EOS in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach

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QCD Equation of State on the lattice

Most studies done with staggerd-type quarks

- less computational costs
- a part of chiral sym. preserved ...

  → $N_f=2+1$, almost physical quark mass, $\mu \neq 0$

- 4th-root trick to remove unphysical "tastes"

  → non-locality "universality is not guaranteed"

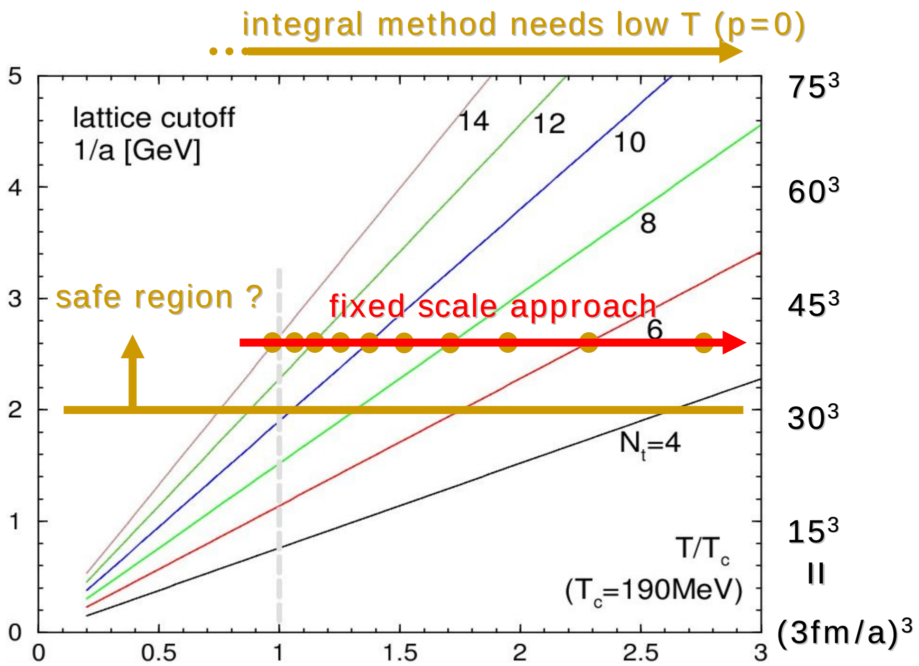
It is important to cross-check with
theoretically sound lattice quarks like Wilson-type quarks

> Our aim is to investigate
> QCD Thermodynamics with Wilson-type quarks

→ WHOT-QCD Collaboration

Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a



- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm.
  - larger 1/a in whole T region
- **Disadvantages**
  - T resolution by integer N_t
  - Statistics in lower T region
  - coding for odd N_t

T-integration method to calculate the EOS

We propose a new method ("T-integration method")
to calculate the EOS at fixed scales

T.Umeda et al. (WHOT-QCD), Phys.Rev.D79 (2009) 051501(R)

Our method is based on the trace anomaly (interaction measure),

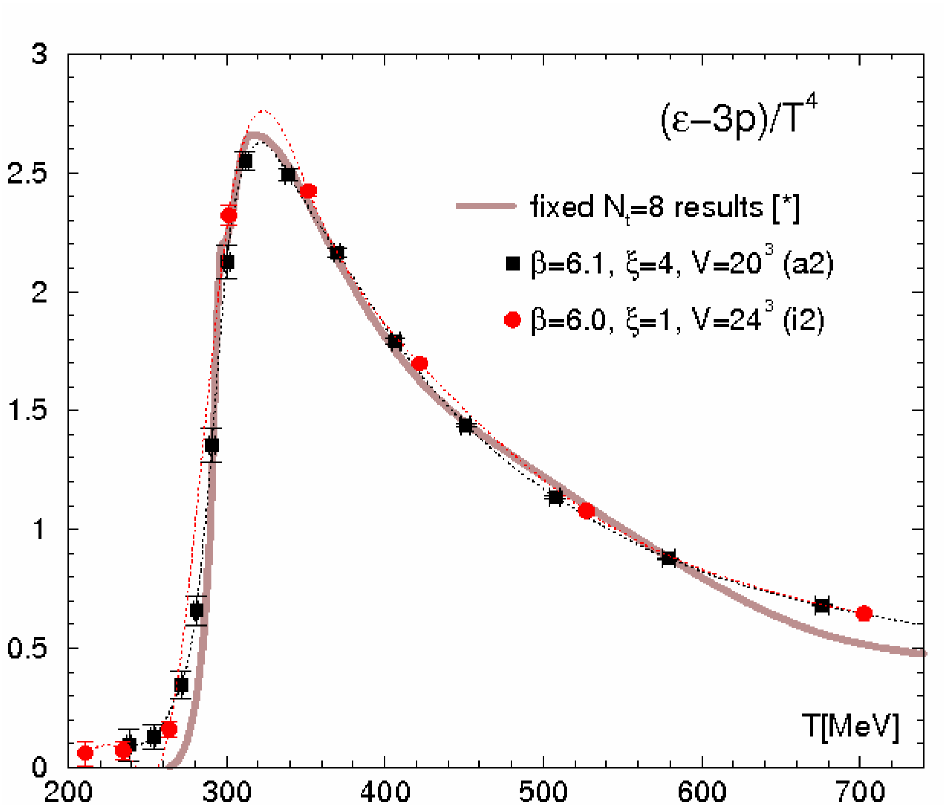
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a \frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

and the thermodynamic relation.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial (p/T^4)}{\partial T}$$

$$\Rightarrow \quad \frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon - 3p}{T'^5}$$

