Phase Shift with LapH Propagators

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Outline

- Introduction
- Distillation
  - single pion correlator construction
- Two particle channel
  - Preliminary partial results for I=2 pi-pi
    - finite momenta (phase shift)
- Other examples
- Future/Summary

Introduction

- Goal: simulate low-lying hadronic spectrum on large, dynamical lattices with "light" dynamical quarks

- In particular, we want to explore the multi-particle spectrum in addition to the single-particle spectrum obtained in
  J.Bulava et al. Phys.Rev.Lett.103:262001,2009
  J.Bulava et al. arXiv:1004.5072 [hep-lat]

- Need interpolating operators with good overlap with multi-particle states

- all-to-all propagators
  - LapH propagators

Distillation

HadSpec: Phys.Rev.D80:054506,2009

Eigenvectors of the 3-D Laplacian smearing function

$$-\vec{\nabla}^2 v^{(i)} = \lambda_i v^{(i)}$$

Smear quarks using these eigenvectors

$$\tilde{\psi} = \sum_{i=0}^{N_{\rm ev}-1} f(\lambda_i) v^{(i)} \otimes \vec{v}^{(i)\dagger} \psi$$

$N_{\rm ev}$ is the number of eigenvectors to keep (32,64,128)

$f(\lambda_i)$ can be chosen to resemble Jacobi smearing

"LapH" Propagators

$f(\lambda_i) = 1$ for $i < N_{\rm ev}$

(Heaviside step-function)

smear the quark fields

$$\tilde{\psi} = \mathbf{V}\mathbf{V}^\dagger \psi = \Box \psi$$

with

$$\mathbf{V}(t) = \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ \vec{v}^{(0)} & \vec{v}^{(1)} & \ldots & \vec{v}^{(N_{ev}-1)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Single pion operator with finite momentum

$$\mathcal{O}_\pi(\vec{p},t) = \sum_{x,y,z} e^{-ipy} \left(\bar{d}_x(t)\Box_{xy}(t)\right)\gamma_5\left(\Box_{yz}u_z(t)\right)$$

Correlation function

$$C_\pi(t,t_0) = \langle \mathcal{O}_\pi(t)\mathcal{O}_\pi^\dagger(t_0)\rangle$$

$$= \mathrm{Tr}\left[\Phi(t)\tau(t,t_0)\Phi(t_0)\tau(t_0,t)\right]$$

Ingredients

$$\Phi_{\alpha\beta}(t) = \mathbf{V}^\dagger(t)\gamma_{5\alpha\beta}\mathbf{V}(t)$$

$$\tau_{\alpha\beta}(t,t_0) = \mathbf{V}^\dagger(t)M^{-1}_{\alpha\beta}(t,t_0)\mathbf{V}(t_0)$$

I=2 pi-pi correlation matrix

$$t: \longleftarrow \pi(-p)\pi(p) \longrightarrow$$

summed over p,q
for S-wave

$$t_0: \leftarrow \pi(-q)\pi(q) \rightarrow$$

$$C_{p,q}(t,t_0) = \langle \sum_{x\,y\,z} e^{ipy}\ \bar{d}_{xt}\Box_{xyt}\ \gamma_5\ \Box_{yzt}u_{zt}$$
$$\sum_{x'y'z'} e^{-ipy'}\bar{d}_{x't}\Box_{x'y't}\ \gamma_5\ \Box_{y'z't}u_{z't}$$
$$\sum_{x''y''z''} e^{iqy''}\ \bar{u}_{x''t_0}\Box_{x''y''t_0}\ \gamma_5\ \Box_{y''z''t_0}d_{z''t_0}$$
$$\sum_{x'''y'''z'''} e^{-iqy'''}\bar{u}_{x'''t_0}\Box_{x'''y'''t_0}\ \gamma_5\ \Box_{y'''z'''t_0}d_{z'''t_0}\ \rangle$$

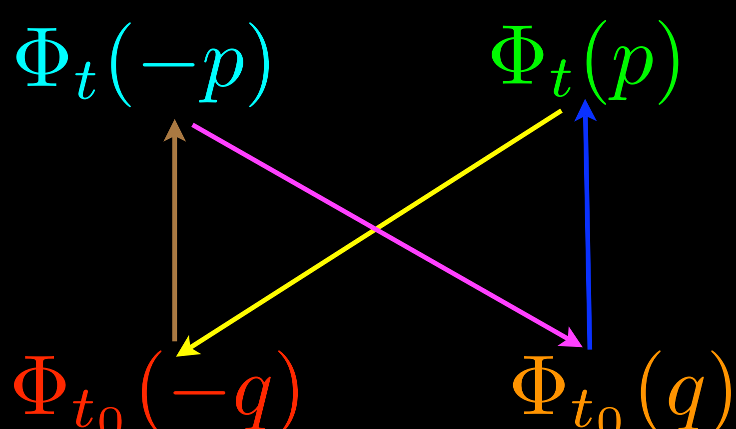
$$C_{p,q}(t,t_0) = \sum_{x'''y'''z'''}\ \sum_{x''y''z''}\ \sum_{x'y'z'}\ \sum_{xyz}$$

$$\left(V^\dagger_{z',t} M_u^{-1}(z',t;x''',t_0)\, V_{x'''t_0}\right)\left(V^\dagger_{y'''t_0}V_{y'''t_0}\right)e^{-iqy'''}$$

$$\left(V^\dagger_{xt}\, M_d^{-1}(x,t;z''',t_0)\, V_{z'''t_0}\right)^\dagger \left(V^\dagger_{yt}\, V_{yt}\right)e^{ipy}$$

$$\left(V^\dagger_{zt}\, M_u^{-1}(z,t;x'',t_0)\, V_{x''t_0}\right)\left(V^\dagger_{y''t_0}V_{y''t_0}\right)e^{iqy''}$$

$$\left(V^\dagger_{x't}\, M_d^{-1}(x',t;z'',t_0)\, V_{z''t_0}\right)^\dagger\left(V^\dagger_{y',t}V_{y',t}\right)e^{-ipy'}$$



+ the C_π^2 type diagram

Construct the matrix of correlation functions:

$$C_{ij}(t,t_0) = \langle \mathcal{O}_{\pi\pi}(\vec{p}_{1i}, -\vec{p}_{2i}, t)\mathcal{O}^\dagger_{\pi\pi}(\vec{q}_{1j}, -\vec{q}_{2j}, t_0)\rangle$$

Solve the generalized eigenvalue problem

$$C_{ij}(t)w_j = \lambda(t,t_0)C_{ij}(t_0)w_j$$

to compute the energy shift/momenta of the pions in the finite box

Use both fixed coefficient optimized correlators and the optimized correlators with fixed $t - t_0$

(Alpha Collaboration JHEP 0904:094,2009)

Simulation Parameters

Nf=2+1 anisotropic lattices

R.Edwards, B.Joo and H-W.Lin

Phys.Rev.D78 014505

as/at=3.5

$16^3 \times 128$ Mpi $\simeq$ 380 MeV mL $\simeq$ 4

32 eigenvectors

$20^3 \times 128$ Mpi $\simeq$ 380 MeV mL $\simeq$ 5

64 eigenvectors

r0/as=3.221(25)

