Semileptonic decays of K and D mesons in $2+1$ flavor QCD

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for FNAL/MILC collaboration

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1. Introduction: semileptonic decays

Extraction of CKM matrix elements

$$\frac{d}{dq^2}\Gamma(P_1 \to P_2 l\nu) \propto |V_{ab}|^2 |f_+^{P_1\to P_2}(q^2)|^2 \qquad\qquad q = p_{P_2} - p_{P_1}$$

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CLEO-c, Besson et al PRD80 (2009)

$$|V_{cs}| f_+(0)^{D\to K} = 0.719(\pm 0.8\% \pm 0.7\%)$$
$$|V_{cd}| f_+(0)^{D\to \pi} = 0.150(\pm 3\% \pm 0.7\%)$$

Aubin et al. PRL94(2005)

$f_+(0)^{D\to K,latt}$: 11% error
$f_+(0)^{D\to \pi,latt}$: 10% error

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$|V_{cs}| f_+(0)^{D\to K} = 0.717(\pm 0.8\% \pm 0.7 \pm 0.7\%)$ (last error from $B(D^0 \to K^-\pi^+)$)

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Experimental average, Antonelli et al. (Flavianet), arXiv:1005.2323

$|V_{us}| f_+(0)^{K\to\pi} = 0.2163(\pm 0.23\%)$ $\qquad$ $f_+(0)^{K\to\pi,latt}$: 0.6% error

1. Introduction: semileptonic decays

* Check unitarity in the first row of CKM matrix.

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0001(6)$$ **M. Antonelli et al**, arXiv:1005.2323

fits to K_{l3}, K_{l2} exper. data and lattice results for $f_+(0)^{K\to\pi}$ and f_K/f_π

$\to \mathcal{O}(10\ \mathrm{TeV})$ bound on the scale of new physics.

2. $D \to \pi$: Extraction of $|V_{cd}|$

\# Validate method in D sem. decays

$\rightarrow$ use same method for other processes like $B \to \pi l \nu$ or $B \to K l \bar{l}$

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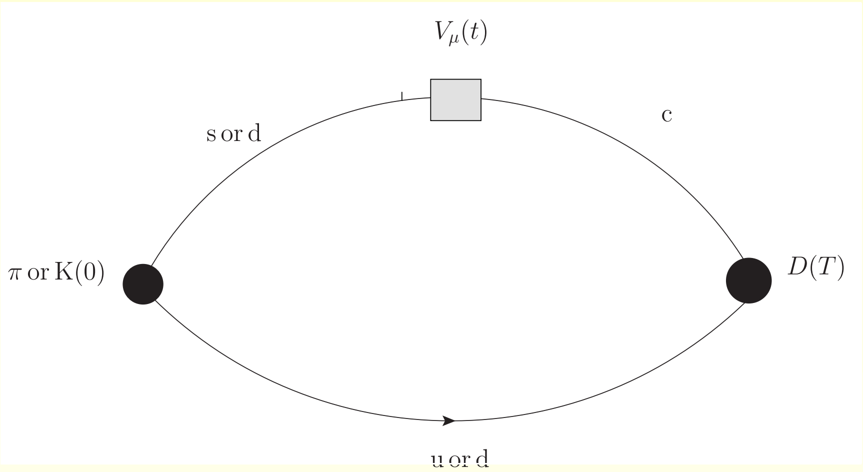

$$\langle P|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_P^\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right]$$

$$+ f_0(q^2)\frac{m_D^2 - m_P^2}{q^2} q^\mu$$

* Work on D rest frame

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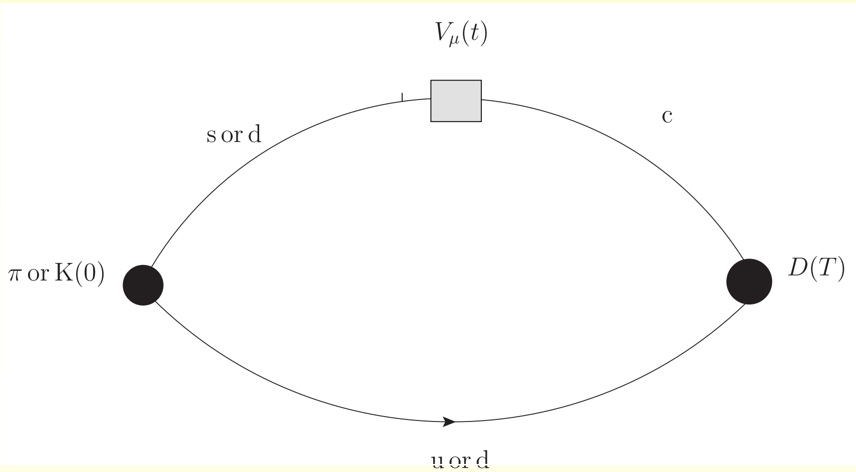

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* Work on D rest frame

A more convenient choice of parameters is

$$\langle P|V_\mu|D\rangle = \sqrt{2m_D}\left[v_\mu f_\parallel^{D\to P}(q^2) + p_{\perp\mu} f_\perp^{D\to P}(q^2)\right]$$

with $f_\parallel^{D\to P}(q^2) = \frac{\langle\pi|\mathcal{V}^0|D\rangle}{\sqrt{2m_D}}$ and $f_\perp^{D\to P}(q^2) = \frac{\langle\pi|\mathcal{V}^i|D\rangle}{\sqrt{2m_D}}\frac{1}{p_\pi^i}$ (D rest frame)

2.1 Simulation details: Lattice actions

Sea quarks: $N_f = 2+1$ **MILC** configurations with improved staggered Asqtad u, d and s sea quarks, and improved glue

* Asqtad: Tree-level order a^2 effects removed
  $\rightarrow$ leading errors are $\mathcal{O}(\alpha_s a)$ $\mathcal{O}(a^4)$

Naik, NPB 1989; **Lepage**, PRD 1999; **Bernard et al**, PRD 1998

* One-loop Symanzik-improved gauge action: **Weisz**, NPB 1983; **Curci et al.**, PLB 1983; **Weisz and Wohlert**, NPB 1984; **Luscher and Weisz**, PLB and CMP 1985; **Alford et al.**, PLB 1995; **Bernard et al.**, PRD 1998

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Valence c: Fermilab action **El-Khadra et al**, PRD 1997

* Fermilab action: Clover with Fermilab interpretation via HQET

** Tune hopping parameter and clover coefficient to eliminate discretization effects through `NLO`

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Valence light quarks: Asqtad action.

