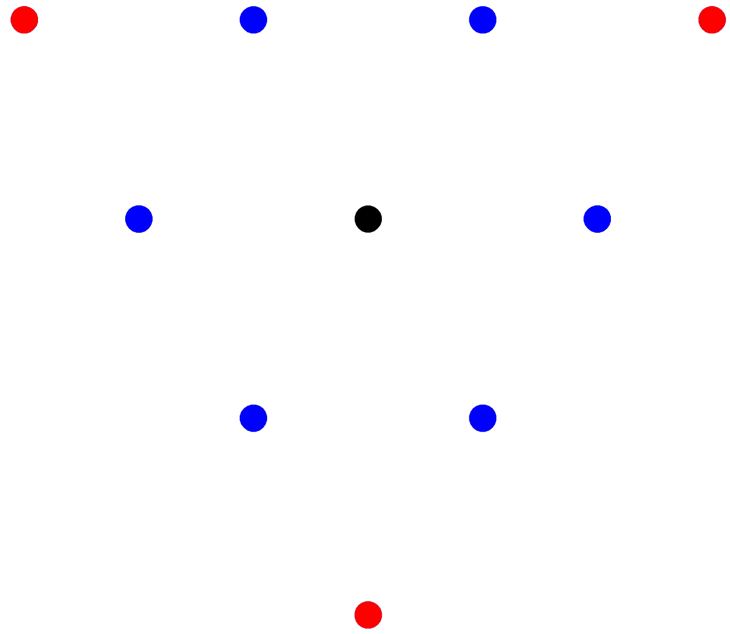
Singlet Quantities





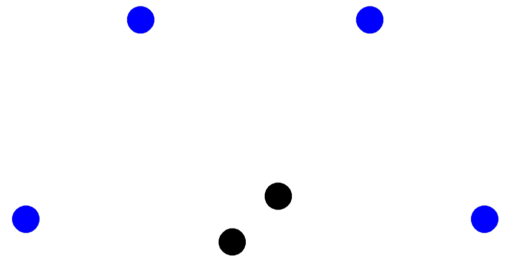
Singlet Quantities

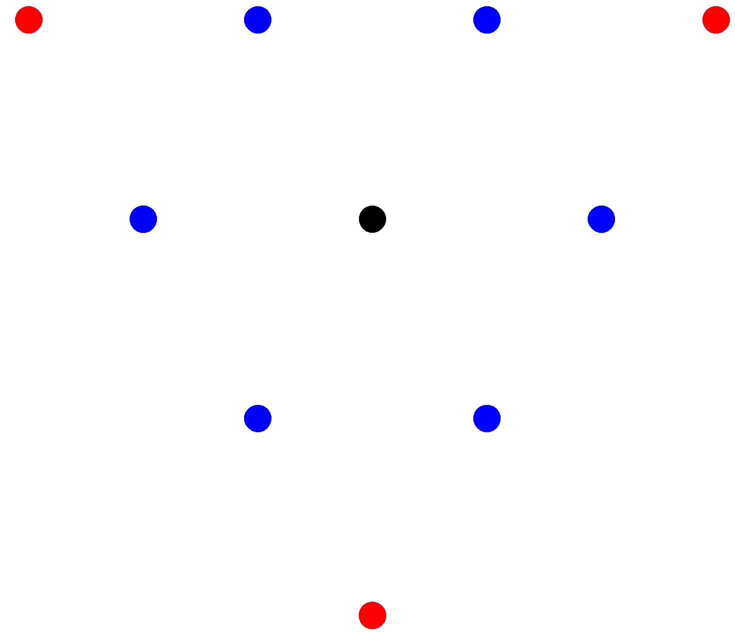




Permutation sets

Singlet Quantities



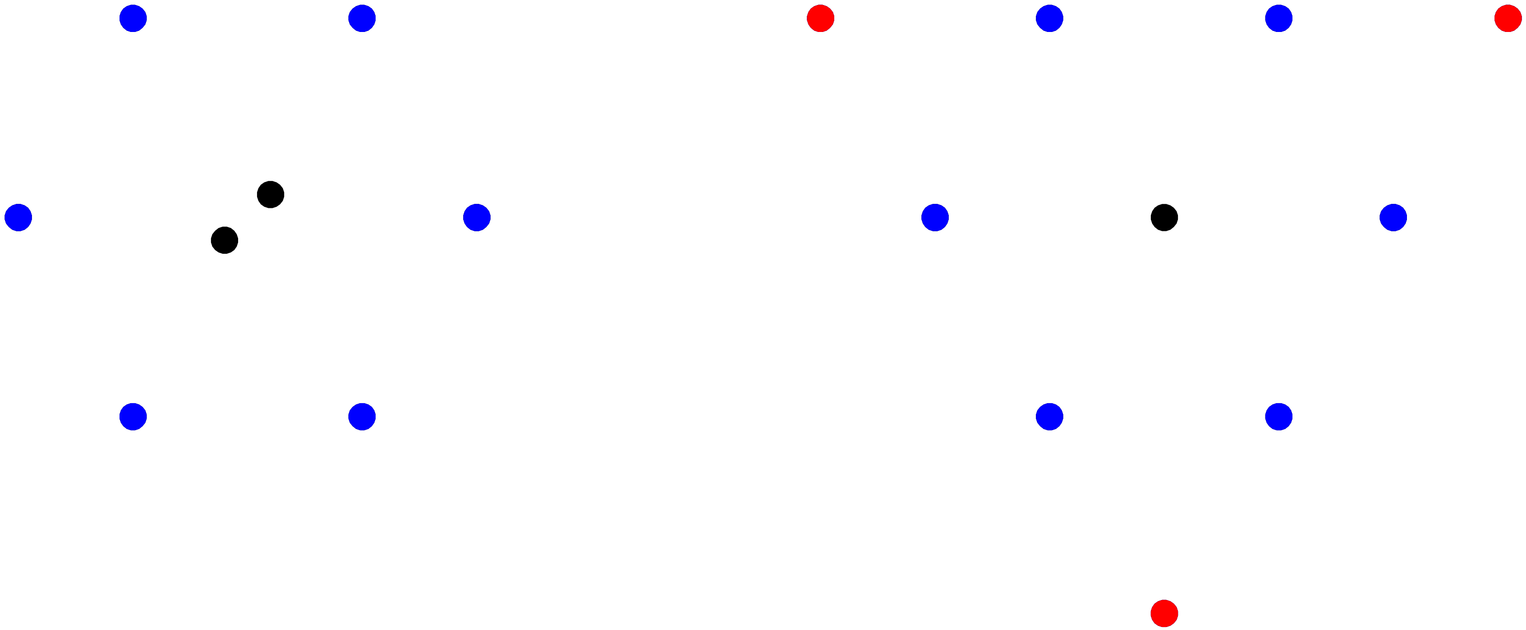


$2M_\Delta + M_\Omega$

$2(M_N + M_\Sigma + M_\Xi)$ $\quad$ $2(M_\Delta + M_{\Sigma^*} + M_{\Xi^*})$

