The continuum limit of 2+1 flavor DWF ensembles

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Introduction

- Previous calculations of $24^3 \times 64$ ensembles-
  $1^{\rm st}$ $2+1f$ dynamical fermion results for
  -f_π, f_K, quark masses [arXiv:0804.0473],
  -B_K [arXiv:hep-ph/0702042].
- Errors dominated by 4% discretisation error, e.g.

$$\begin{array}{lclllll} B_K & = & 0.524 & (10)_{\rm stat} & (21)_{a^2} & (19)_{\rm other\,sys} & \\ f_\pi & = & 124 & (4)_{\rm stat} & (5)_{a^2} & (5)_{\rm other\,sys} & \rm MeV \end{array}$$

- RBC&UKQCD now have simulations at 2 lattice spacings.
- Continuum limit results now possible.

Lattice Overview

$24^3 \times 64,\ L_s = 16,\ \beta = 2.13\,(a^{-1} \approx 1.73\ \text{GeV})$

- 2 ensembles (each ~ 200 configs): $am_l = 0.005, 0.01$.
- Fixed $am_h = 0.04$.
- Lightest unitary $m_\pi \sim 330\text{MeV}$, $m_\pi L \sim 4.6$
- Datasets over doubled in size since PRL! (Argonne BG/P via SciDAC)
- Reweighting $m_h^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) \sim 90 \ldots 110$ MeV

$32^3 \times 64,\ L_s = 16,\ \beta = 2.25\,(a^{-1} \approx 2.28\ \text{GeV})$

- 3 ensembles (each ~ 300 configs): $am_l = 0.004, 0.006, 0.008$.
- Fixed $am_h = 0.03$.
- Lightest unitary $m_\pi \sim 290\text{MeV}$, $m_\pi L \sim 4.1$
- Reweighting $m_h^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) \sim 90 \ldots 110$ MeV

The ideal scaling trajectory

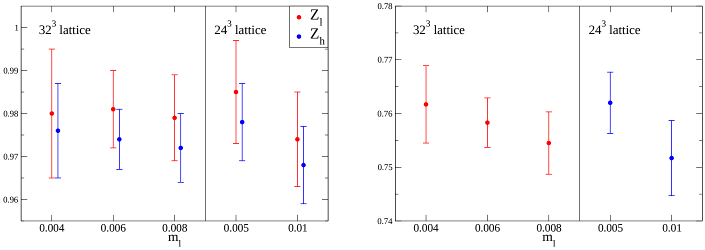
- Must define *scaling curve* - $m_{u/d}(\beta)$, $m_s(\beta)$.
- Scaling curve not unique, differ by $\mathcal{O}(a^2)$.
- Ideally choose $m_{u/d}(\beta)$, $m_s(\beta)$ s.t. m_π / m_Ω and m_K / m_Ω equal phys. values.
- Back in the real world - more precise to 'match' ensembles at simulated masses.
- Decouples ensemble matching from mass extrapolation and experimental input.
- Details [arXiv:0911.1309].

Matching at simulated masses

- CK. presented at Lat2009.
- Find scaling curve $m_l(\beta)$, $m_h(\beta)$ that passes through simulated data.
- Achieve by finding quark masses s.t. $m_\pi^{ll}/m_\Omega^{hhh}$ and m_K^{lh}/m_Ω^{hhh} same at both βs.
- Use m_Ω^{hhh} to find ratio of scales: $R_a = \frac{(am_\Omega^{hhh})(32)}{(am_\Omega^{hhh})(24)}$
- From quark masses at match point form: $Z_l = \frac{1}{R_a}\frac{(a\tilde{m}_l)(32)}{(a\tilde{m}_l)(24)}$, $Z_h = \frac{1}{R_a}\frac{(a\tilde{m}_h)(32)}{(a\tilde{m}_h)(24)}$
- $\tilde{m} = m + m_{\mathrm{res}}$ is DWF PCAC mass.

What does this mean in practise?