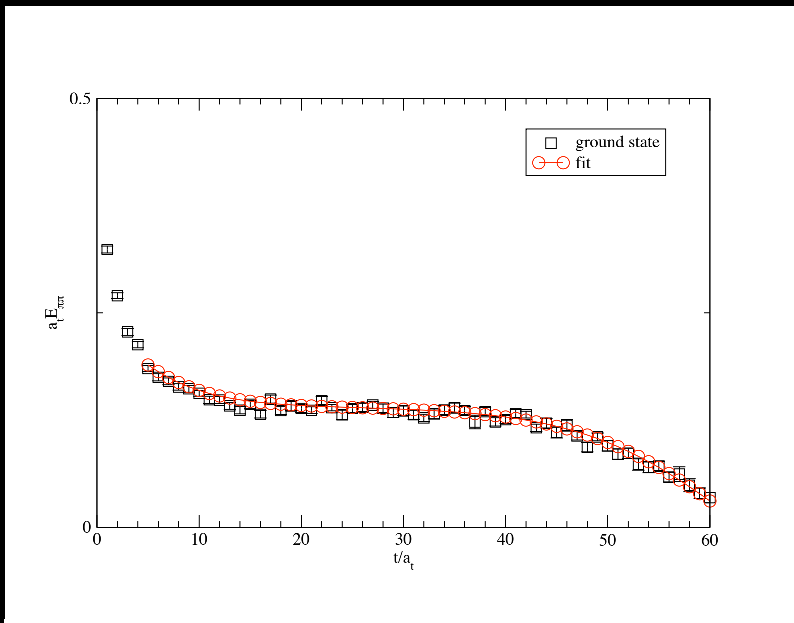
90 - 100 configs

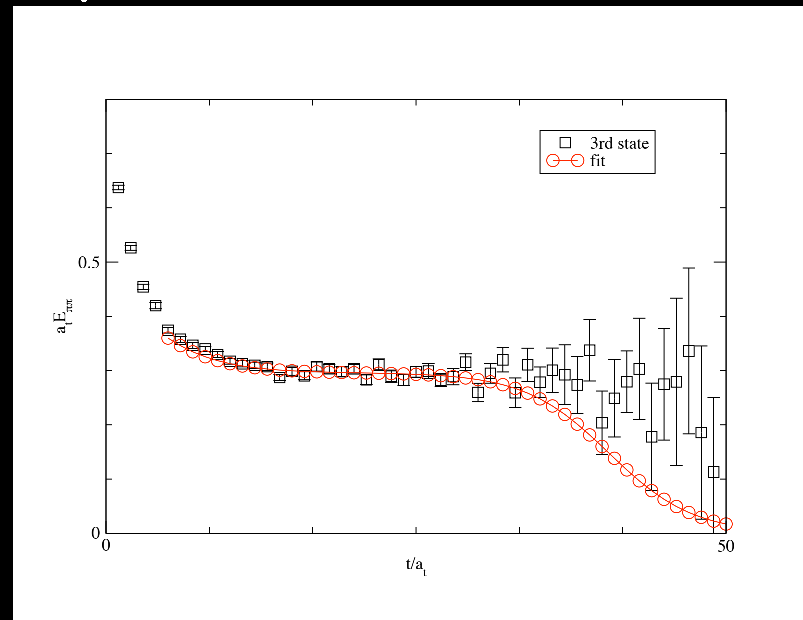
(40 trajectory sep.)

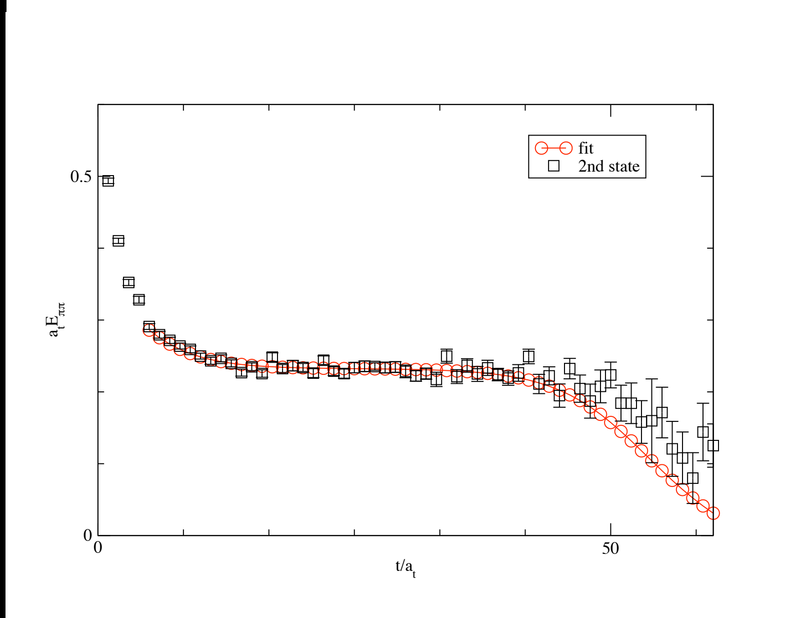
5 of the lowest momenta operators were used

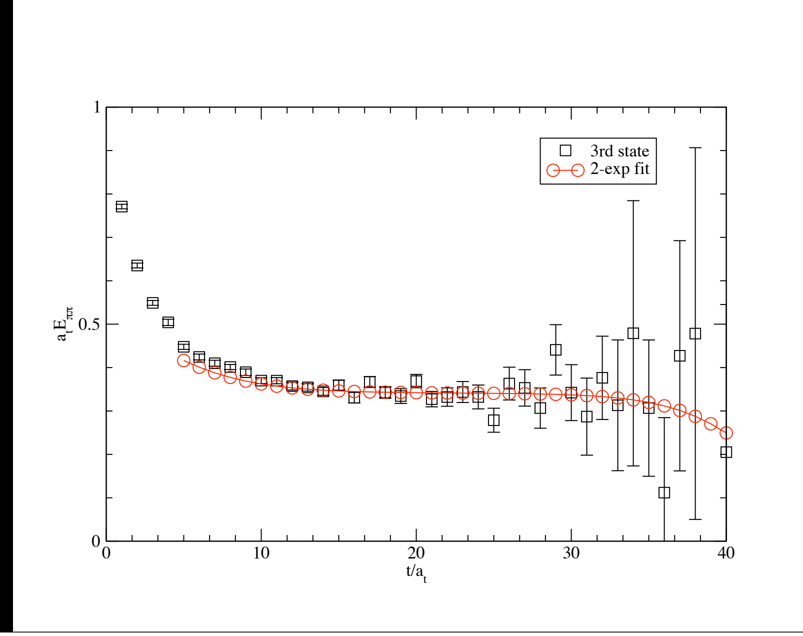
Diagonal Effective Masses

$V=20^3$ t0=15, t*=25









Phase Shift

B.DeWitt, Phys.Rev.103 (1956)
M.Luscher, Com.Math.Phys.(1986)Nucl.Phys.B354(1991)
L.Lellouch, M.Luscher, Com.Math.Phys.219 (2001)
K.Rummukainen, S.Gottlieb, Nucl. Phys.B450 (1995)
X.Feng et al., Phys.Rev.D70 (2004)

