Spatial diquark correlations in a hadron

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Diquarks

- Diquarks are two-quark systems.
- They are colored objects and so cannot be studied in isolation.
- The diquarks considered here are created by operators of the form $\overline{q_C}\Gamma q$, where $\overline{q_C} = q^T C = iq^T\gamma_0\gamma_2$ and $\Gamma = 1, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$. Their color structure combines in the $\overline{\mathbf{3}_c}$ antitriplet representation.

Good and bad diquarks

- The lowest energy diquarks are the spin 0, flavor antisymmetric "good" diquarks $\overline{q_C}\gamma_5 q$ and $\overline{q_C}\gamma_5\gamma_0 q$, and the spin 1, flavor symmetric "bad" diquarks $\overline{q_C}\gamma_i q$ and $\overline{q_C}\sigma_{0i} q$. Both of these are even parity, color $\overline{\mathbf{3}}_c$.
- One gluon exchange in a quark model predicts that the bad diquarks have higher energy by $\sim$ 200 MeV.
- Instanton interactions also favor good diquarks.
- The remaining $\overline{q_C}\Gamma q$ diquarks have odd parity and higher energy.

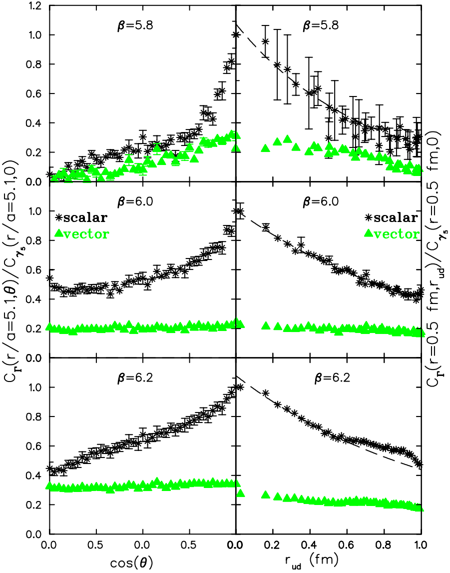
Previous studies

- C. Alexandrou, Ph. de Forcrand and B. Lucini, Phys. Rev. Lett. **97**, 222002 (2006) [arXiv:hep-lat/0609004].
- Color antitriplet diquark combined with a static quark to form a color singlet.
- Measured two-quark density correlator:

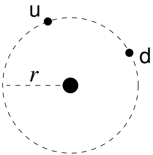
$$C_\Gamma(\mathbf{r}_u, \mathbf{r}_d, t) = \langle 0| J_\Gamma(\mathbf{0}, 2t) J_0^u(\mathbf{r}_u, t) J_0^d(\mathbf{r}_d, t) J_\Gamma^\dagger(\mathbf{0}, 0)|0\rangle$$

where $J_0^f = \overline{f}\gamma_0 f$ and $J_\Gamma = \epsilon^{abc}\left[u_a^T C\Gamma d_b \pm d_a^T C\Gamma u_b\right] s_c$.

Spatial correlations



To isolate the intrinsic diquark correlations, the authors looked at spherical shells $|\mathbf{r}_u| = |\mathbf{r}_d| = r$.



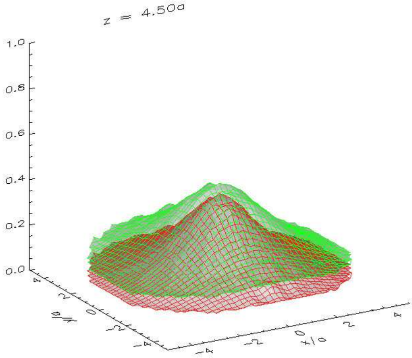
A different approach

- R. Babich, N. Garron, C. Hoelbling, J. Howard, L. Lellouch and C. Rebbi, Phys. Rev. D **76**, 074021 (2007) [arXiv:hep-lat/0701023].
- Using gauge-fixed lattices and finite mass strange quarks (degenerate with u and d), the zero-momentum correlator

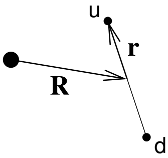
$$G(\vec{r}_u, \vec{r}_d, t) = \sum_{\vec{r}} \langle u(\vec{r} + \vec{r}_u, t) d(\vec{r} + \vec{r}_d, t) s(\vec{r}, t) \bar{u}(\vec{0}, 0) \bar{d}(\vec{0}, 0) \bar{s}(\vec{0}, 0) \rangle$$

  was computed for the Λ, Σ, and Σ^* baryons.
- This was used to define a wave function $\Psi(\vec{r}_u, \vec{r}_d) = \frac{G(\vec{r}_u, \vec{r}_d, t)}{\sum_{\vec{r}_u, \vec{r}_d} |G(\vec{r}_u, \vec{r}_d, t)|^2}$.