$M_\Sigma + M_\Lambda$ $\quad$ M_{Σ^*}

Singlet Quantities



$4M_K^2 + 2M_\pi^2$

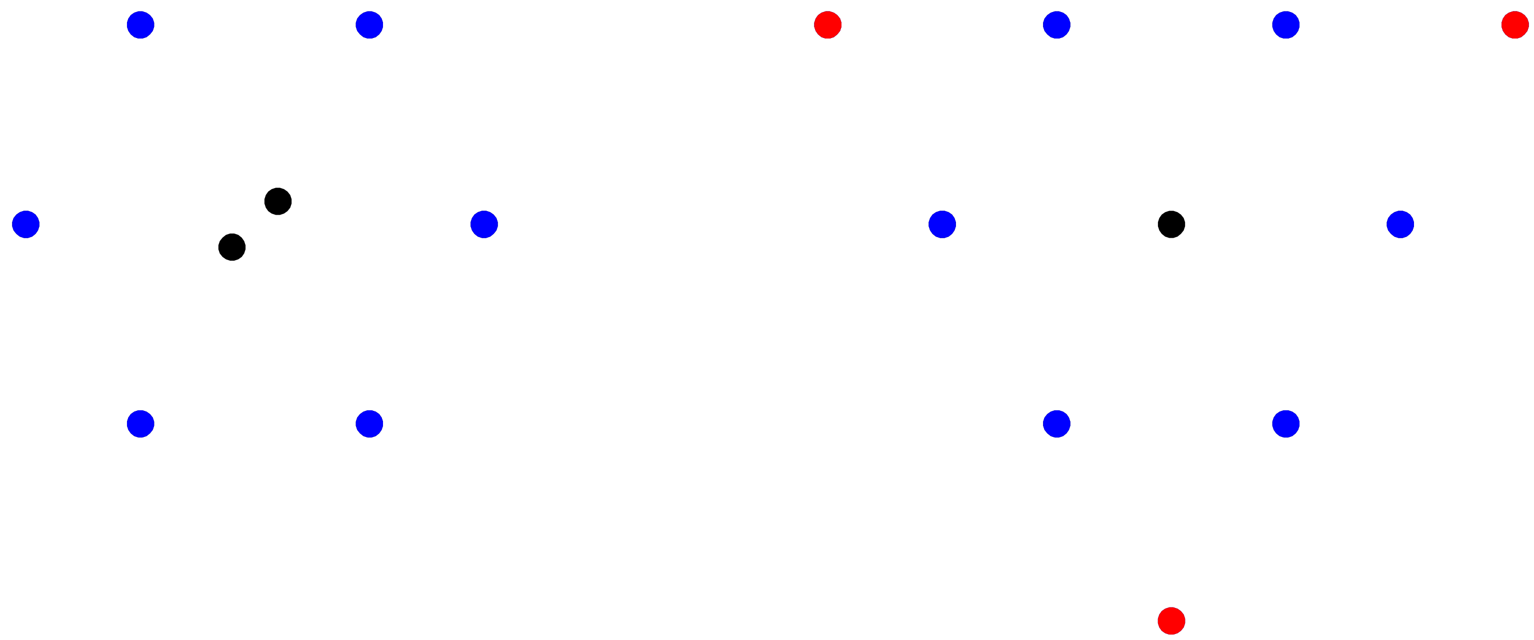
$M_\pi^2 + M_\eta^2$

Singlet Quantities

Use singlets to locate the starting point of our path to physics

$$\frac{2M_K^2 + M_\pi^2}{M_N + M_\Sigma + M_\Xi} = \text{physical value}$$

SU(3) classification



The permutation group yields a lot of useful relationships, but can't capture the entire structure. No connection between Δ^{++}, uuu and Δ^{+}, uud.

SU(3) classification

- Classify physical quantities by $SU(3)$ and permutation group (which is a subgroup of SU(3)).
- Classify quark mass polynomials in same way.
- Taylor expansion about (m_0, m_0, m_0) strongly constrained by symmetry.