Spectrum of the 2-particle state in a finite "box" → phase shift in infinite V

Phase Shift

$$\tan\delta(p_n) = \frac{\pi^{3/2}\sqrt{\tilde{n}}}{\mathcal{Z}_{00}(1;\tilde{n})}$$

$$(a_s p)^2 = \left(\frac{2\pi}{L/a_s}\right)^2 \tilde{n}$$ defines $\tilde{n}$

two-particle energies $a_t E_{\pi\pi}(\vec{p})$

$$(a_s p)^2 = \xi^2 (a_t m)^2 \left[\left(\frac{(a_t E_{\pi\pi})}{2(a_t m)}\right)^2 - 1\right]$$

determines momentum p

more recent, direct dynamical phase shift calculations

- NPLQCD Phys.Rev.D73 (2006)
- CP-PACS Phys.Rev.D70 (2004)

$$\tan\delta(p_n) = \frac{\pi^{3/2}\sqrt{\tilde{n}}}{\mathcal{Z}_{00}(1;\tilde{n})}$$

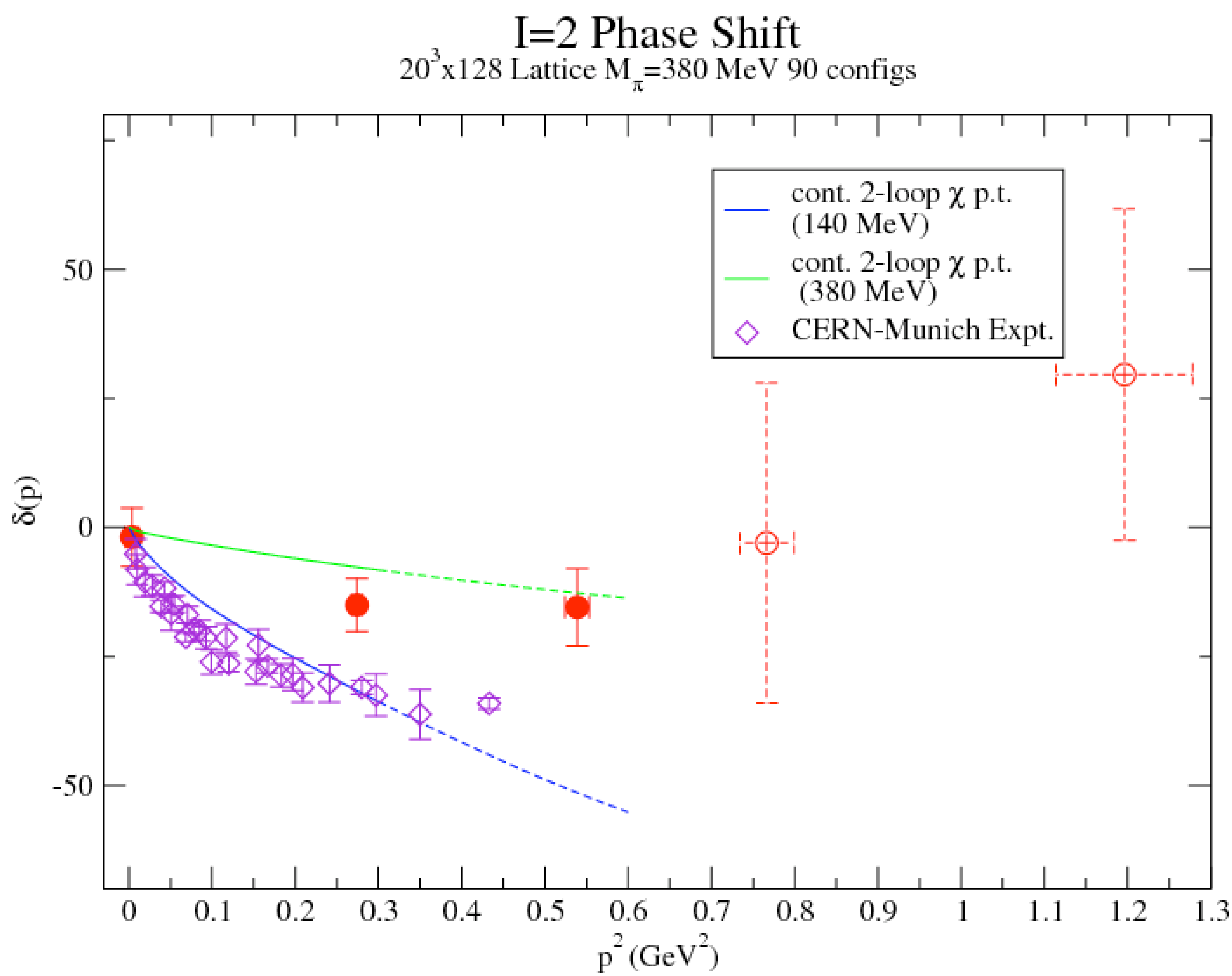
$$\mathcal{Z}_{00}(1,\tilde{n}) = \frac{1}{\sqrt{4\pi}}\left[\sum_{\vec{m}\in Z^3}\frac{e^{-(\vec{m}^2-\tilde{n})}}{(\vec{m}^2-\tilde{n})} + \sum_{l=0}^{\infty}\frac{\pi^{3/2}\tilde{n}^l}{(l-1/2)l!} + \int_0^1 \mathrm{d}t e^{\tilde{n}t}\left(\frac{\pi}{t}\right)^{3/2}\sum_{\vec{m}\in Z^3}{}' e^{\pi^2 m^2/t}\right]$$

Phase Shift

Excited state spectra of the 2-particle state

$$(a_s p)^2 \equiv \left(\frac{2\pi}{L/a_s}\right)^2 \tilde{n} \qquad \tan\delta(p_n) = \frac{\pi^{3/2}\sqrt{\tilde{n}}}{\mathcal{Z}_{00}(1;\tilde{n})}$$

$a_t E_{\pi\pi}^{(i)}$	$a_s p^{(i)}$	$\tilde{n}$	$\delta(p^{(i)})$
0.150(3)	0.04(4)	0.015(33)	-1.8(57)
0.275(4)	0.326(4)	1.078(26)	-15(5)
0.368(4)	0.457(6)	2.118(58)	-15(7)
0.447(9)	0.54(1)	3.01(13)	-3(30)
0.52(5)	0.68(2)	4.7(3)	30(30)


I=2 Phase Shift
20^3x128 Lattice Mπ=380 MeV 90 configs
cont. 2-loop χ p.t. (140 MeV)
cont. 2-loop χ p.t. (380 MeV)
CERN-Munich Expt.
δ(p)
50
0
-50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
p^2 (GeV^2)

Comments

- L=16 volume is too small for this mass
  Is L=20 OK?

- pions are still too heavy for a detailed numerical comparison with ChiPT

- need lighter pions

  .. need bigger boxes

But the technology is working !

What about I=0 and decays ?