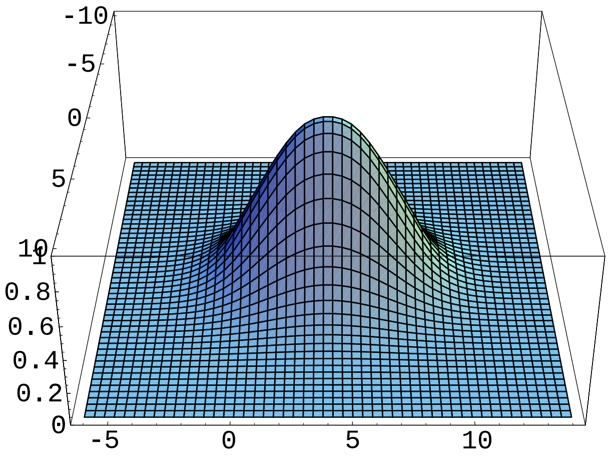
Diquark wavefunction



Λ (red) and Σ^* (green) in the Coulomb gauge, at $R/a = 4.5$ (left) and $R/a = 2.25$ (right).



Correlation function



- Uncorrelated $\rho_2(\mathbf{r}_1, \mathbf{r}_2) = \rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)$ plotted this way can give the appearance of a diquark.
- Want to show only the clustering induced by the diquark interaction.

Correlation function

$$C(\mathbf{r}_1, \mathbf{r}_2) = \frac{\rho_2(\mathbf{r}_1, \mathbf{r}_2) - \rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)}{\rho_1(\mathbf{r}_1)\rho_1(\mathbf{r}_2)}$$

- Is zero if there is no diquark interaction.
- Denominator compensates for presence of static quark at $\mathbf{r} = 0$.

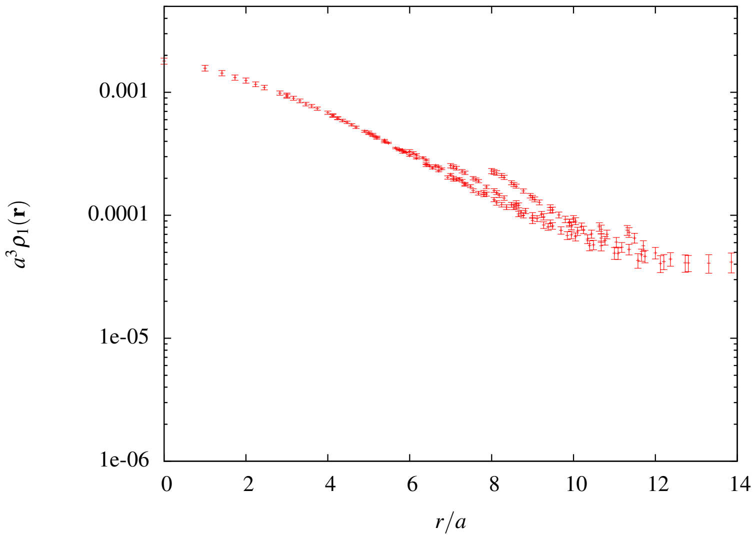
Measurements

$$\rho_2(\mathbf{r}_1,\mathbf{r}_2) \propto \langle 0| J_{\gamma_5}(\mathbf{0},t_f) J_0^u(\mathbf{r}_1,t) J_0^d(\mathbf{r}_2,t) \overline{J_{\gamma_5}}(\mathbf{0},t_i)|0\rangle$$

$$\rho_1(\mathbf{r}) \propto \langle 0| J_{\gamma_5}(\mathbf{0},t_f) J_0^u(\mathbf{r},t) \overline{J_{\gamma_5}}(\mathbf{0},t_i)|0\rangle$$

