# Sigma term and strange content of the nucleon

[Preliminary results with a subset of the Budapest-Marseille-Wuppertal ensembles]

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# Motivation

## Definitions

$$\sigma_{\pi N} = \hat{m}\langle N(p) \mid (\bar{u}u + \bar{d}d)(0) \mid N(p)\rangle$$

$$\sigma_{ksN} = m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle$$

$$y = \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle}$$

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## Feynman-Hellman theorem

$$\frac{\partial E(\lambda)}{\partial \lambda} = \langle \lambda \mid \frac{\partial H(\lambda)}{\partial \lambda} \mid \lambda \rangle$$

If applied to QFT

$$\hat{m}\frac{\partial M_N}{\partial \hat{m}} = \hat{m}\langle N \mid \frac{\partial H_{QCD}}{\partial \hat{m}} \mid N\rangle = \sigma_{\pi N}$$

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## Important for

- Hadron spectrum
- The strangeness content of the nucleon ($\rightarrow$ Detection of dark matter).
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes

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# Simulation details

- Gauge Action: Symanzik $\mathcal{O}(a^2)$ improved.
- Fermion Action: $2+1$ Wilson fermions. Tree level $\mathcal{O}(a)$ improved.
- 6 levels of `stout` smearing.
- Algorithm: HMC (for 2 degenerate quarks), RHMC (for the s quark).
- Volume: Spatial size L big so that data can be corrected with (small) finite volume effects. ($M_\pi L \geq 4$).
- Three values of the lattice spacing. ($a \approx 0.124 fm, 0.083 fm, 0.063 fm$).
- Pion masses range: $190 MeV \lesssim M_\pi \lesssim 460 MeV$.

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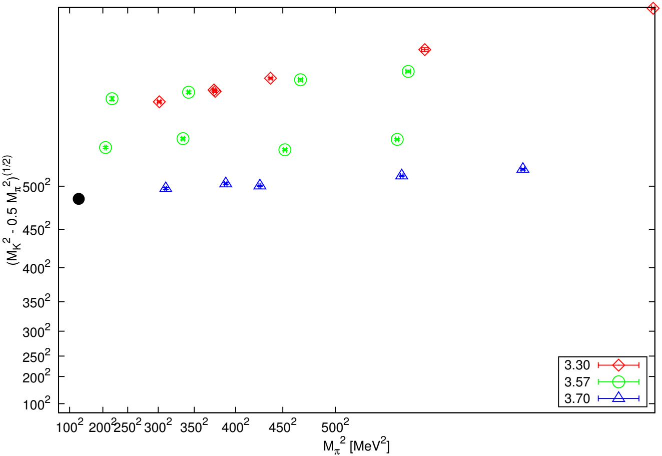
# Simulation details

Figure: Dataset used in [S. Dürr *et al* Science 322 (2008)], [S. Dürr *et al* Phys.Rev.D 81 (2010)]



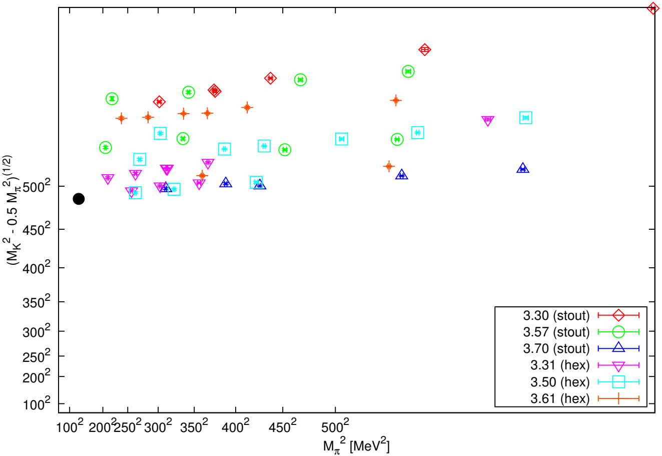
# Simulation details

BMW also has ensembles generated with 2 steps of HEX smearing. [T. Kurth and C. Hoelbling talks].

- Three values of the lattice spacing ($a \approx 0.11 fm, 0.085 fm, 0.07 fm$)

# Simulation details

Figure: Landscape



# Chiral extrapolation.

Having data $190MeV < M_\pi < 460MeV$ we need to extrapolate our data to the physical point.