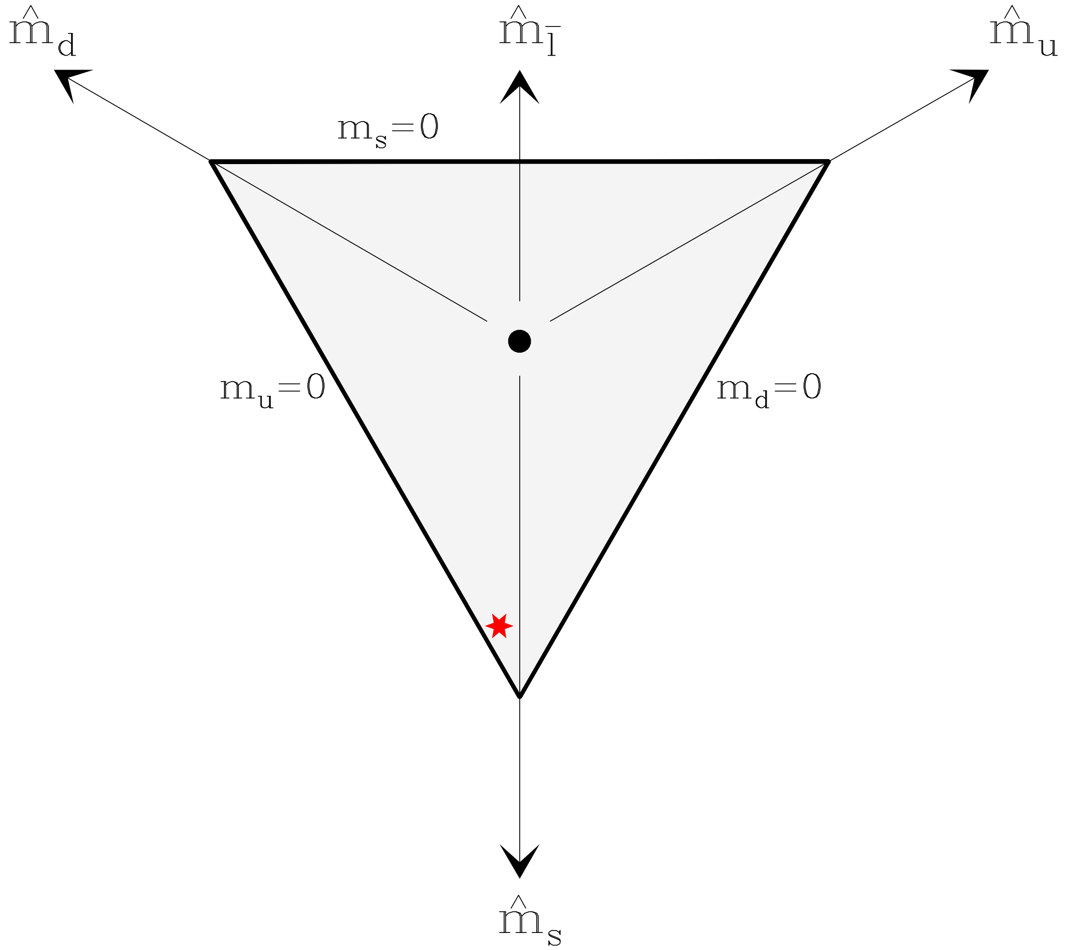
SU(3) classification

Polynomial		S_3	$SU(3)$		
1	✓	A_1	1		
$(\overline{m} - m_0)$		A_1	1		
$\hat{m}_s$	✓	E^+		8	
$(\hat{m}_u - \hat{m}_d)$	✓	E^-		8	
$(\overline{m} - m_0)^2$		A_1	1		
$(\overline{m} - m_0)\hat{m}_s$		E^+		8	
$(\overline{m} - m_0)(\hat{m}_u - \hat{m}_d)$		E^-		8	
$\hat{m}_u^2 + \hat{m}_d^2 + \hat{m}_s^2$	✓	A_1	1		27
$3\hat{m}_s^2 - (\hat{m}_u - \hat{m}_d)^2$	✓	E^+		8	27
$\hat{m}_s(\hat{m}_d - \hat{m}_u)$	✓	E^-		8	27

