Charmonium correlation and spectral functions at finite temperature

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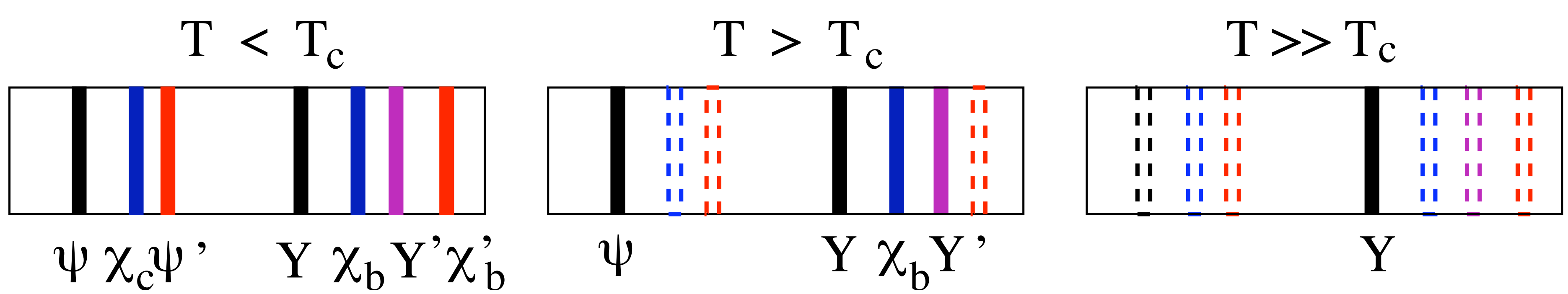
- Heavy quarkonium are useful probes of deconfinement of QCD at finite temperature [Matusi & Satz '86]

- Crucial to investigate the properties of heavy quarkonium at finite temperature from the first principle



- Physical observable: dilepton production

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{1}{\omega^2(e^{\omega/T}-1)} \sigma_V(\omega,\vec{p},T)$$

- Heavy quark diffusion constant

$$D = \frac{\pi}{3\chi^{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\sigma_V^{ii}(\omega,\vec{p}=0,T)}{\omega}$$

charmonium correlation and spectral functions

- Spectral function

$$\sigma(\omega) = \frac{D^{>}(\omega) - D^{<}(\omega)}{2\pi} = \frac{1}{\pi}\mathrm{Im}D_R(\omega)$$

- Correlation function

$$G_H(\tau, T) = \sum_{\vec{x}} \left\langle J_H(\tau, \vec{x})\, J_H^{\dagger}(0, \vec{0}) \right\rangle$$

$$J_H(\tau, \vec{r}) = \bar{q}(\tau, \vec{r})\, \Gamma_H\, q(\tau, \vec{r})$$

Γ_H	${}^{2S+1}L_J$	J^{PC}	$c\bar{c}$
γ_5	${}^1S_0$	0^{-+}	η_c
γ_μ	${}^3S_1$	1^{--}	J/ψ
1	${}^3P_0$	0^{++}	χ_{c0}
$\gamma_5\gamma_\mu$	${}^3P_1$	1^{++}	χ_{c1}

- Integral representation

$$G(\tau, T) = D^{>}(-i\tau) \qquad D^{>}(t) = D^{<}(t + i/T)$$

$$\boxed{G_H(\tau, T) = \int_0^{\infty} \mathrm{d}\omega\, K(\tau, \omega, T)\, \sigma_H(\omega, T); \qquad K(\tau, \omega, T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh(\frac{\omega}{2T})}}$$

A closer look at the integral kernel

- the very high frequency behavior [Le Bellac, Thermal field theory]

$$K(\tau,\omega) = e^{-\omega\tau}$$

correlator is insensitive to high frequency part of spf

- the very low frequency behavior [Aarts et al., PRL99(07)022002]

$$K(\tau,\omega) = \frac{2T}{\omega} + (\frac{1}{6T} - \tau + T\tau^2)\omega + \mathcal{O}[\omega]^3,$$

- the trivial temperature dependence [Datta et al., PRD69(04)094507]

$$\frac{G(\tau,T)}{G_{rec}(\tau,T)} = \frac{\int_0^\infty \mathrm{d}\omega \, \boxed{\sigma(\omega,T)} \, \boxed{K(\omega,\tau,T)}}{\int_0^\infty \mathrm{d}\omega \, \boxed{\sigma(\omega,T')} \, \boxed{K(\omega,\tau,T)}}$$

- the useful exact relation

$$\boxed{\sum_{\tau'=\tau;\ \tau'+=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh(\omega N'_\tau/2)} \equiv \frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh(\omega N_\tau/2)}}$$

$$T' = (aN'_\tau)^{-1},\ \ T = (aN_\tau)^{-1},\ \ \tau' \in [0,\ N'_\tau - 1],\ \ \tau \in [0,\ N_\tau - 1],\ \ N'_\tau = m\ N_\tau,\ \ m \in \mathbb{Z}^+$$

An alternative (better) way to evaluate G_{rec}

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh(\omega N_\tau/2)} \equiv \sum_{\tau'=\tau;\ \tau'+=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh(\omega N'_\tau/2)}$$