# Chiral extrapolation.

## Taylor based extrapolations

The nucleon mass has an expansion around a regular point
$(M_\pi^{\exp})^2 = \frac{1}{2}[(M_\pi^{\Phi})^2 + (M_\pi^{\max})^2]$; $M_{ks}^2 = M_K^2 - \frac{1}{2}M_\pi^2$

$$M_N = \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\exp})^2\right]^i + \sum_{i=1}^{N_\pi} \beta_i' \left[M_{ks}^2 - \left(M_{ks}^{\Phi}\right)^2\right]^j$$

In particular at the physical point

$$M_N^{\Phi} = \sum_{i=1}^{N_\pi} \alpha_i' \left[(M_\pi^{\Phi})^2 - (M_\pi^{\exp})^2\right]^i + \sum_{i=1}^{N_\pi} \beta_i' \left[(M_{ks}^{\Phi})^2 - \left(M_{ks}^{\exp}\right)^2\right]^j$$

# Chiral extrapolation.

## Taylor based extrapolations

Analytical expressions in quark masses.

$$
\begin{aligned}
(aM_N) &= (aM_N^{\Phi})\left\{1+\sum_{i=1}^{N_\pi}\alpha_i\left[\left(\frac{aM_\pi}{aM_N^{\Phi}}\right)^2-\left(\frac{M_\pi^{\Phi}}{M_N^{\Phi}}\right)^2\right]^i+\right.\\
&+\left.\sum_{j=1}^{N_K}\beta_j\left[\left(\frac{aM_{ks}}{aM_N^{\Phi}}\right)^2-\left(\frac{M_{ks}^{\Phi}}{M_N^{\Phi}}\right)^2\right]^j\right\}
\end{aligned}
$$

# Chiral extrapolation.

## Taylor based extrapolations

Pade expressions to control higher order terms

$$
\begin{aligned}
(aM_N) \quad &= \quad (aM_N^{\Phi}) \left\{ 1 + \sum_{i=1}^{N_\pi} \alpha_i \left[ \left( \frac{aM_\pi}{aM_N^{\Phi}} \right)^2 - \left( \frac{M_\pi^{\Phi}}{M_N^{\Phi}} \right)^2 \right]^i + \right. \\
&+ \quad \left. \sum_{j=1}^{N_K} \beta_j \left[ \left( \frac{aM_{ks}}{aM_N^{\Phi}} \right)^2 - \left( \frac{M_{ks}^{\Phi}}{M_N^{\Phi}} \right)^2 \right]^j \right\}^{-1}
\end{aligned}
$$

# Chiral extrapolation.

## Three flavour BChPT

For a better control of the chiral extrapolation, three flavour covariant ChPT formulas have been computed [See S. Dürr poster].

# Continuum extrapolation

- Formally the two different actions receive different cutoff corrections.
- Although our actions formally have $\mathcal{O}(a\alpha_s)$ corrections, data suggests cutoff effects quadratic in $a$ [BMW. Phys.Rev.D. **79** (2009)].

## Parametrisation of cutoff effects

Each of the "chiral LEC" get an $a$ dependence.

$$p \longrightarrow p(1 + \mathcal{O}(a^n))$$

or

$$p \longrightarrow p(1 + \mathcal{O}(\alpha_s a^n))$$

## No cutoff effects

Data consistent with no cutoff effects!.

# Finite volume

In all our ensembles the bound $M_\pi L \gtrsim 4$ is maintained.

## Parametrisation of finite volume effects

We parametrise the finite volume effects as

$$\propto e^{-M_\pi L}$$

## No finite volume effects

Data consistent with no finite volume effects.