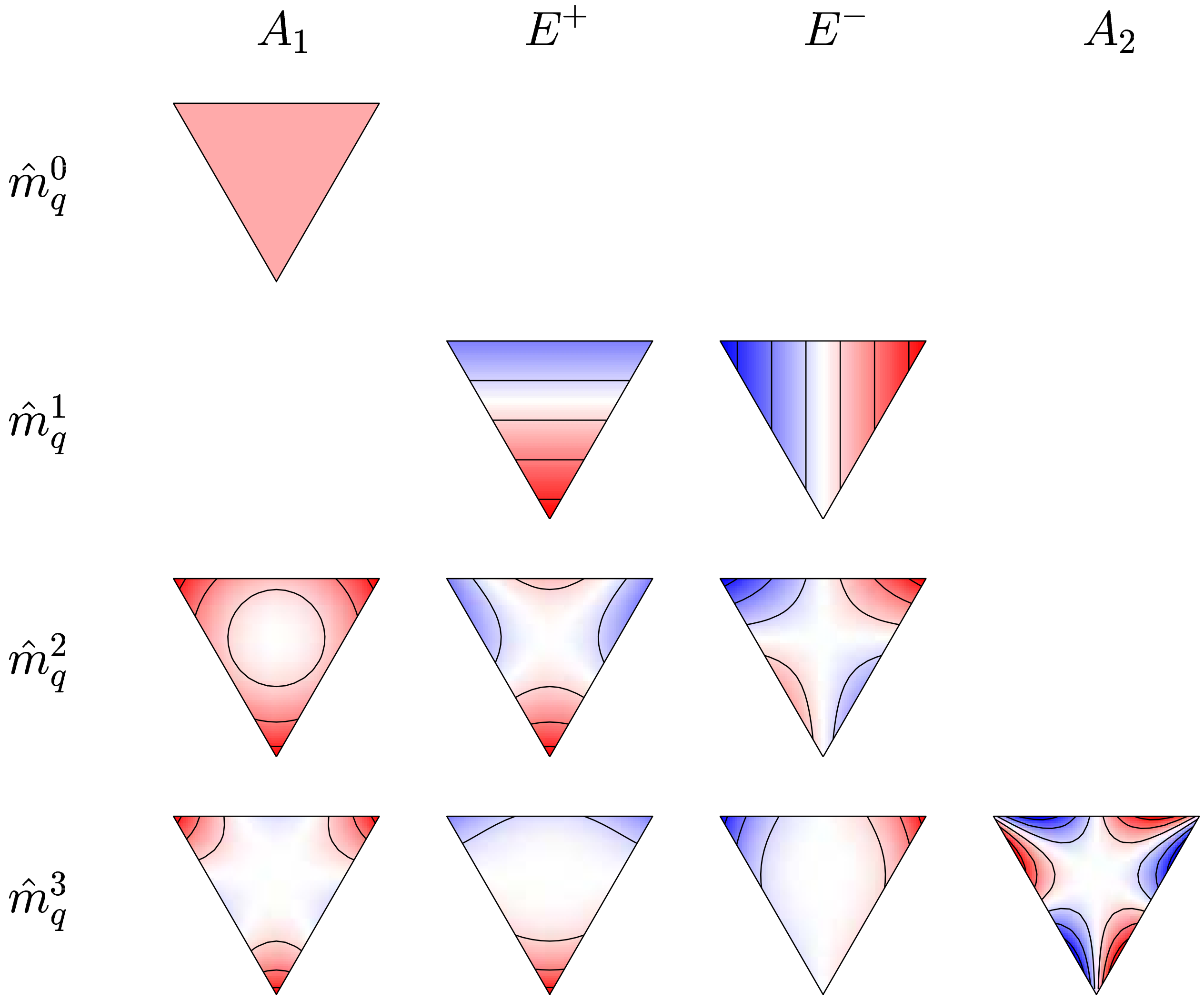
SU(3) classification

Polynomial		S_3	$SU(3)$					
$(\overline{m}-m_0)^3$		A_1	1					
$(\overline{m}-m_0)^2\hat{m}_s$		E^+		8				
$(\overline{m}-m_0)^2(\hat{m}_u-\hat{m}_d)$		E^-		8				
$(\overline{m}-m_0)(\hat{m}_u^2+\hat{m}_d^2+\hat{m}_s^2)$		A_1	1				27	
$(\overline{m}-m_0)\left[3\hat{m}_s^2-(\hat{m}_u-\hat{m}_d)^2\right]$		E^+		8			27	
$(\overline{m}-m_0)\hat{m}_s(\hat{m}_d-\hat{m}_u)$		E^-		8			27	
$\hat{m}_u\hat{m}_d\hat{m}_s$	✓	A_1	1				27	64
$\hat{m}_s(\hat{m}_u^2+\hat{m}_d^2+\hat{m}_s^2)$	✓	E^+		8			27	64
$(\hat{m}_u-\hat{m}_d)(\hat{m}_u^2+\hat{m}_d^2+\hat{m}_s^2)$	✓	E^-		8			27	64
$(\hat{m}_s-\hat{m}_u)(\hat{m}_s-\hat{m}_d)(\hat{m}_u-\hat{m}_d)$	✓	A_2			10	$\overline{10}$		64

SU(3) classification



SU(3) classification



SU(3) classification

The only quantities with a non-zero slope at symmetric point are flavour octet quantities.
(only applies on $m_u + m_d + m_s = const$ line.)
Often slopes highly constrained:
Decuplet baryons - 4 particles; but 1 slope parameter.
Octet baryons - 4 particles; but 2 slopes.
Octet mesons - 3 particles; but 1 slope parameter.