2.1 Simulation details: Lattice actions

Improved vector currents.

* Rotate heavy quark field to remove $\mathcal{O}(1/m_c)$ errors:

$$\psi_c \rightarrow \Psi_c = \left(1 + a d_1 \vec{\gamma} \cdot \vec{D}_{lat}\right) \psi_c$$

** d_1 its fixed to its tadpole-improved tree-level value.

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Renormalization: Partially non-perturbative.

$$Z_{V_\mu}^{hl} = \rho_{V_\mu}^{hl} \sqrt{Z_V^{hh} Z_V^{ll}}$$

* Z_V^{hh} and Z_V^{ll} determined non-perturbatively

* $\rho_{V_\mu}^{hl}$ perturbative correction (one-loop). Small correction.

2.1 Simulation details: Parameters

	$\approx a$ (fm)	am_l/am_s	Volume	N_{conf}	$am_l^{\rm valence}$
coarse	0.12	0.02/0.05	$20^3 \times 64$	2052	0.005, 0.007, 0.01,
		0.01/0.05	$20^3 \times 64$	2259	0.02, 0.03, 0.0415,
		0.007/0.05	$20^3 \times 64$	2110	0.05; 0.0349
		0.005/0.05	$24^3 \times 64$	2099	
fine	0.09	0.0124/0.031	$28^3 \times 96$	1996	0.0031, 0.0047, 0.0062,
		0.0062/0.031	$28^3 \times 96$	1946	0.0093, 0.0124, 0.031;
		0.00465/0.031	$32^3 \times 96$	983	0.0261
		0.0031/0.031	$40^3 \times 96$	1015	
superfine	0.06	0.0072/0.018	$48^3 \times 144$	593	0.0036, 0.0072, 0.0018,
		0.0036/0.018	$48^3 \times 144$	668	0.0025, 0.0054, 0.0160;
		0.0025/0.018	$56^3 \times 144$	800	0.0188
		0.0018/0.018	$64^3 \times 144$	826	

* Valence charm quark mass: tuned to physical value

* Current analysis: $m_l^{\rm valence} = m_l^{\rm sea}$

2.2 Correlation functions

Generate 3-point and 2-point correlation functions

$$C_{3,\mu}^{D\to\pi}(t,T;\vec{p}_\pi) = \sum_{\vec{x},\vec{y}} e^{i\vec{p}_\pi\cdot\vec{y}} \langle \mathcal{O}_\pi(t_{source},\vec{0}) V_\mu(t,\vec{y}) \mathcal{O}_D^\dagger(T,\vec{x})\rangle$$

$$C_2^\pi(t;\vec{p}_\pi) = \sum_{\vec{x}} e^{i\vec{p}_\pi\cdot\vec{x}} \langle \mathcal{O}_\pi(t_{source},\vec{0}) \mathcal{O}_\pi^\dagger(t,\vec{x})\rangle$$

$$C_2^D(t) = \sum_{\vec{x}} \langle \mathcal{O}_D(t_{source},\vec{0}) \mathcal{O}_D^\dagger(t,\vec{x})\rangle$$

* D rest frame: $\vec{p}_\pi = (0,0,0),(1,0,0),(1,1,0),(1,1,1),(2,0,0)$.

* Randomizing spatial location of the sources $\rightarrow$ decreases autocorrelations.

* Smearing: D-`meson` interpolating operators are smeared with a $1S$ charmoniun wavefunction.

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* Randomizing spatial location of the sources $\rightarrow$ decreases autocorrelations.

* Smearing: D-`meson` interpolating operators are smeared with a $1S$ charmoniun wavefunction.

Build combinations of $C_3^{D\to\pi}(t,T)$ and $C_3^{D\to\pi}(t,T+1)$ to suppress contributions from opposite parity (staggered artifacts) and excited states

$\rightarrow$ fit to a plateau or plateau + dominant oscillating contamination

2.3 Chiral and continuum extrapolation

Partially-quenched heavy-meson staggered ChPT **Aubin & Bernard**,PRD76(2007)

* Use complete NLO + analytic NNLO expressions.

$$f_{\parallel} = \frac{c_0}{f_\pi}\left[1 + \mathrm{logs} + c_1 m_l + c_2 E_\pi + c_3 (E_\pi)^2 + c_4 a^2 + \mathrm{NNLO\,analy.\,terms}\right]$$

$$f_{\perp} = \frac{c_0' g_\pi}{f_\pi}\left\{\frac{1}{E_\pi + \Delta_l^* + \mathrm{logs}} + \frac{1}{E_\pi + \Delta_l^*}\left[\mathrm{logs} + c_1' m_l + c_2' E_\pi \right.\right.$$
$$\left.\left. + c_3' (E_\pi)^2 + c_4' a^2 + \mathrm{NNLO\,analy.\,terms}\right]\right\}$$

