Baryon axial charges and quark momentum fractions with $N_f = 2 + 1$ dynamical fermions

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Outline

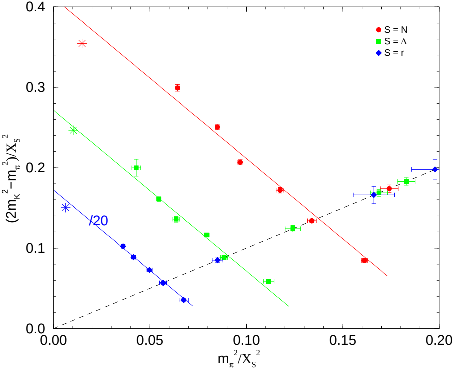
Simulation parameters

- $N_f = 2 + 1$ quark flavours
  2 mass degenerate light quarks
  1 strange quark
- Singlet quark mass fixed

$$\bar{m}^R = \frac{1}{3}(2m_l^R + m_s^R) = \text{const.}$$

- $m_K \nearrow m_K^*$ $\quad m_\pi \searrow m_\pi^*$

→ P. Rakow [Tue Parallel33, 15:30 PM]
→ R. Horsley [Tue Parallel33, 15:50 PM]



κ_l	κ_s	$m_\pi\,[GeV]$	$m_K\,[GeV]$	L/a	$m_\pi L$	N_{conf}
0.120830	0.121040	0.459	0.401	24x48	4.63	2500
0.120900	0.120900	0.424	0.424	24x48	4.28	4000
0.120950	0.120800	0.395	0.439	24x48	3.99	2500
0.121000	0.120700	0.359	0.450	24x48	3.63	2500
0.121040	0.120620	0.333	0.462	24x48	3.37	2500
$a = 0.083$fm						

Fermion action

- $O(a)$ improved Wilson fermion action $\quad c_{\rm sw} = 2.65$
- fat links $\tilde{U}$ applied (one iteration of stout smearing with $\alpha = 0.1$)

$$S_F = \sum_x \Big\{ \kappa \sum_\mu [\overline{q}(x)(\gamma_\mu - 1)\tilde{U}_\mu(x) q(x + a\hat{\mu}) - \overline{q}(x)(\gamma_\mu + 1)\tilde{U}^\dagger_\mu(x - a\hat{\mu}) q(x - a\hat{\mu})] + \overline{q}(x)q(x) - \frac{1}{2}\kappa a c_{sw} \sum_{\mu\nu} \overline{q}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)q(x) \Big\}$$

Gluon action

- tree-level Symanzik action (plaquette + rectangle):

$$S_G = \frac{6}{g_0^2} \Big\{ c_0 \sum_{Plaquette} \frac{1}{3} \mathrm{Re\,Tr}(1 - U_{Plaquette}) + c_1 \sum_{Rectangle} \frac{1}{3} \mathrm{Re\,Tr}(1 - U_{Rectangle}) \Big\}$$

$c_0 = 20/12$, $c_1 = -1/12$ and $\beta = 10/g_0^2$.

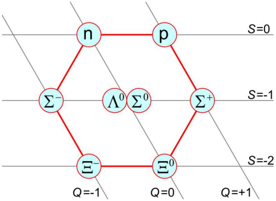
Axial charges

- Govern neutron β-decay
- Quantitative measure of spontaneous chiral symmetry breaking
- Definition: (Baryon B, vanishing 4-momentum q)

$$g_{\mathrm{A,BB}} = G_A(q^2 = 0)$$

- Isovector axial current

$$A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$$



N (lll), Σ (sll), Ξ (ssl)

- Matrix element

$$\langle B|A_\mu^{u-d}|\bar{B}\rangle = \bar{u}(p', s')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2m_N} G_P(q^2)\right] u(p,s)$$

- HBχPT Hyperon axial charges in terms of SU(3) constants

$$g_A = D + F + \sum_n C_N^{(n)} x^n \qquad g_{A,\Sigma\Sigma} = F + \sum_n C_\Sigma^{(n)} x^n \qquad g_{A,\Xi\Xi} = F - D + \sum_n C_\Xi^{(n)} x^n$$

Momentum fractions

- Momentum fraction, baryon B, quark flavour q

$$\langle B|\bar{q}\mathcal{O}_{\langle x\rangle}q|\bar{B}\rangle = 2\,\langle x\rangle_q^B\,p$$