Gell Man Okubo relations

$$
\begin{aligned}
4M_\Delta + 3M_{\Sigma^*} + 2M_{\Xi^*} + M_\Omega &= 13.821 \text{ GeV} \\
-2M_\Delta \qquad\qquad + M_{\Xi^*} + M_\Omega &= 0.742 \text{ GeV} \\
4M_\Delta - 5M_{\Sigma^*} - 2M_{\Xi^*} + 3M_\Omega &= -0.044 \text{ GeV} \\
-M_\Delta + 3M_{\Sigma^*} - 3M_{\Xi^*} + M_\Omega &= -0.006 \text{ GeV}
\end{aligned}
$$

Hierarchy:

$$1, \qquad 8, \qquad 27, \qquad 64.$$

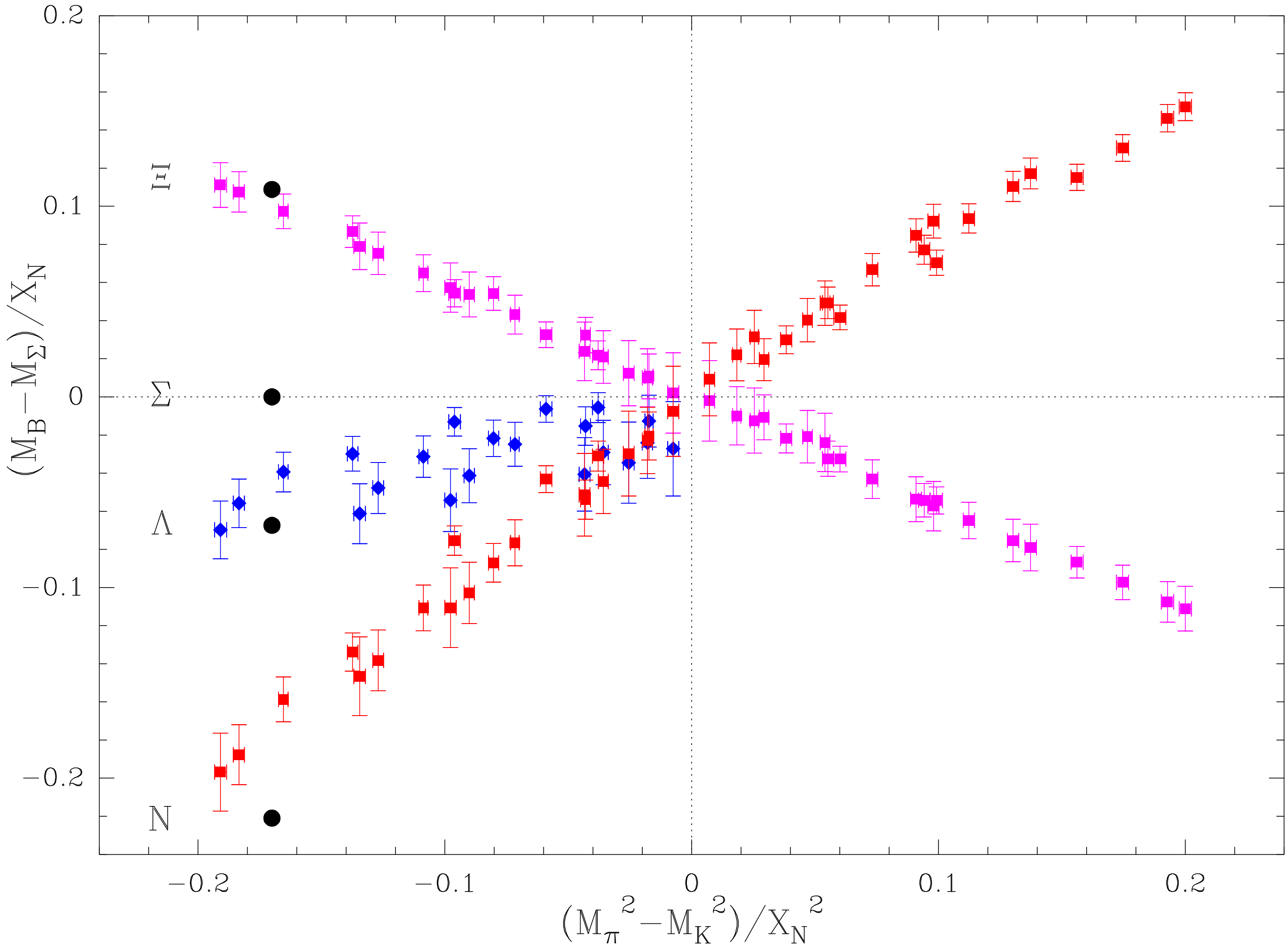
$$(m_s - m_l)^0, \quad (m_s - m_l)^1, \quad (m_s - m_l)^2, \quad (m_s - m_l)^3$$

