N_F = 3 Critical Point from Canonical Ensemble

- Finite Density Algorithm with Canonical Approach and Winding Number Expansion
- Update on N_F = 4, and 3 with Clover Fermion
- New Algorithm Aiming at Large Volumes

χ QCD Collaboration:
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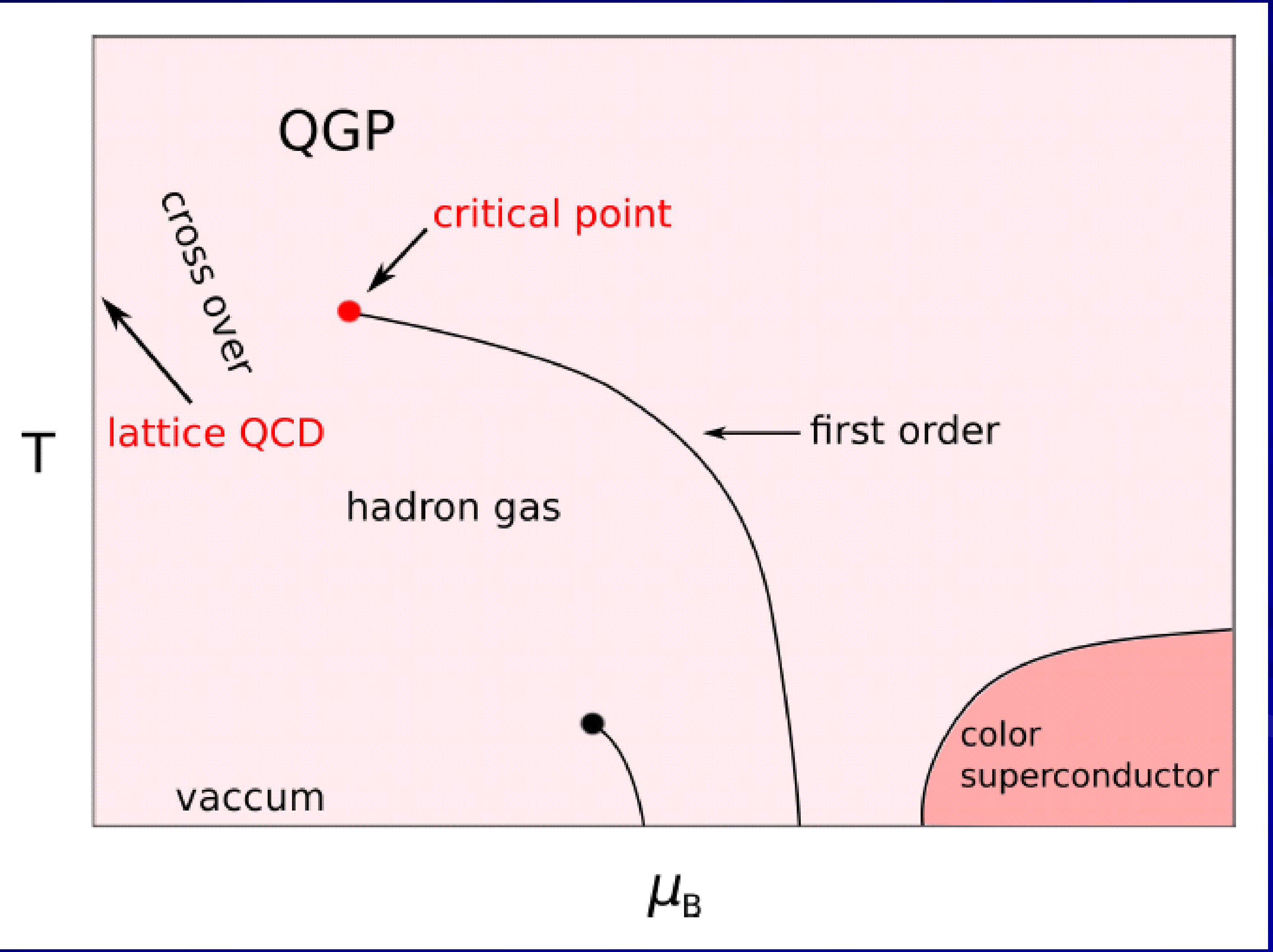
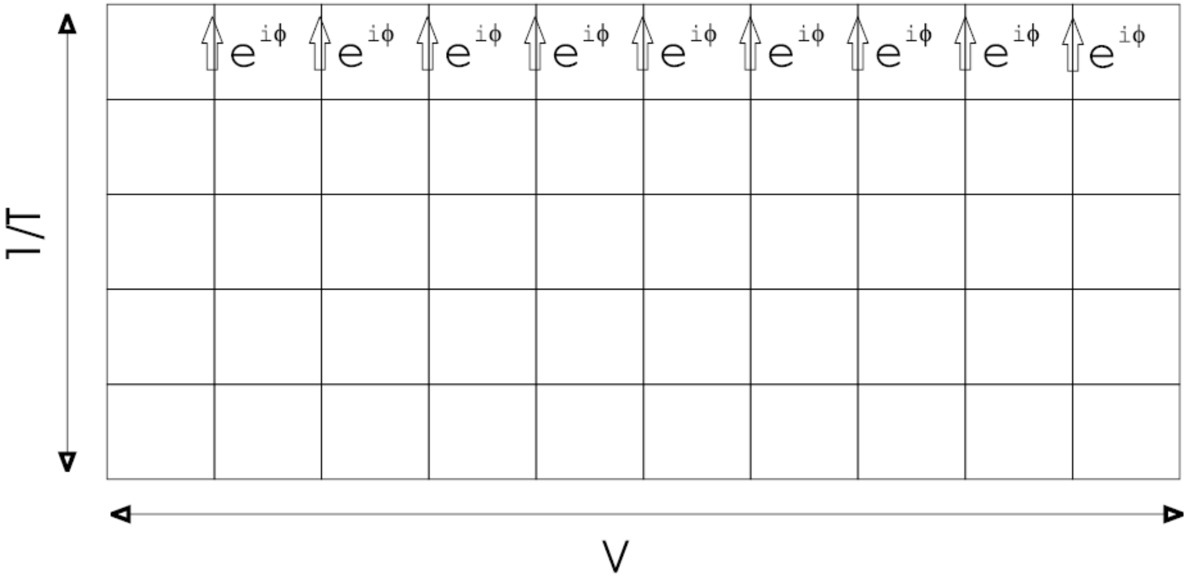
A Conjectured Phase Diagram



Canonical partition function



Using the fugacity expansion $Z_{GC}(V,\mu,T) = \sum_{k=-4V}^{k=4V} Z_C(V,k,T)e^{\frac{\mu}{T}k}$ we get

$$Z_C(V,k,T) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-ik\varphi} Z_{GC}(V,\mu = i\varphi T, T)$$

Canonical approach

K. F. Liu, *QCD and Numerical Analysis* Vol. III (Springer,New York, 2005),p. 101.
Andrei Alexandru, Manfried Faber, Ivan Horva´th,Keh-Fei Liu, *PRD* 72, 114513 (2005)

Canonical ensembles

$$Z_C(V,T,k) = \int \mathcal{D}\mathrm{U}\, e^{-S_g(U)} \widetilde{\det}_k M^2(U) =$$

$$\underbrace{\int DU e^{-S_g(U)} \det M^2(U)}_{\text{Standard HMC}} \underbrace{\frac{|\mathrm{Re}\,\widetilde{\det}_k M^2(U)|}{\det M^2(U)}}_{\text{Accept/Reject}} \underbrace{\frac{\widetilde{\det}_k M^2(U)}{|\mathrm{Re}\,\widetilde{\det}_k M^2(U)|}}_{\text{Phase}}$$

Fourier transform

$$\widetilde{\det}_k M^2(U) \equiv \frac{1}{2\pi}\int d\phi\, e^{-ik\phi} \det M^2(U_\phi)$$

Real due to C or T, or CH

$$\det M^2(U_\phi) = e^{2log \det M(U_\phi)}$$

$$log \det M(U_\phi)$$

WNEM → Continues Fourier transform
Useful for large k

Winding number expansion (I)