Test in quenched QCD



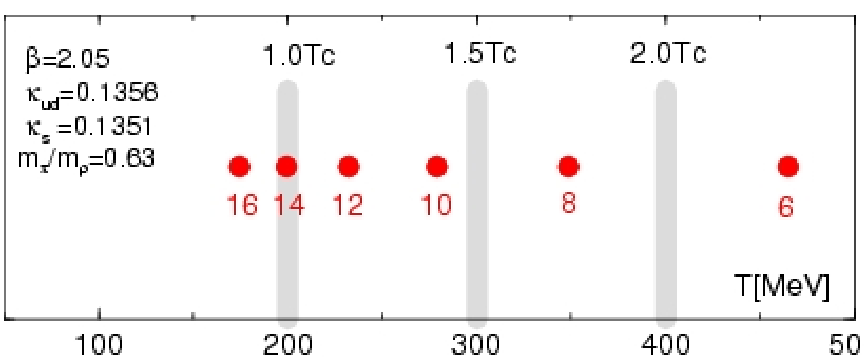
[*] G. Boyd et al., NPB469, 419 (1996)

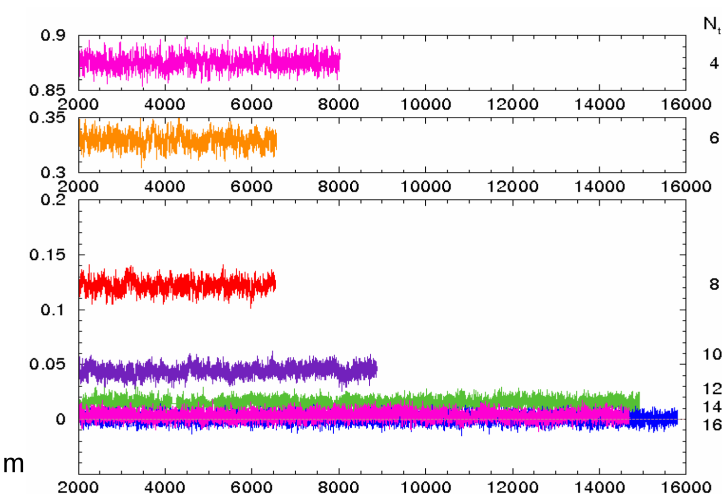
- Our results are roughly consistent with previous results.
- Our results deviate from the fixed N_t=8 results [*] at higher T (aT~0.3 or higher)
- Trace anomaly is sensitive to spatial volume at lower T (below T_c). $V \gtrsim (2fm)^3$ is ncessarry.

Lattice setup

- T=0 simulation: on $28^3 \times 56$ by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502*
  - RG-improved Iwasaki glue + NP-improved Wilson quarks
  - $\beta = 2.05$, $\kappa_{ud} = 0.1356$, $\kappa_s = 0.1351$
  - $V \sim (2\ \mathrm{fm})^3$, $a = 0.07$ fm, $\left(m_\pi \sim 634\mathrm{MeV},\ \frac{m_\pi}{m_\rho} = 0.63,\ \frac{m_{\eta_s s}}{m_\phi} = 0.74 \right)$
  - configurations available on the ILDG/JLDG
- T>0 simulations: on $32^3 \times N_t$ (N_t=4, 6, ..., 14, 16) lattices

RHMC algorithm, same parameters as T=0 simulation





Formulation for N_f=2+1 improved Wilson quarks

$$S = S_g + S_q$$

$$S_g = -\beta \left\{ \sum_{x,\mu>\nu} c_0 W^{1\times 1}_{\mu\nu}(x) + \sum_{x,\mu,\nu} c_1 W^{1\times 2}_{\mu\nu}(x) \right\}$$

$$S_q = \sum_{f=u,d,s} \sum_{x,y} \bar{q}^f_x D_{x,y} q^f_y$$

$$\beta = \frac{6}{g^2}$$

$$D_{x,y} = \delta_{x,y} - \kappa_f \sum_{\mu} \{(1-\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}\} - \delta_{x,y} c_{SW} \kappa_f \sum_{\mu>\nu} \sigma_{\mu\nu} F_{\mu\nu}$$

$$c_{SW}(\beta) = 1 + 0.113 g^2 + 0.0209 g^4 + 0.0049 g^6$$

Phys. Rev. D73, 034501
CP-PACS/JLQCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a\frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a\frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a\frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right)$$