# Excited states

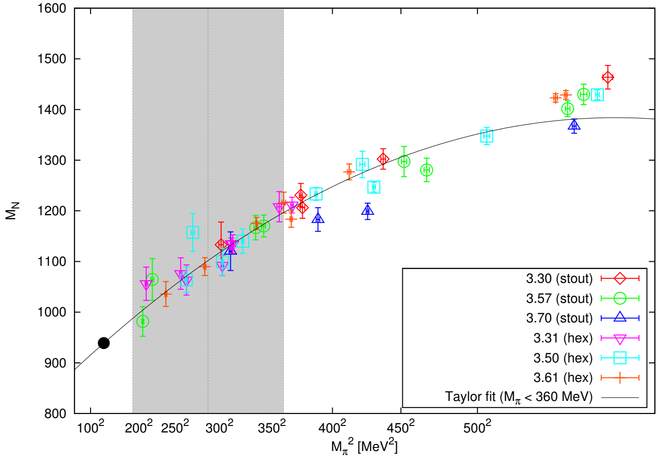
For each value of beta we use 2-3 values for the fitting interval strongly dominated by the ground state.

**Estimate excited states contributions**

Use the total of 144 possible combinations of fitting intervals.

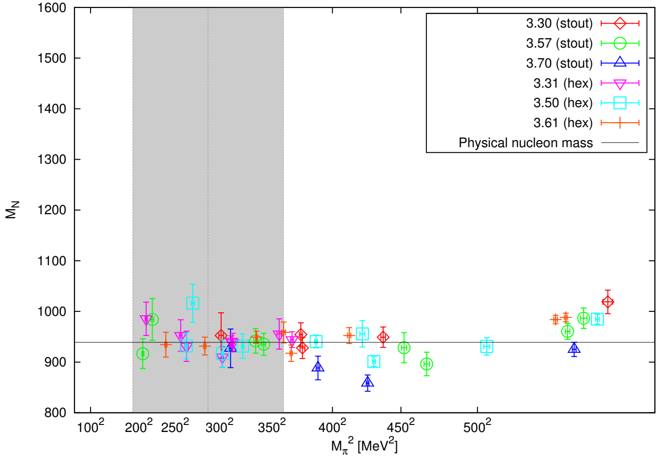
# A typical fit

Figure: A taylor fit with $M_\pi < 360 MeV$. $\chi^2/dof = 17.3/17$



# A typical fit

Figure: Residuals after the fit



# Preliminary Results

## Systematics *a là* BMW

- 2 formulas to extrapolate to the physical point: Taylor, Padé.
- 2 pion mass cuts: $M_\pi < 360 MeV$ and $M_\pi < 460 MeV$.
- No cutoff effects or finite volume corrections (consistent with our data).
- 144 fitting intervals to estimate excited estates contributions.

This amounts a total of $144 \times 2 \times 2 = 576$ analysis.

# Preliminary Results

**Central idea of the analysis.**

Use all of them, and let data decide.

- use each of the 576 values, weighted by the quality of the fit, to construct a distribution of values.
- The final result is the median.
- The systematic error are given by the 16th and 84th percentiles.
- All the process is bootstrapped and the deviation over bootstraps samples taken as the statistical error.
- The total error is the sum by quadratures of the systematic and the statistical error.

# Preliminary Results

### Following the procedure

- $\sigma_{\pi N} = 49(10)_{\text{stat}}(11)_{\text{sys}}$ MeV
- $\sigma_{ksN} = 49(37)_{\text{stat}}(26)_{\text{sys}}$ MeV
- $y = 0.08(7)_{\text{stat}}(4)_{\text{sys}}$