In QCD

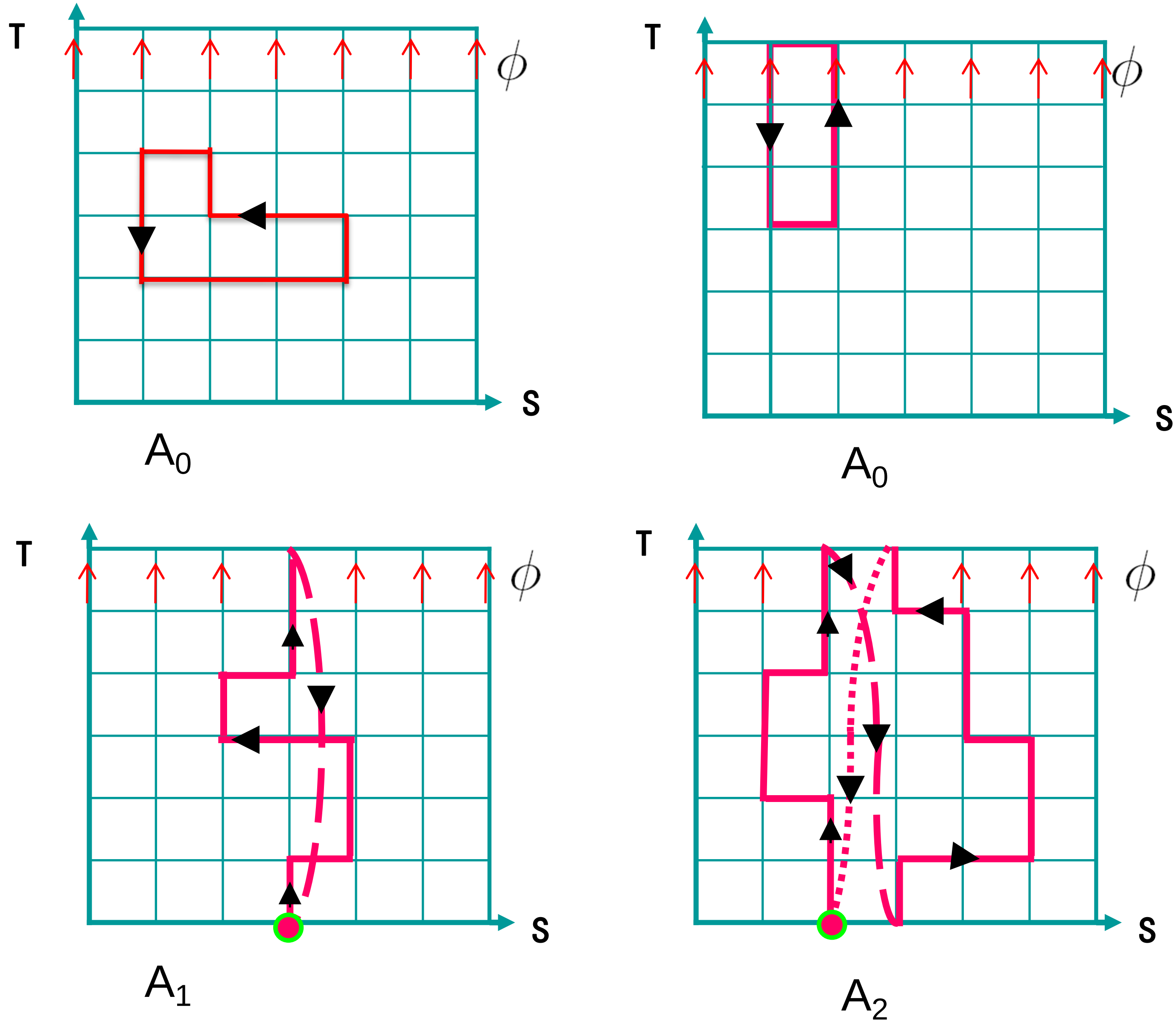
Tr log $\longrightarrow$ loop $\longrightarrow$ loop expansion

In particle number space

$$\begin{aligned} TrlogM(U,\phi) &= A_0(U) + \Sigma loop(U,\phi) \\ &= A_0(U) + [\sum_k e^{ik\phi} W_k(U) + e^{-ik\phi} W_k^\dagger(U)] \end{aligned}$$

Where $W_n = \frac{1}{2\pi}\int_0^{2\pi} e^{-ik\phi} trlogM(U,\phi)$

$$\begin{aligned} TrlogM(U,\phi) &= A_0(U) + [\sum_k e^{ik\phi} W_k(U) + e^{-ik\phi} W_k^\dagger(U)] \\ &= A_0(U) + \sum_k A_k \cos(k\phi + \delta_k)|_{A_k = 2|W_k|, \delta_k = \delta_{W_k}} \end{aligned}$$


T
S
ϕ
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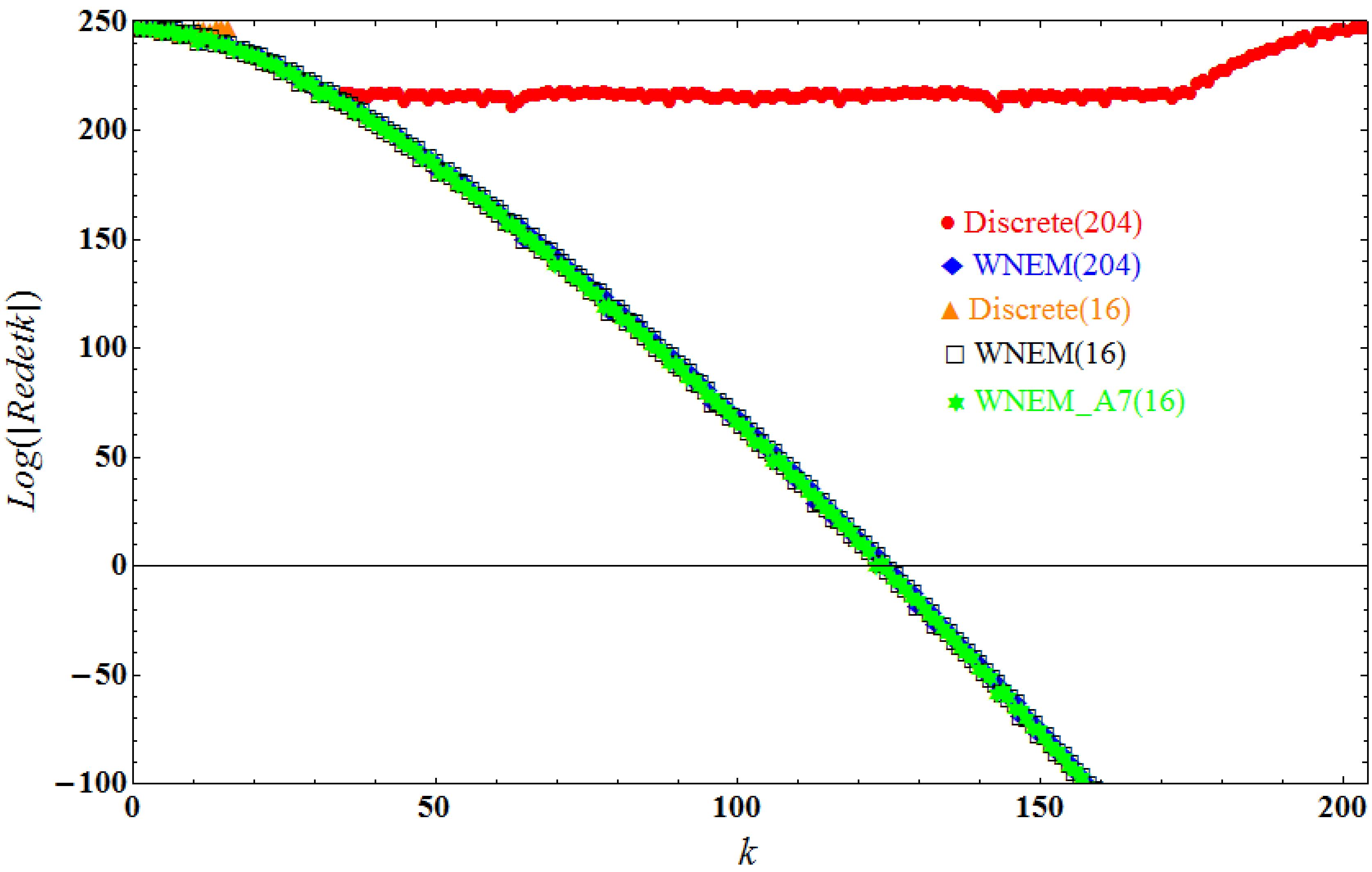
A_2



250
200
150
100
50
0
−50
−100
Log(|Redetk|)
0
50
100
150
200
k
Discrete(204)
WNEM(204)
Discrete(16)
WNEM(16)
WNEM_A7(16)

Observables

Polyakov loop

$$\langle|P|\rangle_{k'} = \frac{\langle R(U,k')|P(U)|\rangle_0}{\langle R(U,k')\rangle_0} \qquad R(U,k') = \frac{\widetilde{\det}_k M^2(U)}{|\mathrm{Re}\,\widetilde{\det}_k M^2(U)|}$$