- Operator

$$\mathcal{O}_{\langle x\rangle} = \gamma_4 D_4 - \frac{1}{3}\left(\gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3\right)$$

  - Doubly represented quark:

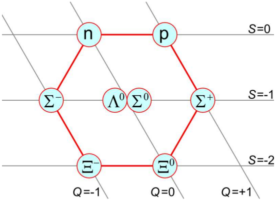
$$\langle x\rangle_D^N\,,\ \langle x\rangle_D^\Sigma\,,\ \langle x\rangle_D^\Xi$$

  - Singly represented quark:

$$\langle x\rangle_S^N\,,\ \langle x\rangle_S^\Sigma\,,\ \langle x\rangle_S^\Xi$$

  - Cancelation of disconnected contributions:

$$\langle x\rangle_{D-S}^N\,,\ \langle x\rangle_{D-S}^\Sigma\,,\ \langle x\rangle_{D-S}^\Xi$$



N (lll), Σ (sll), Ξ (ssl)

Quark level

$$C_{\text{3pt}} = \langle B_\alpha | \mathcal{O} | \overline{B}_\beta \rangle$$

$$B_\alpha = \epsilon^{abc} \, \psi_\alpha^{(l)a} \, \psi_\beta^{(l)b} \, C_{\beta\gamma} \, \psi_\gamma^{(s)c}$$

- Quark line connected terms only
- Sequential source

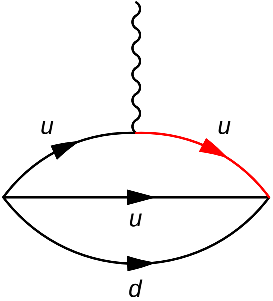
$$C_{\text{3pt}} = \langle \Sigma_\Gamma(x,0) | \mathcal{O}(y) | G(y,0) \rangle$$

$$\Sigma_\Gamma(x,0) = \sum_y S_\Gamma(x,0) G(x,y)$$

$$M\Sigma_\Gamma = S_\Gamma(x,0)$$

- Jacobi smearing at source and sink

$$\kappa_{\text{Jacobi}} = 0.21 \qquad N_{\text{Jacobi}} = 65$$



Hadron level

- 3pt correlation functions $C_{\mathrm{3pt}} = \langle B|\mathcal{O}|\overline{B}\rangle$

$$C_{\mathrm{3pt}}(t,\tau,p,p') = \sum_{\vec{x},\vec{y}} \sum_{s,s'} e^{E_{p'}(t-\tau)} e^{-E_p \tau} e^{i\vec{p}'(\vec{y}-\vec{x})} e^{-i\vec{p}\vec{x}} \times \langle 0|B|p',s'\rangle \langle p',s'|\mathcal{O}|p,s\rangle \langle p,s|\bar{B}|0\rangle$$

- 2pt correlation functions $C_{\mathrm{2pt}} = \langle B|\overline{B}\rangle$

$$C_{\mathrm{2pt}}(t,p) = \sum_{\vec{x},s} e^{E_p t} e^{-i\vec{p}\vec{x}} \langle 0|B|p,s\rangle \langle p,s|\bar{B}|0\rangle$$

- Ratios 3pt / 2pt

$$R(t,\tau,p,p') = \frac{C_{\mathrm{3pt}}(t,\tau,p,p')}{C_{\mathrm{2pt}}(t,p)}$$

- Axial charge

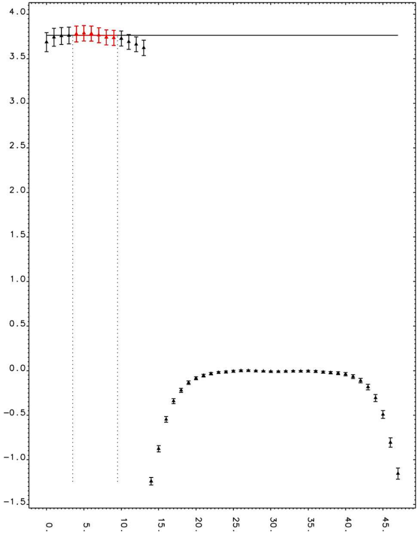
$$R_{g_A} = i \frac{m}{E_{\vec{p}}} g_A$$