Stochastic LapH

Quark Propagator

$$Q = \langle\langle D_j S\phi \ (D_k S\rho)^\dagger \rangle\rangle$$

or

$$Q = \langle\langle \tilde{\phi} \ \tilde{\rho}^\dagger \rangle\rangle$$

where $\tilde{\phi} = D_j S\phi$

$\tilde{\rho} = D_j S\rho$

(smeared and displaced fields)

One last step ...

we don't actually want a noisy laph

(Diluted) Stochastic LapH

Use dilution on the noise vectors

Dilution $\rho = \sum_i \mathcal{P}^{(i)} \rho$

insert $1 = \sum_i \mathcal{P}^{(i)} \mathcal{P}^{(i)\dagger}$

$$1 = \langle\!\langle \rho \rho^\dagger \rangle\!\rangle$$

$$Q = D_j S \left(M\right)^{-1} \sum_i \mathcal{P}^{(i)} \langle\!\langle \rho \rho^\dagger \rangle\!\rangle \mathcal{P}^{(i)\dagger} S D_k^\dagger$$

$$= \sum_i \left(D_j S \left(M\right)^{-1} \mathcal{P}^{(i)} \rho \right) \left(D_k S \mathcal{P}^{(i)} \rho \right)^\dagger$$

(Diluted) Stochastic LapH

Avoid putting noise into the distilled propagators

Introduce noise only in the LapH subspace

- time, spin, eigenmode index

Projection operators

time $$\mathcal{P}^{(B)}_{\alpha j;\beta k}(t;t') = \delta_{jk}\delta_{\alpha\beta}\delta_{Bt}\delta_{Bt'}$$

spin $$\mathcal{P}^{(B)}_{\alpha j;\beta k}(t;t') = \delta_{jk}\delta_{B\alpha}\delta_{B\beta}\delta_{tt'}$$

eigen mode $$\mathcal{P}^{(B)}_{\alpha j;\beta k}(t;t') = \delta_{Bj}\delta_{Bk}\delta_{\alpha\beta}\delta_{tt'}$$

(Diluted) Stochastic LapH

$$Q = D_j S \left(M\right)^{-1} S D_k^{\dagger}$$

$$Q = D_j S \left(M\right)^{-1} V_s V_s^{\dagger} D_k^{\dagger}$$

$$Q = D_j S \left(M\right)^{-1} V_s \sum_a \mathcal{P}^{(a)} \mathcal{P}^{(a)\dagger} V_s^{\dagger} D_k^{\dagger}$$

$$Q = \sum_a D_j S \left(M\right)^{-1} V_s \, \mathcal{P}^{(a)} \langle\!\langle \varrho \varrho^{\dagger} \rangle\!\rangle \mathcal{P}^{(a)\dagger} \, V_s \, D_k^{\dagger}$$

$$Q = \sum_a \langle\!\langle D_j S \left(M\right)^{-1} V_s \mathcal{P}^{(a)} \, \varrho \varrho^{\dagger} \mathcal{P}^{(a)\dagger} V_s \, D_k^{\dagger} \rangle\!\rangle$$

$$Q = \sum_a \langle\!\langle \, D_j S \left(M\right)^{-1} V_s \mathcal{P}^{(a)} \varrho \ . \left(D_k V_s \mathcal{P}^{(a)} \varrho \right)^{\dagger} \rangle\!\rangle.$$

(Diluted) Stochastic LapH

$$Q = \sum_a \langle\!\langle \left(D_j S \left(M\right)^{-1} V_s \mathcal{P}^{(a)} \varrho \right) \left(D_k V_s \mathcal{P}^{(a)} \varrho \right)^\dagger \rangle\!\rangle.$$

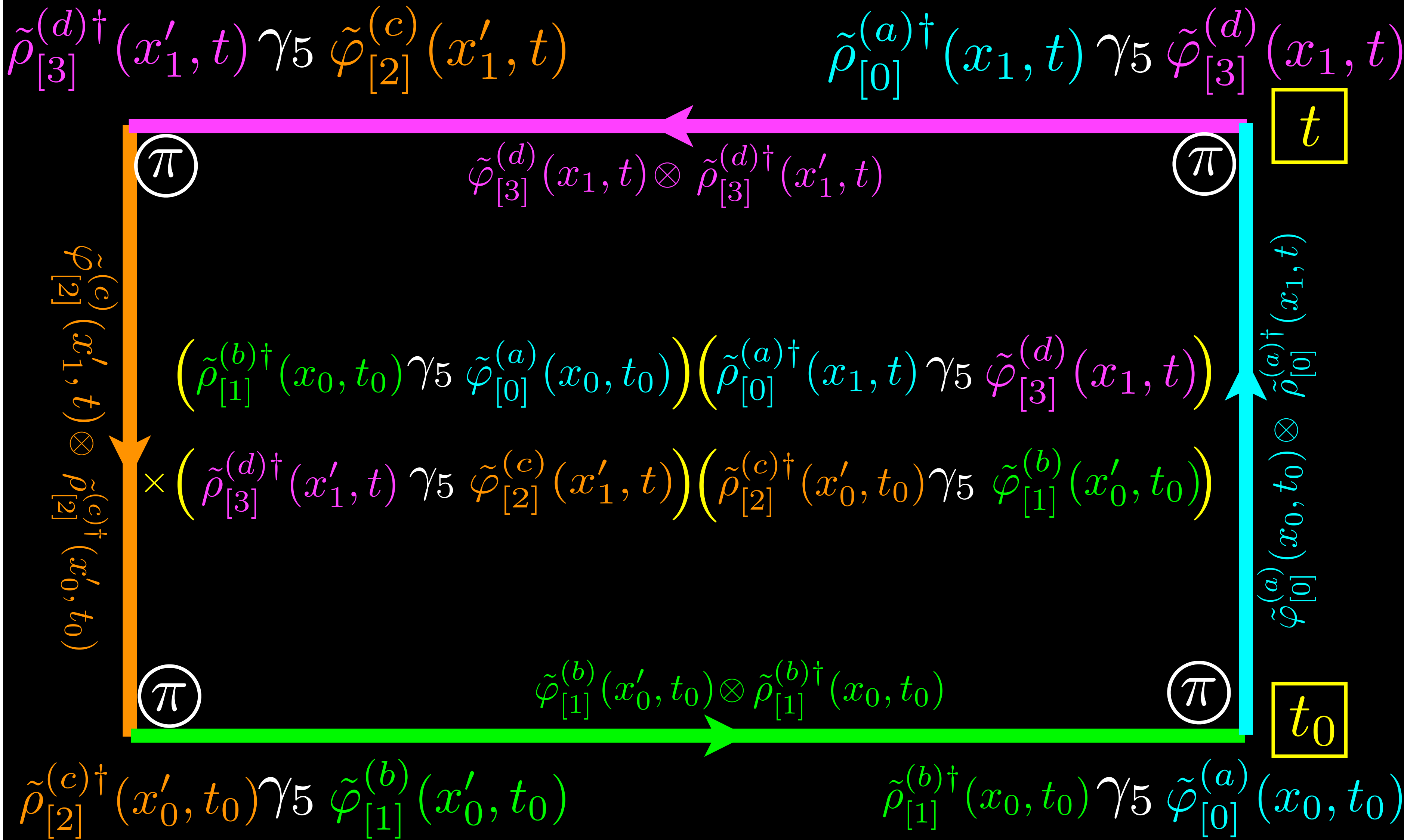
sources $\tilde{\varrho}_j^{(a)} = D_j V_s \mathcal{P}^{(a)} \varrho$

solutions $\tilde{\varphi}_j^{(a)} = D_j S \left(M\right)^{-1} \left(V_s \mathcal{P}^{(a)} \varrho \right).$