$T' = (aN'_\tau)^{-1}, T = (aN_\tau)^{-1}$,
$\tau' \in [0,\ N'_\tau - 1], \quad \tau \in [0,\ N_\tau - 1]$,
$N_\tau = m\ N'_\tau, \quad m \in \mathbb{Z}^+$

$$\int_0^\infty \mathrm{d}\omega\ \sigma(\omega, T')\ \frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh(\omega N_\tau/2)} \equiv \sum_{\tau'=\tau;\ \tau'+=N_\tau}^{N'_\tau - N_\tau + \tau} \left(\int_0^\infty \mathrm{d}\omega\ \sigma(\omega, T')\ \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh(\omega N'_\tau/2)} \right)$$

$$G_{rec}(\tau, T;\ T') \equiv \sum_{\tau'=\tau;\ \tau'+=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau, T')$$

T=4T':

$$G_{rec}(\tau, T; T') = G(\tau, T') + G(\tau + N_\tau, T') + G(\tau + 2N_\tau, T') + G(\tau + 3N_\tau, T')$$

T=3T':

$$G_{rec}(\tau, T; T') = G(\tau, T') + G(\tau + N_\tau, T') + G(\tau + 2N_\tau, T')$$

T=2T': [Meyer, arXiv:1002.3343]

$$G_{rec}(\tau, T; T') = G(\tau, T') + G(\tau + N_\tau, T')$$

Extract spf by Maximum Entropy Method [Asakawa 2001]

- Very hard to extract spectral function

$$G(\tau, T) = \int_0^\infty \mathrm{d}\omega \, K(\tau, \omega, T) \; \sigma(\omega, T)$$

Discretized $\mathcal{O}(10)$ Continuous $\mathcal{O}(10^3)$

- MEM: find the most probable spf which maximizes P[σ|Gm]

Bayesian theorem: $P[\sigma|Gm] \propto P[G|\sigma] \; P[m] \equiv \exp(-\frac{\chi^2}{2} + \alpha S)$

Shannon-Jaynes entropy: $S = \int_0^\infty \mathrm{d}\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \, \ln \left(\frac{\sigma(\omega)}{m(\omega)} \right) \right]$

Default Model (DM): m(ω), provides the prior information

- Results in principle should be independent of DMs

- Nothing beats good data in solving ill-posed problem

Prior information

- high frequency behavior of spf should resemble the non-interacting spf
  - ⋆ free lattice spf rather than free continuum spf [HTD et al.,'09]
- low frequency behavior of spf: zero mode contribution [Umeda, PRD 75(2007)094502]

I: Non-interacting case: [Karsch et al., PRD 68(2003)014504]

$$\sigma_H(\omega,T) = \frac{N_c}{8\pi^2}\Theta(\omega^2-4m^2)\omega^2 tanh\left(\frac{\omega}{4T}\right) \times\sqrt{1-\left(\frac{2m}{\omega}\right)^2}\left[a_H+\left(\frac{2m}{\omega}\right)^2 b_H\right] + \boxed{\frac{N_c}{3}\frac{T^2}{2}f_H\omega\delta(\omega)}.$$

- $\omega\delta(\omega)$ term corresponds to a τ independent constant in the correlator
- No zero mode contribution in PS channel
- f_H =1 in Vector channel

II: Interacting case: [Aarts & Martinez-Resco '02, Petreczky & Teaney '06]

$$\sigma_V^{\mathrm{low}}(\omega) = \frac{3}{\pi}\frac{T\chi_{00}}{M}\frac{\omega\eta}{\omega^2+\eta^2}, \qquad \eta=\frac{T}{MD}$$

M: mass of heavy quark, D: heavy quark diffusion constant, χ_{00} : quark number susceptibility