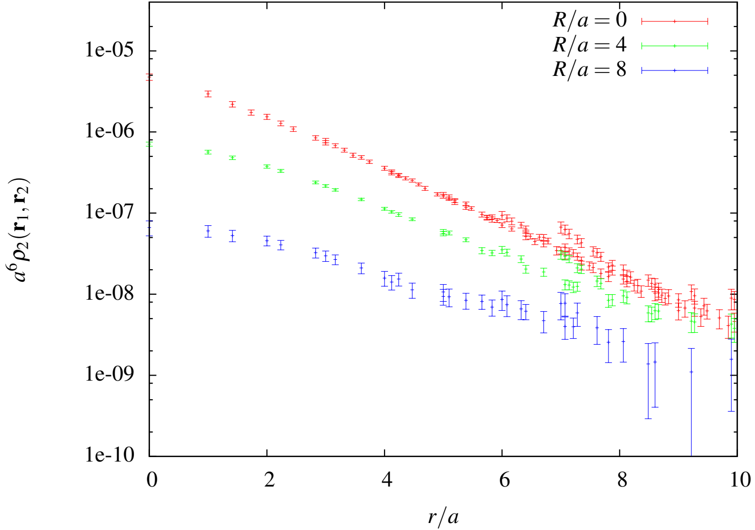
- $16^3 \times 32$, $\beta = 6.0$, quenched, $a = 0.093$ fm, 200 configurations (NERSC OSU_Q60a).
- $m_\pi = 893$ MeV, Wilson fermions.
- Measurements averaged over 2 timeslices and seven static quark lines.
- ρ_1 and ρ_2 normalized so that $\sum_{\mathbf{r}} \rho_1(\mathbf{r}) = 1$ and $\sum_{\mathbf{r}_1,\mathbf{r}_2} \rho_2(\mathbf{r}_1,\mathbf{r}_2) = 1$.
- Also computed $\overline{q_C}\gamma_i q$ bad diquark case.

Image effects



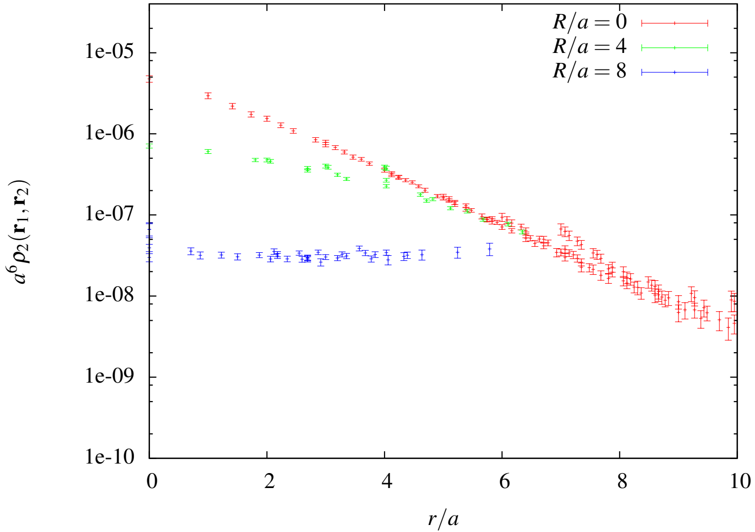
$\rho_1(r)$

Image effects



ρ_2 with R fixed and $\mathbf{R} \perp \mathbf{r}$

Image effects



ρ_2 with R fixed and $\mathbf{R} \parallel \mathbf{r}$

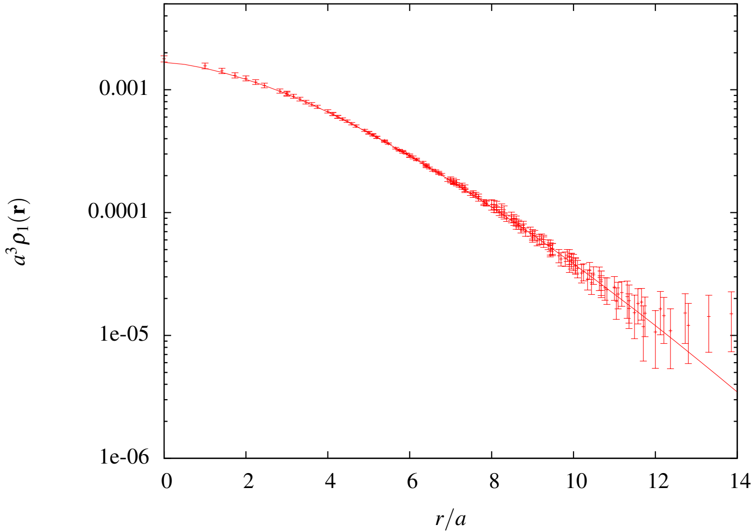
Image correction

- Deal with image effects by fitting a parameterized function to the data.
- Instead of fitting $f(\mathbf{r}_1, \mathbf{r}_2)$, fit