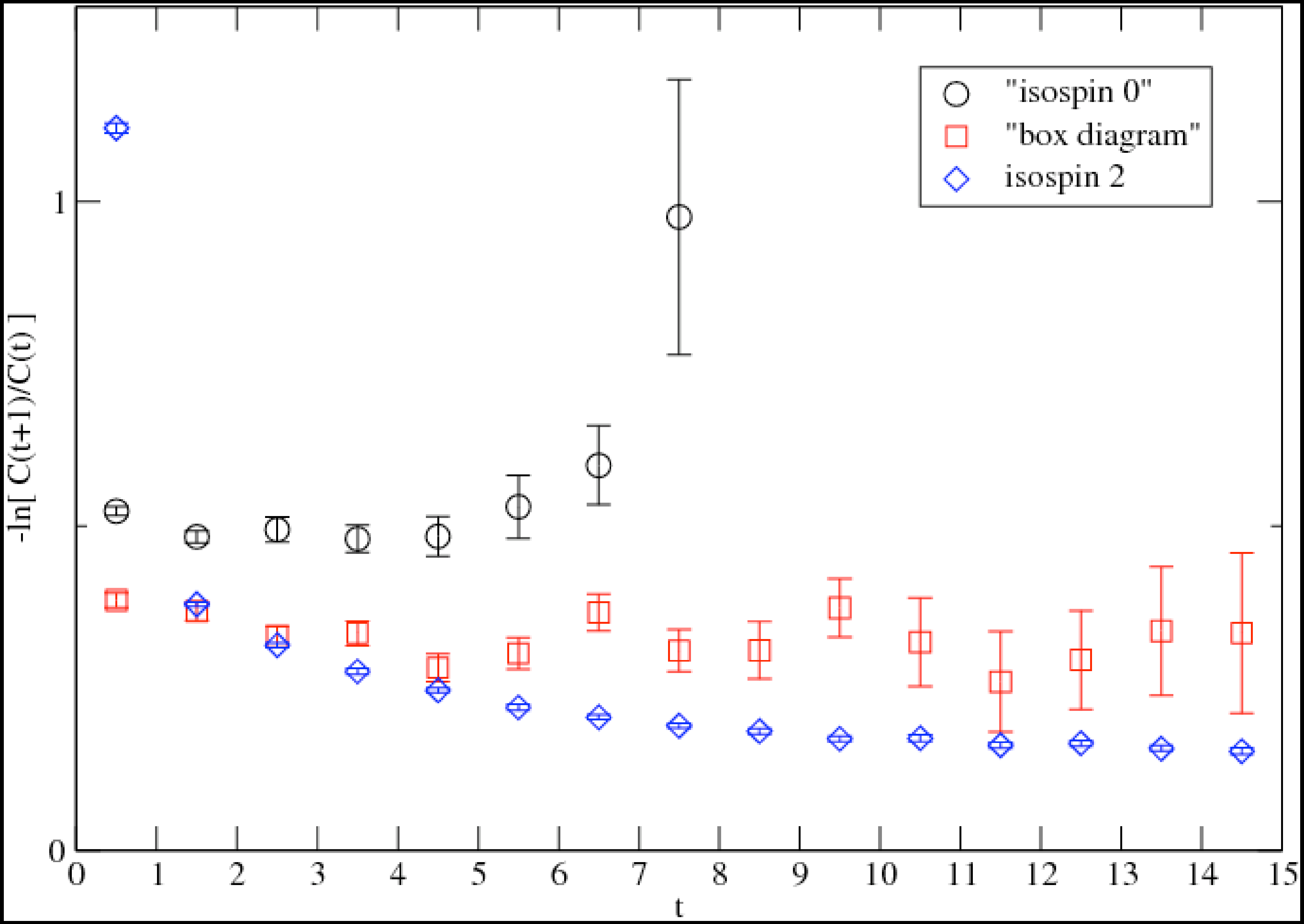
$$Q_{jk} = \sum_a \tilde{\varphi}_j^{(a)} \tilde{\varrho}_k^{(a)\dagger}$$

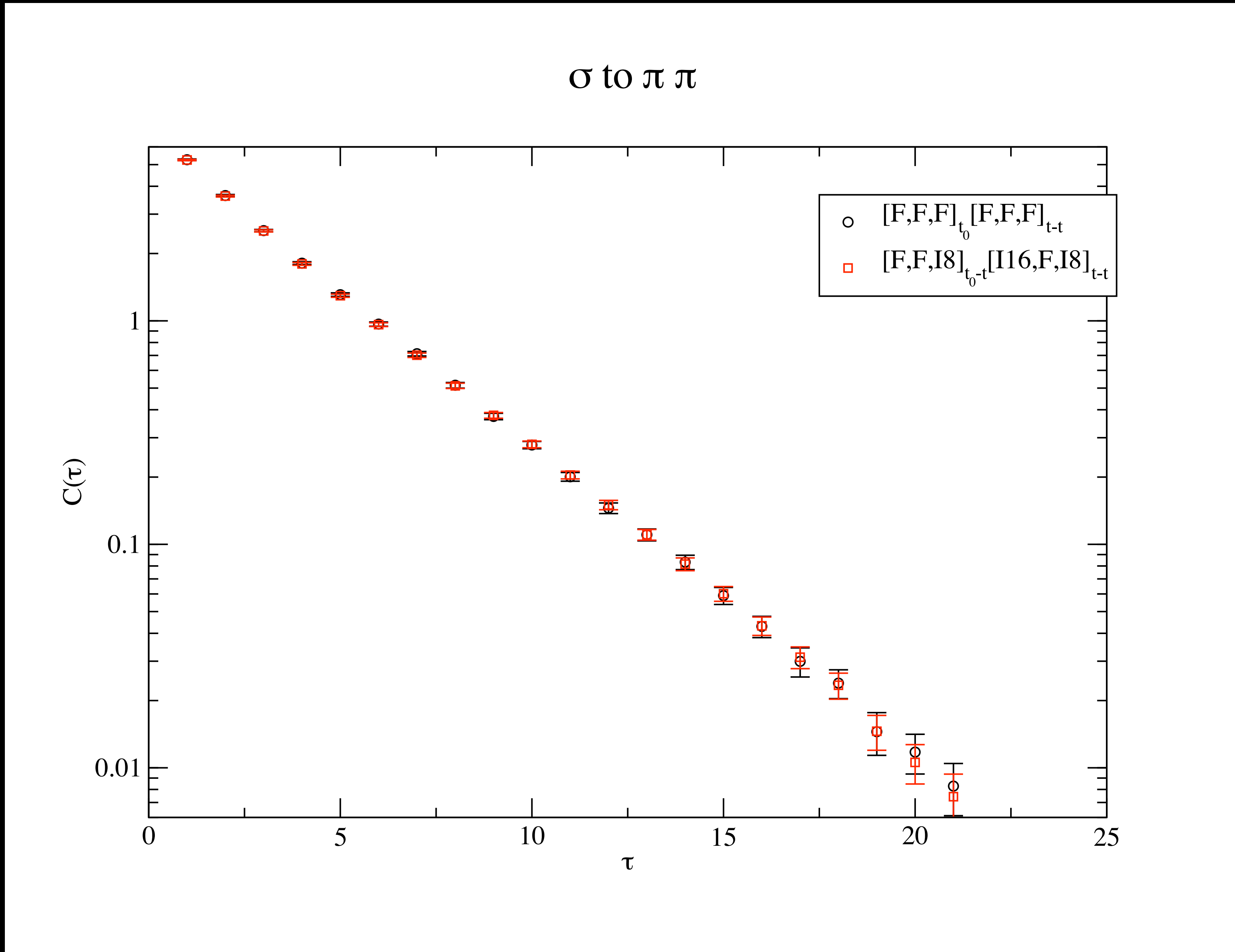
$$\overline{Q}_{jk} = \sum_a \gamma_5 \gamma_4 \tilde{\varrho}_k^{(a)} \tilde{\varphi}_j^{(a)\dagger} \gamma_4 \gamma_5$$