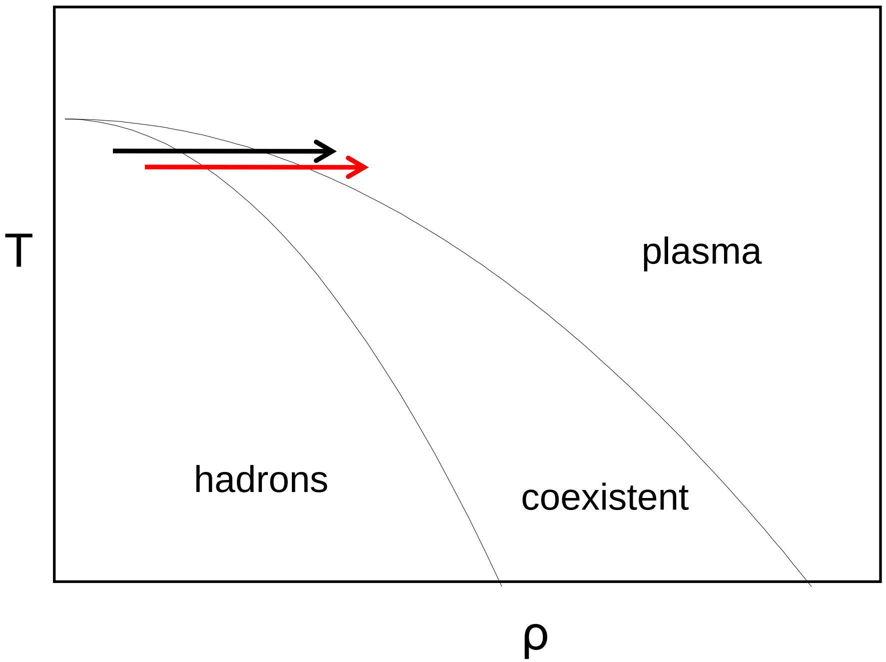
Phase

Baryon chemical potential

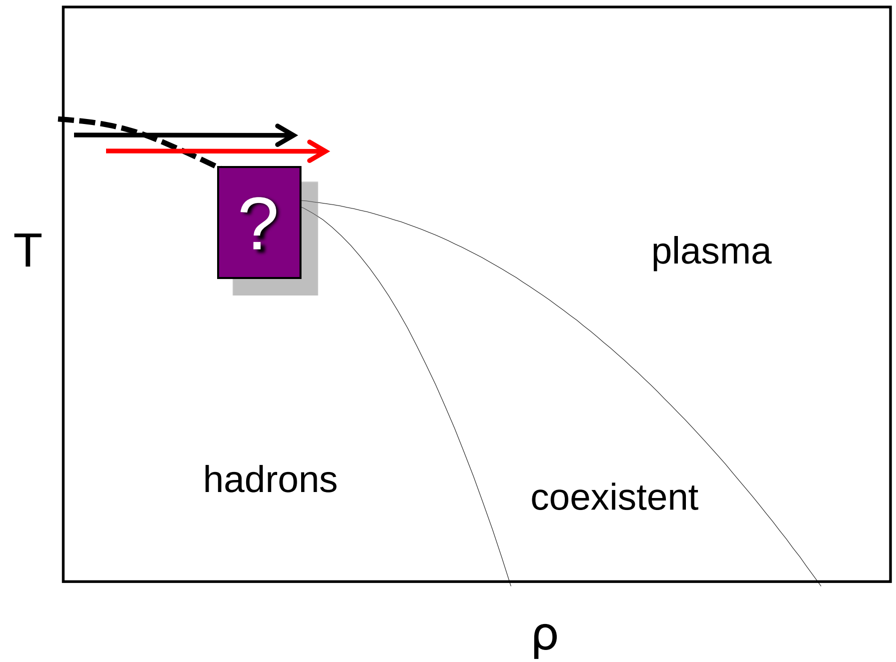
$$\begin{aligned}
\langle\mu\rangle_{n_B} &\equiv \frac{F(n_B+1)-F(n_B)}{(n_B+1)-n_B} \\
&= -\frac{1}{\beta}\ln\frac{\widetilde{Z}_C(3n_B+3)}{\widetilde{Z}_C(3n_B)} \\
&= -\frac{1}{\beta}\ln\frac{1}{\widetilde{Z}}\int \mathcal{D}U e^{-S_g(U)}|\mathrm{Re}\,\widetilde{\det}_{3n_B}M^2(U)|\frac{\mathrm{Re}\widetilde{\det}_{3n_B+3}M^2(U)}{|\mathrm{Re}\widetilde{\det}_{3n_B}M^2(U)|}
\end{aligned}$$

Phase diagram

Four flavors

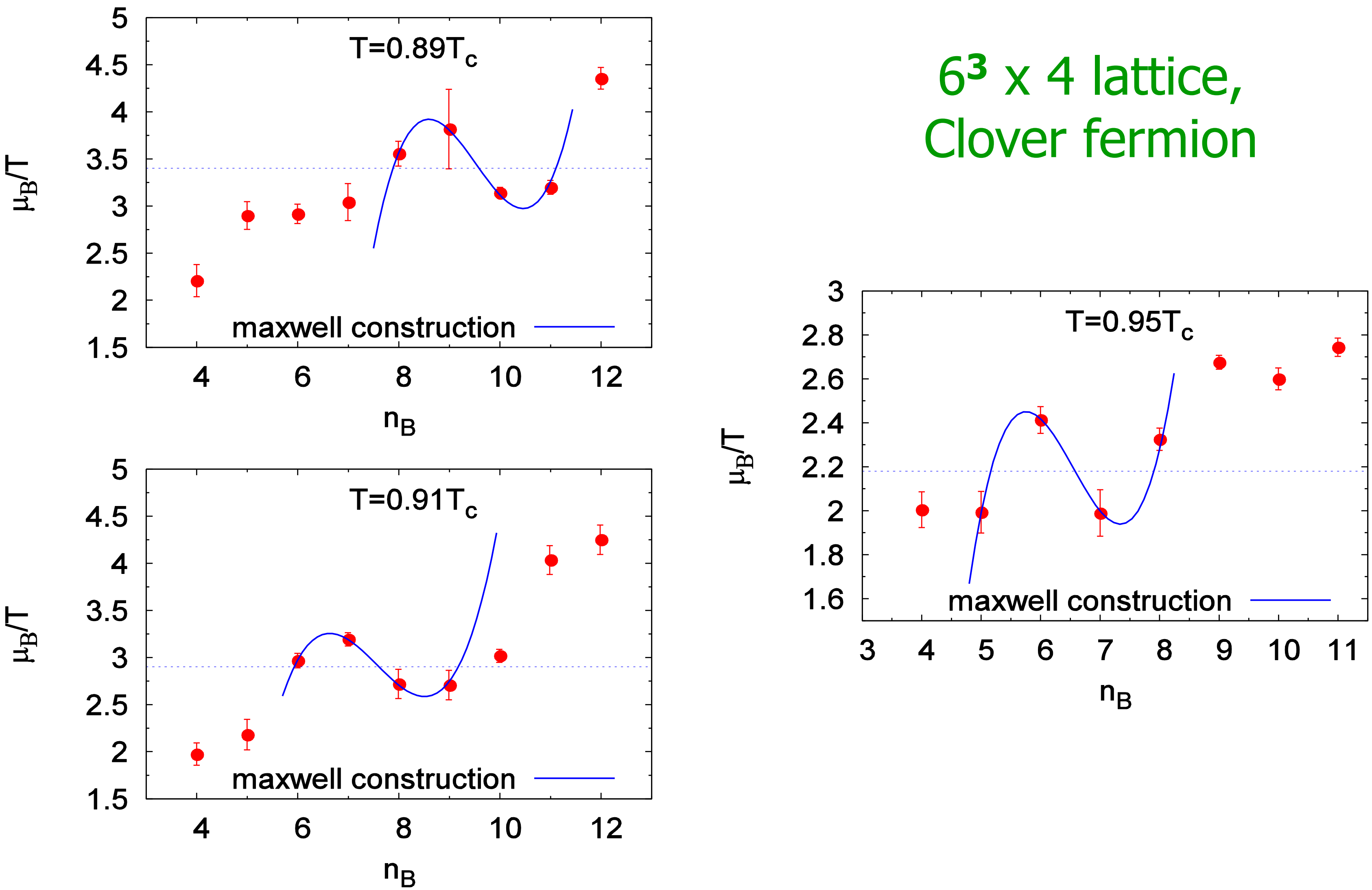


Three flavors



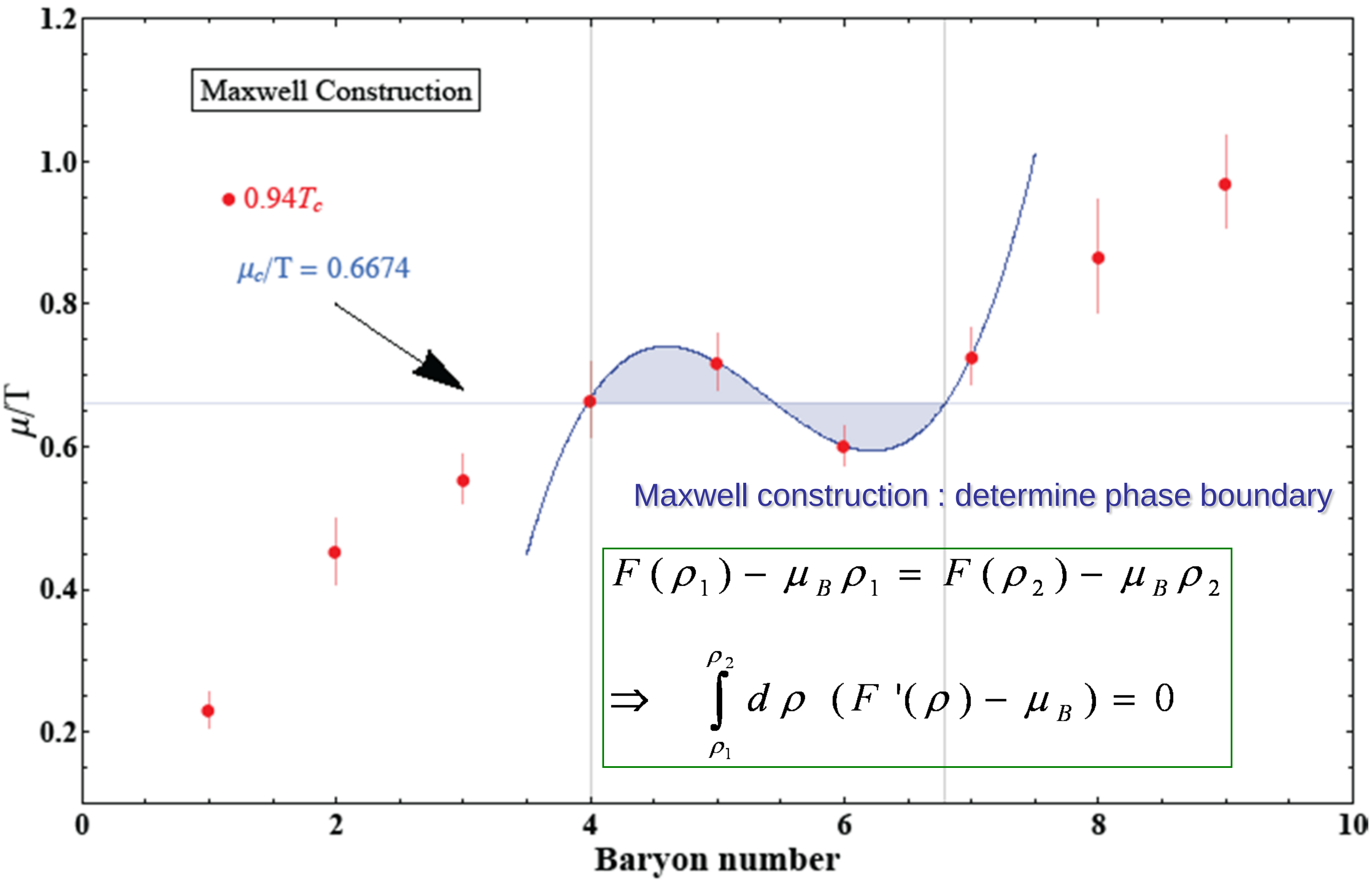
Baryon Chemical Potential for $N_f = 4$ ($m_\pi \sim 0.8$ GeV)