$$f_{\text{img}}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{n_1^i, n_2^i = -1,0,1} f(\mathbf{r}_1 + \mathbf{n}_1 L, \mathbf{r}_2 + \mathbf{n}_2 L)$$

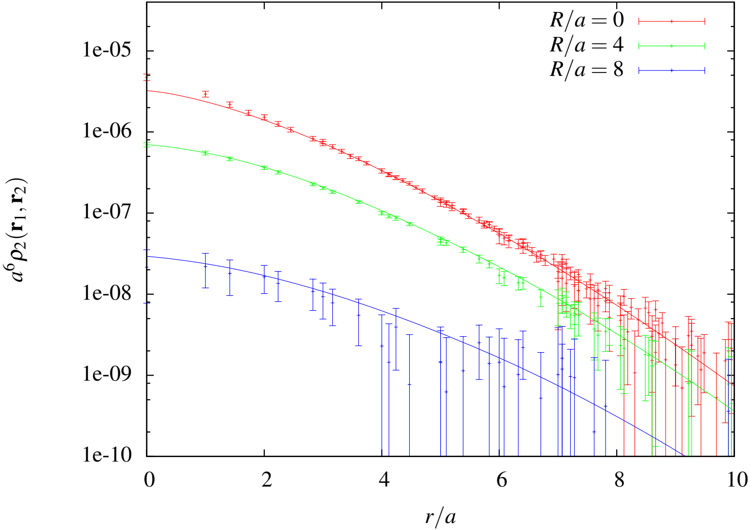
- Given a good fit, the image effects can be subtracted off.
- Data points most affected by images were excluded from the fit shown here.
- 11 parameter fit, $\frac{\chi^2}{\text{dof}} \simeq 0.25$

