Recent results on excited hadrons in 2-flavor QCD

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Outline

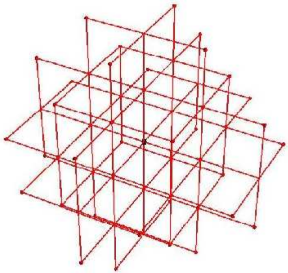
- Setup of the simulation
- Methods in lattice hadron spectroscopy
- Results on the hadron spectrum
- Conclusion and outlook

- The **Ginsparg-Wilson** (GW) equation is the condition for chiral symmetry on the lattice:

$$\mathbf{D}\,\gamma_5 + \gamma_5\,\mathbf{D} = \mathbf{a}\,\mathbf{D}\,\gamma_5\,\mathbf{D}$$

- Chirally Improved Dirac operator:
General ansatz for bilinear fermion action ($\bar{\psi}_n D_{nm} \psi_m$):
(Gattringer, PRD63(2001)114501)

$$D_{nm} = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in \mathcal{P}^\alpha_{m,n}} c^\alpha_p \prod_{l \in p} U_l \, \delta_{n,m+p}$$



- Plug it in the GW-equation.
- Truncate the length of the contributions (to, e.g., 4) and compare the coefficients.
- Leads to a set of (e.g. 50) coupled algebraic equations, which can be solved numerically.

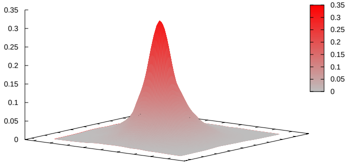
- Chirally improved fermions (D_{CI}), $n_f = 2$ light quarks
- Fermion action includes a level of stout smearing.
- Lüscher-Weisz gauge action
- Hybrid Monte Carlo simulation

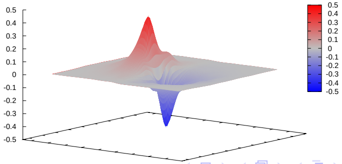
- Three ensembles for $\mathbf{16^3} \times \mathbf{32}$:

set	$\mathbf{m_\pi}$**[MeV]**	a[fm]	β_{LW}	m_0	m_{AWI}[MeV]	configs
A	**525(7)**	0.151(2)	4.70	-0.050	43.0(4)	100
B	**470(4)**	0.150(1)	4.65	-0.060	35.1(2)	200
C	**322(5)**	0.144(1)	4.58	-0.077	15.0(4)	200

- Coarse lattices possible due to improved action.
- For details of the simulation: Gattringer et al., PRD79(2009)054501

- Extended quark sources show better overlap with physical states.
- Allow for a larger basis in the variational method.
- We use 3 types:
  - Gaussian narrow (ψ_n)
  - Gaussian wide (ψ_w)
  - Derivative (ψ_{∂_i}): applied on wide source
- Gauge-covariant construction by Jacobi-smearing (using link-variables as gauge-transporter).
  Güsken et al, PLB227(1989)266; Best et al, PRD56(1997)2743; Gattringer et al, PRD78(2008)034501





- Use particular combinations of smeared quarks and Dirac structures.
- Construct interpolators with **quantum numbers of a physical state**.

- Mesons: isotropic spatial structure allows for spin $J = 0, 1$ only.
- Derivative sources ($\in T_1$) yield non-trivial spatial structure.
- **Decomposition** of space $\otimes$ Dirac to **irreducible representations**:

$$T_1 \otimes T_1 = A_1 \oplus T_1 \oplus T_2 \oplus E$$

- Allows for spin $\mathbf{J} \geq \mathbf{2}$ and exotic quantum numbers.
- E.g. 2^{++} (a_2): T_2 interpolator $|\epsilon_{ijk}|\bar{a}_{\partial_k}\gamma_j b_n$
- Lacock et al. PRD54(1996)6997; Liao/Manke PRD72(2005)094506; Dudek et al. PRD77(2008)034501

- **Variational Method**: In each channel, compute all cross-correlations, obtaining the **correlation matrix**:

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger|0\rangle \;= \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle$$

- Solve the generalized eigenvalue problem:

$$C(t)\,\vec{v}_k = \lambda_k(t)\,C(t_0)\,\vec{v}_k$$

projecting to zero momentum, it follows:

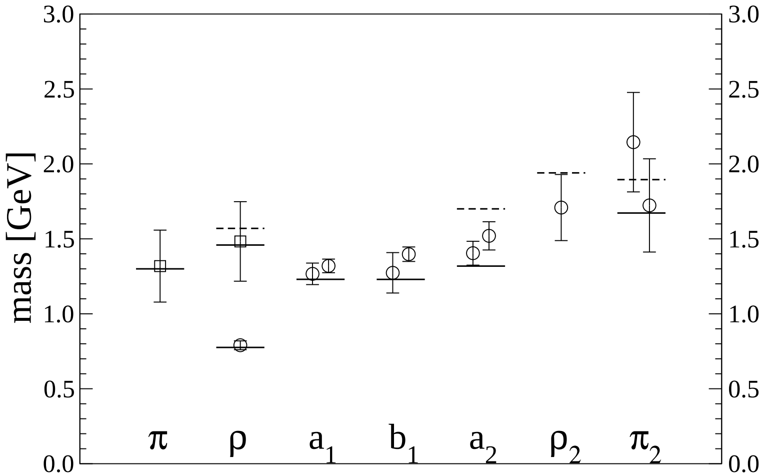
$$\lambda_k(t,t_0) \propto e^{-t\,m_k}\left(1+\mathcal{O}(e^{-t\,\Delta m_k})\right)$$

- Each eigenvalue is related only to a single mass at large time separations.
- Extraction of **excited states** possible.
- Corresponding eigenvectors are “fingerprints” of the states.
- A good basis of different interpolators is crucial.
- Michael NPB259(1985)58; Lüscher/Wolff NPB339(1990)222; Blossier et al. JHEP(2009)0904:094

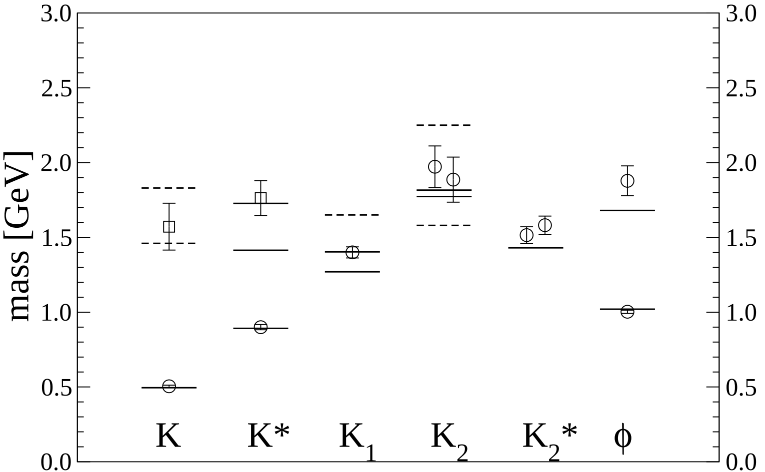
- Estimate of the energy level of two free hadrons:

$$E\left(A(\vec{p}),B(-\vec{p})\right)=\left(\sqrt{m_A^2+|\vec{p}|^2}+\sqrt{m_B^2+|\vec{p}|^2}\right)(1+\mathcal{O}(ap))$$