- Zero momentum transfer $p = 0$

$$R_{g_A} = i g_A$$

- Fitting example: g_A (up-part)

$m_\pi [GeV]$	$m_K [GeV]$	L/a	$m_\pi L$
0.459	0.401	24x48	4.63
0.424	0.424	24x48	4.28
0.395	0.439	24x48	3.99
0.359	0.450	24x48	3.63
0.333	0.462	24x48	3.37



Ratio momentum fraction $R_{<x>}$

- Momentum fraction

$$R_{<x>} = -\frac{E_{\vec{p}}^2 + \frac{1}{3}\vec{p}^2}{E_{\vec{p}}} < x >$$

- Zero momentum transfer $p = 0$

$$R_{<x>} = m < x >$$

- Fitting example: Nucleon $< x >_u$

$m_\pi [GeV]$	$m_K [GeV]$	L/a	$m_\pi L$
0.459	0.401	24x48	4.63
0.424	0.424	24x48	4.28
0.395	0.439	24x48	3.99
0.359	0.450	24x48	3.63
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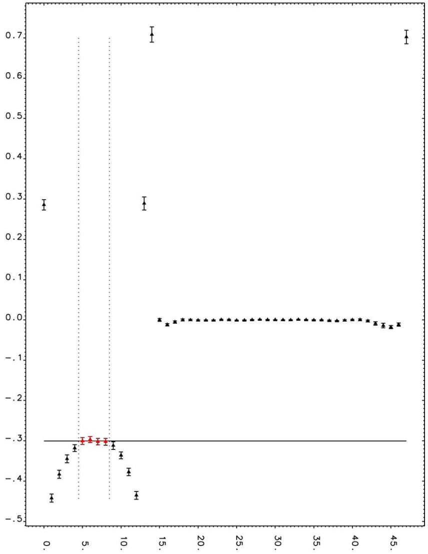
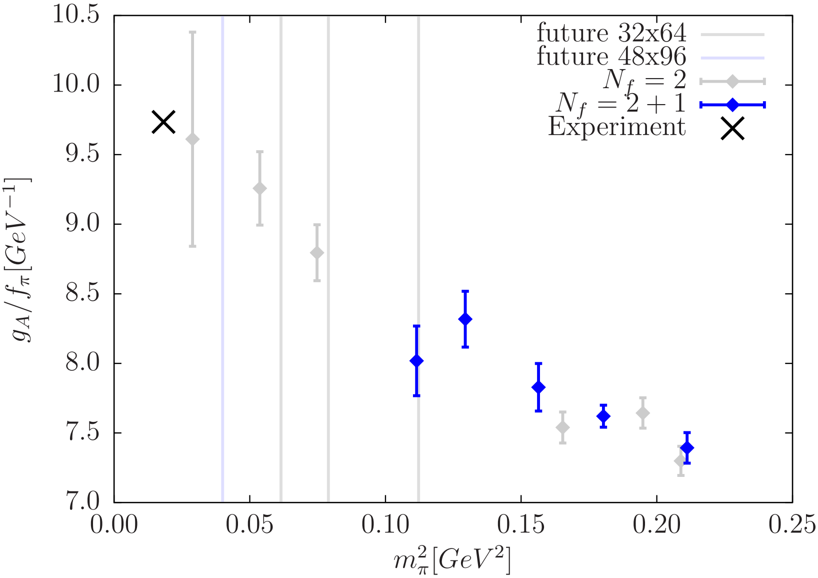


No renormalisation yet

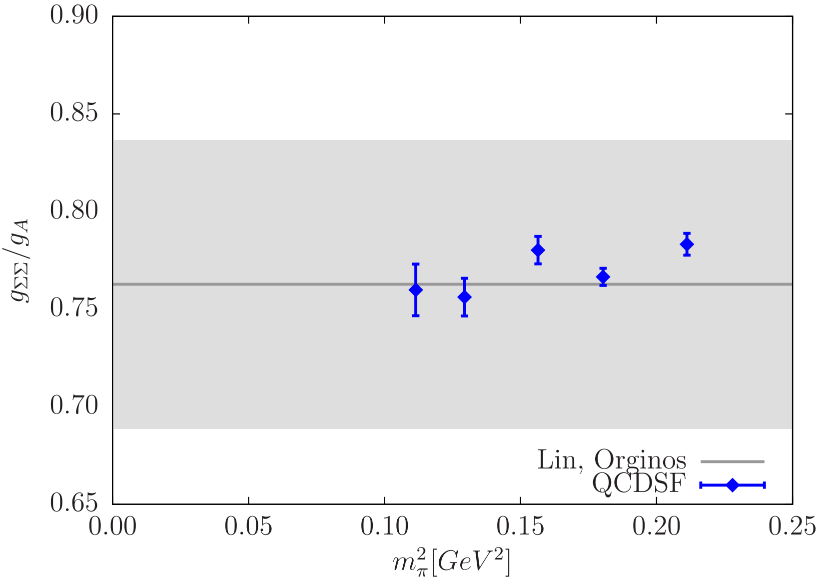
- No renormalisation constant Z for $N_f = 2 + 1$ yet.
  $\rightarrow$ no direct comparision to experiment