$6^3 \times 4$ lattice, Clover fermion



Phase Boundaries from Maxwell Construction

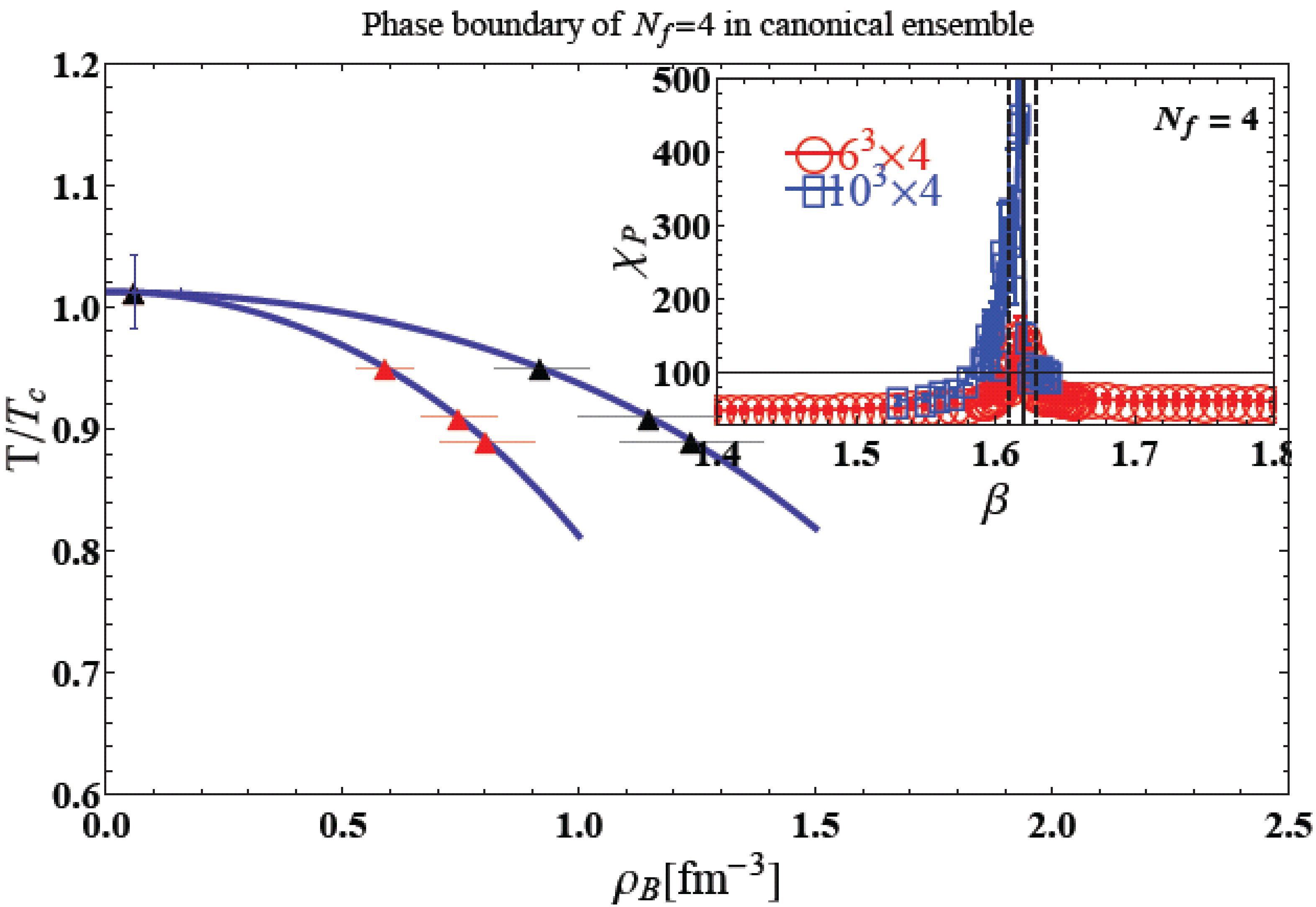
$N_f = 4$ Wilson gauge + fermion action



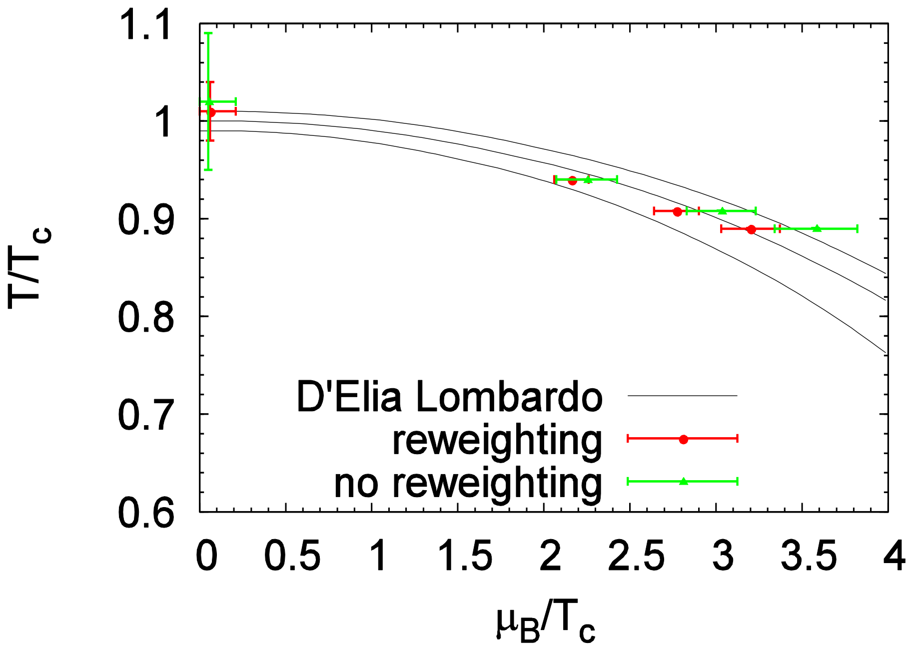
$$F(\rho_1) - \mu_B \rho_1 = F(\rho_2) - \mu_B \rho_2$$

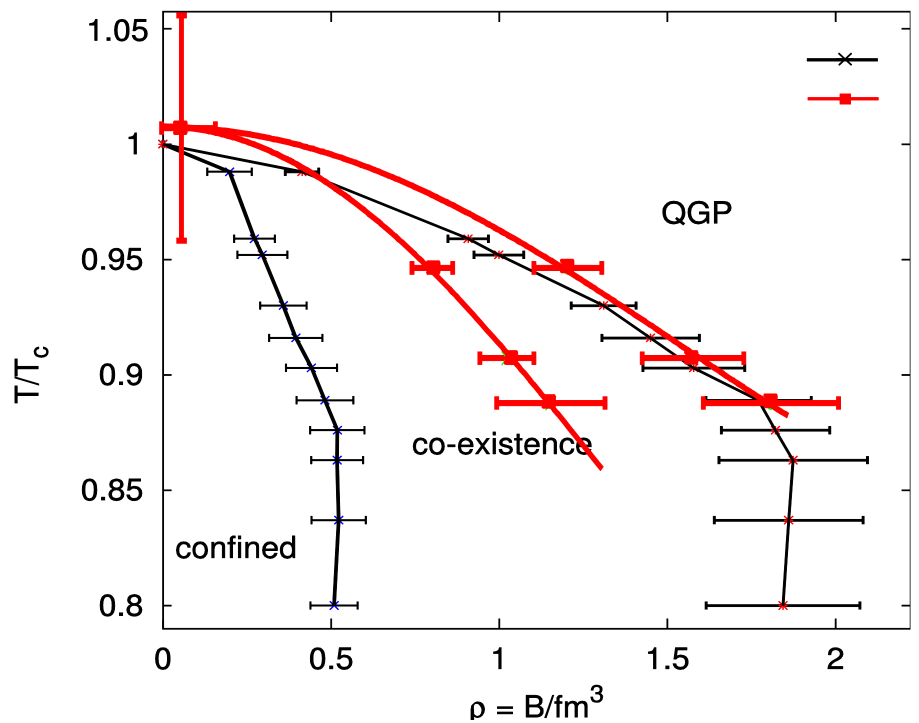
$$\Rightarrow \int_{\rho_1}^{\rho_2} d\rho \, (F'(\rho) - \mu_B) = 0$$