- Defines $m_l(\beta)$, $m_h(\beta)$ s.t. m_π^{ll}, m_K^{hl} and m_Ω^{hhh} at these unphysical masses are artefact free - no cutoff dependence.
- No formal guarantee about other quark masses away from match point.
- Double expansion in a^2 and $\delta m_q = m_q - m_{u/d}(\beta)$.
- Scaling imperfection at nearby masses $\propto \delta m_q \times a^2 \leftarrow$ ignore.
- Numerical evidence:



Chiral/continuum extrapolation

- Use Z_l and Z_h to relate 24^3 quark masses to equivalent 32^3 masses.
- Combined chiral/continuum fit over both latt.
- Double expansion in a^2, $\tilde{m}_q$ - truncate 2^{nd} order terms:

$$A + Ba^2 + C\tilde{m}_q + \cancel{D\tilde{m}_q a^2 + \ldots}$$

- Simultaneously fit m_π, m_K, f_π, f_K and m_Ω (B_K separate)
- Match onto continuum:
  Find physical m_l, m_h and latt. spacings s.t. predicted m_π, m_K and m_Ω match physical values.
- Use $R_a = a(32)/a(24)$ from matching analysis.
- Investigate multiple chiral ansatze.

ChPT ansatze

- NLO $SU(2)$ PQChPT for chiral extrapolation - NLO $SU(3)$ not good description of 24^3 data.
- Couple kaons as independent heavy fields.
- Add a^2 term for f_π and f_K, eg.

$$f_{ll} \quad = \quad f\left[1 + c_f a^2\right] + f \cdot \left\{ \frac{8}{f^2}(2l_4 + l_5)\chi_l - \frac{\chi_l}{8\pi^2 f^2} \log \frac{\chi_l}{\Lambda_\chi^2} \right\}$$

- $\chi_l = 2Bm_l$.
- Fit form for m_π and m_K standard.
- Linear form for m_Ω:

$$m_{hhh} \quad = \quad m^{(\Omega)} + m^{(\Omega)}\, c_{m_\Omega, m_l}\, \chi_l \,.$$

- Also use finite-volume PQChPT to estimate FV errors.

Analytic ansatz

- Also investigate analytic ansatz - Taylor expansion about unphysical quark mass, keep linear terms:

$$m_{xy}^2 = C_0^{m_\pi} + C_1^{m_\pi} \left(\frac{1}{2}(\tilde{m}_x + \tilde{m}_y) - \tilde{m}^m \right) + C_2^{m_\pi} (\tilde{m}_l - \tilde{m}^m) \,,$$

- Rewrite as

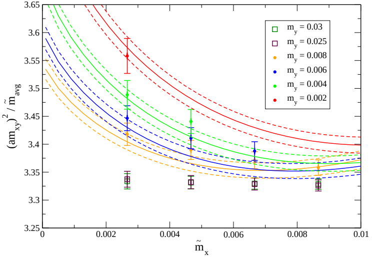
$$m_{xy}^2 = C_0^{m_\pi} + \frac{1}{2} C_1^{m_\pi} (\tilde{m}_x + \tilde{m}_y) + C_2^{m_\pi} \tilde{m}_l \,.$$

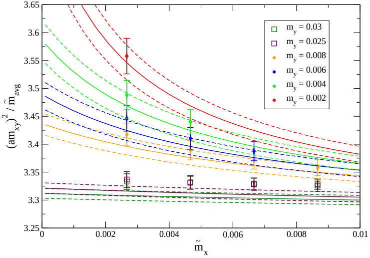
- Similarly for scale-dependent f_π:

$$f_{xy} = C_0^{f_\pi} [1 + C_f a^2] + \frac{1}{2} C_1^{f_\pi} (\tilde{m}_x + \tilde{m}_y) + C_2^{f_\pi} \tilde{m}_l \,.$$

m_π partially-quenched fits

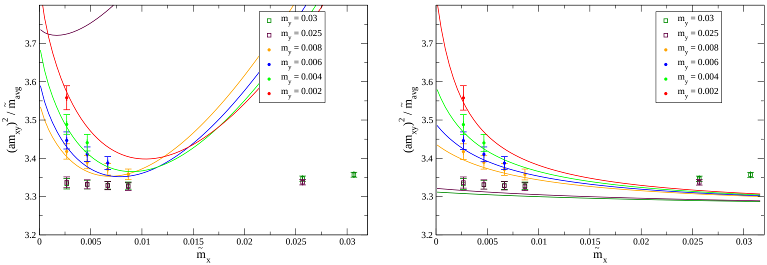
- Partially-quenched $2m_{xy}^2/(\tilde{m}_x + \tilde{m}_y)$,
  32^3 $m_l = 0.004$ ensemble: ChPT (left), analytic(right).