- Looking at ratios instead, Z cancels

$$g_{A,\Xi\Xi} = \frac{Z_A\, g_{A,\Xi\Xi}^{\mathrm{lat}}}{Z_A\, g_A^{\mathrm{lat}}} g_A^{\mathrm{exp}} \qquad\qquad g_{A,\Sigma\Sigma} = \frac{Z_A\, g_{A,\Sigma\Sigma}^{\mathrm{lat}}}{Z_A\, g_A^{\mathrm{lat}}} g_A^{\mathrm{exp}}$$

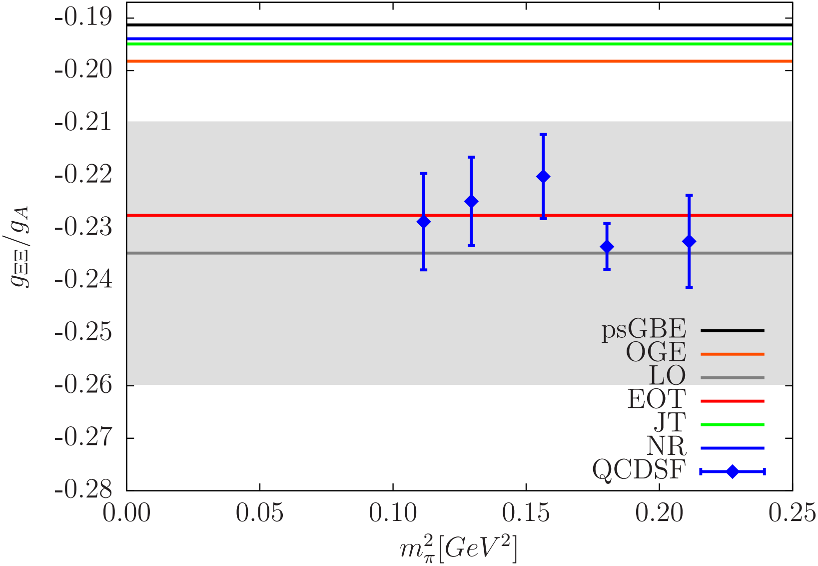


$\rightarrow$ D. Pleiter [Mon Parallel 1, 14:30 PM]



$g_{\Sigma\Sigma}/g_A * g_A^{\text{exp.}} \approx 0.46(5)$

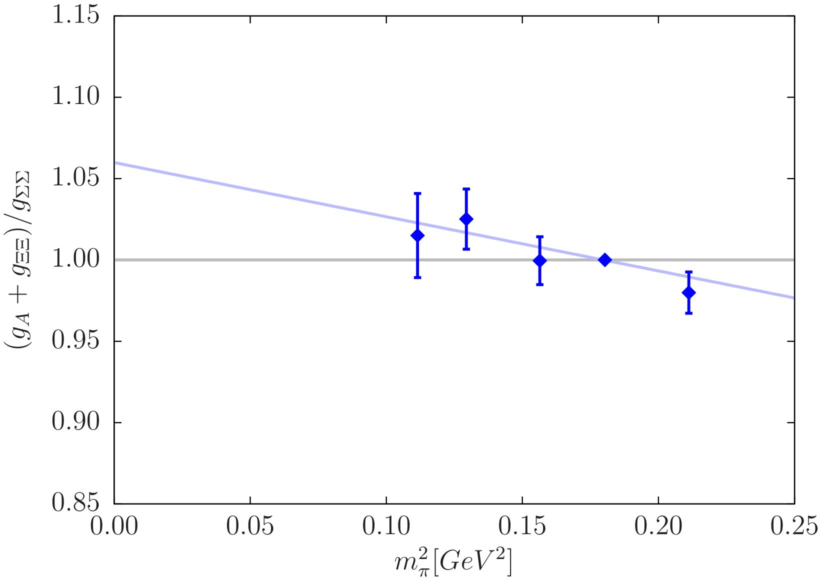
$g_{\Sigma\Sigma}^{\text{L.O.}} = 0.450(21)$