Box Diagram (I=0)

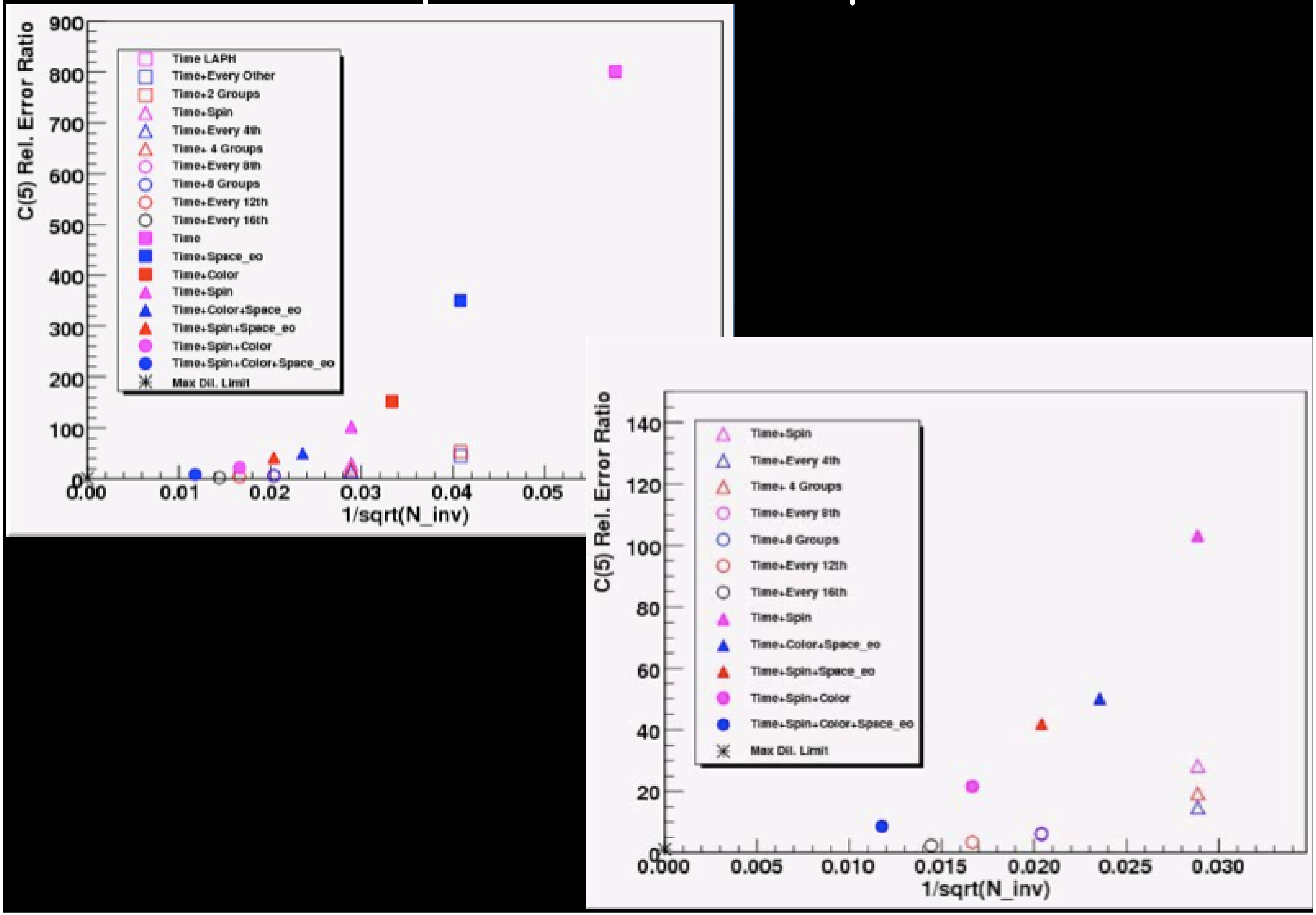


Effective Masses

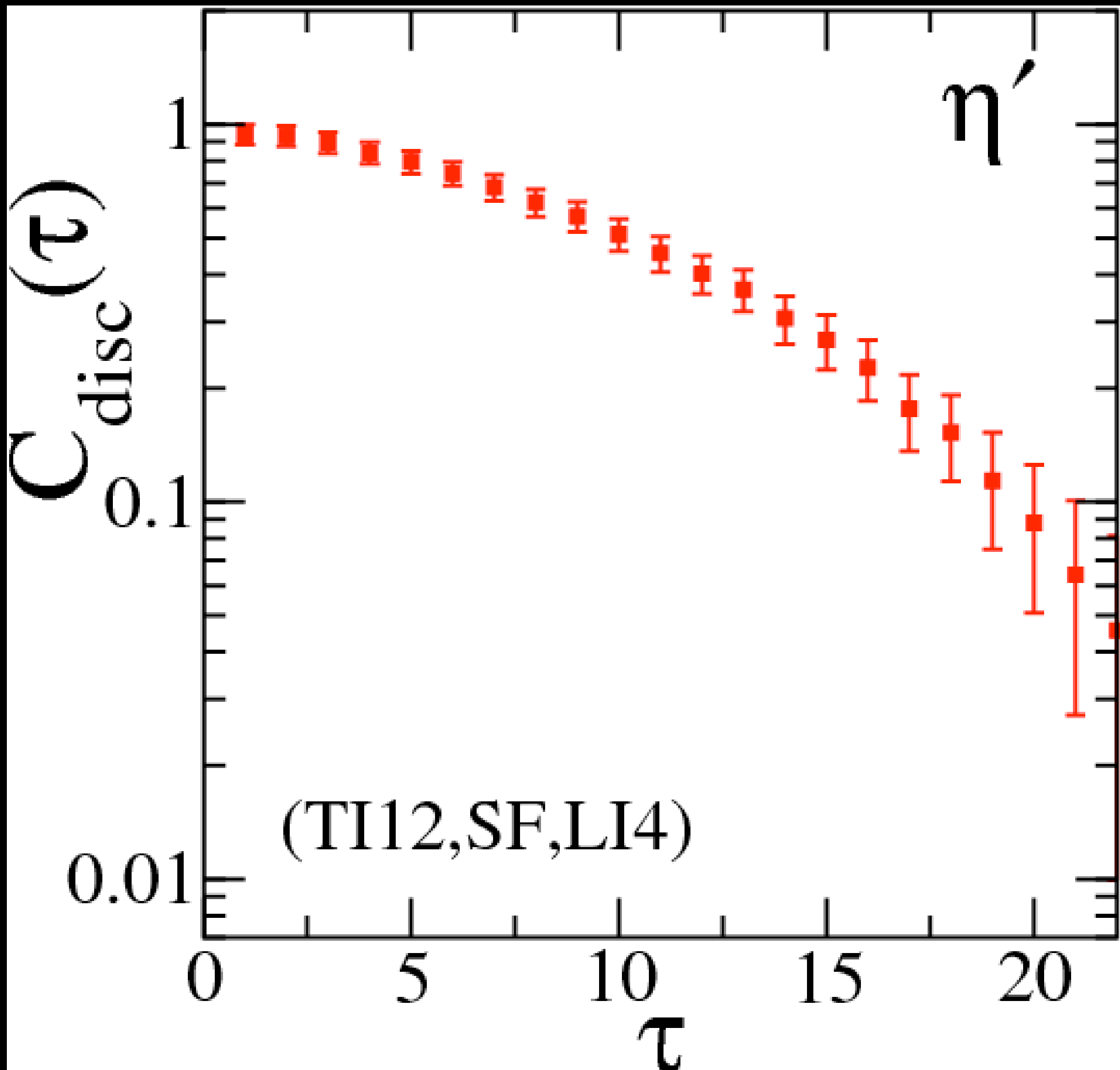



σ to π π
C(τ)
τ
1
0.1
0.01
0
5
10
15
20
25
[F,F,F]$_{t_0}$[F,F,F]$_{t-t}$
[F,F,I8]$_{t_0-t}$[I16,F,I8]$_{t-t}$