$$\left\langle \frac{\partial S}{\partial \beta} \right\rangle = N_s^3 N_t \left(- \left\langle \sum_{x,\mu>\nu} c_0 W^{1\times 1}_{\mu\nu}(x) + \sum_{x,\mu,\nu} c_1 W^{1\times 2}_{\mu\nu}(x) \right\rangle + N_f \frac{\partial c_{SW}}{\partial \beta} \kappa_f \left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right)$$

$$\left\langle \frac{\partial S}{\partial \kappa_f} \right\rangle = N_f N_s^3 N_t \left(\left\langle \sum_{x,\mu} \mathrm{Tr}^{(c,s)} \{(1-\gamma_\mu)U_{x,\mu}(D^{-1})_{x+\hat{\mu},x} + (1+\gamma_\mu)U^\dagger_{x-\hat{\mu},\mu}(D^{-1})_{x-\hat{\mu},x}\} \right\rangle + c_{SW} \left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right)$$

Noise method (#noise = 1 for each color & spin indices)

Beta-functions from CP-PACS/JLQCD results

Trace anomaly needs Beta-functions in $N_f=2+1$ QCD

$$\frac{\epsilon-3p}{T^4}=\frac{N_t^3}{N_s^3}\left(a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S}{\partial\beta}\right\rangle_{sub}+a\frac{\partial\kappa_{ud}}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_{ud}}\right\rangle_{sub}+a\frac{\partial\kappa_s}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_s}\right\rangle_{sub}\right)$$

Direct fit method   Phys. Rev. D64 (2001) 074510

fit β, κ_{ud}, κ_s as functions of (am_ρ), $\left(\frac{m_\pi}{m_\rho}\right)$, $\left(\frac{m_{\eta ss}}{m_\phi}\right)$

$$\begin{pmatrix}\beta\\ \kappa_L\\ \kappa_S\end{pmatrix}=\vec{c}_1+\vec{c}_2(am_\rho)+\vec{c}_3(am_\rho)^2+\vec{c}_4\left(\frac{m_\pi}{m_\rho}\right)+\vec{c}_5\left(\frac{m_\pi}{m_\rho}\right)^2+\vec{c}_6(am_\rho)\left(\frac{m_\pi}{m_\rho}\right)$$
$$+\vec{c}_7\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)+\vec{c}_8\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2+\vec{c}_9(am_\rho)\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)+\vec{c}_{10}\left(\frac{m_\pi}{m_\rho}\right)\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$$

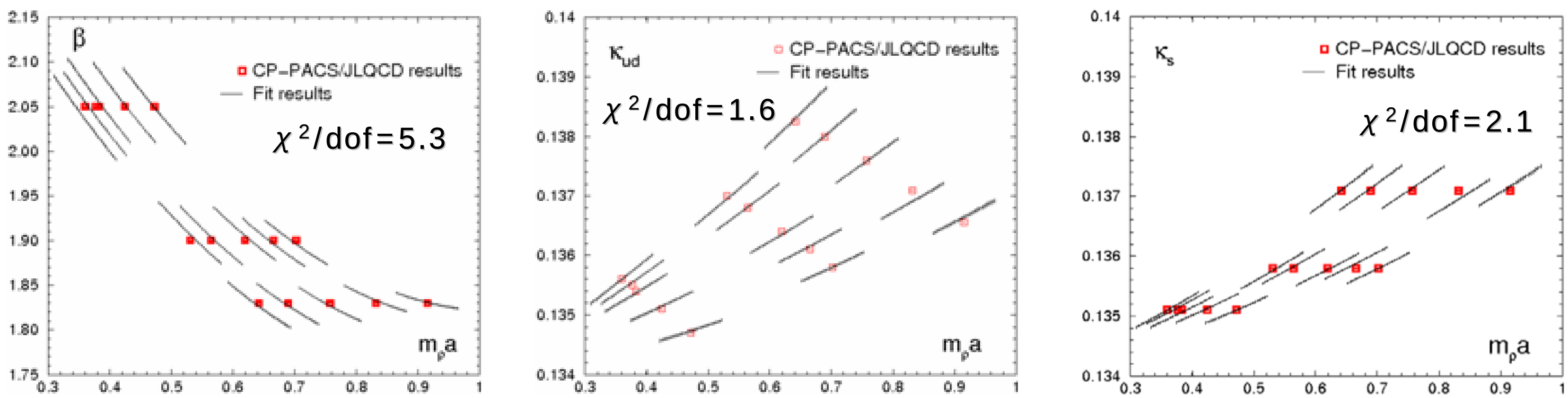
$$am_\rho\frac{\partial X}{\partial(am_\rho)}$$ with fixed $\left(\frac{m_\pi}{m_\rho}\right)$, $\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$ $(X=\beta,\kappa_{ud},\kappa_s)$

