Rho decays widths from the lattice

Preliminary results from a subset of BMW configurations

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Lattice 2010 International Symposium

Outline

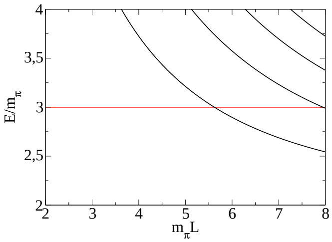
Basics on avoided crossings

If ρ were not coupled to $\pi\pi$

- m_ρ constant up to exponentials,
- $\pi\pi$ are quantified with momenta $2\pi/L \cdot \vec{n}$,
- For some box sizes, ρ and $\pi\pi$ will cross.

Turning interaction on

- Avoided crossing
- Information on coupling strength
- ρ and $\pi\pi$ fully mix.



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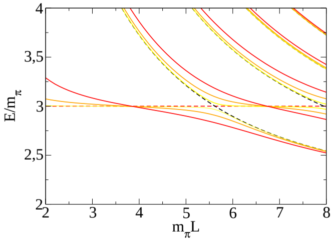
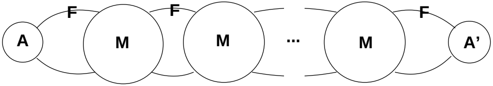


Lüscher's Formula



$$\rightarrow AF\frac{1}{1-MF}A'$$

Interactions move F poles to $1-MF$ zeros. See [Kim'05]

Lüscher's Formula

Under the 4π inelastic threshold quantification condition reads (up to exponentially small terms in $m_\pi L$) :

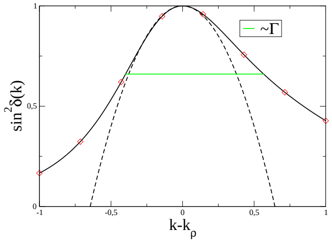
$$\Phi(q) = n\pi - \delta(q)$$

$$q = kL/2\pi \quad , \quad \delta \text{ is } I = J = 1 \text{ phase shift}$$
$$\Phi = \text{kinematical function of } q \text{ only.}$$

The g-coupling model

Why we need a model

- phase shift at each k would cost too much
- need some inter/extrapolations $(m_{phys}, a \to 0, L \to \infty, \dots)$



Effective range

$$\frac{k^3}{W}\cot\delta = b(k^2 - k_\rho^2) \qquad \text{where} \quad b = -\frac{4k_\rho^3}{m_\rho^2\Gamma_\rho}$$

The g-coupling model(2)

Effective lagrangian

$$\mathcal{L}_{int} = g\epsilon_{abc}\rho^a_\mu \pi^b \partial^\mu \pi^c \qquad \text{gives} \quad \Gamma_\rho = \frac{g^2}{6\pi} \cdot \frac{k^3_\rho}{m^2_\rho}$$

experimentally $g \simeq 6.0$

Validity

- Effective lagrangian is just $\Gamma \leftrightarrow g$
- Approximations, both empirically good :
  - $g \approx cst$
  - Effective range

Results with single operator

A simple formula

$$m_\rho^2 = W^2 - \frac{g^2}{6\pi}\frac{k^3}{W}\cot\Phi(kL/2\pi)$$

Contradictory needs

- Far away from the crossing no signal for width
- On the crossing, no precise $m_\rho \Rightarrow$ no signal for width
- Must compute both mass and crossing size at the same time.

Illustrative results from 6-stout

In combined fits $\delta M_\rho \sim 5\%$, $g = 9.5 \pm 4.6$ (physical value 6.0).

Generalized Eigenvalue Problem

Exact case

If exactly N eigenstates, energies can be computed from a set of N cross-correlators :

$$C_{ij} = \langle 0 \mid \mathcal{O}_i(t)\mathcal{O}_j(0) \mid 0 \rangle = \sum_n e^{-E_n t} \langle 0 \mid \mathcal{O}_i \mid n \rangle \langle n \mid \mathcal{O}_j \mid 0 \rangle$$

using eigenvalues of the matrix $C(t)C^{-1}(t_0)$

$$\lambda_i = e^{-E_i(t-t_0)}$$

if and only if the operators are linearly independent.

Generalized Eigenvalue Problem(2)

Higher exponentiels

- With N-by-N cross correlators the energies are computed with errors $\exp[(E_N - E_{N+1})t]$, $\exp[(E_N - E_{N+1})t_0]$
- t **and** t_0 must be large
- but $t - t_0$ can be either small or large

Suppressing them : a little trick

- new operators : time-shifted copies of basical operators
- Mix GEVP and black-box methods
- Not a huge improvement but absolutely no cost

Operators and contractions

ρ operator

$$\rho^i = \bar{q}\gamma^i q$$

$\pi\pi$ operator

$$\mathcal{O}_i(\vec{p},\vec{q}) = (p_i - q_i)\left[\pi^+(\vec{p})\pi^-(\vec{q}) - \pi^-(\vec{p})\pi^+(\vec{q})\right]$$