Lattice setup

★ non-perturbatively clover improved Wilson fermions

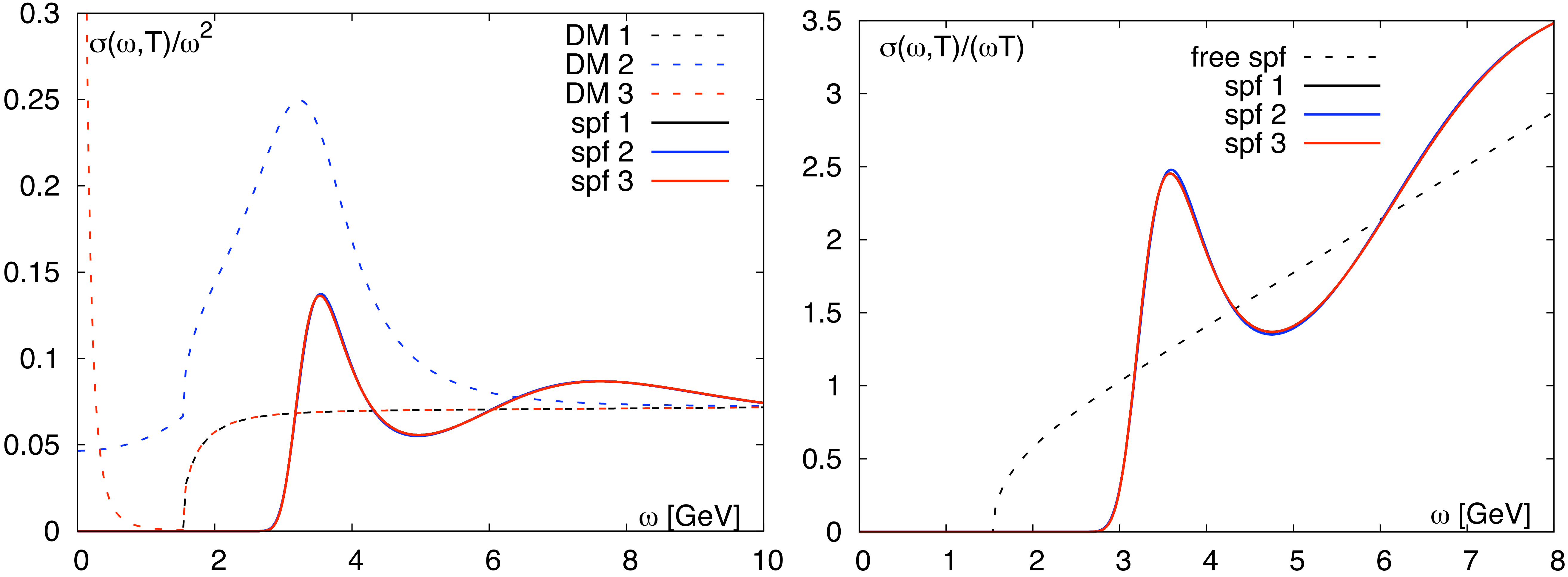
★ isotropic quenched lattice

β	κ	a [fm]	$N_\sigma^3 \times N_\tau$	T/T_c	# of conf.
7.793	0.13200	0.010	$128^3 \times 96$	0.75	155
			$128^3 \times 48$	1.5	471
7.457	0.13179	0.015	$128^3 \times 64$	0.75	179
			$128^3 \times 32$	1.5	250
6.872	0.13035	0.031	$128^3 \times 32$	0.75	126
			$128^3 \times 16$	1.5	198

HTD et al., Lattice 09

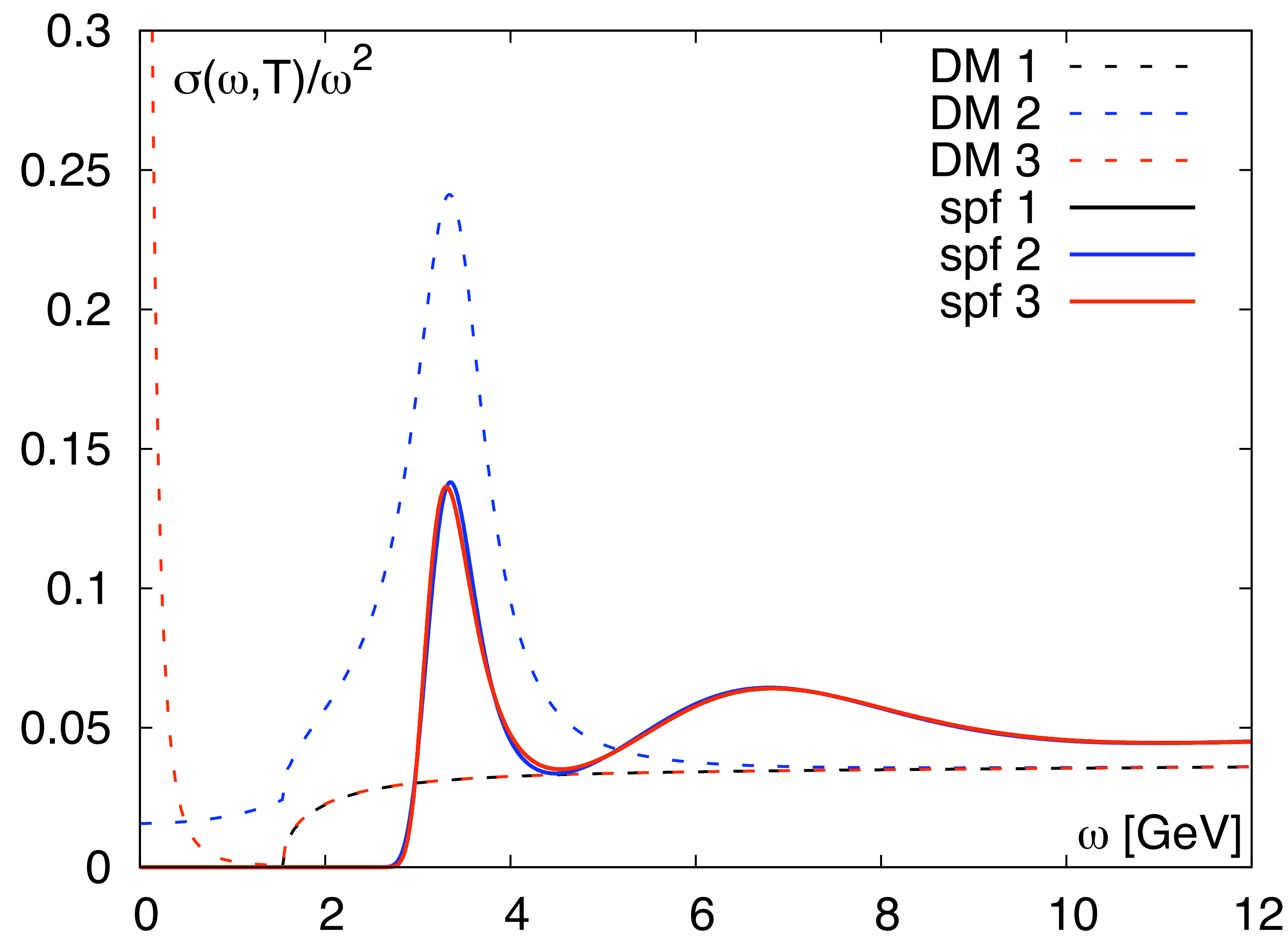
- mass tuning on fine lattice: $M_{J/\psi} = 3.48(1)\ \mathrm{GeV},\ \ M_{\eta_c} = 3.35(1)\ \mathrm{GeV}$
- fine lattice spacing: $m_c a \approx 0.0659 \ll 1$
- temporal extent: $\tau_{max} \approx 0.5\mathrm{fm}\ (0.75T_c)$