** No sea quark mass dependence is included (m_s^{sea} similar in all ensembles and $m_l^{val.} = m_l^{sea}$)

** No a^4 term is included (only two lattice spacings).

** logs are known non-analytical functions of $m_{K,\pi}$ containing dominant taste-breaking effects

$\rightarrow$ remove the dominant light discretization errors

2.3 Chiral and continuum extrapolation

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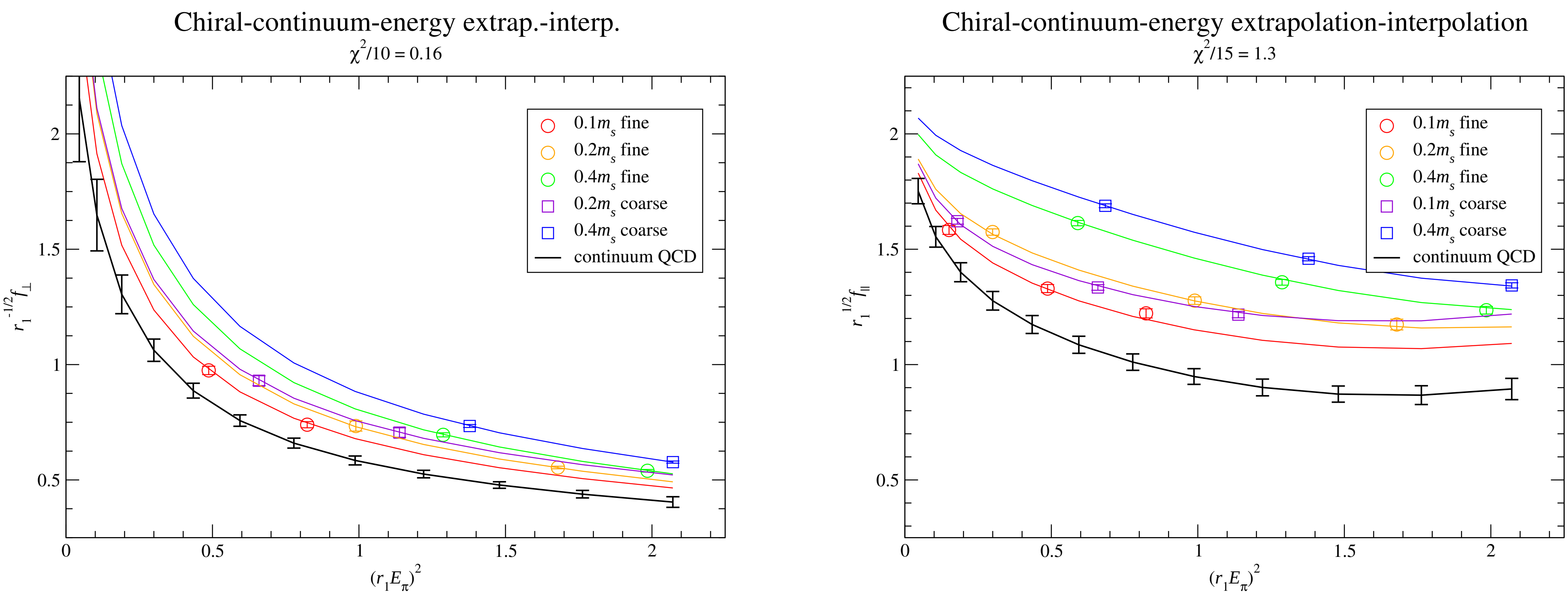
* Future plan: Include also heavy-quark discretization errors from the action and the current to improve errors estimates

  ** Corrections given by higher order operators in the HQET expansion. Known functions of am_c included in the fits with $\mathcal{O}(1)$ coefficients to be determined by the fit.

  $\rightarrow$ incorporates power counting fixing hadronic scales more systematically

2.3 Chiral and continuum extrapolation

Preliminary results



ChPT not reliable for $\chi_\pi = \frac{\sqrt{2} E_\pi}{4\pi f_\pi} > 1 \rightarrow \vec{p}_\pi = (1,1,1), (2,0,0)$ not used in the fits.

$\rho^{hl}_{V_\mu}$ factors not included yet in the renormalization.

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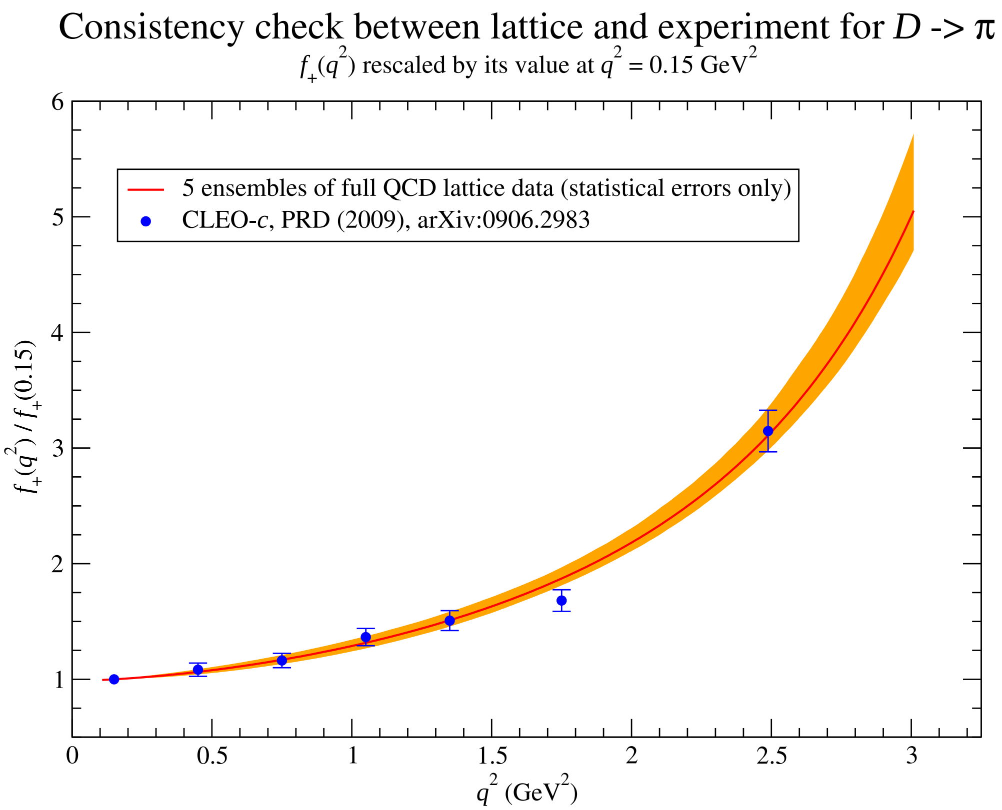