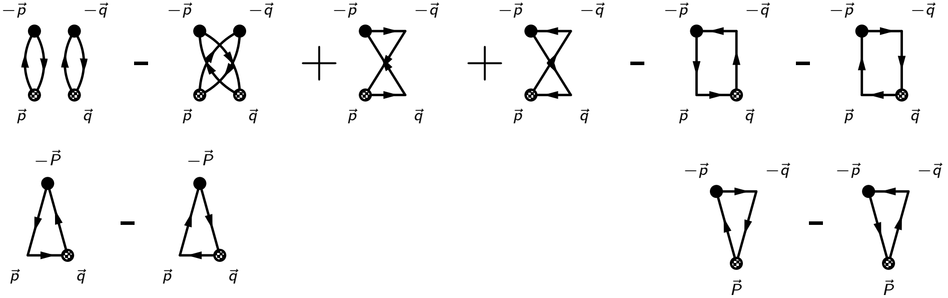
Stochastic propagators

As in [Aoki'07] we use two kinds of stochastic propagators :

$$Q(\vec{x},t \mid \vec{q},t_s,\xi_j) = \sum_{\vec{y}} D^{-1}(\vec{x},t;\vec{y},t_s)\cdot[e^{i\vec{p}\cdot\vec{y}}\xi_j(\vec{y})]$$

$$W(\vec{x},t \mid \vec{k},t_1 \mid \vec{q},t_s) = \sum_{\vec{z}} D^{-1}(\vec{x},t;\vec{z},t_1)\cdot[e^{i\vec{k}\cdot\vec{z}}\gamma_5 Q(\vec{z},t_1 \mid \vec{q},t_s)]$$

Operators and contractions(2)



$$G^{1st}_{\pi\pi\to\rho} = \sum_{j,\vec{x}} e^{-i\vec{P}\vec{x}} \left\langle Q(\vec{x},t \mid \vec{0},t_s,\xi_j) W^\dagger(\vec{x},t \mid -\vec{p},t_s \mid -\vec{q},t_s,\xi_j)\gamma_5\gamma_3 \right\rangle$$

Our lattices

Action

- 2+1 full QCD
- tree-level $O(a^2)$-improved Lüscher-Weisz gauge action
- tree-level clover-improved Wilson fermions
- 2 steps of HEX smearing

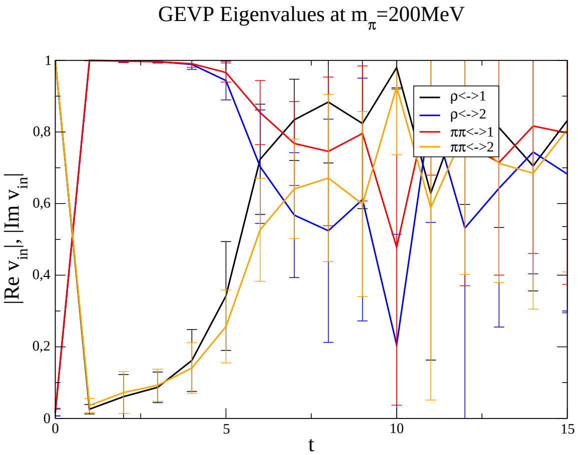
200MeV point

- $\beta = 3.31,\ a = 0.116\ \mathrm{fm}$
- $32^3 x48$
- 130 configurations analyzed
- $\vec{P} = (0,0,0)$

340MeV point

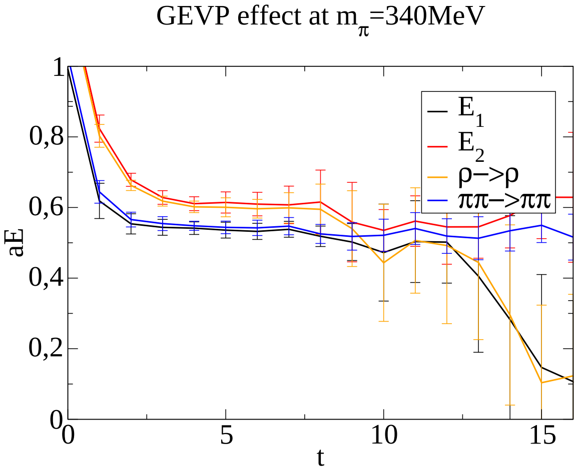
- $\beta = 3.31,\ a = 0.116\ \mathrm{fm}$
- $24^3 x48$
- 77 configurations analyzed
- $\vec{P} = (0,0,1)$

Putting it all together



- For $t < 6$ higher states contamination
- $6 \leq t \leq 9$ has some signal

Putting it all together(2)



- For the point 200MeV : $g = 5.5 \pm 2.9$
- For the point 340MeV : $g = 6.6 \pm 3.4$
- Combining both : $g = 6.0 \pm 2.2$

Conclusion

- $g = 6.0 \pm 2.2$ (central value gives $\Gamma = 150\mathrm{MeV}$, compatible with experiment)
- Preliminary results
  - More configurations are available and need to be analyzed. Could use several noises/cfg
  - Needs for other β,a,...
- Method now well-known, can lead to interesting results
- Lowest m_π up to my knowledge $\Rightarrow$ Γ of the physical order.
- Approximations good up to our precision
  - Effective range
  - $g \sim cst$