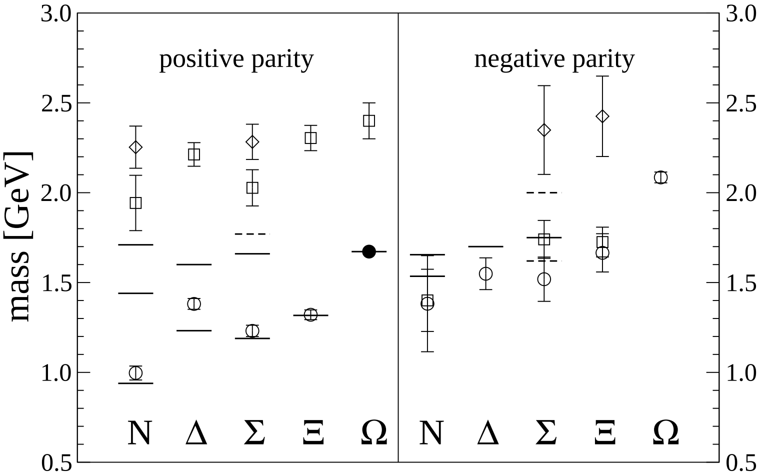
- **Weak coupling** of 1-particle interpolators **to many-particle states**.
- Our results are compatible with the picture:
  - *S* wave states contribution possible (e.g. $\pi\eta'$ in the a_0 channel).
  - *P* wave states suppressed (e.g. $\pi\pi$ in the ρ channel).
    Possible reason: Small scattering amplitude at small momenta.
- 2-particle interpolators in the basis desirable but cross-correlators expensive.



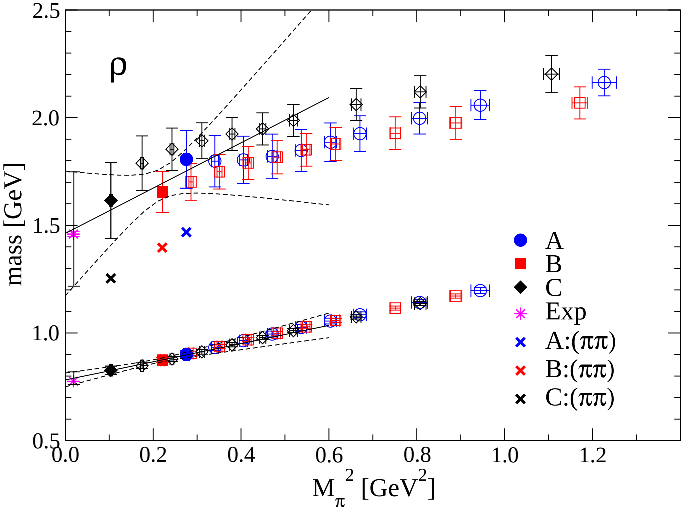
- Chiral extrapolation linear in m_π^2
- Agreement with experiment within error bars.
- Sizable statistical uncertainty in case of radial excitations and spin 2 mesons.



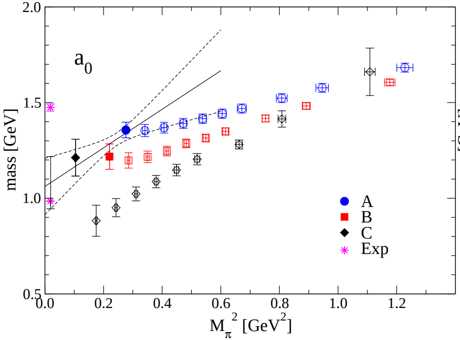
- Error bar smaller due to larger quark mass.
- Mixing of ($C \approx +$) and ($C \approx -$) not explicitely considered, may be important for small pion masses.

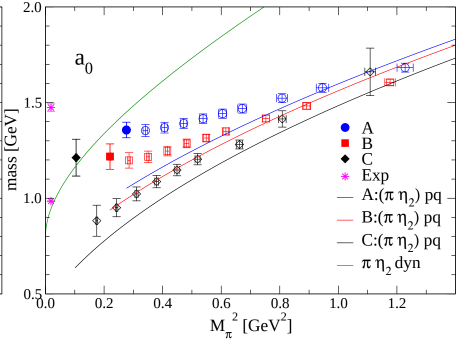


- Excited baryons too high, maybe due to finite volume effects.
- Ground states of negative parity baryons too low (due to scattering states?).