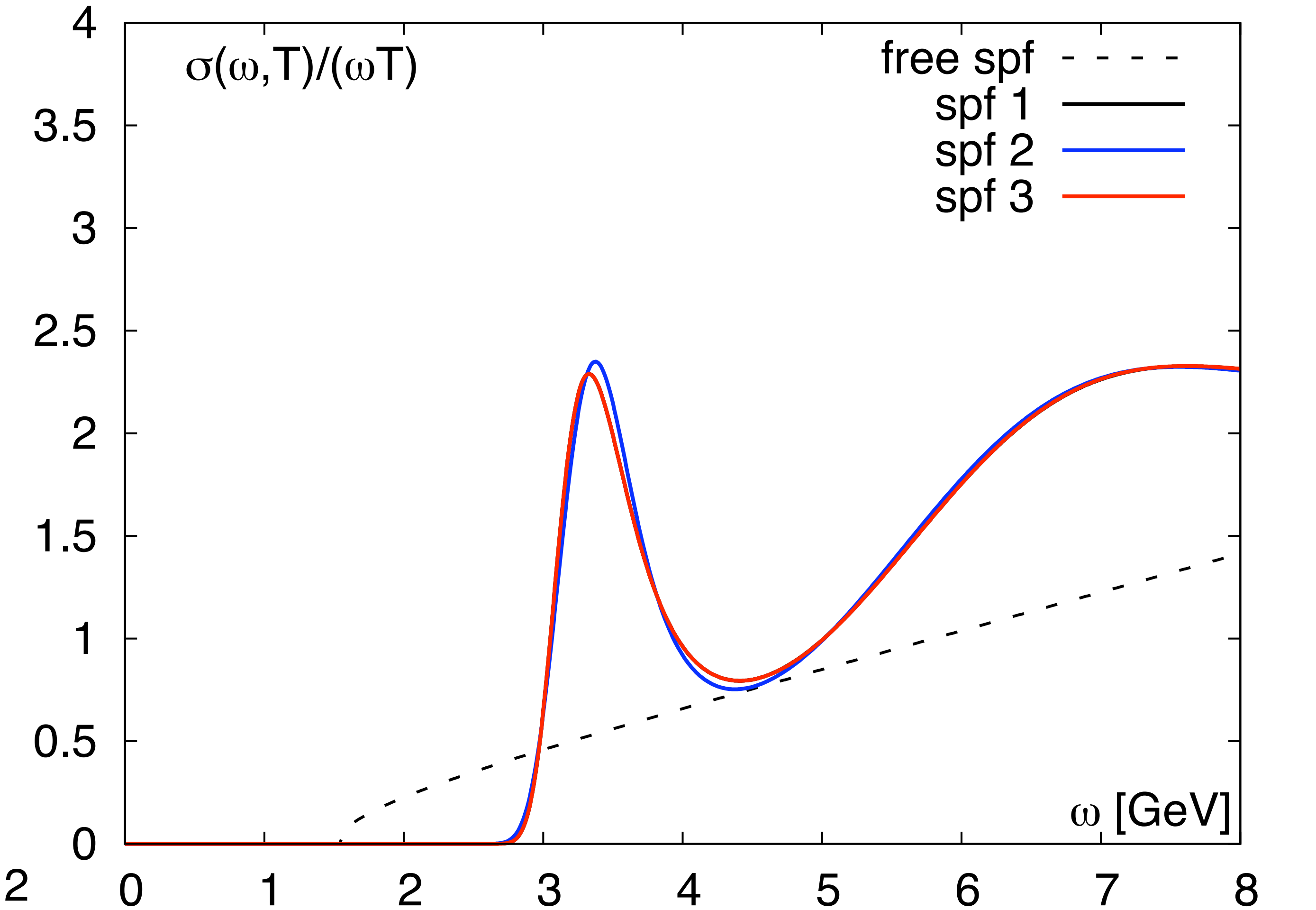
Vector spectral function at 0.75 T_c



- Small default model dependence
- Ground state remains robust
- Below Tc no zero mode contribution found