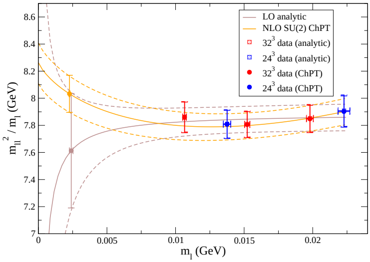
- Traditional plot format for enhancing chiral curvature.
- Note: Non-linearity in analytic plot is artefact of plot format.
- Linear analytic fits describe data well.

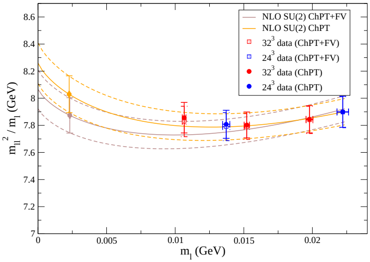
m_π partially-quenched fits - II



- Analytic appears to describe data over larger range.
- ChPT always seems to break down just beyond $\tilde{m}_x$ mass cut.
- Goldstone's theorem
  $\Rightarrow \quad m_\pi = 0$ when valence quarks massless,
  $\Rightarrow \quad C_2^{m_\pi} = 0$
- Fit gives $C_2^{m_\pi} = 0.43(8)$.
- This is OK but indicates curvature <u>must</u> appear somewhere in PQ direction.

m_π continuum extrapolation





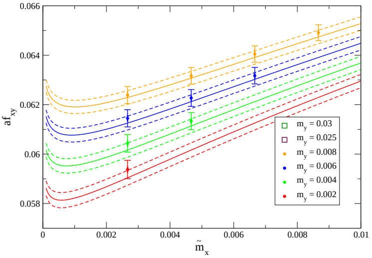
- Small finite-volume effects.
- Analytic error blows up as allow small constant term in fit form.
- $C_0^{m_\pi} = -0.001(1)$ *is* consistent with Goldstone's theorem.
- Don't even need curvature at present precision.

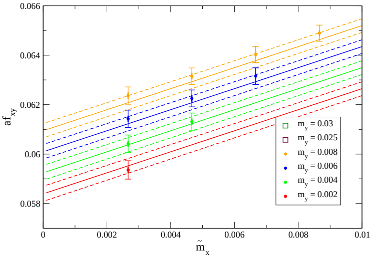
Requirement for chiral curvature

- Goldstone's theorem in unitary direction satisfied, need no χ-curvature.
- Not satisfied in PQ direction - does need χ-curvature.
- Consistency not necessary for extracting quantities at physical quark masses.
- Not clear where breakdown of analytic behaviour occurs.

f_π partially-quenched fits

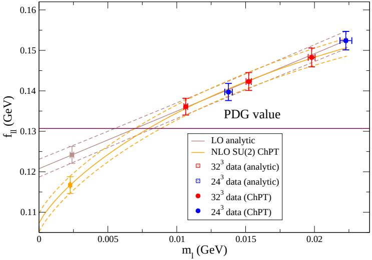
- Partially-quenched f_{xy},
  32^3 $m_l = 0.004$ ensemble: ChPT (left), analytic(right).

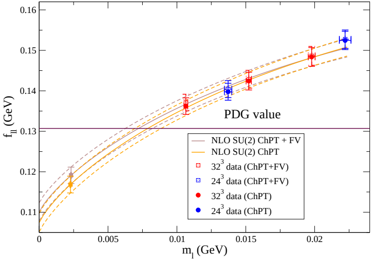




- No evidence for chiral curvature in data.
- But no inconsistency between data and chiral form.

f_π continuum extrapolation

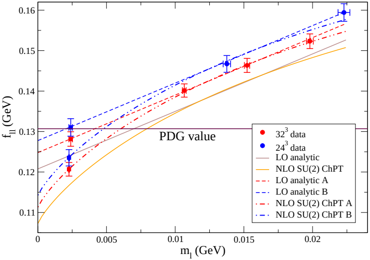




- ChPT too low by $\sim 12\%$.
- FV effects $\sim 2\%$.
- Estimate NNLO effects: $\text{NLO}^2 \sim 5 - 15\%$. Size consistent with discrepancy
- Try NNLO - introduction of significant model dependence necessary to fit data.

f_π continuum extrapolation - II

- Analytic borderline consistent $\sim 3-4\%$.
- Analytic extrapolations of data on both latt. consistent with physical point:



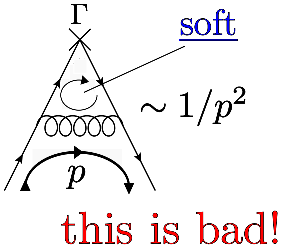
- Continuum extrapolation pulls result down.
- ChPT finite-β extrapolations not consistent with physical point.