Suggests short Taylor series may work well all the way from symmetry point (m_0, m_0, m_0) to physical point.

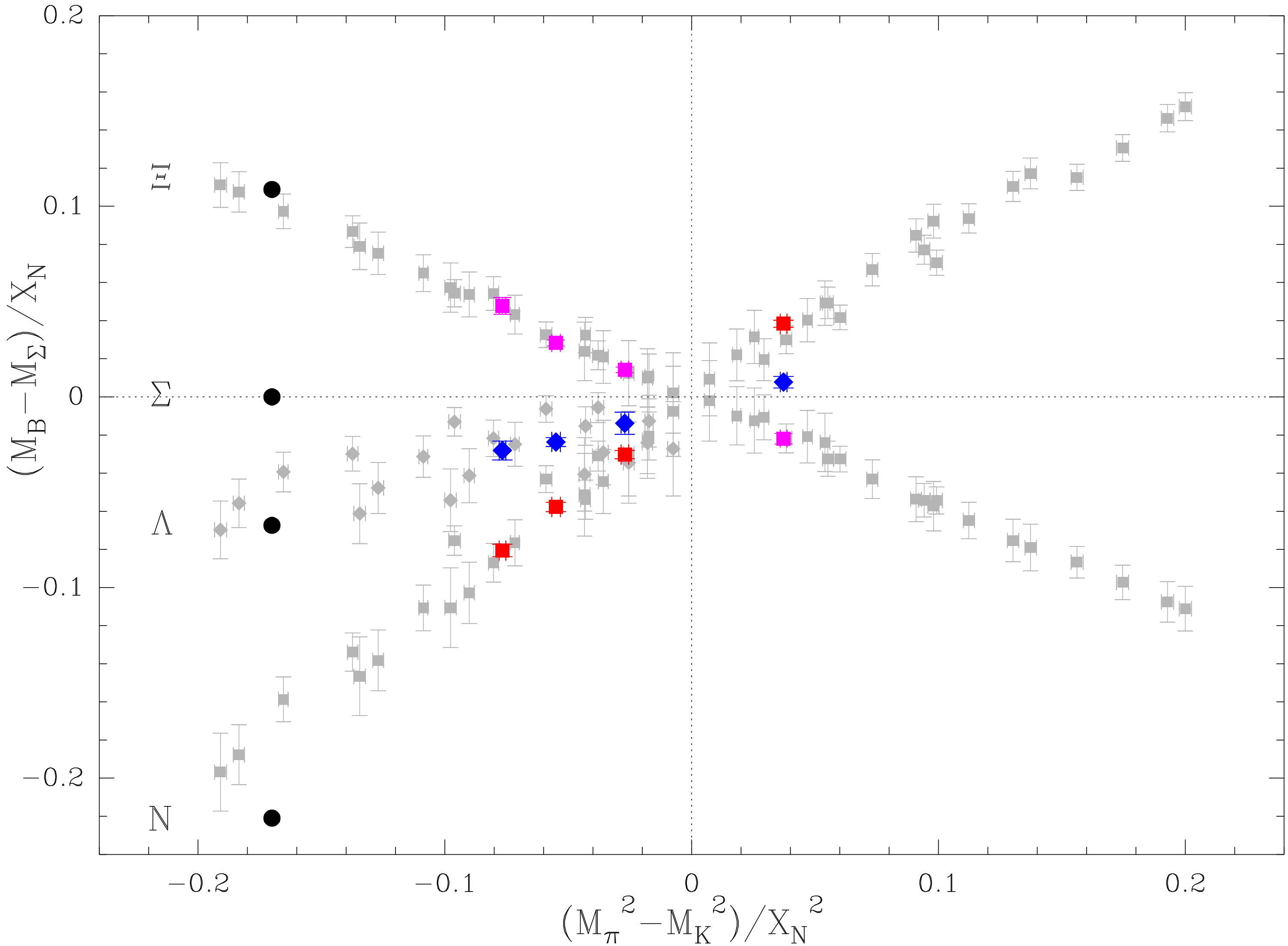
Partial Quenching

Partial quenching (making measurements with valence quarks which have masses different from the sea quarks used to generate a configuration) works well along the line $\overline{m}_{sea} = const$. The argument is very similar to the one we gave earlier, the effects of making the u_{sea} and d_{sea} lighter is largely cancelled by the effect of making the s_{sea} heavier. The cancellation is perfect at the symmetric point. On our trajectory, the error from partial quenching is quadratic in the quark mass; normally partial quenching errors are linear in m_q.

Partial Quenching



Partial Quenching



Partial Quenching

As an example, let us look at the Ω made with three quarks with $\kappa_{val} = 0.12080$.
We have measured this combination on 4 different backgrounds, 3 with the same value for $(m_u + m_d + m_s)_{sea}$, and one lying off the trajectory, with a larger value of $(m_u + m_d + m_s)_{sea}$.

κ_l^{sea}	κ_s^{sea}	κ^{val}	aM_Ω	
0.12100	0.12070	0.12080	$0.610(7)$	PQ
0.12095	0.12080	0.12080	$0.605(4)$	full
0.12090	0.12090	0.12080	$0.608(7)$	PQ
0.12080	0.12080	0.12080	$0.642(10)$	full

On trajectory, PQ and full results agree, but not off the trajectory.

Partial Quenching

Partial Quenched mass formulae. $(m_u + m_d + m_s)_{sea}$ held constant - no constraint on valence masses.