Pseudo-scalar spectral function at 0.75 T_c





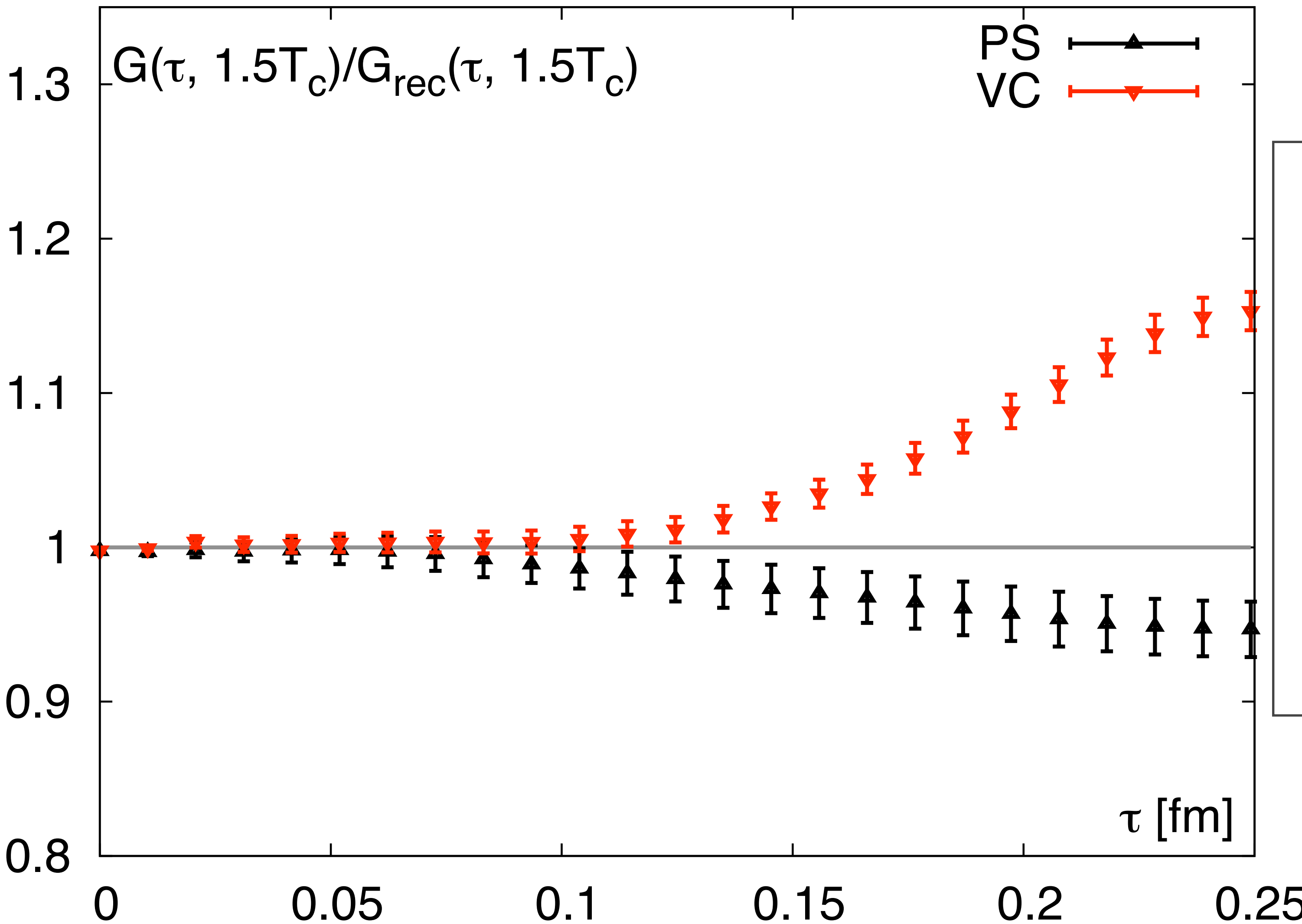
- Small default model dependence
- Ground state remains robust
- Below T_c no zero mode contribution found
- No zero mode contribution is expectable at temperature above T_c

Temperature dependences of charmonia

$$\frac{G(\tau,T)}{G_{rec}(\tau,T)} = \frac{\int_0^\infty \mathrm{d}\omega\ \sigma(\omega,T)\ K(\omega,\tau,T)}{\int_0^\infty \mathrm{d}\omega\ \sigma(\omega,T')\ K(\omega,\tau,T)}$$

Deviation of the ratio from unity indicates thermal modifications

- Evaluate $G(\tau,T)/G_{rec}(\tau,T)$ directly from correlator data without MEM



- The big rise in VC channel can be understood as mainly from zero mode contribution [HTD et al.,'09, Petreczy et al., '09]
- Petreczky EPJC62(2009)85: The deviation from PS channel could be due to negative zero mode contribution. ?

How much can we learn at the correlator level ?

Estimation of low frequency info. of vector spf

- No zero mode contribution in VC and PS channels at 0.75 T_c
- Assume $G(\tau,1.5T_c)=G^{low}(\tau,1.5T_c)+G^{high}(\tau,1.5T_c)$

$$G^{\rm low}(\tau,T) = \int_0^\infty {\rm d}\omega \frac{\cosh\left(\omega(\tau-1/2T)\right)}{\sinh(\omega/2T)}\,\frac{3}{\pi}\,\frac{T\chi_{00}}{M}\,\frac{\omega\eta}{\omega^2+\eta^2}$$