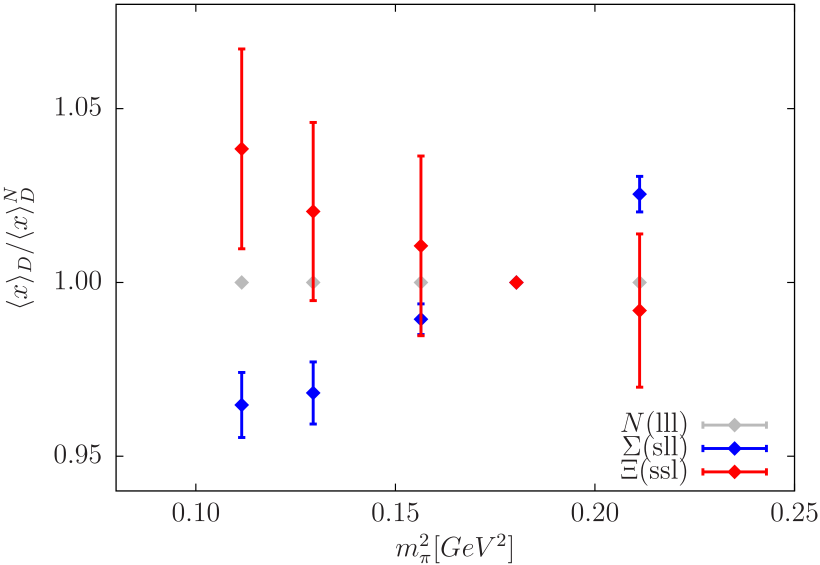


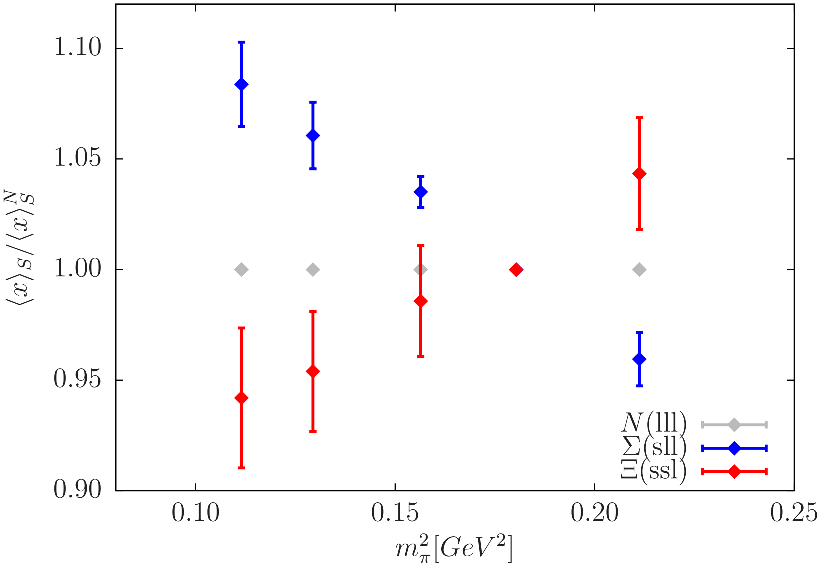
$g_{\Xi\Xi}/g_A * g_A^{\text{exp.}} \approx -0.28(3)$