Image correction



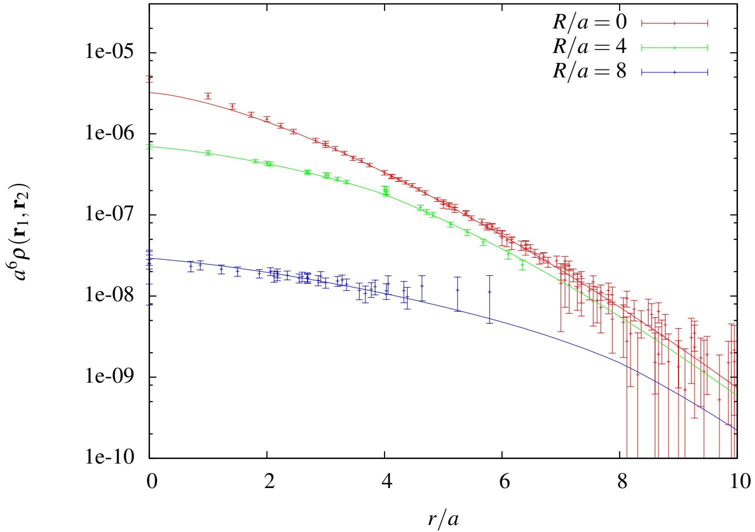
$\rho_1(r)$

Image correction



ρ_2 with R fixed and $\mathbf{R} \perp \mathbf{r}$

Image correction

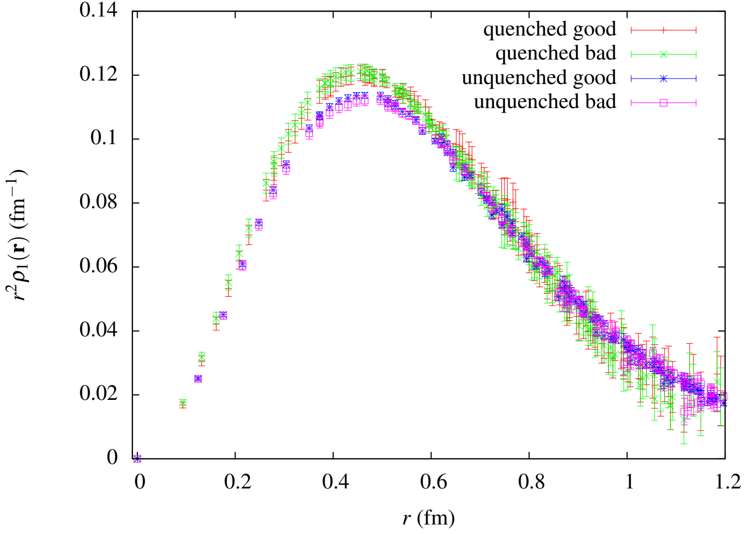


ρ_2 with R fixed and $\mathbf{R} \parallel \mathbf{r}$

Unquenched ensemble

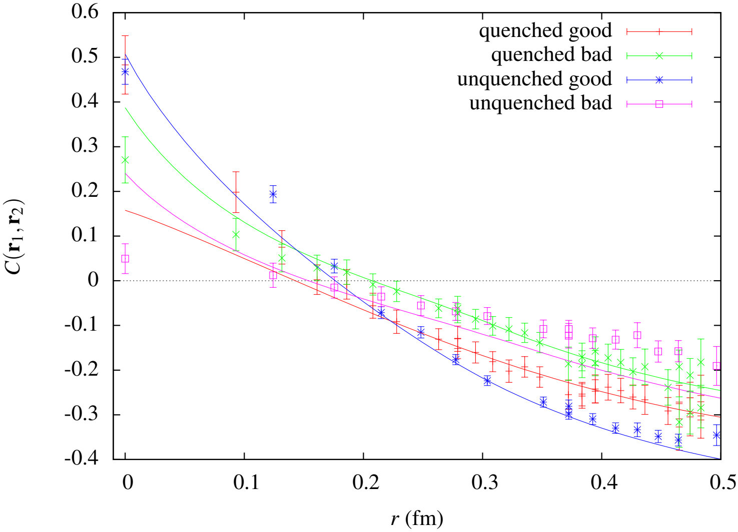
- MILC gauge configurations with domain wall valence quarks.
- $20^3 \times 64$, $a = 0.1241$ fm, $m_\pi = 293$ MeV
- 8 measurements per configuration
- 453 configurations.