$$G^{\rm high}(\tau,T) = (1+k(T))\,G_{rec}(\tau,T)$$

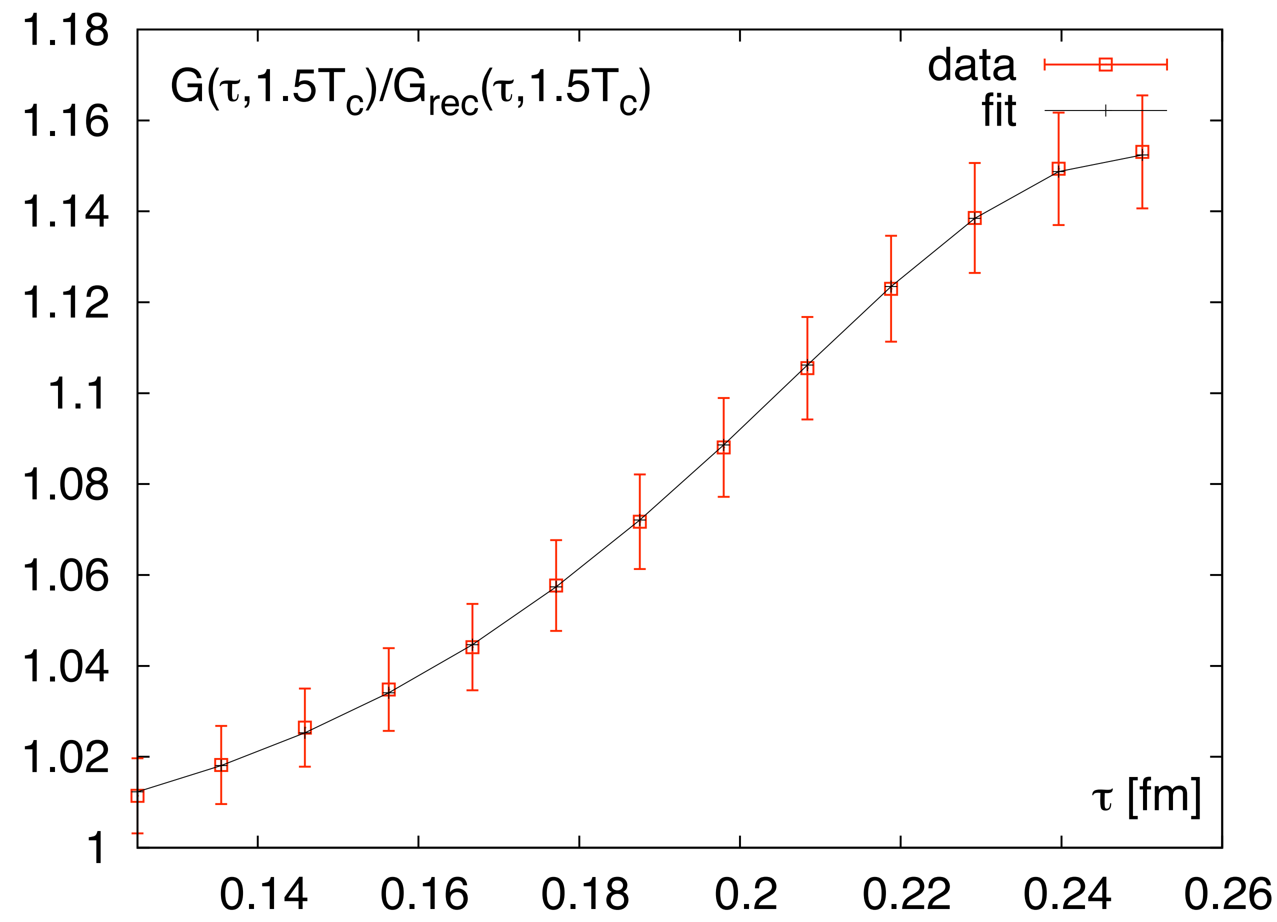
3 parameters (M, η, k(T)) to be determined

- Fitting to $G(\tau,1.5T_c)/G_{rec}(\tau,1.5T_c)$

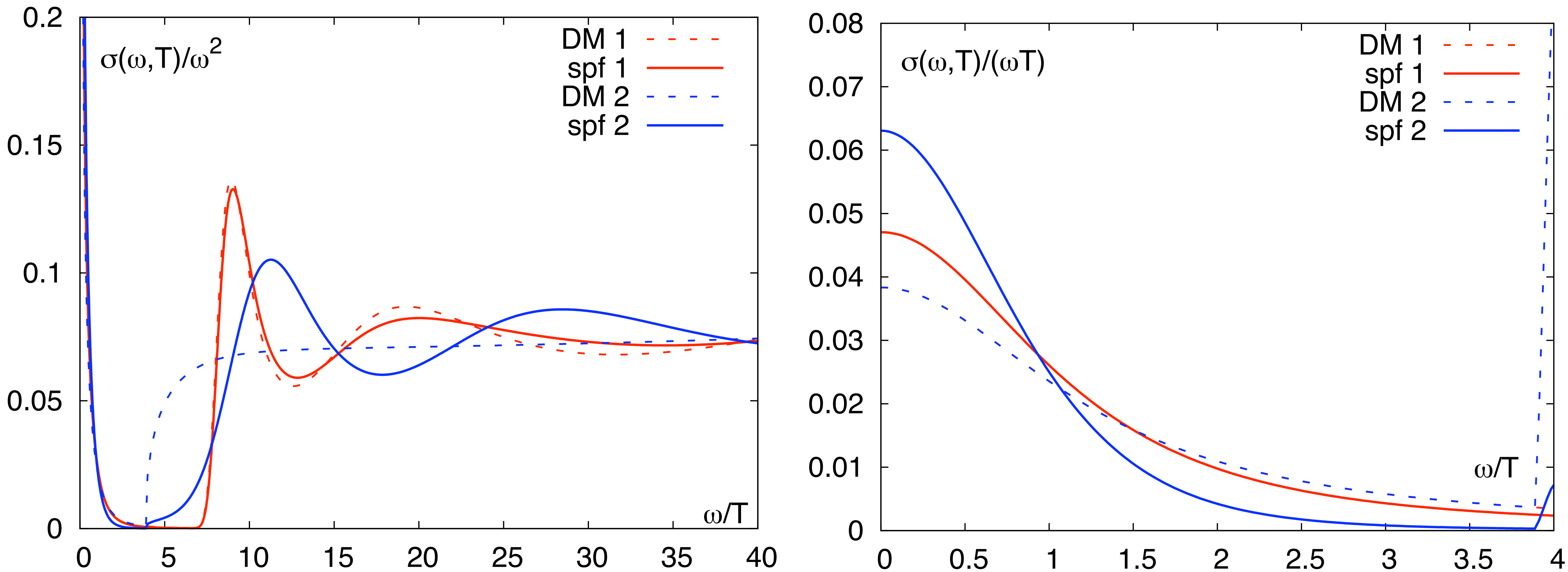
$\chi_{00}/(M\eta) \approx 0.04$

$k(T) \approx -0.006239$

Additional information is needed to constrain the parameters, e.g. thermal moments [see next talk by F. Karsch]

