From loops to surfaces

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Outline

Introduction

At $N = \infty$ $SU(N)$ pure YM simplifies, but this has not yet been turned into a quantitative tool. Our objective is to change this state of affairs.

Let $\mathcal{O}$ be an observable characterized by one single scale l.

For $l\Lambda_N \ll 1$, $\langle \mathcal{O} \rangle_{N=\infty}$ can be computed by summing up contributions from planar Feyman diagrams. Moreover, there are good reasons to believe that the series in $\frac{1}{\log(l\Lambda_N)}$ converges.

For $l\Lambda_N \gg 1$, for a class of specific $\mathcal{O}$'s, $\langle \mathcal{O} \rangle_{N=\infty}$ can be expanded in $\frac{1}{l\Lambda_N}$ using an effective theory based on free strings.

How are the two regimes connected ?

Some $\mathcal{O}$'s have a narrow crossover as l changes from $l\Lambda_N < 1$ to $l\Lambda_N > 1$ at finite N, becoming a "phase transition" at $N = \infty$.

Such large N phase transitions tend to fall in Random Matrix universality classes. The hope is to exploit this to connect field theoretical perturbation theory to effective string theory.

Wilson loop operator - ignoring renormalization

- $\mathcal{P}e^{i\oint_{\mathcal{C}} A_R\cdot dx} \equiv \Omega_R(\mathcal{C})$, $R = SU(N)$-representation. $W_R(\mathcal{C}) = \mathrm{tr}\langle\Omega_R(\mathcal{C})\rangle/d_R$ are Wilson loops.
- Restrict to totally antisymmetric representations, $R = k, N-k$; $k = 1, ..., \left[\frac{N}{2}\right]$. The generating function for the W_k is $\langle\det(z + \Omega_f(\mathcal{C}))\rangle = Q(z, \mathcal{C})$, a palindromic polynomial of rank N in z.
- As $N \to \infty$ a continuum density of roots of $Q(z, \mathcal{C})$ develops, supported on $|z| = 1$, gapped at $z = 1$ for small loops, and uniform for infinite loops.
- As the loop $\mathcal{C}$ is dilated, its minimal area A grows and one can extract k-string tensions σ_k from $W_k(\mathcal{C}) \sim \exp(-\sigma_k A)$. Hence, for very large loops the deviation of the density from uniformity is controlled by $[N/2]$ exponents $e^{-\sigma_k A}$, dominated by $e^{-\sigma_f A}$.

Large N phase transition

- Separates small from large loops and occurs in D=2,3,4.
- Close to critical loop-size, and for z close to 1, there is a universal description common to all dimensions.
- The case $D = 2$ is exactly soluble so universal form is known.
- This "phase transition" is seen only in $Q(z, \mathcal{C})$, but not in the individual W_k's.
- The universality provides an economic parametrization of the short-scale to long-scale crossover in $Q(z, \mathcal{C})$ for $1 \ll N < \infty$.
- For this to be meaningful, need to renormalize $Q(z, \mathcal{C})$.

Renormalization of Wilson loops

- $Q(z,\mathcal{C}) = \langle \det(1 + z\Omega_f^\dagger(\mathcal{C}))\rangle$.
- $\det[1 + z\Omega_f^\dagger(\mathcal{C})] = \int [d\bar\psi d\psi] e^{\int_0^l d\sigma \bar\psi(\sigma)[\partial_\sigma - \mu - ia(\sigma)]\psi(\sigma)}$.
- $z = e^{-\mu l}$, σ parametrizes $\mathcal{C}$ by $x(\sigma)$, $[\partial_\sigma x_\mu(\sigma)]^2 = 1$.
- l is the length of $\mathcal{C}$.
- $\bar\psi(\sigma), \psi(\sigma)$ obey a.p.b.c.
- $a(\sigma) = A_\mu(x(\sigma))\frac{\partial x_\mu(\sigma)}{d\sigma}$.
- $Q(z,\mathcal{C}) = \langle \int [d\bar\psi d\psi] e^{\int_0^l d\sigma \bar\psi(\sigma)[\partial_\sigma - \mu - ia(\sigma)]\psi(\sigma)}\rangle$
- $[\sigma] = -1 \Rightarrow [\bar\psi, \psi] = 0$
- Non-redundant ct-s: $[\bar\psi\psi]^k,\ k = 1, ..N$
- $\psi \to \bar\psi,\ \ \bar\psi \to \psi,\ \ A_\mu \to A_\mu^* \Rightarrow$ number of ct-s is $\left[\frac{N}{2}\right]$.
- Can make $\frac{1}{d_R} W_R(\mathcal{C}) \leq 1$ for all antisymmetric R.
- $\Rightarrow Q(z,\mathcal{C})$ is palindromic & all roots on $|z| = 1$.
- Divergences are linear in 4D and logarithmic in 3D.