# Preliminary Results

Figure: Distribution of values

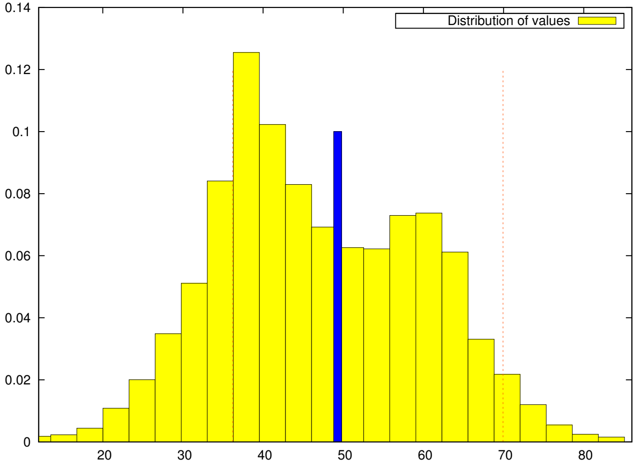
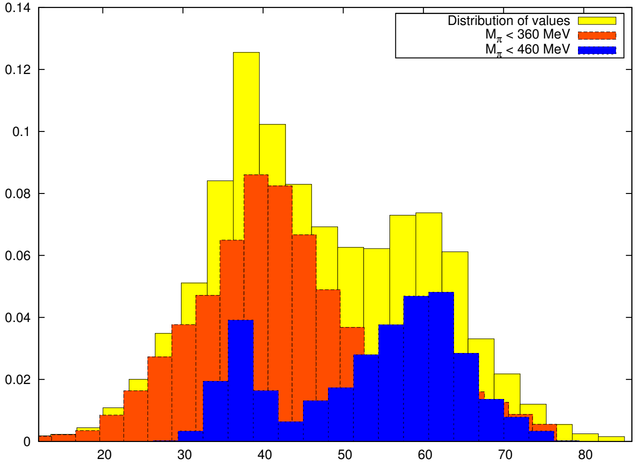


Figure: Different $M_\pi$ cuts is the main source of systematic error



# Future

$M\pi = 134$ MeV just by chance!!

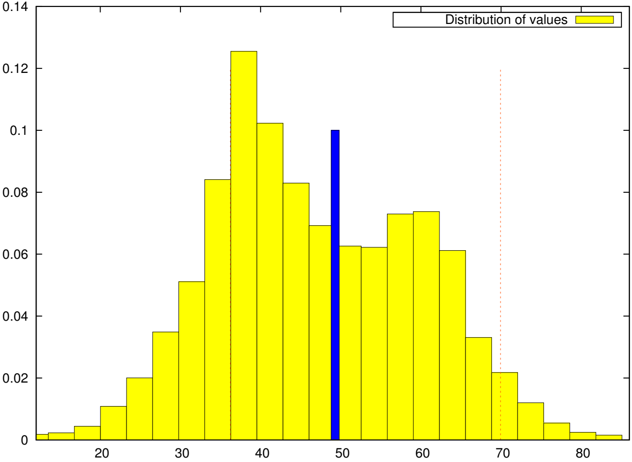
## Imaginary world

Repeat the analysis in an imaginary world with a heavier $m_{ud}$.

- $M_\pi = 200$ MeV
- $M_K = 505$ MeV
- $M_N = 1000$ MeV

# Future

Figure: Distribution of values in imaginary world.



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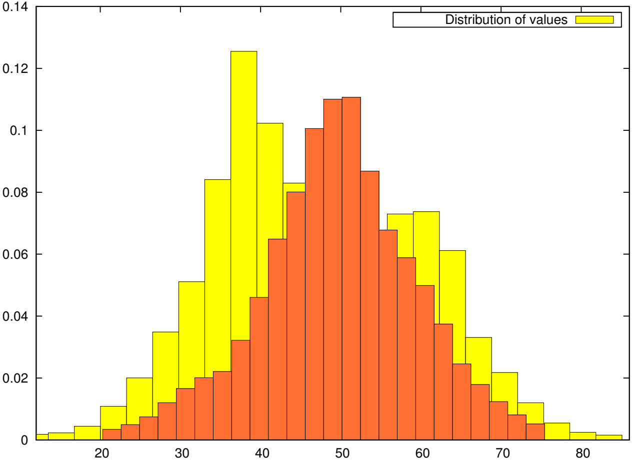

# Future

## Sigma term in imaginary world

$$\sigma_{\pi N} = XX(7)_{\text{stat}}(6)_{\text{sys}}$$

Both systematic and statistical errors reduced roughly by a 70%.

# Conclusions

- Using a sub-sample of the BMW ensemble:
  - 2 (similar) actions.
  - 6 values of $\beta$.
  - Large volumes ($M_\pi L \gtrsim 4$).
  - Data in the range $190 MeV \lesssim M_\pi \lesssim 460 MeV$.
- We have presented preliminary results for the nucleon sigma term, obtaining $\sigma_{\pi N} = 49 \pm 15$ MeV.
- For a better control over chiral extrapolation, and for a better sensitivity to the strange content, still need to work in the three flavour CBChPT [see S.Dürr poster].
- Computing sigma terms of all the octet.

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