Phase boundary



Phase Boundaries



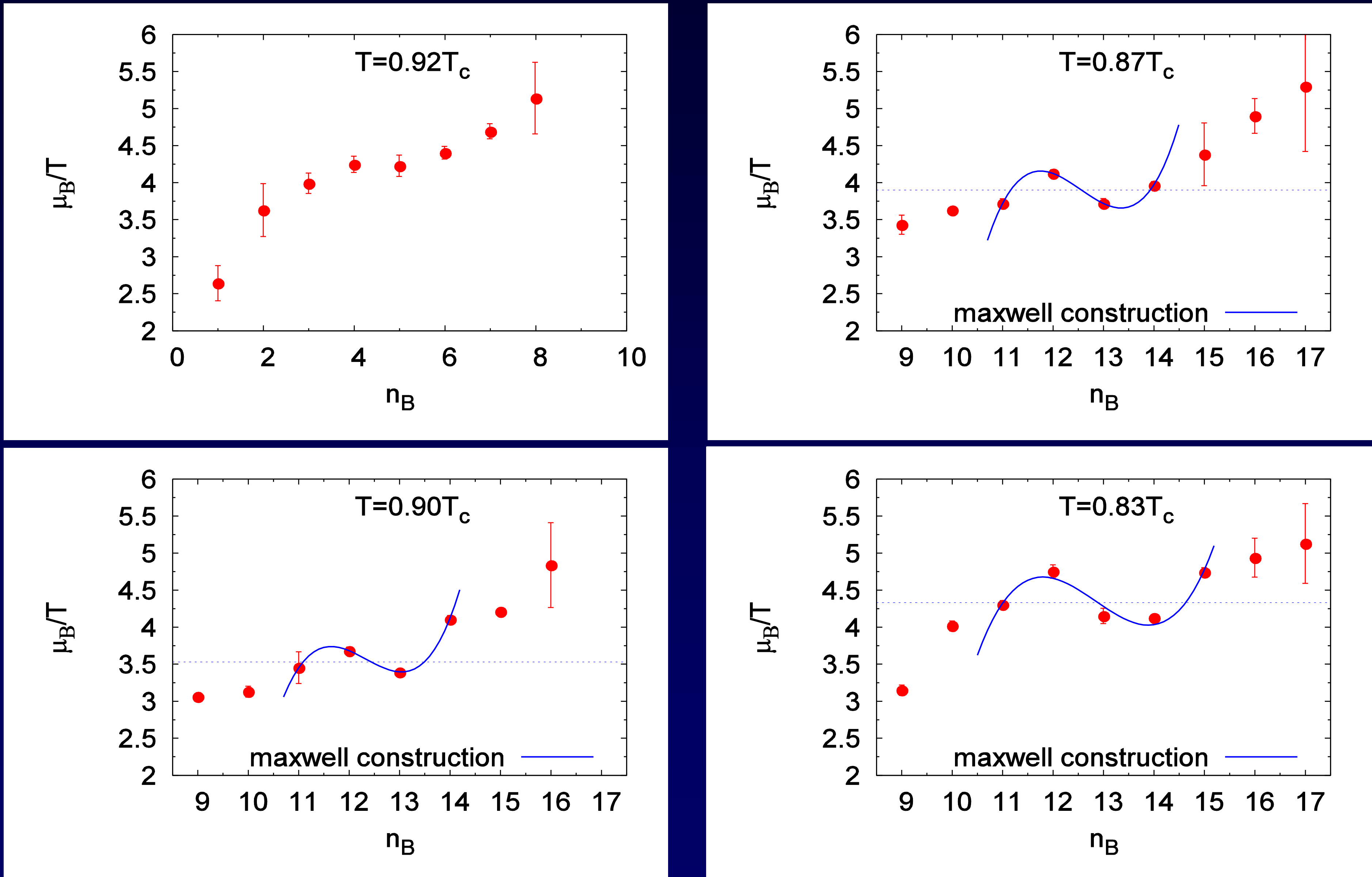


Ph. Forcrand,S.Kratochvila, Nucl. Phys. B (Proc. Suppl.) 153 (2006) 62

4 flavor (taste) staggered fermion

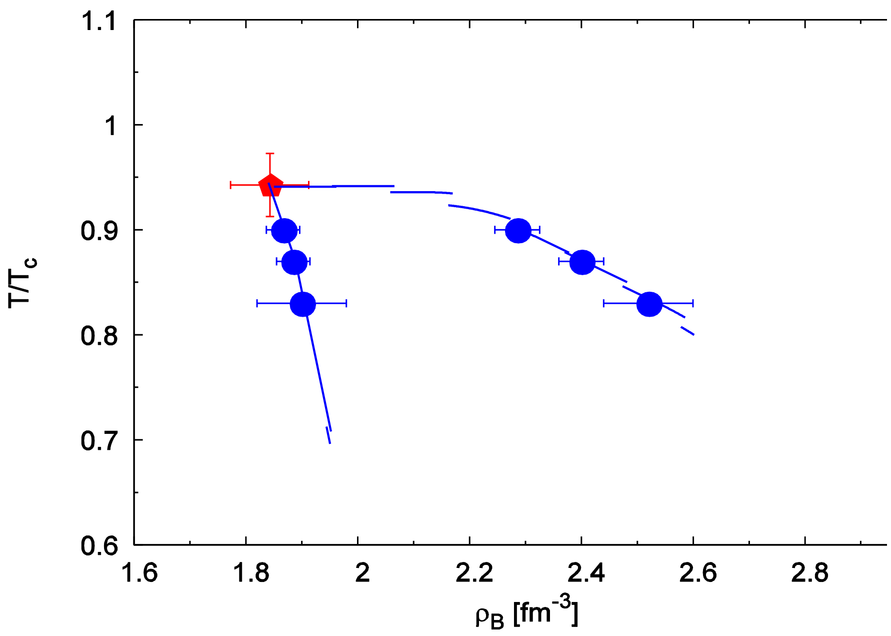
Three flavor case ($m_\pi \sim 0.8$ GeV)