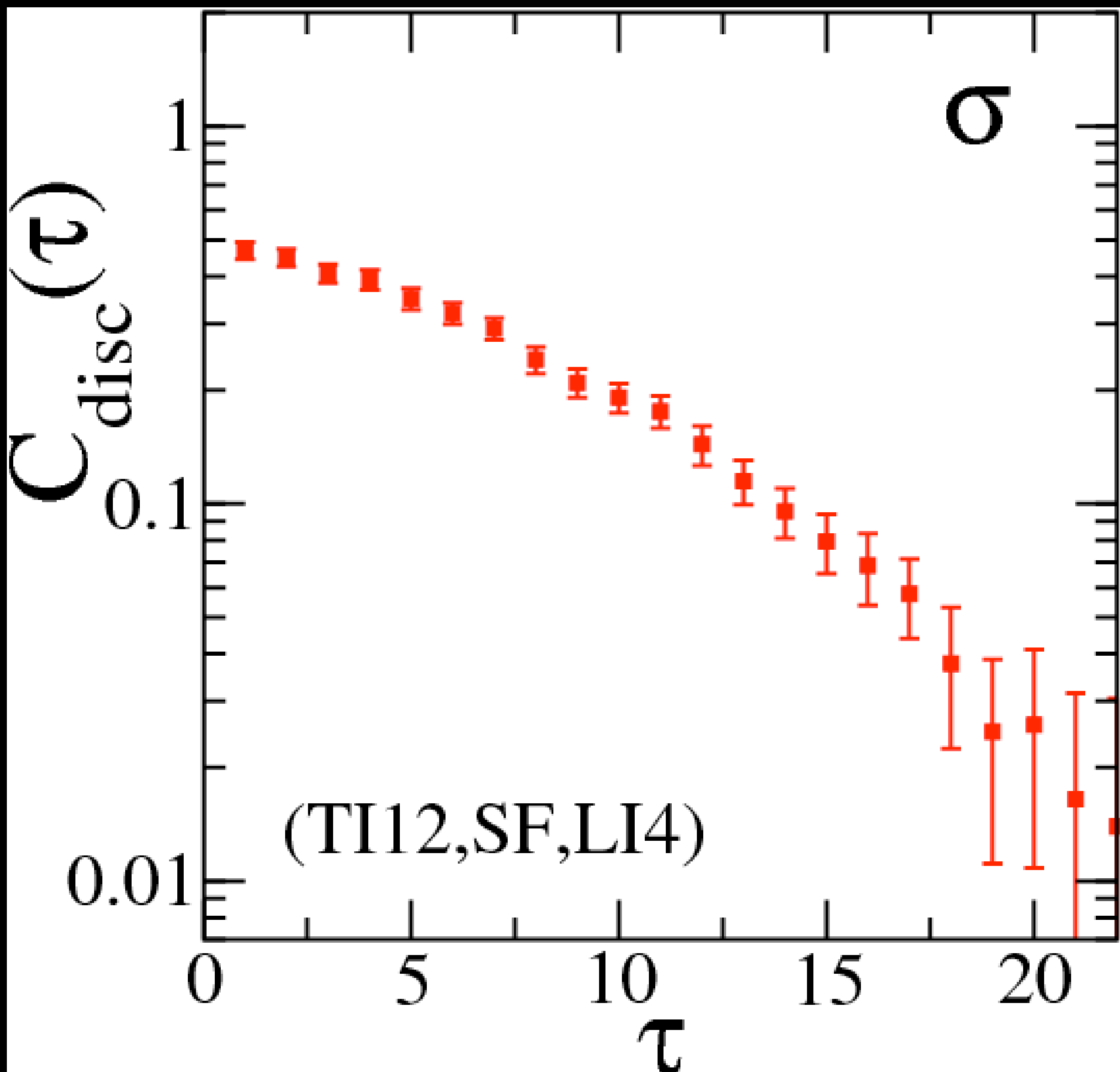
Example Nucleon TDT operator



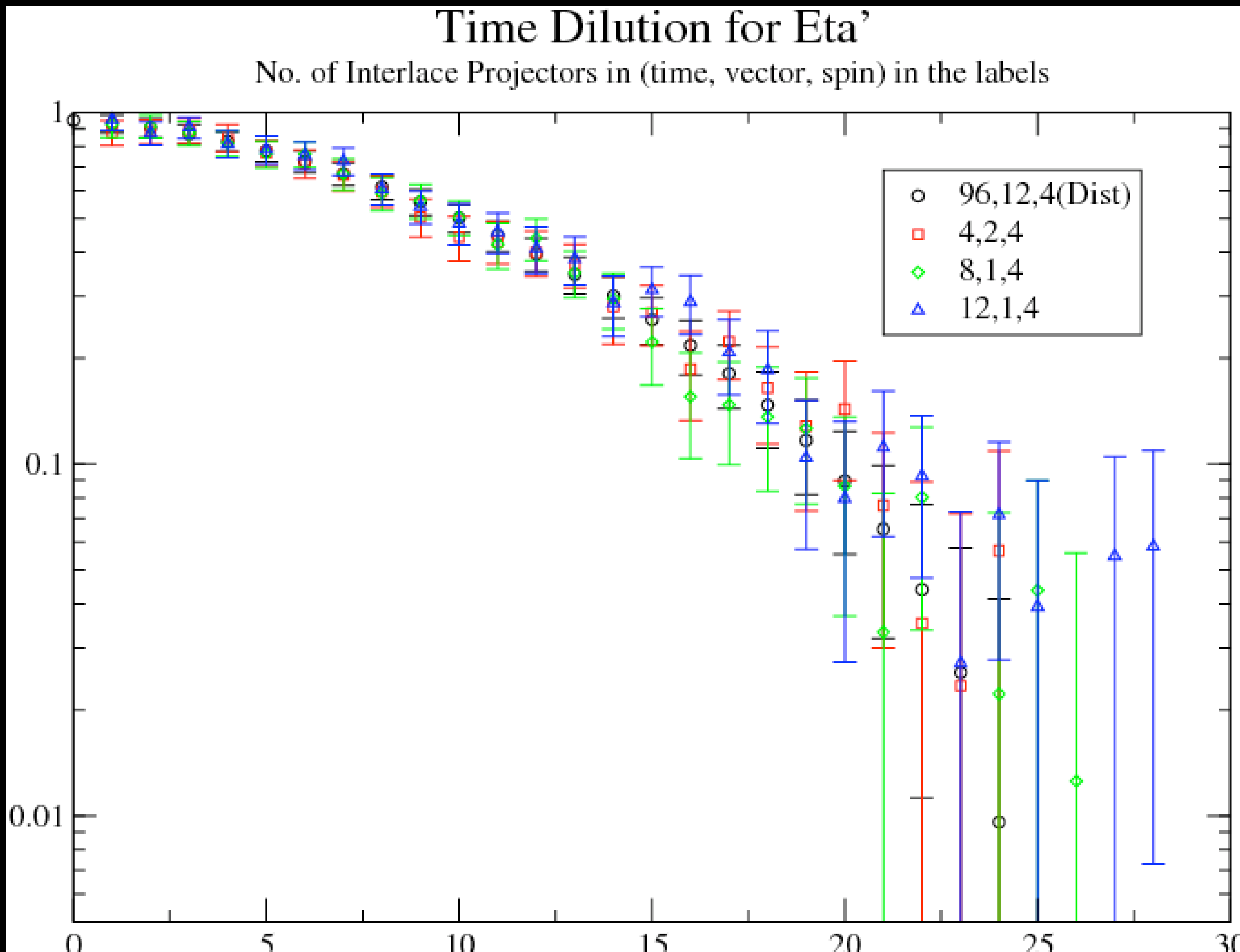
Example: eta′



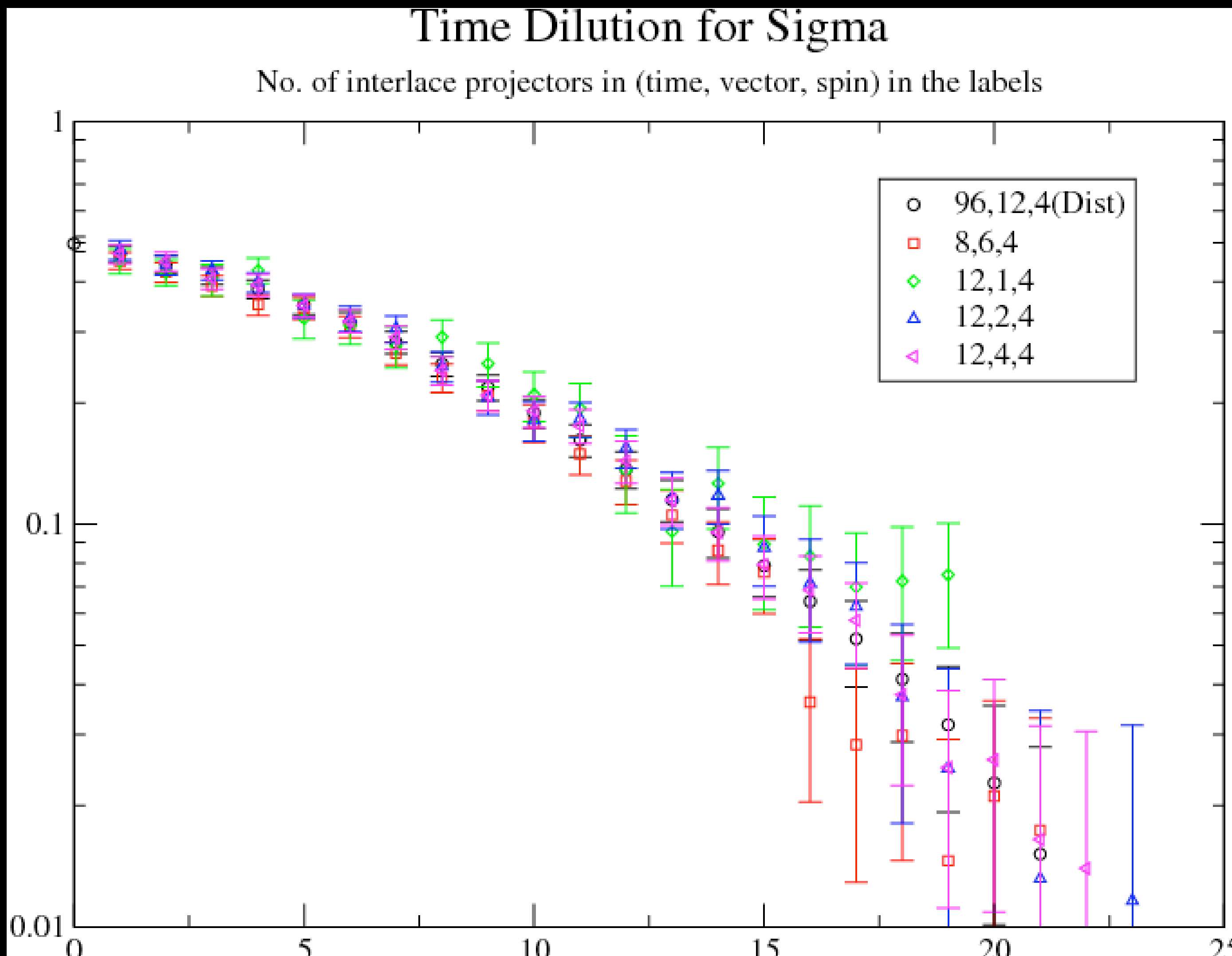
Example: sigma



Comparing schemes: eta'



Comparing schemes: sigma



Conclusions

- Distillation/LapH method works well
  - single particle excited states
  - two-particle states
    - finite momenta operators
    - phase shifts are do-able (if V is big enough)
- stochastic LapH with dilution works well
  - mixing between single-particle and two-particle
  - t-to-t diagrams do not appear to be a problem

Excitation spectra with multi-particle states within reach ...