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502.*

3 (β) x 5 (κ_{ud}) x 2 (κ_s) = 30 data points

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$



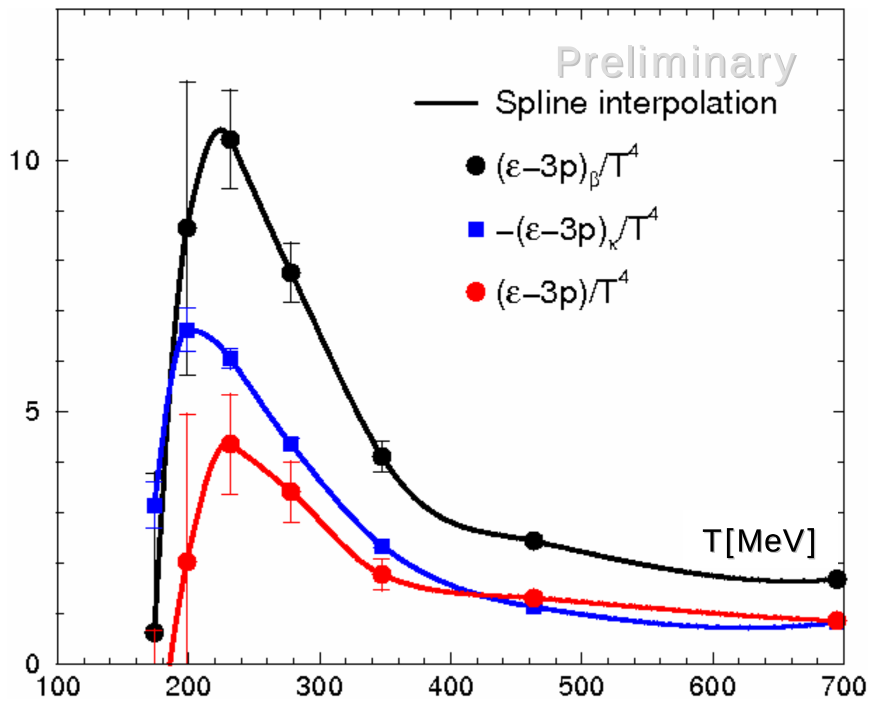
$$\left(a\frac{\partial\beta}{\partial a},\ a\frac{\partial\kappa_{ud}}{\partial a},\ a\frac{\partial\kappa_s}{\partial a} \right)_{\text{simulation point}}$$

$$= (-0.334(4),\ 0.00289(6),\ 0.00203(5))$$

only statistical error

Trace anomaly in N_f=2+1 QCD

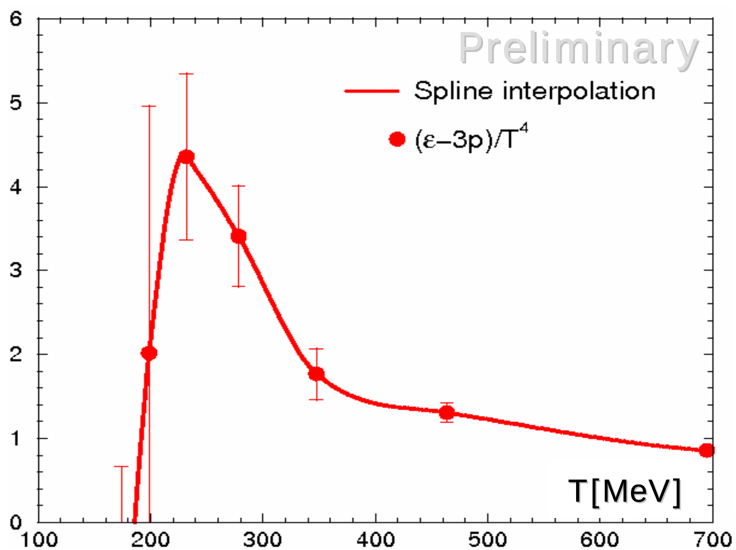
$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3}\left(\underbrace{a\frac{\partial \beta}{\partial a}\left\langle \frac{\partial S}{\partial \beta}\right\rangle_{sub}}_{(\epsilon-3p)_\beta/T^4} + \underbrace{a\frac{\partial \kappa_{ud}}{\partial a}\left\langle \frac{\partial S}{\partial \kappa_{ud}}\right\rangle_{sub} + a\frac{\partial \kappa_s}{\partial a}\left\langle \frac{\partial S}{\partial \kappa_s}\right\rangle_{sub}}_{(\epsilon-3p)_\kappa/T^4} \right) \qquad S = S_g + S_q$$



Nt	config. (x 5MD traj.) S_g	S_q
56	1300(*)	143
16	1542	208
14	1448	192
12	1492	336
10	863	240
8	628	320
6	657	160
4	802	143

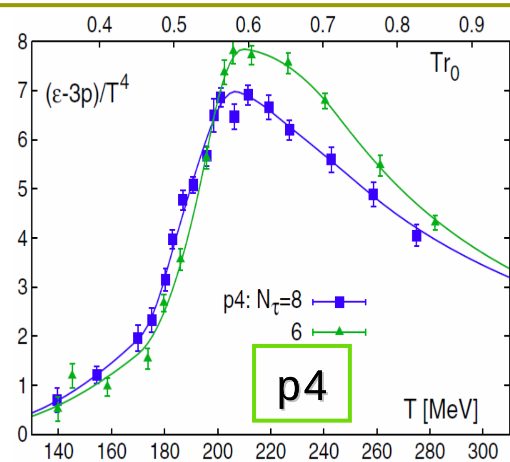
(*) T=0 (Nt=56) by CP-PACS/JLQCD
S_g calculated with 6500traj.