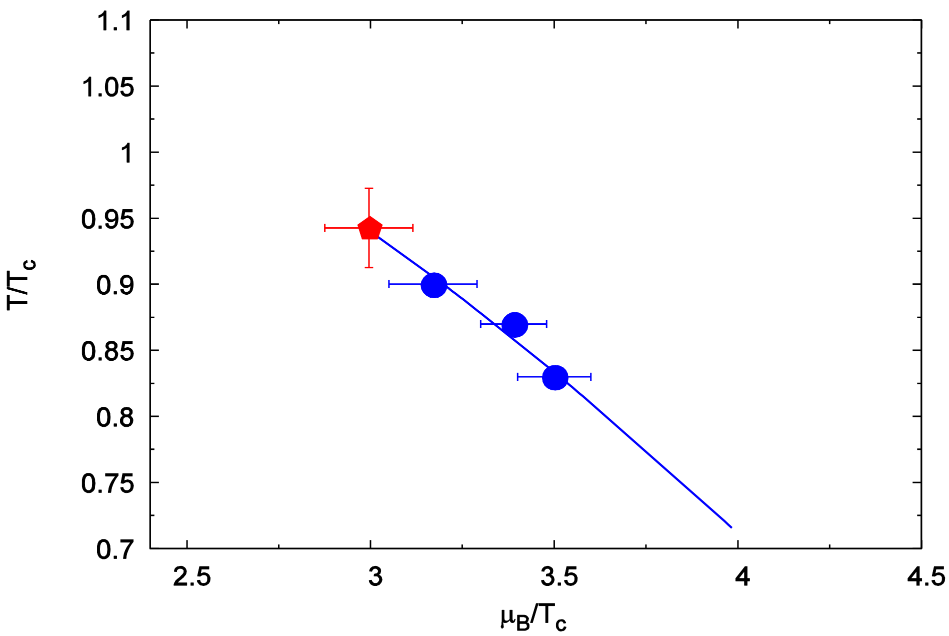
6^3 x 4 lattice,
Clover fermion



Critial Point of N_f = 3 Case

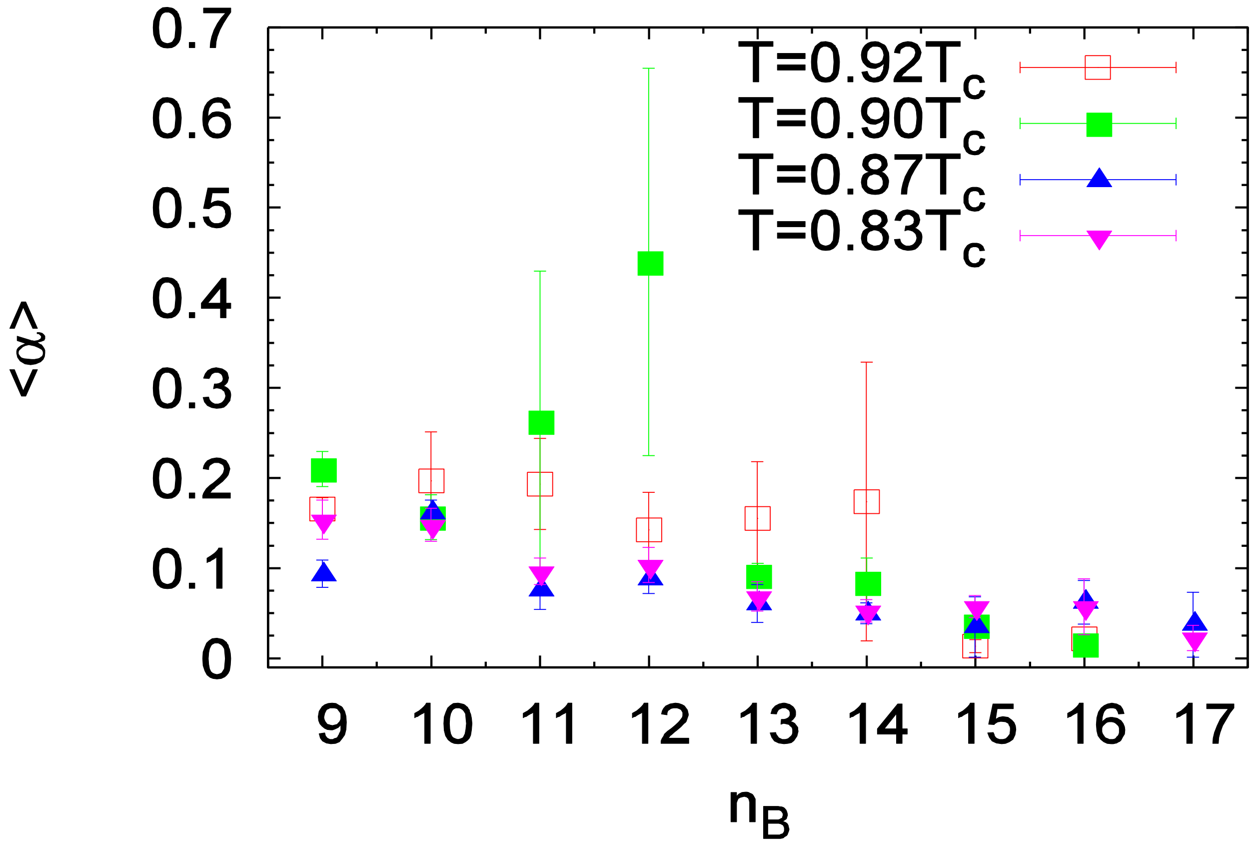
m_π ~ 0.8 GeV, 6^3 x 4 lattice





$T_E = 0.94(4)\ T_C$, $\mu_E = 3.01(12)\ T_C$

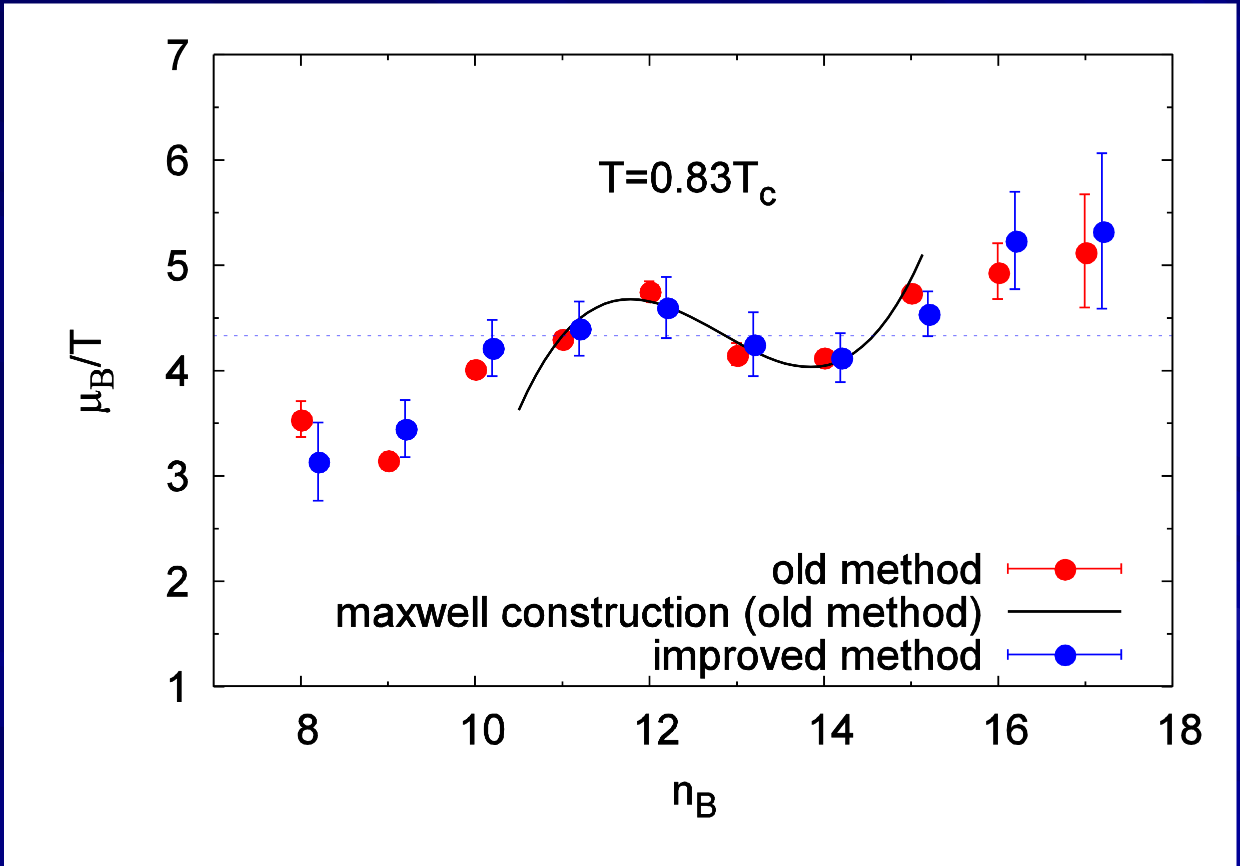
Sign Problem?