$$\hat{\mu}_f \equiv m_f^{val} - m_0$$

Can rotate valence masses independently of sea masses. Sea masses - singlet polynomials.

Constraint: Ω mass independent of m_u^{val}, m_d^{val}.

Second order polynomials - singlet coefficients fixed by 8-plet and 27-plet.

Partial Quenching

Decuplet

$$q_1 \; q_2 \; q_3$$

$$\hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3$$

Second order

$$\hat{\mu}_1^2 + \hat{\mu}_2^2 + \hat{\mu}_3^2$$

$$(\hat{\mu}_1 - \hat{\mu}_2)^2 + (\hat{\mu}_1 - \hat{\mu}_3)^2 + (\hat{\mu}_2 - \hat{\mu}_3)^2$$

(or linear combinations, eg

$$\hat{\mu}_1\hat{\mu}_2 + \hat{\mu}_1\hat{\mu}_3 + \hat{\mu}_2\hat{\mu}_3$$

Partial Quenching

$$\begin{aligned}
M_\Delta &= M_0 + 3A\hat{\mu}_l + B_0\hat{m}_l^2 + 3B_1\hat{\mu}_l^2 \\
M_{\Sigma^*} &= M_0 + A(2\hat{\mu}_l + \hat{\mu}_s) + B_0\hat{m}_l^2 \\
&\quad +B_1(2\hat{\mu}_l^2 + \hat{\mu}_s^2) + B_2(\hat{\mu}_s - \hat{\mu}_l)^2 \\
M_{\Xi^*} &= M_0 + A(\hat{\mu}_l + 2\hat{\mu}_s) + B_0\hat{m}_l^2 \\
&\quad +B_1(\hat{\mu}_l^2 + 2\hat{\mu}_s^2) + B_2(\hat{\mu}_s - \hat{\mu}_l)^2 \\
M_\Omega &= M_0 + 3A\hat{\mu}_s + B_0\hat{m}_l^2 + 3B_1\hat{\mu}_s^2
\end{aligned}$$

Conclusions

- Extrapolating from lattice simulations to the physical quark masses is made much easier by keeping $m_u + m_d + m_s$ constant.
- Flavour SU(3) analysis strongly constrains Taylor expansions in quark masses.
- See next talk for how well the idea works in practice.

Extra

Allowed Region