$g_{\Xi\Xi}^{\text{L.O.}} = -0.277(15)$

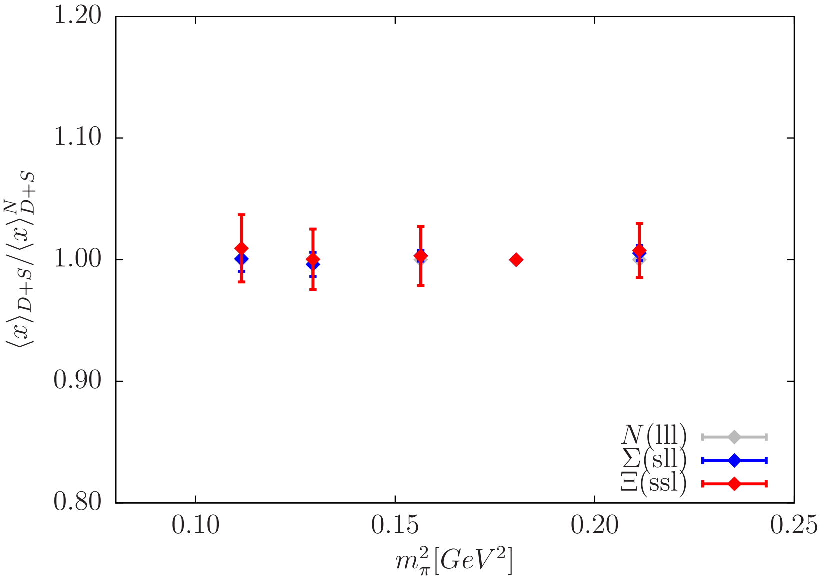
$(g_A + g_{\Xi\Xi})/g_{\Sigma\Sigma}$

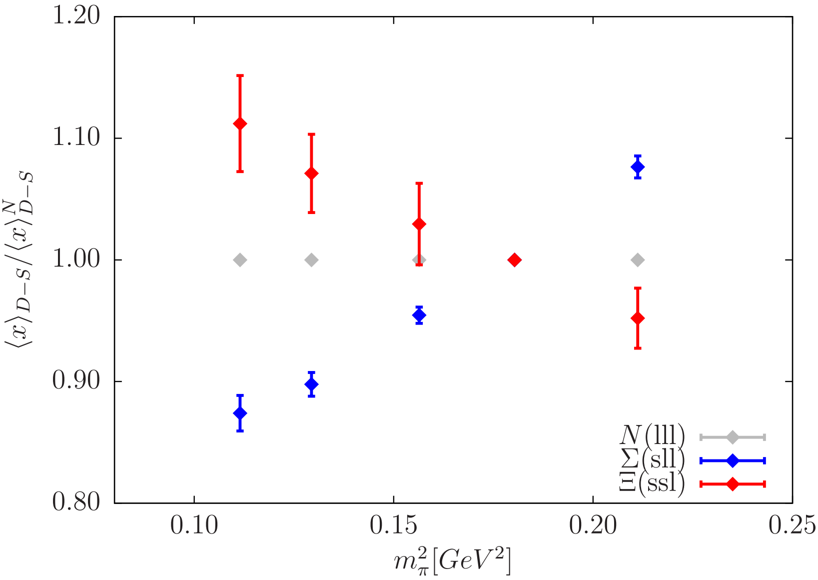



1.05
1.00
0.95
$\langle x \rangle_D / \langle x \rangle_D^N$
0.10
0.15
0.20
0.25
$m_\pi^2 [GeV^2]$
N(lll)
Σ(sll)
Ξ(ssl)



1.10
1.05
1.00
0.95
0.90
$\langle x \rangle_S / \langle x \rangle_S^N$
0.10
0.15
0.20
0.25
$m_\pi^2 [GeV^2]$
N(lll)
Σ(sll)
Ξ(ssl)


1.20
1.10
1.00
0.90
0.80
0.10
0.15
0.20
0.25
$\langle x \rangle_{D+S} / \langle x \rangle^{N}_{D+S}$
$m_\pi^2 [GeV^2]$
N(lll)
Σ(sll)
Ξ(ssl)


1.20
1.10
1.00
0.90
0.80
0.10
0.15
0.20
0.25
$\langle x \rangle_{D-S} / \langle x \rangle^{N}_{D-S}$
$m_\pi^2 [GeV^2]$
N(lll)
Σ(sll)
Ξ(ssl)

Summary and Outlook

- QCDSF $N_f = 2 + 1$ results agree with
  - $N_f = 2$ results
  - results from other collaborations
- Signs of SU(3) flavour symmetry breaking
- For ratios FSE effects seem to be small

- Simulation at lighter pion masses, larger lattices
- Renormalisation
- FSE analysis
- χPT extrapolation