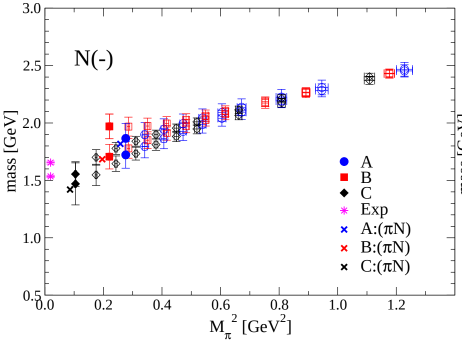
- Accurate ground state, first excitation compatible with experiment.
- *P* wave scattering state $\pi\pi$ not observed.

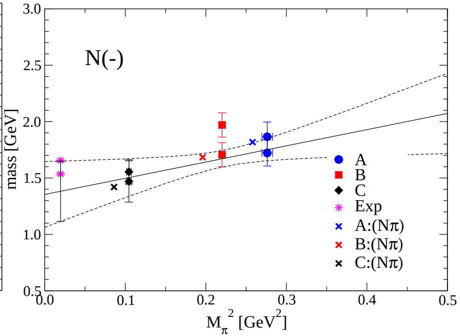




- Partially quenched data do not smoothly extrapolate to dynamical data.
- Agreement with partially quenched prediction of the *S* wave scattering state $\pi\eta_2$ (Prelovsek et al. PRD70(2004)094503).
- Interpret as large contribution from $\pi\eta_2$ at partially quenched data.
- Particle content at dynamical point unclear, there may be contributions from a_0 and from $\pi\eta_2$.

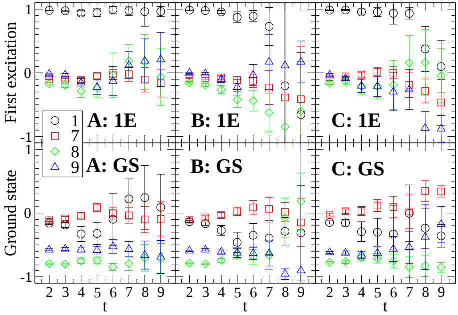
Results: Nucleon negative parity: Masses





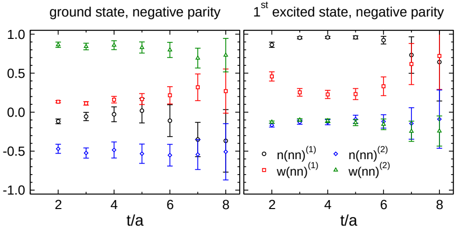
- Results too low at small pion masses.
- Would be compatible with the *S* wave scattering state $N\pi$.
- Suggests a level crossing between 300 and 550 MeV, but picture not confirmed by eigenvectors.

Dynamical results



Quenched results

Burch et al. PRD74(2006)014504



- Ground state dominated by Dirac structure χ_2, first excitation by χ_1:
  - 1E: $N^{\chi_1} = \epsilon_{abc} \mathbb{1} u_a (u_b^T C \gamma_5 d_c - d_b^T C \gamma_5 u_c)$
  - GS: $N^{\chi_2} = \epsilon_{abc} \gamma_5 u_a (u_b^T C d_c - d_b^T C u_c)$

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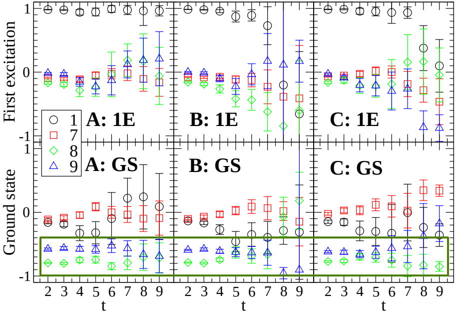

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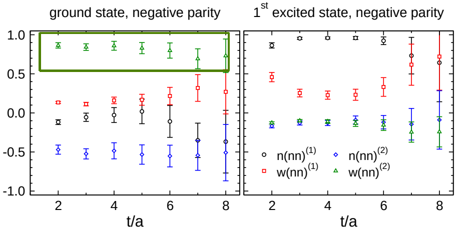