Large N transition and a Dirac operator

- The $\left[\frac{N}{2}\right]$ ct-s are necessary to eliminate the $\left[\frac{N}{2}\right]$ perimeter divergences associated with the distinct N-ality representations, not counting conjugate ones.
- Physically, the ct-s represent the arbitrary amounts of thickening the distinct k-strings need.
- On the lattice they are implemented by coarsening (via a continuous version of smearing/cooling/...) the gauge fields the fermions see.
- The spectrum of $D_1(\mathcal{C}) \equiv \partial_\sigma - ia(\sigma)$ will have a gap for small loops and will be gap-less for large loops.
- There is an analogy to spontaneous chiral symmetry breaking and its connection to chiral random matrix theory.

Surface observable

- Replace the curve by a 2-dimensional surface Σ, described by $x_\mu(\sigma)$. Put massive Dirac fermions on Σ, which is characterized by a single scale l.
- At zero mass, one has chiral symmetry and the fermionic determinant is the exponent of the Polyakov-Wiegmann action.
- The gauge connection on Σ is $a_\alpha = A_\mu(x(\sigma))\frac{\partial x_\mu}{\partial \sigma_\alpha}$.
- The massless Dirac operator is $D_2(\Sigma) = \gamma_\alpha[\partial_{\sigma_\alpha} - i a_\alpha(\sigma)]$.
- $Q(\mu, \Sigma) = \langle \int [d\bar\psi d\psi] e^{\int_\Sigma d^2\sigma \bar\psi(\sigma)[D_2(\Sigma)-\mu]\psi(\sigma)} \rangle$
- Currents: $J^j_\alpha(\sigma) = \bar\psi(\sigma)\gamma_\alpha T^j \psi(\sigma)$, $J_\alpha = \bar\psi(\sigma)\gamma_\alpha\psi(\sigma)$.
- Only two ct-s are $\mathcal{L}_1 = J^j_\alpha J^j_\alpha, \quad \mathcal{L}_2 = J_\alpha J_\alpha$.
- At $N = \infty$ a SχSB transition is possible, depending on l.
- In 4D one has logarithmic divergences, and $\mathcal{L}_1$ will be generated, but in 3D no ct-s are generated because the theory is superrenormalizable.

Results in 3D

- No renormalization necessary; set $\mu = 0$ and $N = \infty$.
- For Σ an infinite plane: $\langle \bar{\psi}\psi \rangle_{N=\infty} = 0.29(1)\sqrt{\sigma_f}$
- For Σ a cylinder with square base of side s: spontaneous chiral symmetry breaking at $s > s_c$ and preserved chiral symmetry for $s < s_c$.
- For $s > s_c$ with $(s - s_c)/s_c \ll 1$ we have $\langle \bar{\psi}\psi \rangle_{N=\infty} \propto (s - s_c)^{1/2}$
- $s_c = 1.4(1)/\sqrt{\sigma_f}$

Outlook

- Generalize to 4D.
- Find analytical ways to get $\langle\bar{\psi}\psi\rangle$ and s_c in 3D.

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