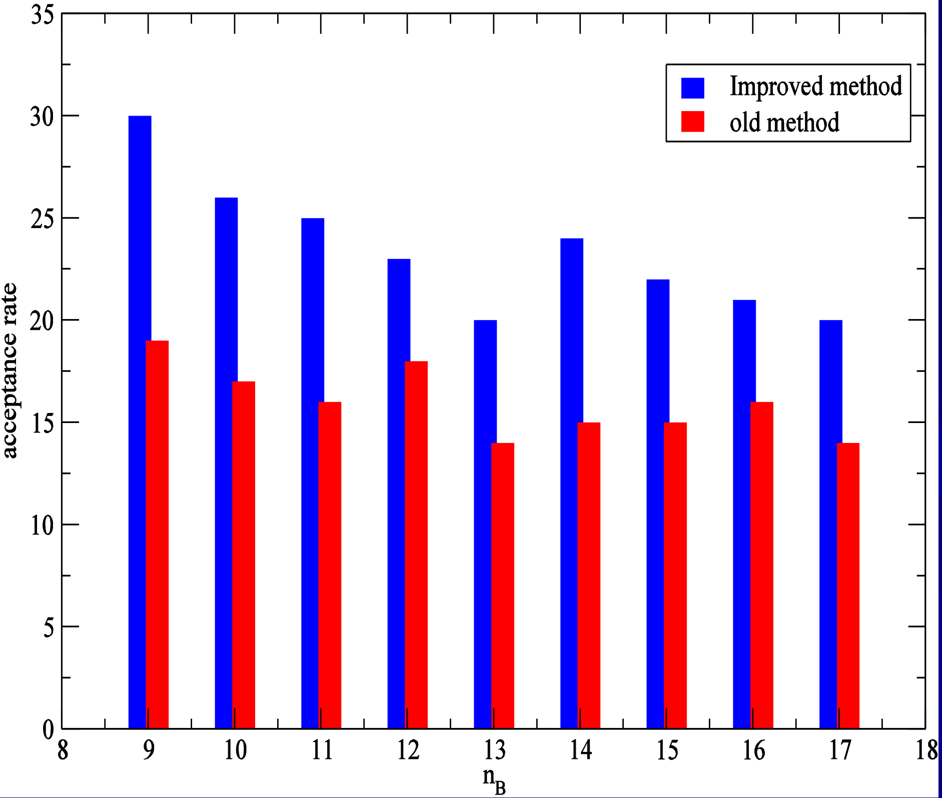


Update

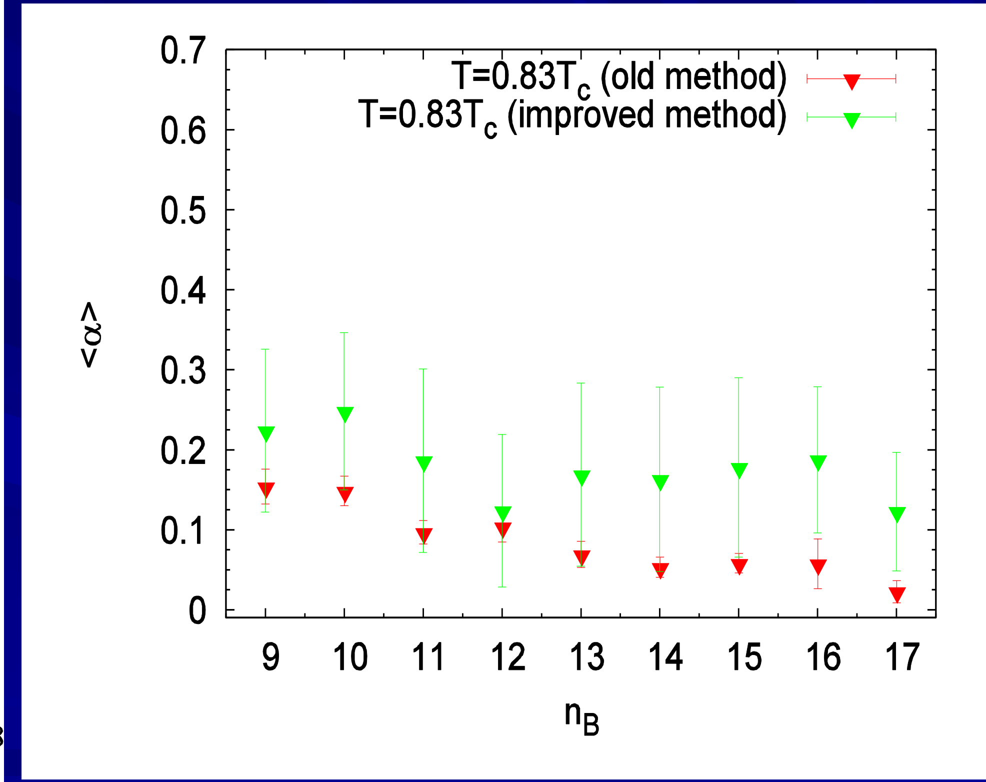
- Previous results were based on accepted configurations with 16 discrete Φ's in the WNEM and reweighting with \`exact' FT for $\det_k$ with sufficient Φ's [16, 128] so that A_{16}/A_1 is less than 10^{-15}.
- Updated results on $N_F = 3$ is from accepted configurations with sufficient Φ's for \`exact' $\det_k$.

Chemical Potential at T = 0.83 T_c





Improved method
old method
acceptance rate
n_B



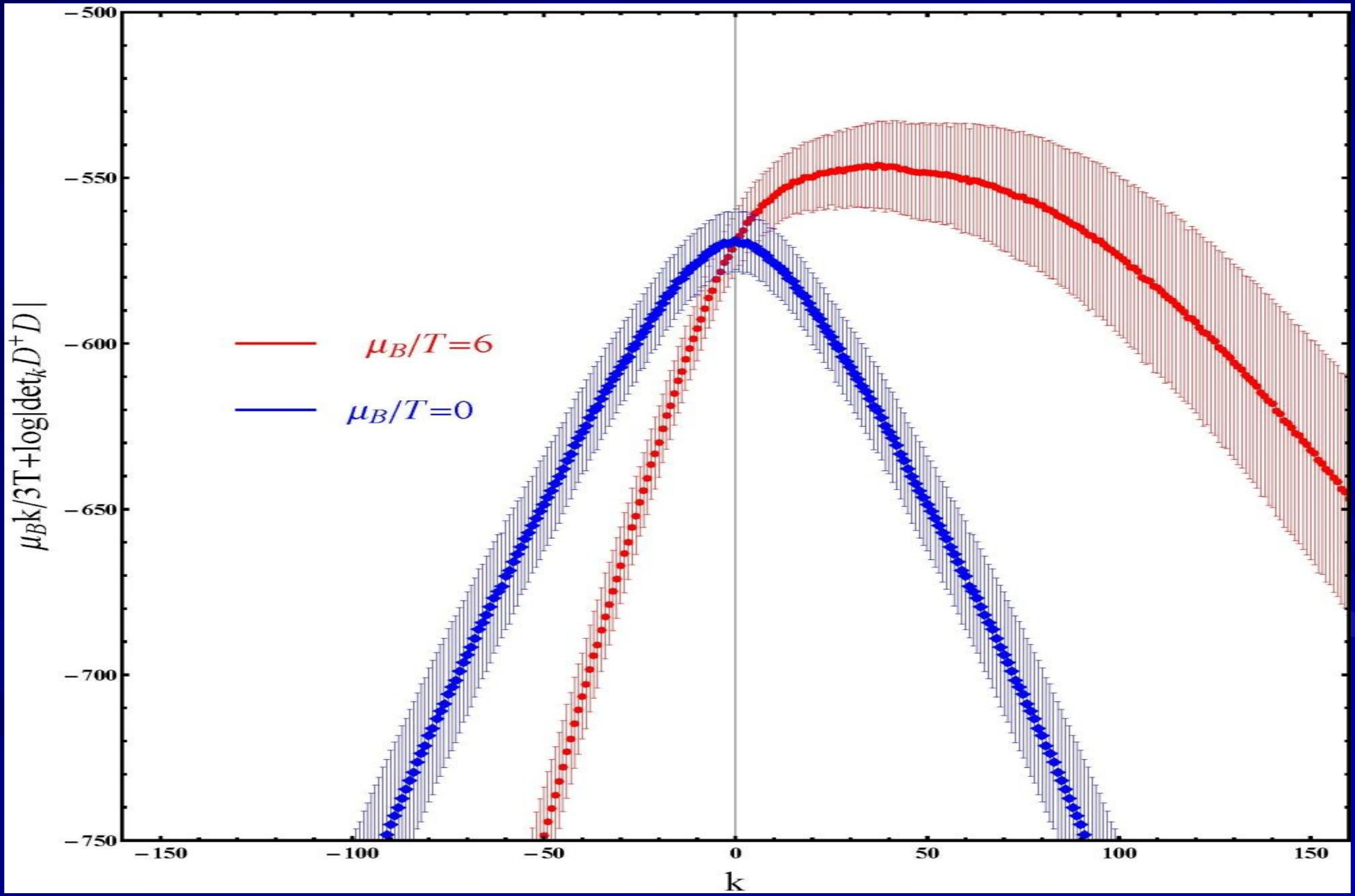
T=0.83T$_c$ (old method)
T=0.83T$_c$ (improved method)
<α>
n_B

Challenges for more realistic calculations

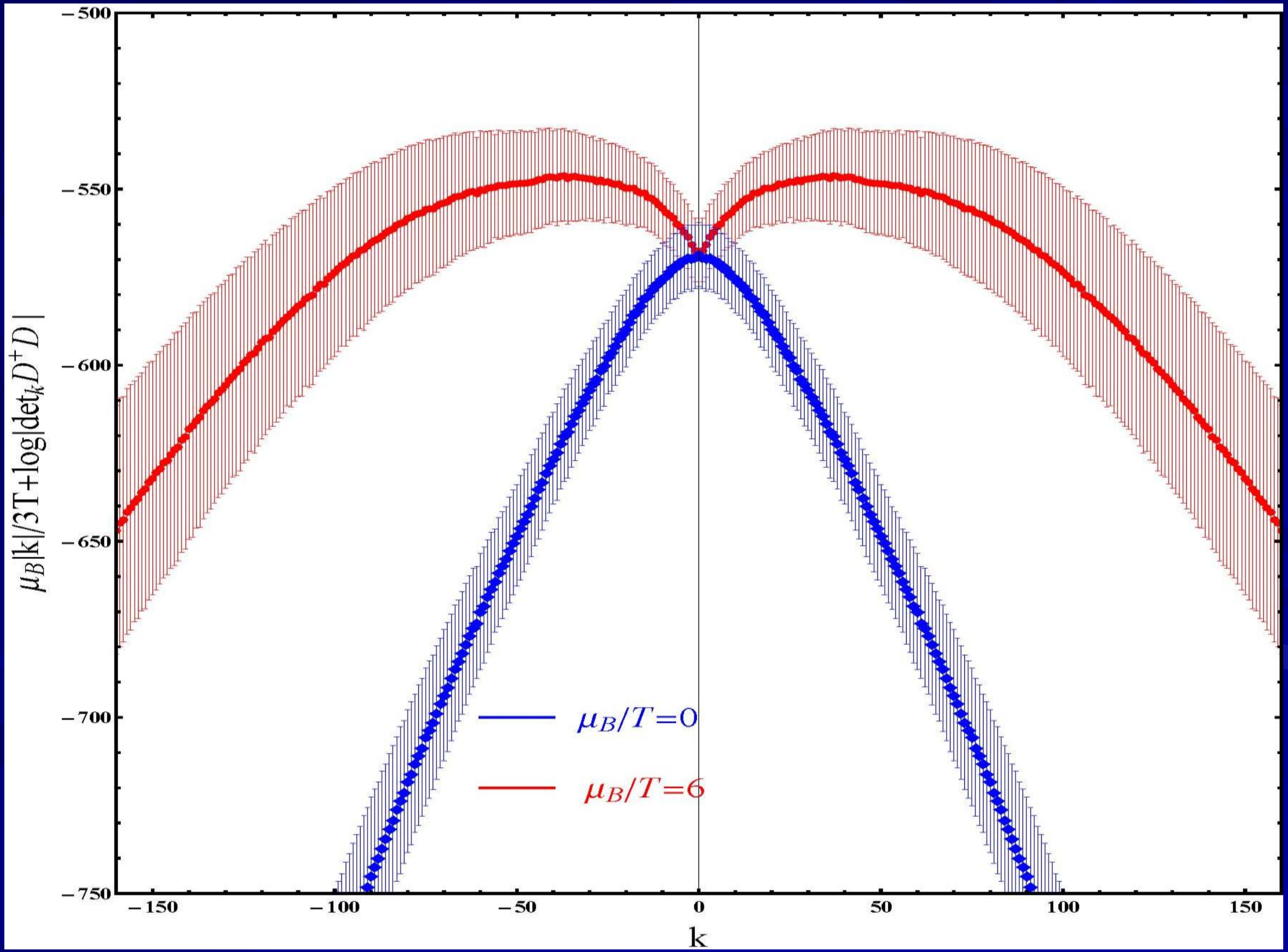
- Smaller quark masses: HYP smearing, larger volume.
- Larger volume:
  - larger quark number k to maintain the same baryon density → numerically more intensive to calculate more Φ's.
  - Any number of Φ's in one stroke mehod for the Wilson-Clover fermion is developed by Urs Wenger.
  - Acceptance rate and sign problem → isospin chemical potential in HMC and A/R with $\det_k$.