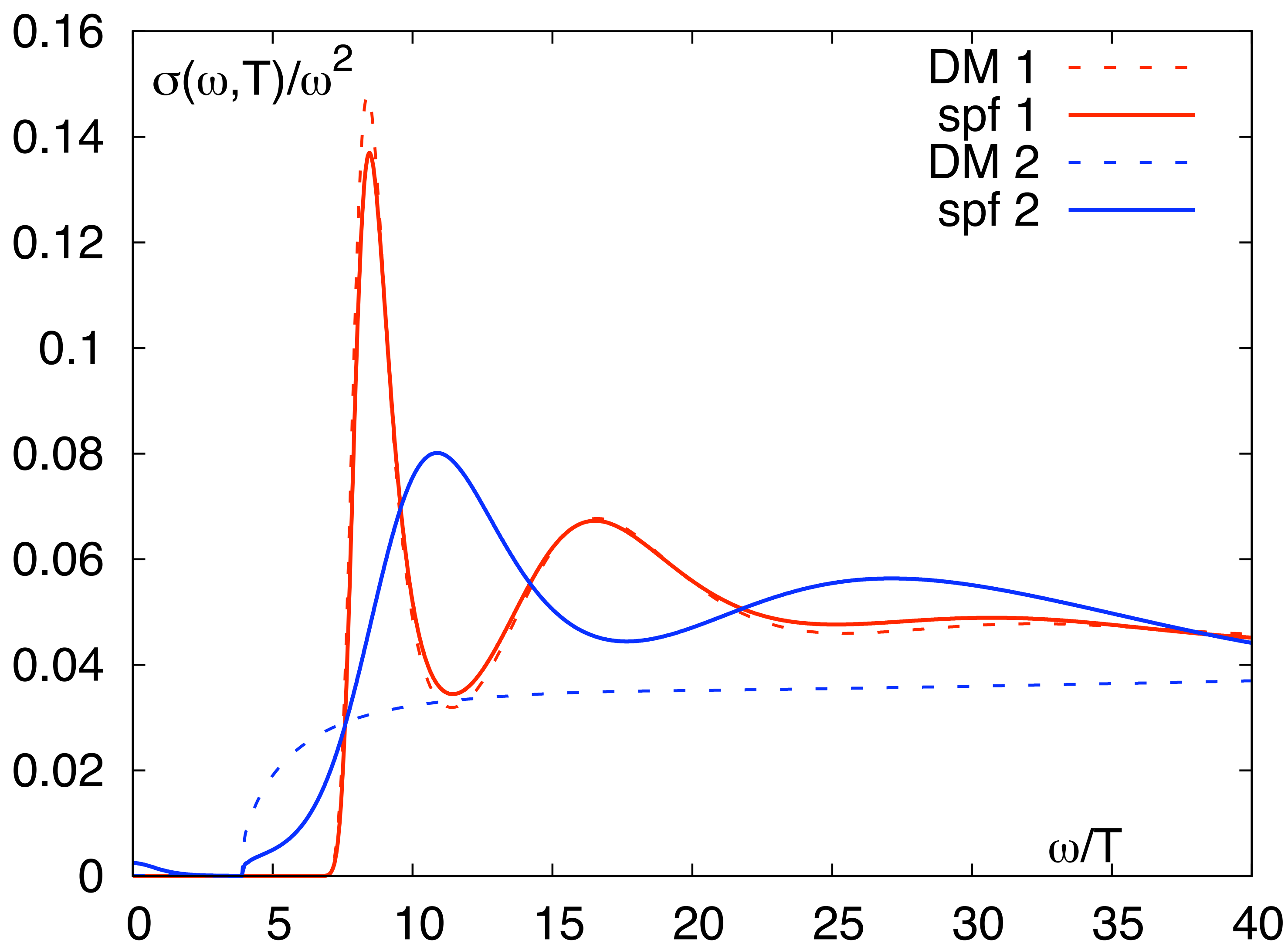


Vector spectral funtion at 1.5 T_c



- DM 1: spectral function at 0.75 T_c with transport peak
- DM 2: free lattice spectral function with transport peak
- The fate of J/psi at 1.5 T_c is not certain

Pseudo-scalar spectral funtion at 1.5 T_c



- DM 1: spectral function at 0.75 T_c
- DM 2: free lattice spectral function
- The fate of η_c at 1.5 T_c is not certain

Extended Maximum Entropy Method [Laue et al., J. Magn. Res., 63(1995)418]

Spectral function analysis on G-G$_{rec}$ to cancel the large ω spf and enhance the low frequency part : σ(ω,T)-σ(ω,T') might be negative somewhere. MEM can only deal with positive-definite spectral functions.