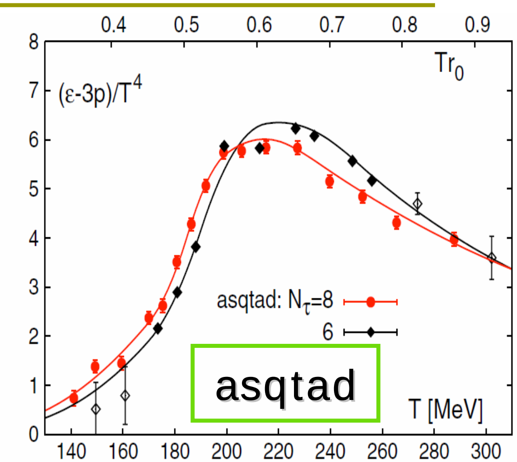
Trace anomaly in N_f=2+1 QCD



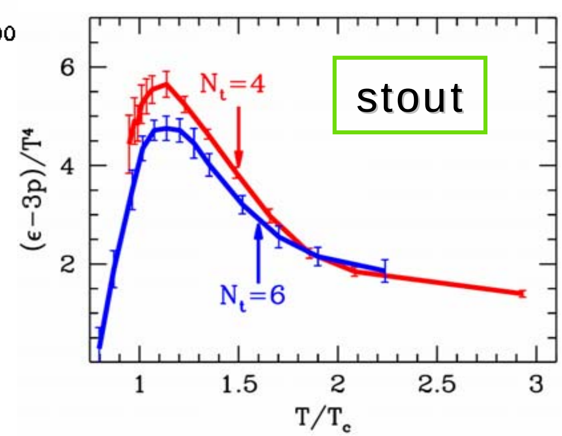
- peak height ~ 5-7
  in KS results
  (m_q ~ $m_q^{phys.}$)

- peak height → small
  as N_t → large

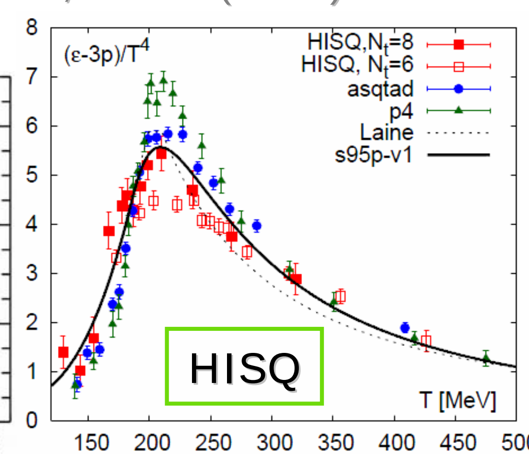




HotQCD PRD80,014504(2009)

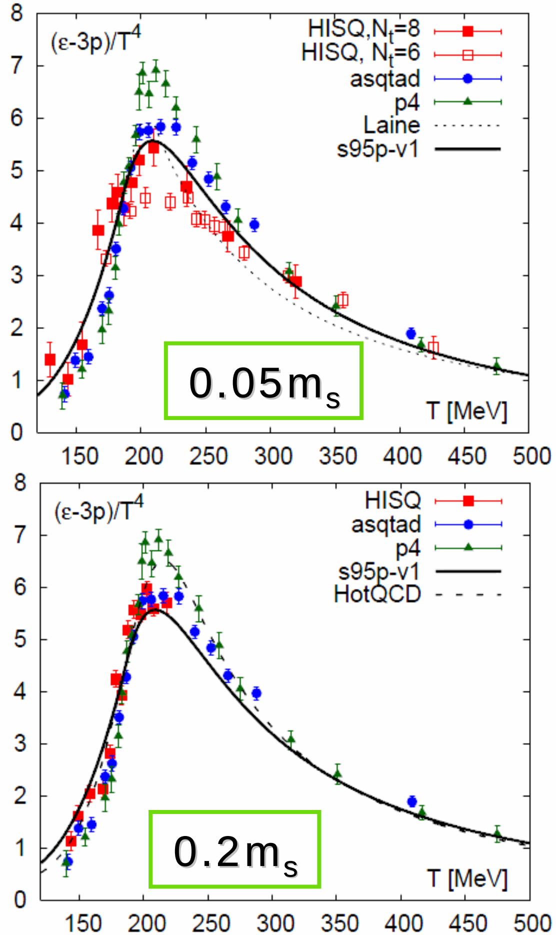


Aoki et al. JHEP01,089(2006)