Predictions

- Chiral curvature must exist somewhere, but less than NLO ChPT implies for fits to present mass range.
- Need lighter quark masses
  - see R.Mawhinney's talk about new ensembles.
- Chiral extrapolation systematic as diff. of analytic and ChPT-fv.
- Allows for possibility of χ-curvature above physical point.
- Take central value as average of analytic and ChPT+FV results.
- Use ChPT+FV - ChPT to estimate FV effects.
- $f\pi = 122(2)_{\rm stat}(5)_\chi(2)_{\rm FV}$ MeV.
- $f_K = 147(2)_{\rm stat}(4)_\chi(1)_{\rm FV}$ MeV.
- $f_K/f_\pi = 1.208(8)_{\rm stat}(23)_\chi(14)_{\rm FV}$ MeV.

Non-perturbative renormalisation

- Quark masses and B_K need renormalisation.
- Previously used RI/MOM NPR scheme.
- 'Exceptional kinematics'- Enhances χSB at large p^2:



- Now use non-exceptional $p_{in} \neq p_{out}$ kinematics.
- 'Symmetric' $p_{in}^2 = p_{out}^2 = q^2$: RI/SMOM schemes.
- Use volume sources
  - greatly improved stat. error.
  - reduces localised source systematics.
- Investigate several SMOM schemes: better estimate truncation err.

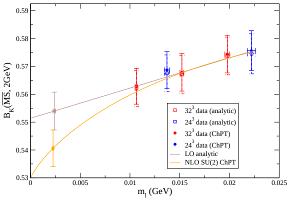
Quark masses

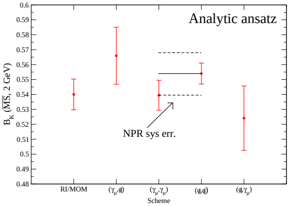
- Quark masses already in scheme where $Z_{24} = Z_l$, $Z_{32} = 1$.
- Extrapolate Z_m/Z_l to continuum, renormalise quark masses in continuum limit.
- Sys. error breakdown similar to B_K - discuss shortly.
- Results:

$$m_{u/d}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 3.65(20)_{\mathrm{stat}}(13)_{\mathrm{sys}}(8)_{\mathrm{ren}}\,\mathrm{MeV}$$
$$m_{s}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 97.3(1.4)_{\mathrm{stat}}(0.2)_{\mathrm{sys}}(2.1)_{\mathrm{ren}}\,\mathrm{MeV}$$

B_K

- Renormalise B_K in RI/MOM and 4 RI/SMOM schemes - fit to renormalised data. Example:





- Z_{B_K} systematics:
  - *spread* - $O(4)$ symmetry breaking under discretisation.
  - *slope* - truncation of perturbative expansion -dominant.
  - χSB - residual χ-symmetry breaking.
  - m_s - non-zero strange mass.
- $SMOM(\not{q}, \not{q})$ best described by PT [arXiv:1006.0422] - has smallest slope err.
- NPR sys err. from $SMOM(\not{q}, \not{q}) - SMOM(\gamma^\mu, \gamma^\mu)$.
- $B_K(\overline{\mathrm{MS}}, 2\,\mathrm{GeV}) = 0.546(7)_{\mathrm{stat+spread}}(16)_\chi(3)_{FV}(14)_{\mathrm{ren}}$.

Summary and Outlook

- Simultaneous chiral/continuum extrapolation with 2 βs.
- Alternative scaling trajectory running through simulated data.
- Analytic, $SU(2)$ ChPT(+FV) chiral ansatze.
- Analytic describes data well, but χ-curvature needed in PQ direction - χ-extrap. sys. error to encompass both.
- Quark masses and B_K NPR using multiple RI/SMOM schemes with non-exceptional kinematics.
- NPR error dominated by PT truncation.

Immediate future:

- Third sim. using DSDR action:
  - $\Rightarrow$ Additional lattice spacing in cont. extrap.
  - $\Rightarrow$ Lighter pions - beat down χ-systematic.
- NPR with Twisted-BCs:
  - $\Rightarrow$ Remove $O(4)$-breaking systematic.
  - $\Rightarrow$ Better estimate of PT-truncation errs.