$$\det D^2(\mu;U) = \sum_k e^{\frac{\mu k}{T}} \det{}_k D^2(U) = \sum_k e^{\frac{\mu k}{T} + \log \det_k D^2(U)}$$

$$\frac{d}{dk}\left(\frac{\mu k}{T} + \log \det{}_k D^2(U)\right) = 0$$

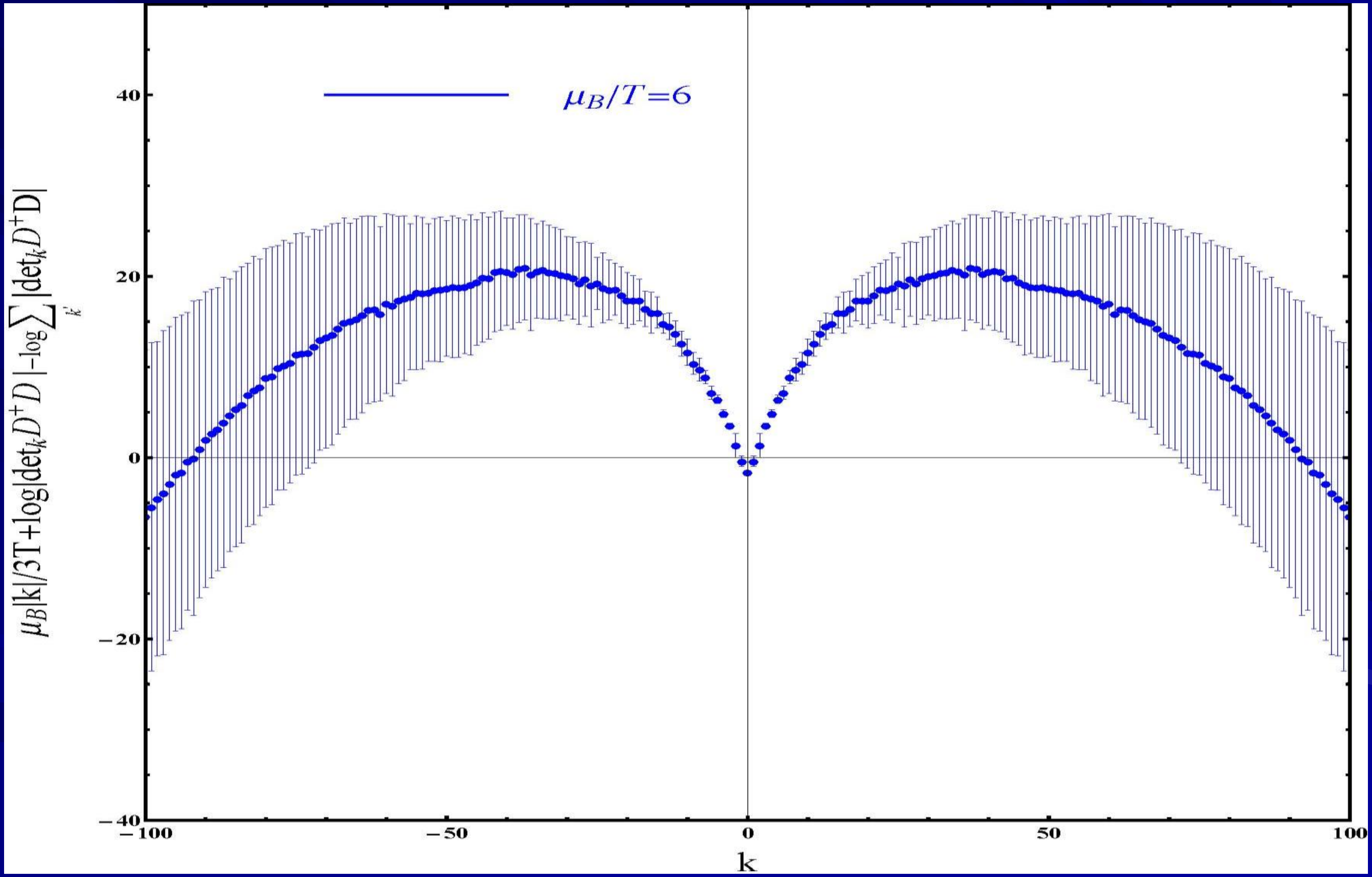


$$\det D^{\dagger}D(\mu_I;U) = \sum_k e^{\frac{\mu|k|}{T}} \det_k D^{\dagger}D(U) = \sum_k e^{\frac{\mu|k|}{T} + \log\det_k D^{\dagger}D(U)}$$



$$\frac{\det D^{\dagger}D(\mu_I;U)}{\det D^{\dagger}D(U)} = \sum_k e^{\frac{\mu|k|}{T}} \frac{\det_k D^{\dagger}D(U)}{\det D^{\dagger}D(U)}$$

$$= \sum_k e^{\frac{\mu|k|}{T} + \log\det_k D^{\dagger}D(U) - \log\det D^{\dagger}D(U)}$$

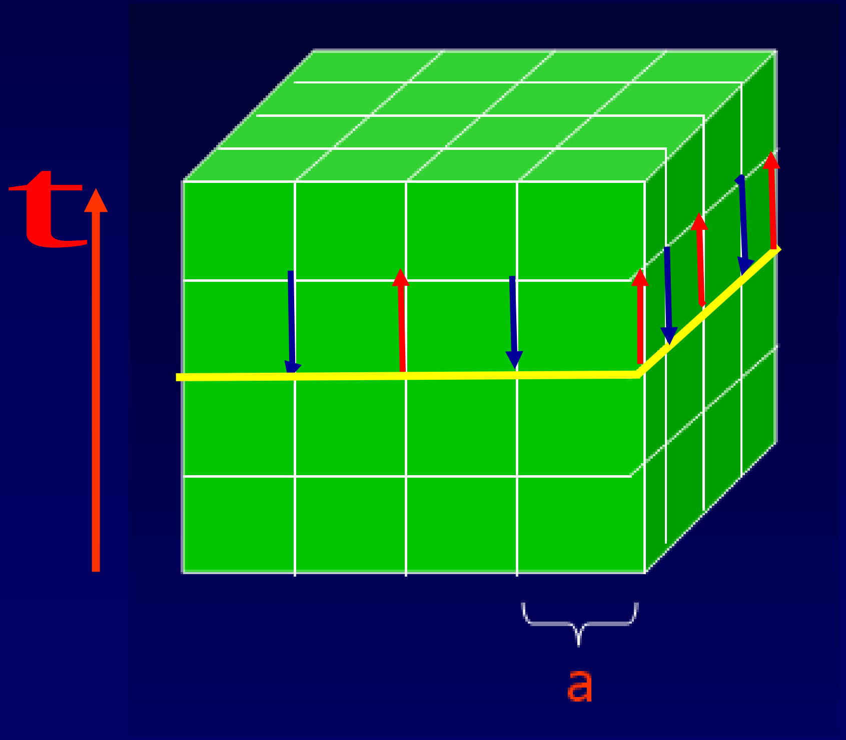


- Canonical Ensemble Approach:

$$Z_B(T,V) = \frac{\beta}{2\pi}\int_0^{2\pi/\beta} d\mu\, Z_{GC}(i\mu) e^{-i\beta\mu B}$$

$$= \int DU\, e^{-S_g} \int_0^{2\pi} d\varphi / 2\pi\, e^{-i3B\varphi} \det M(\varphi);$$

$$M(\theta)_{m,n} = \delta_{m,n} - \kappa[(1+\gamma_4)U_4^+(n) e^{i\phi}\delta_{m,n+4} + (1-\gamma_4)U_4 e^{-i\varphi}(m)\delta_{m+4,n} + \ldots]$$



$$\det M = e^{Tr \log M(\theta)}$$

is real

Winding number expansion (II)

For

$$detM(U,\phi) = \exp(TrlogM(U,\phi))$$

So

$$\log detM(U,\phi) = A_0 + A_1\cos(\phi+\delta_1) + A_2\cos(2\phi+\delta_2) + ...$$

$$detM(U,\phi) = \exp[A_0 + A_1\cos(\phi+\delta_1) + A_2\cos(2\phi+\delta_2) + ...]$$

The first order of winding number expansion

$$detM(U,\phi)_{k=1} = \exp(A_1\cos(\phi+\delta_1))$$

Here the important is that the FT integration of the first order term has analytic solution

$$\int_0^{2\pi}\frac{d\phi}{2\pi}e^{-ik\phi}e^{[A_1\cos(\phi+\delta_1)]} = e^{ik\delta_1}I_k(A_1)$$

$I_k(x)$ *is Bessel function of the first kind .*