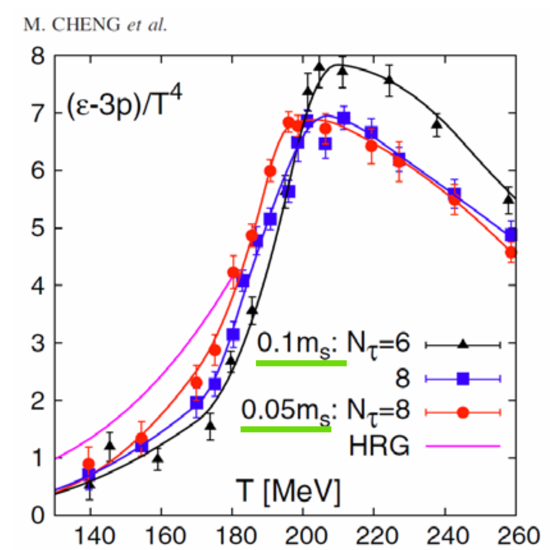


HotQCD arXiv1005.1131

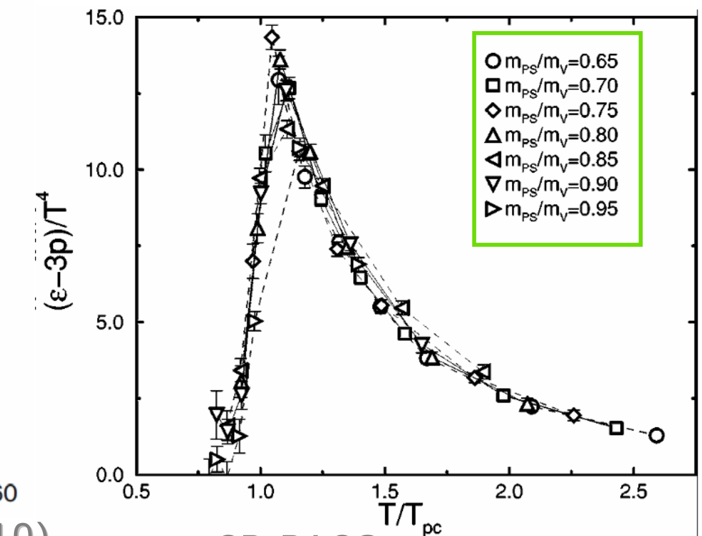
Quark mass dependence of Trace anomaly



HotQCD arXiv1005.1131



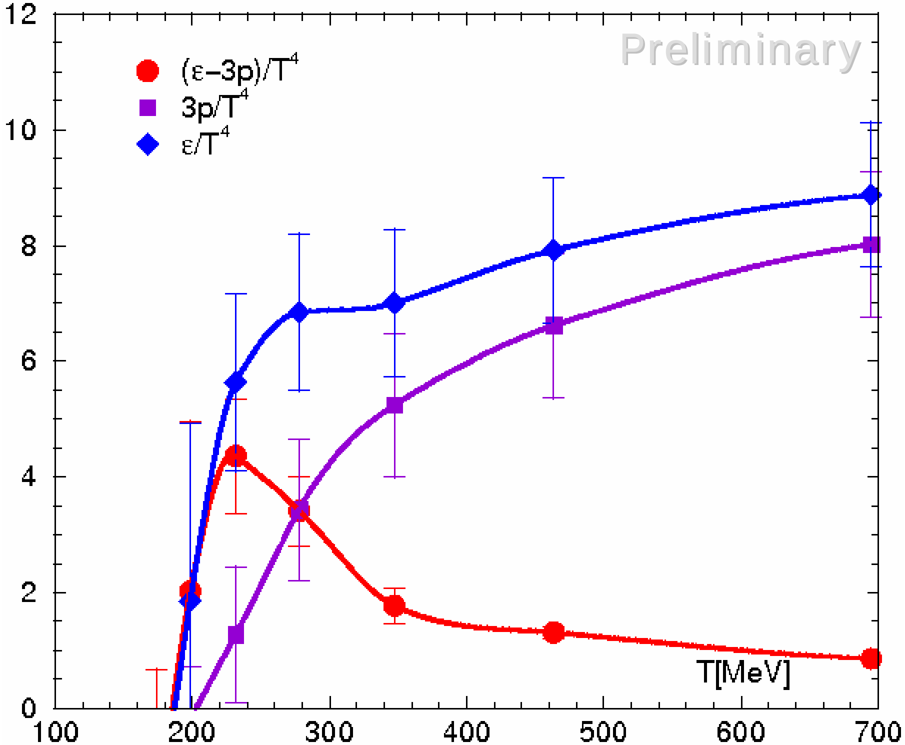
HotQCD PRD91,054504(2010)



CP-PACS PRD64,074510 (2001)

- peak height of the Trace anomaly
  → small quark mass dependence
- peak position shifts slightly.
- Our result seems to be reasonable !

Equation of State in N_f=2+1 QCD



- T-integration

$$\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}$$

  is performed by the trapezoidal rule.

- ε / T^4 is calculated from

$$\frac{\epsilon}{T^4} = \frac{\epsilon - 3p}{T^4} + \frac{3p}{T^4}$$

- Large error in whole T region

Summary & outlook

We reported on EOS in N_f=2+1 QCD using improve Wilson quarks

- Beta functions

  More work needed

  Reweighting method ?