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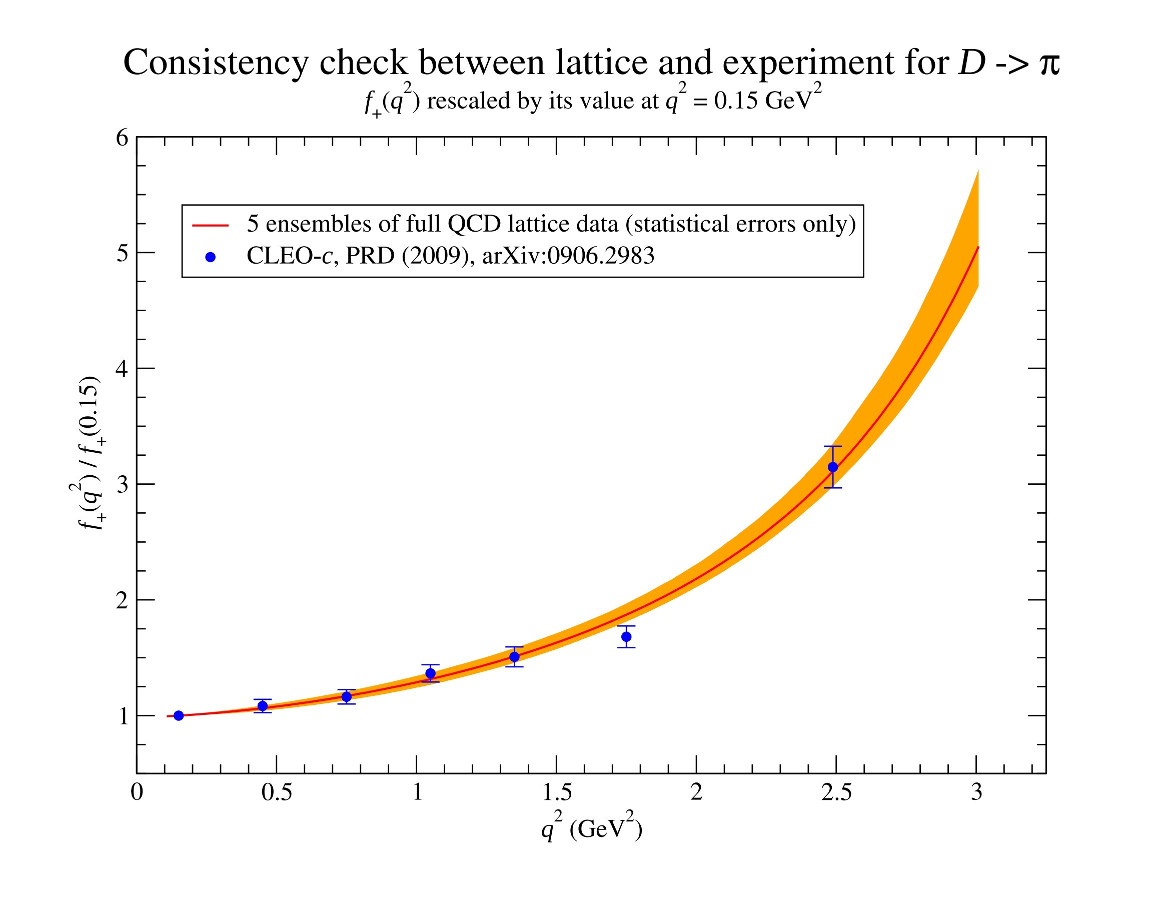

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Plan: Use $z-$**expansion** to extract $|V_{ud}|$ in a model-independent way from a simultaneous fit of lattice and experimental data over q^2

$z-$**expansion** based on unitarity, analyticity, crossing symmetry, and heavy-quark symmetry

3.1. $K \to \pi l \nu$: Methodology

HPQCD method for semileptonic decays (see **H. Na** talk)

* In the continuum, the Ward identity ($S = \bar{d}s$)

$$q^\mu \langle \pi | V_\mu^{cont.} | K \rangle = (m_s - m_q) \langle \pi | S^{cont} | K \rangle$$

relates matrix elements of vector and scalar currents.

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$$q^{\mu}\langle\pi|V_{\mu}^{lat.}|K\rangle Z = (m_s - m_q)\langle\pi|S^{lat.}|K\rangle$$

Using it together with

$$\langle\pi|V^{\mu}|K\rangle = f_{+}(q^2)\left[p_K^{\mu} + p_{\pi}^{\mu} - \frac{m_K^2 - m_{\pi}^2}{q^2}q^{\mu}\right] + f_0(q^2)\frac{m_K^2 - m_{\pi}^2}{q^2}q^{\mu}$$

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$$f_0(q^2) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle_{q^2} \Longrightarrow \boxed{f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2 = 0}}$$

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Advantages of the method:

* No need of renormalization factors Z.
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Downside: can get $f_+(q^2)$ only at $q^2 = 0$.