★ Extended Maximum Entropy Method:

Consider a distribution **g** as the difference of the two independent positive distributions **f** and **h**: **g= f -h**

The total entropy, could be written as the additive of two Shannon-Jaynes entropies of **f** and **h**:

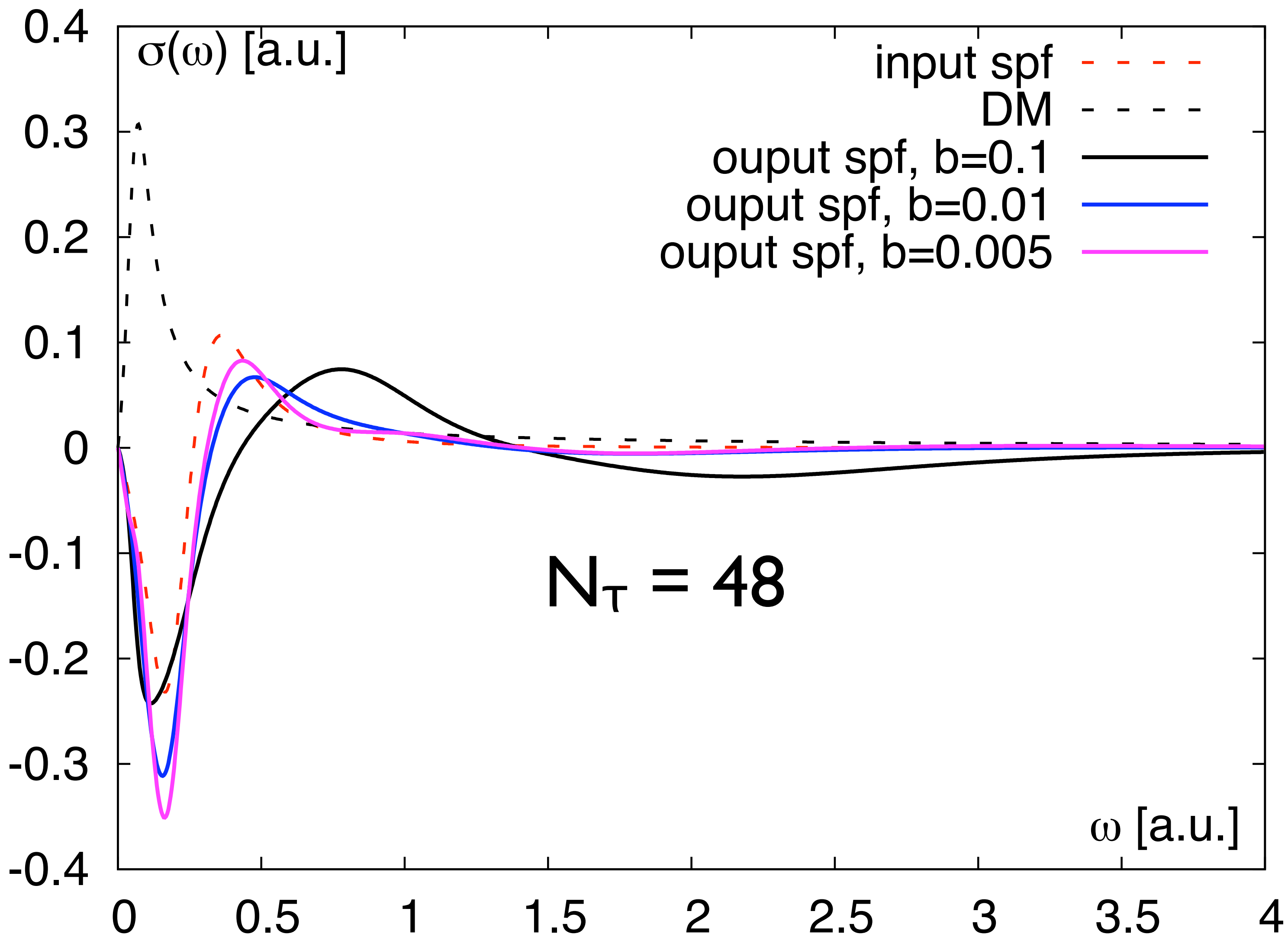
$$\tilde{S}(\mathbf{f},\mathbf{h}) = \sum_i (f_i - m_i - f_i \log(f_i/m_i)) + \sum_i (h_i - m_i - h_i \log(h_i/m_i))$$

As $\frac{\partial \tilde{S}}{\partial \mathbf{f}} = -\frac{\partial \tilde{S}}{\partial \mathbf{h}}$, the entropy can be written in **g** alone:

$$\boxed{\tilde{S}(\mathbf{g}) = \sum_i (\psi_i - 2m_i - g_i \log((\psi_i + g_i)/2m_i)), \;\; \psi_i = (g_i^2 + 4m_i^2)^{1/2}.}$$

the distribution **g** could be either negative or positive.

Mock data test of Extended MEM



$$\sigma(\omega, M, \gamma) = \frac{M}{\pi} \frac{2\omega\gamma}{(\omega\gamma)^2 + (\omega^2 - M^2)^2}$$

$$\sigma(\omega)_{in} = \sigma(\omega, M_1, \gamma_1) - \sigma(\omega, M_2, \gamma_2)$$

$$G(\tau) = \int \mathrm{d}\omega K(\omega, \tau)\sigma_{in}(\omega)$$

$$G^{mock}(\tau) = G(\tau) + b \cdot G(\tau) \cdot \tau$$

- The negative part of the spectral behavior is correctly reproduced
- Increasing statistics leads to better spf closer to the input spf
- Analysis on the real data is in progress

Summary

- A new way to evaluate G/G_{rec} directly from the correlator without MEM
- Extended Maximum Entropy Method can be used to study the difference of correlators in order to cancel the large omega rise in the spf and consequently enhance the signal of low frequency part of spf
- No zero mode contributions are found and ground peaks are stable at 0.75 T_c in PS and VC channels
- In PS channel, the small deviation of G/G_{rec} from unity could be caused by the change in the ground peak rather than "negative zero mode contribution"
- In VC channel, the interplay of the bound state peak and transport peak makes the extraction of physics harder
- The fate of J/ψ and η_c is not certain at 1.5 T_c indicated from the current data