- Equation of state

  More statistics needed in the lower temperature region

- N_f=2+1 QCD just at the physical point

  the physical point (pion mass ~ 140MeV) by PACS-CS

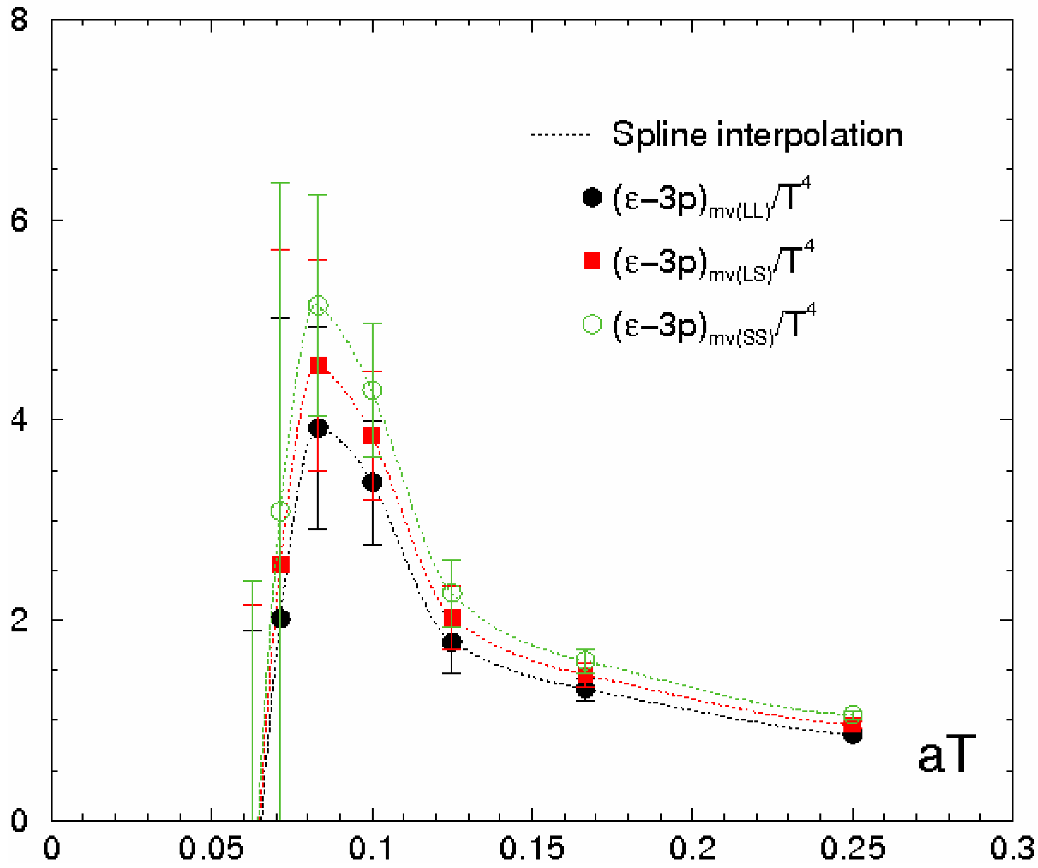
- Finite density

  We can combine our approach with the Taylor expansion method,

  to explore EOS at $\mu \neq 0$

Back-up slides

A systemtic error



To calc. Beta-functions

fit β , κ_{ud}, κ_s as functions of

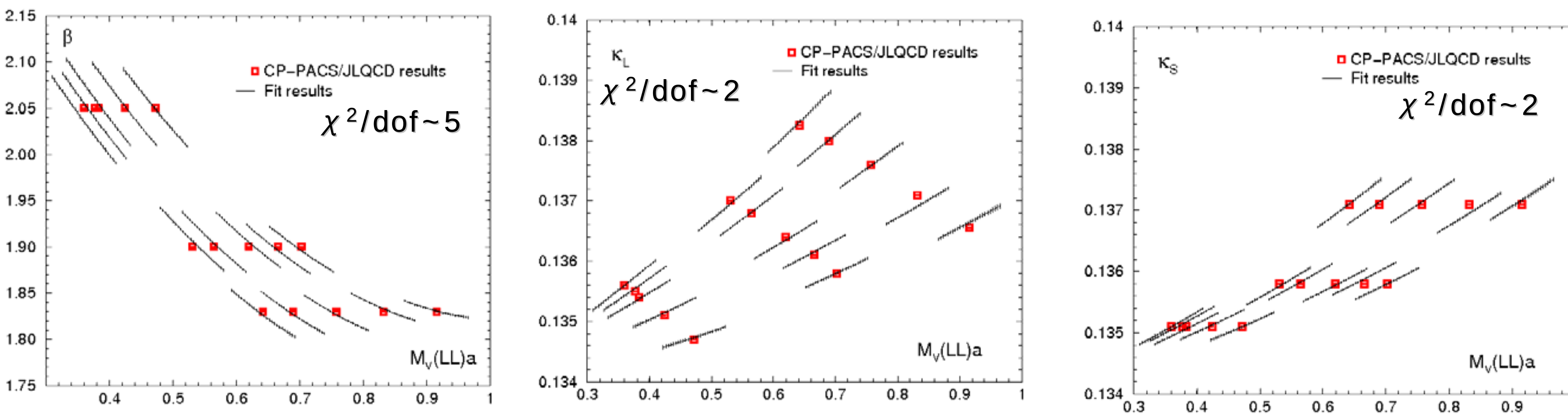
$$(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_\rho \frac{\partial X}{\partial(am_\rho)}$$

$$(am_{K^*}), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_{K^*} \frac{\partial X}{\partial(am_{K^*})}$$

$$(am_\phi), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_\phi \frac{\partial X}{\partial(am_\phi)}$$