3.2. Test run: simulation details

Sea quarks: $N_f = 2 + 1$ **MILC** configurations with improved staggered Asqtad u, d and s sea quarks, and improved glue

Valence quarks: Hisq action **E. Follana et al, HPQCD coll.**, Phys.Rev.D75 (2007)

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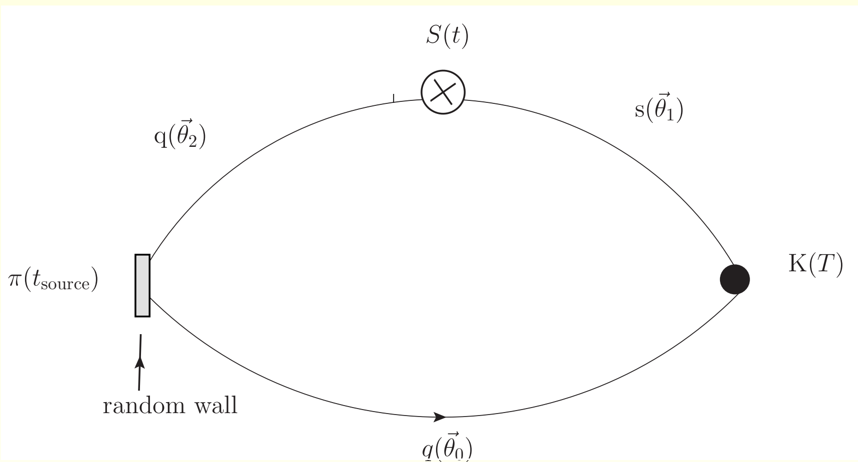

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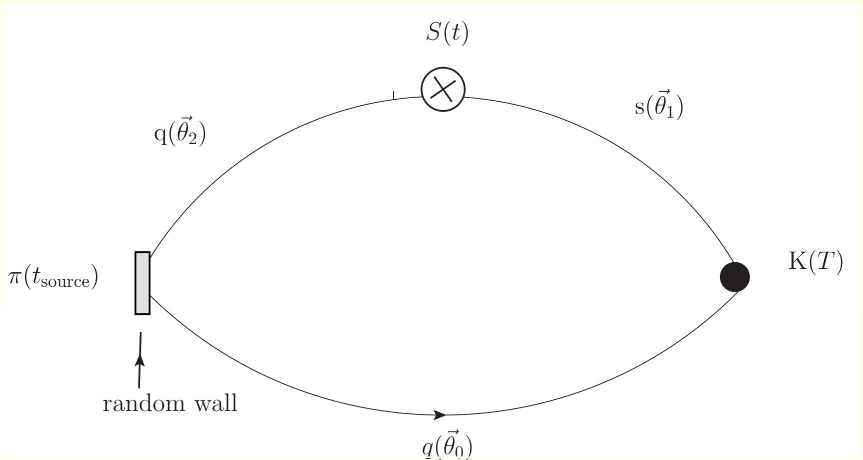

* Random wall sources: reduce stat. errors by a factor of 2-3.

* Twisted boundary conditions: allow to inject arbitrary momentum and simulate at $q^2 = 0$ directly.

** Momentum injected on the K: $\vec{\theta}_0 = \vec{\theta}_2 = 0$, $\vec{\theta}_1 \neq 0$ $\quad \vec{p}_K = \vec{\theta}_1 \frac{\pi}{L}$

** Momentum injected on the π: $\vec{\theta}_0 = \vec{\theta}_1 = 0$, $\vec{\theta}_2 \neq 0$ $\quad \vec{p}_\pi = \vec{\theta}_2 \frac{\pi}{L}$

3.2. Test run: simulation details

$\approx a$ (fm)	am_l^{sea}/am_s^{sea}	N_{conf}	$N_{sources}$	$am_s^{\rm valence}$	$am_l^{\rm valence}$
0.12	0.010/0.050	592	4	0.0546	0.010
0.09	0.0062/0.031	551	4	0.0366	0.0062

Generate $C_3^{K\to\pi}(t,T;\vec{p}_{\pi/K})$ for $T = 18 - 22$ for coarse lattice and $T = 27 - 30$ for fine lattice.

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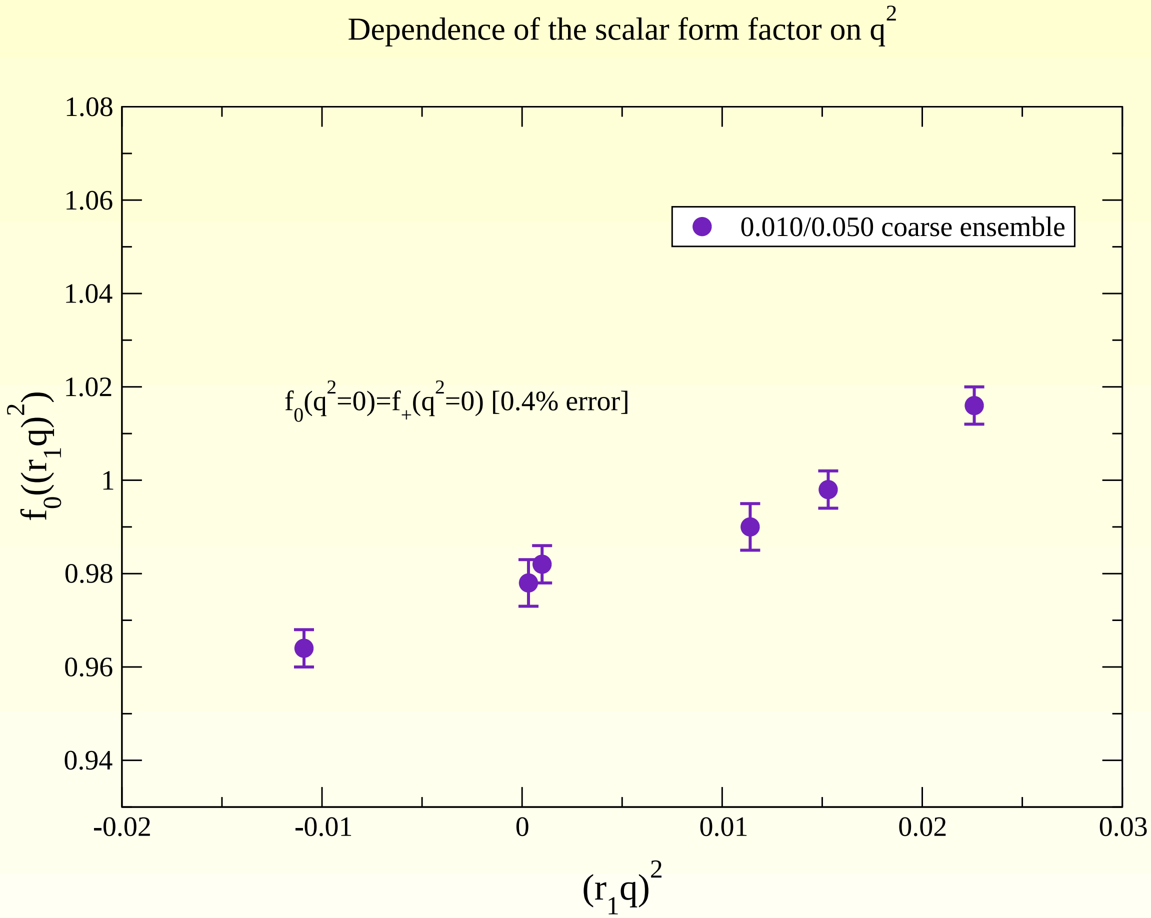
Generate $C_3^{K\to\pi}(t,T;\vec{p}_{\pi/K})$ for $T=18-22$ for coarse lattice and $T=27-30$ for fine lattice.