Comparison between configurations



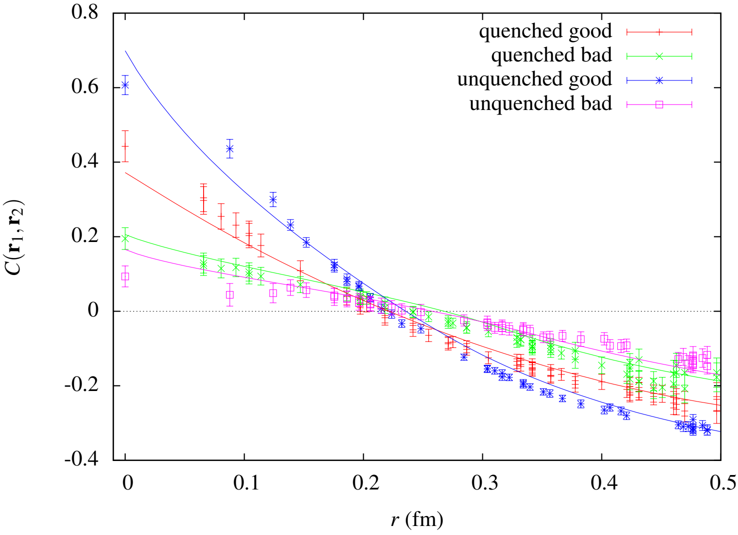
$r^2\rho_1(r)$

Correlation function



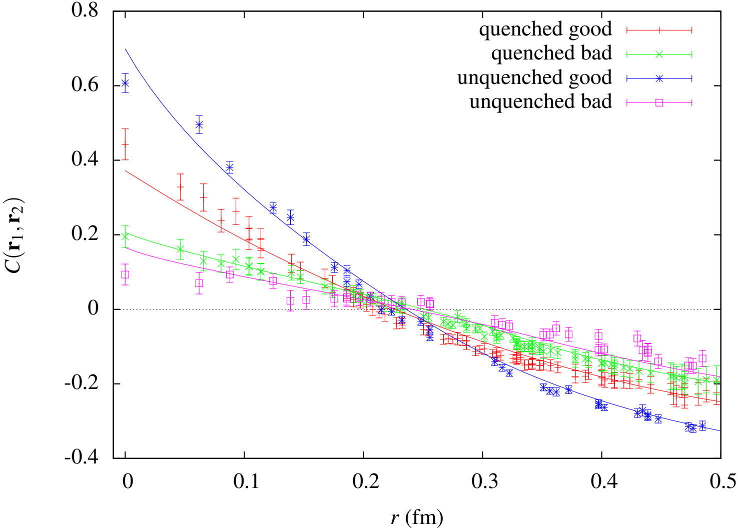
$C(\mathbf{R} + \mathbf{r}, \mathbf{R} - \mathbf{r})$ with $R = 0$

Correlation function



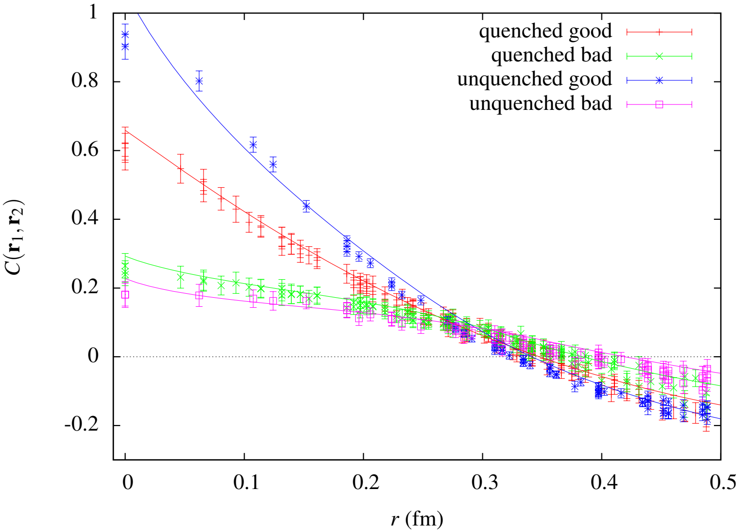
C with $R = 0.2$ fm and $\mathbf{R} \perp \mathbf{r}$

Correlation function



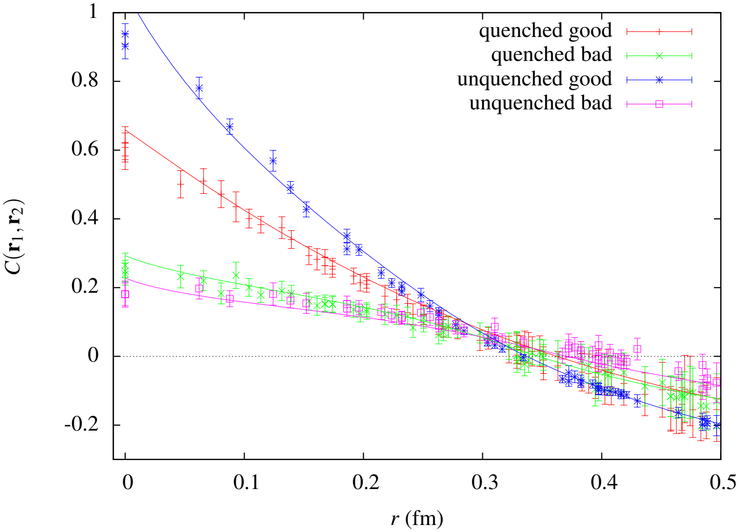
C with $R = 0.2$ fm and $\mathbf{R} \parallel \mathbf{r}$

Correlation function



C with $R = 0.4$ fm and $\mathbf{R} \perp \mathbf{r}$

Correlation function



C with $R = 0.4$ fm and $\mathbf{R} \parallel \mathbf{r}$

Summary

- Good and bad diquark compared at $m_\pi = 893$ MeV and $m_\pi = 293$ MeV.
- Good diquark has stronger correlation as expected.
- Difference between good and bad diquark is greater at smaller pion mass.

References

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