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502.*



fit β, κ_{ud}, κ_s as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$

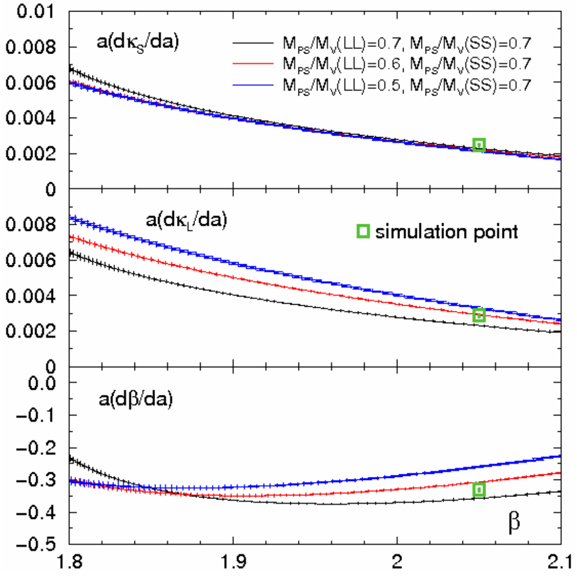
$$\left(a\frac{\partial\beta}{\partial a},\ a\frac{\partial\kappa_{ud}}{\partial a},\ a\frac{\partial\kappa_s}{\partial a} \right)_{\text{simulation point}}$$

$$= (-0.330(3), 0.00288(5), 0.00247(5)) \quad m_\rho$$

$$= (-0.340(3), 0.00286(5), 0.00242(5)) \quad m_{K^*}$$

$$= (-0.345(3), 0.00285(5), 0.00242(5)) \quad m_\phi$$

Beta-functions from CP-PACS/JLQCD results



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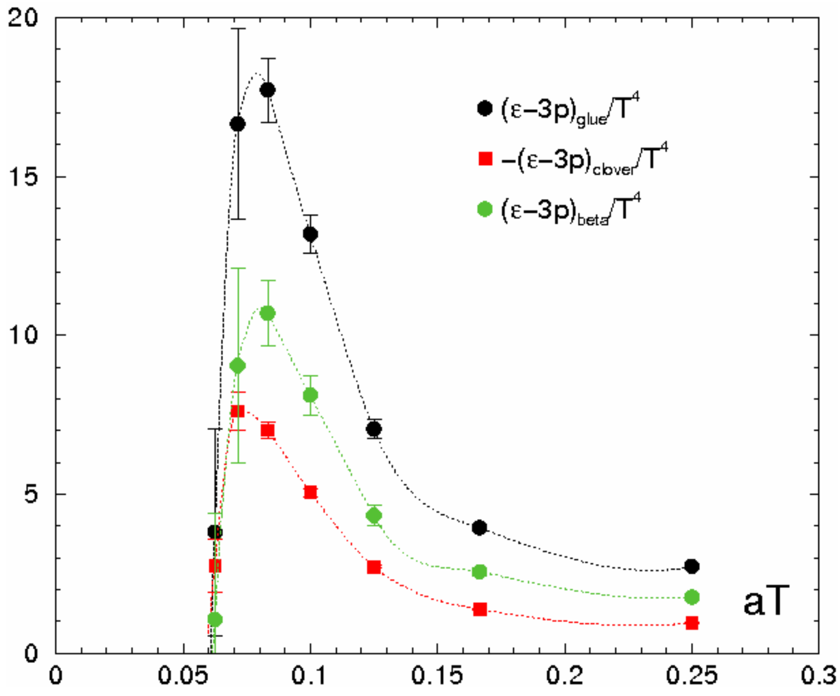
$$= (-0.345(3), 0.00285(5), 0.00242(5)) \quad m_\phi$$

Trace anomaly (β & κ derivative part)

$$\frac{\epsilon-3p}{T^4}=\frac{N_t^3}{N_s^3}\left(a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S}{\partial\beta}\right\rangle_{sub}+a\frac{\partial\kappa_{ud}}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_{ud}}\right\rangle_{sub}+a\frac{\partial\kappa_s}{\partial a}\left\langle\frac{\partial S}{\partial\kappa_s}\right\rangle_{sub}\right)$$

$$(\epsilon-3p)_\beta/T^4=(\epsilon-3p)_{glue}/T^4+(\epsilon-3p)_{clover}/T^4$$



$$(\epsilon-3p)_\kappa/T^4=(\epsilon-3p)_{\kappa_{ud}}/T^4+(\epsilon-3p)_{\kappa_s}/T^4$$