Fit combinations of 3-point functions with $T=19-20$ for coarse and $T=28-29$ for fine, together with corresponding 2-point π and K functions.

* Fit to ground state + dominant oscillating for central values of 3-point amplitudes.

* Checking stability: Fit to ground state and keeping dependence on oscillating and excited states (up to 4 exponentials).

3.2. Test run: simulating at $q^2 \sim 0$

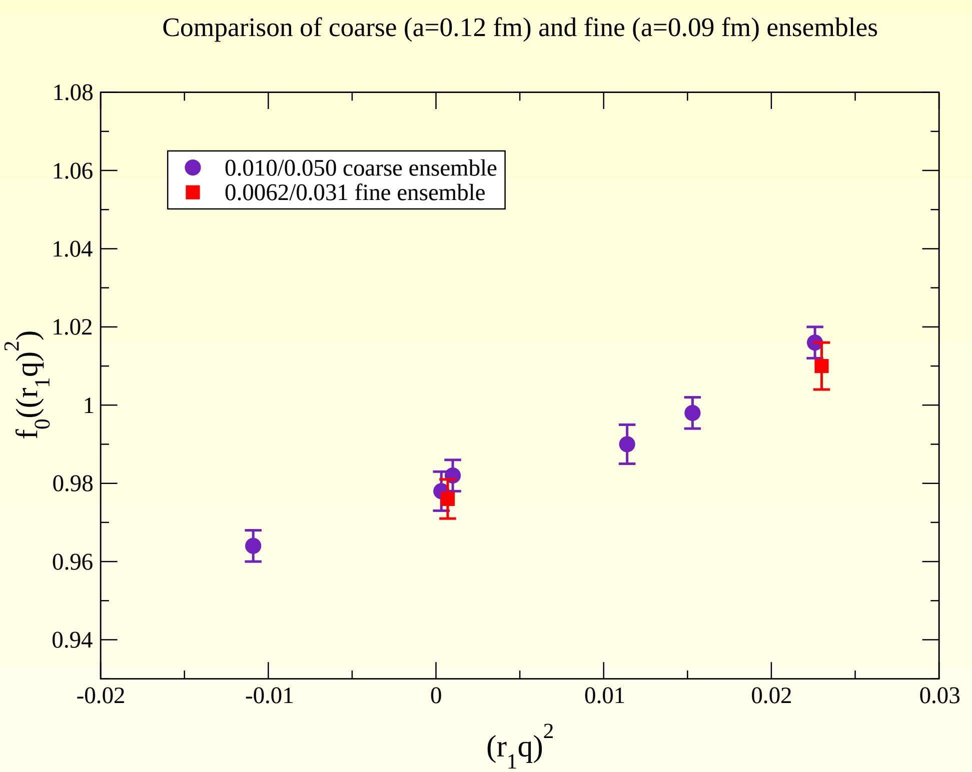


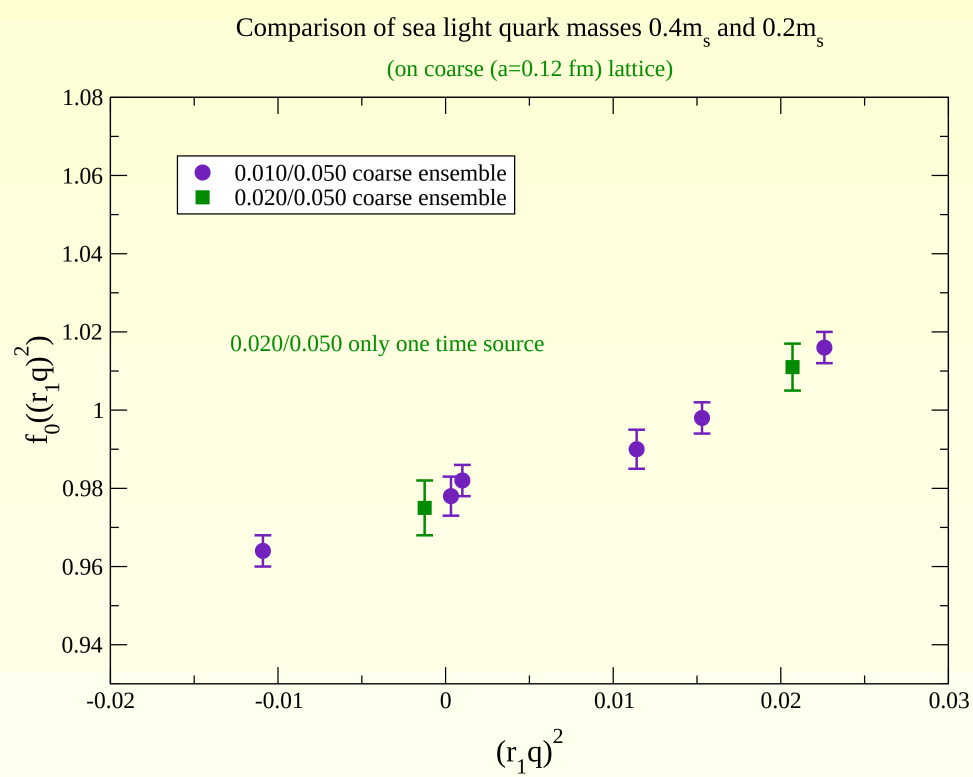
$\lvert\vec{\theta_1}\rvert$	$\lvert\vec{\theta_2}\rvert$	q^2
0	0	0.0227(3)
0	**0.7295**	**0.0011(4)**
0.7295	0	0.0153(3)
0	0.9105	-0.0109(5)
0.9105	0	0.0114(5)
1.2876	**0**	**0.0003(3)**

Goal: Eliminate uncertainty in q^2 interpolation (minimum value of $|(r_1 q)^2|$ available with periodic boundary conditions is $\simeq -0.104$)

$f_+(q^2 \simeq 0)$ calculated with $\sim 0.4\%$ statistical error (momentum in π). 4 times more configurations available! $\rightarrow$ 0.2-0.3% stat. error

3.3. Test run: discretization and sea quark mass effects





Discretization errors: negligible after extrapolation to the continuum.

Sea quark mass effects: negligible after extrapolation to the physical point.

3.4. Future plans: Hisq valence quarks

Use full MILC statistics for medium-coarse ($a \simeq 0.15\ fm$), coarse ($a \simeq 0.12\ fm$), and fine ($a \simeq 0.09\ fm$) ensembles.

* $N_{conf.} \simeq 2000$ in coarse, $N_{conf.} \simeq 1000 - 2000$ in fine, and $N_{conf.} \simeq 600$ in medium-coarse.

* At least 4 different sea quark masses for coarse, fine, and medium-coarse available.

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* At least 4 different sea quark masses for coarse, fine, and medium-coarse available.

Use (partially quenched) Staggered ChPT to extrapolate to the continuum and the physical light masses (valence strange mass tuned to the physical one)

* NLO ($\mathcal{O}(p^4)$) including taste-changing effects plus NNLO ($\mathcal{O}(p^6)$) continuum ChPT

3.4. Future plans: Hisq valence quarks

\# Use full MILC statistics for medium-coarse ($a \simeq 0.15\ fm$), coarse ($a \simeq 0.12\ fm$), and fine ($a \simeq 0.09\ fm$) ensembles.

* $N_{conf.} \simeq 2000$ in coarse, $N_{conf.} \simeq 1000 - 2000$ in fine, and $N_{conf.} \simeq 600$ in medium-coarse.

* At least 4 different sea quark masses for coarse, fine, and medium-coarse available.

\# Use (partially quenched) Staggered ChPT to extrapolate to the continuum and the physical light masses (valence strange mass tuned to the physical one)

* NLO ($\mathcal{O}(p^4)$) including taste-changing effects plus NNLO ($\mathcal{O}(p^6)$) continuum ChPT

\# **Statistitical erorrs**: 0.3-0.2%

\# **Analysis of systematic errors**:

* Chiral and continuum extrap.: one of the dominant uncertainties
* Tuning of bare quark masses
* Finite volume effects

4. Conclusions and future prospects

$\boxed{D \to \pi l \nu\text{:}}$ Complete analysis adding 2 coarse ensembles, 1 fine ensemble, and 1/2 superfine ensembles.

* Add $m_l^{val.} \neq m_l^{sea}$: will help especially with $f_{\|}$ extrapolation.

* Use `z-expansion` to combine lattice and experimental data.

* Updated treatment of correlations **Bernard et al.** PRD80 (2009).

* Incorporates known functional forms for higher order heavy-quark discretization effects in the chiral-continuum extrapolation.

4. Conclusions and future prospects

Projected error budget for $f_+(0)^{D\to\pi}$

source	error (%)	comments/improvements
$\mathrm{Stat.}+\chi\mathrm{PT}$	5	better when including $a = 0.06\ fm$, more m_l^{val}
$g_{D^*D\pi}$	2.2	$=$
r_1	1.5	f_π
$\hat{m}$	0.5	$m_\pi,\ m_K$
m_s	2.0	$=$
m_c	0.2	updated κ_c
HQ	3.9	better estimate from chir+cont fits
Z_V	0.7	statist. dominated
ρ	0.7	pert. error
$L^3 < \infty$	0.5	`ChPT`
Sys.	5.3	
Total	$\mathbf{7.3}$	(previous error 11% **Aubin et al.** PRL94(2005))

4. Conclusions and future prospects

\# $\boxed{D \to \pi l \nu\text{:}}$ Complete analysis adding 2 coarse ensembles, 1 fine ensemble, and 1/2 superfine ensembles.

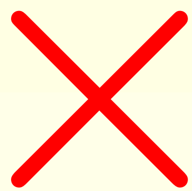
* Add $m_l^{val.} \neq m_l^{sea}$: will help especially with $f_{\|}$ extrapolation.

* Use `z-expansion` to combine lattice and experimental data.

* Updated treatment of correlations **Bernard et al.** PRD80 (2009).

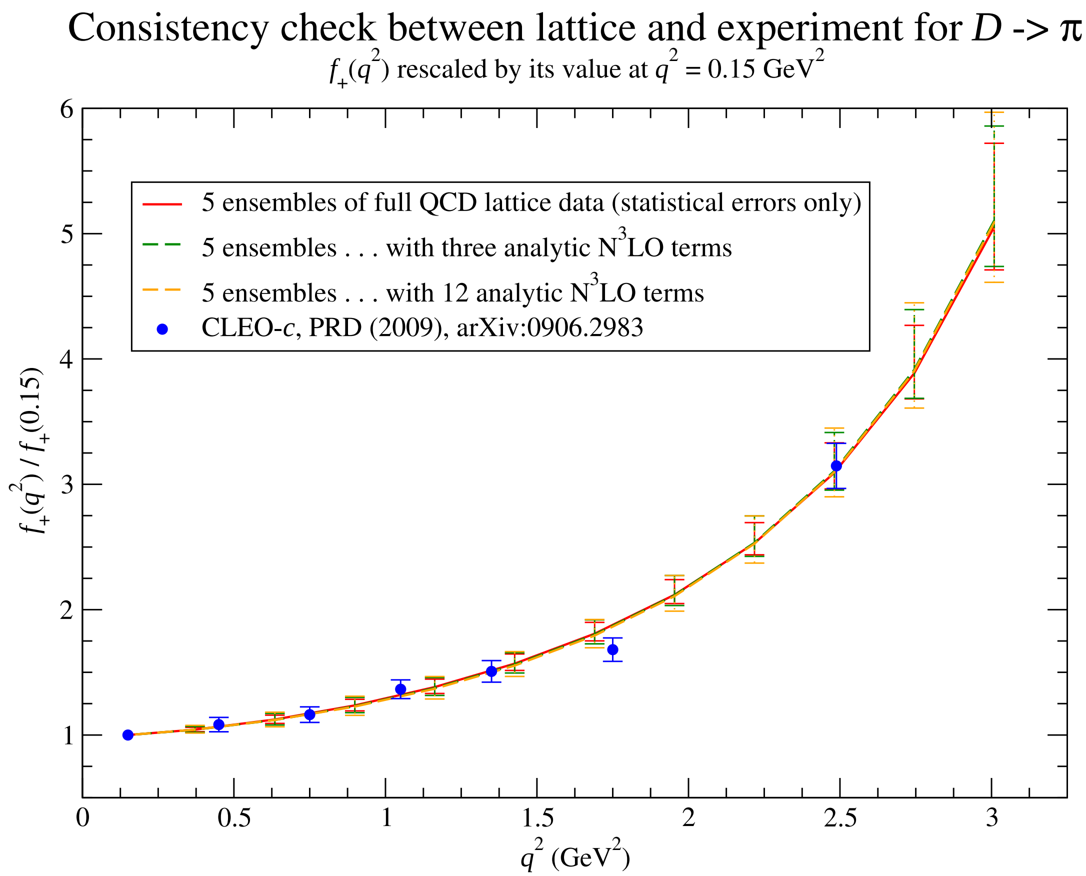
* Incorporates known functional forms for higher order heavy-quark discretization effects in the chiral-continuum extrapolation.

\# $\boxed{D \to K l \nu}$. Expect similar errors.



Iterative Super Average

$$
\begin{aligned}
\bar{C}_{3pt}^{K\to\pi}(t,T) &= \frac{e^{-E_\pi^{(0)}t}\, e^{-m_K^{(0)}(T-t)}}{8} \\
&\times \left[\frac{C_{3pt}^{K\to\pi}(t,T)}{e^{-E_\pi^{(0)}t}e^{-m_K^{(0)}(T-t)}} + \frac{C_{3pt}^{K\to\pi}(t,T+1)}{e^{-E_\pi^{(0)}(t)}e^{-m_K^{(0)}(T+1-t)}}\right. \\
&+ \frac{2\,C_{3pt}^{K\to\pi}(t+1,T)}{e^{-E_\pi^{(0)}(t+1)}e^{-m_K^{(0)}(T-t-1)}} + \frac{2\,C_{3pt}^{K\to\pi}(t+1,T+1)}{e^{-E_\pi^{(0)}(t+1)}e^{-m_K^{(0)}(T-t)}} \\
&+ \left.\frac{C_{3pt}^{K\to\pi}(t+2,T)}{e^{-E_\pi^{(0)}(t+2)}e^{-m_K^{(0)}(T-t-2)}} + \frac{C_{3pt}^{K\to\pi}(t+2,T+1)}{e^{-E_\pi^{(0)}(t+2)}e^{-m_K^{(0)}(T-t-1)}}\right] \\
&\approx A_\mu^{00} e^{-E_\pi^{(0)}t}\, e^{-m_K^{(0)}(T-t)} \\
&+ (-1)^T A_\mu^{11} e^{-E_\pi^{(1)}t} e^{-m_K^{(1)}(T-t)} \left(\frac{\Delta m_K}{2}\right) \\
&+ \mathcal{O}(\Delta E_\pi^2,\, \Delta E_\pi \Delta m_K,\, \Delta m_K^2)
\end{aligned}
$$



Consistency check between lattice and experiment for D -> π
$f_+(q^2)$ rescaled by its value at q^2 = 0.15 GeV^2
5 ensembles of full QCD lattice data (statistical errors only)
5 ensembles . . . with three analytic N^{3}LO terms
5 ensembles . . . with 12 analytic N^{3}LO terms
CLEO-c, PRD (2009), arXiv:0906.2983
$f_+(q^2)$ / $f_+(0.15)$
q^2 (GeV^2)
0
0.5
1
1.5
2
2.5
3
1
2
3
4
5
6

$K \to \pi l \nu$: extraction of $|V_{us}|$

$K \to \pi l \nu$: extraction of $|V_{us}|$

Look for new physics effects in the comparison of $|V_{us}|$ from helicity suppressed $K_{\mu 2}$ versus helicity allowed K_{l3}

$$R_{\mu 23} = \left(\frac{f_K / f_\pi}{f_+(0)} \right) \times \mathrm{experim.\,data\,on}\, K_{\mu 2} \pi_{\mu 2}\, \mathrm{and}\, K_{l3}$$

* In the SM $R_{\mu 23} = 1$. Not true for some BSM theories (for example, charged Higgs)

* Current value $R_{\mu 23} = 0.999(7)$, limited by lattice inputs.