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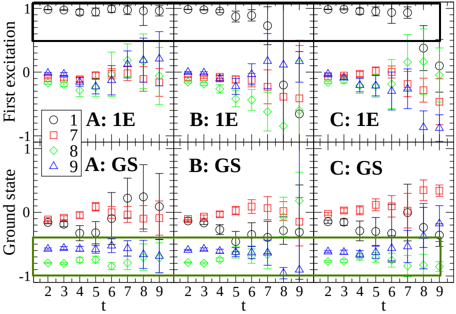

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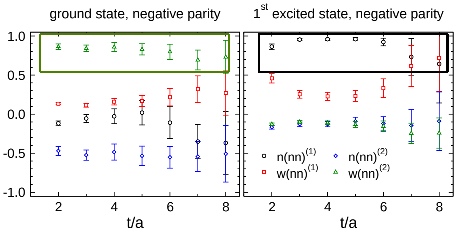

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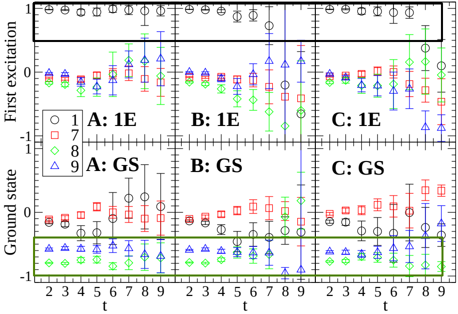

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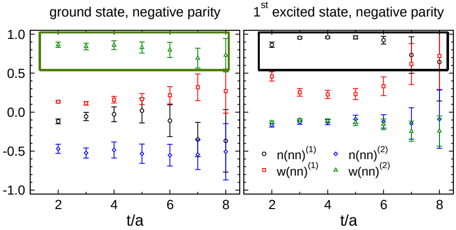

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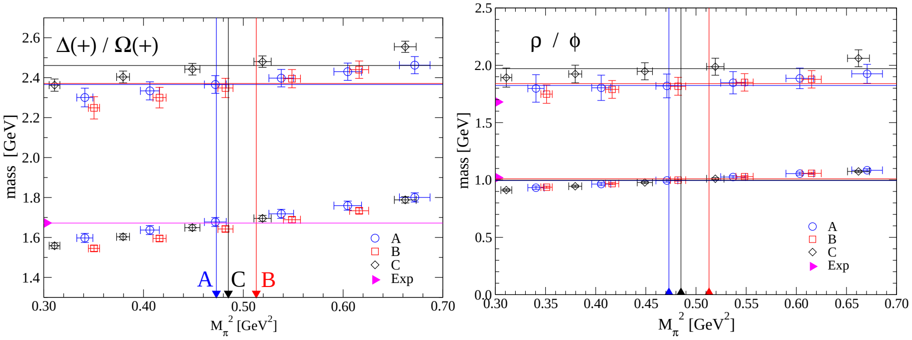
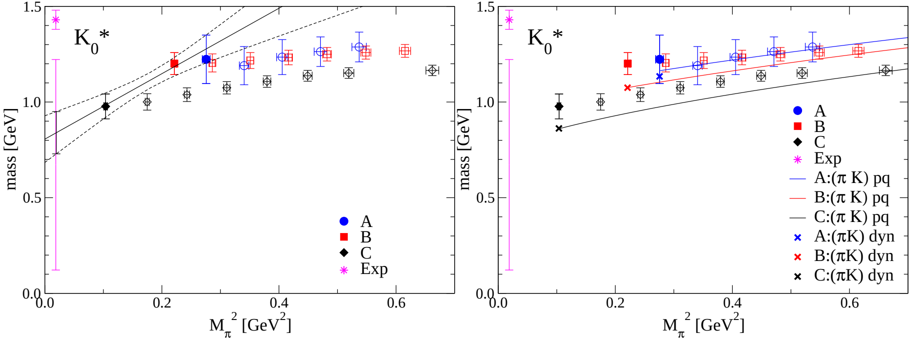
- Also quark smearing similar to quenched.
- Eigenvectors suggest no level crossing and thus no scattering state.

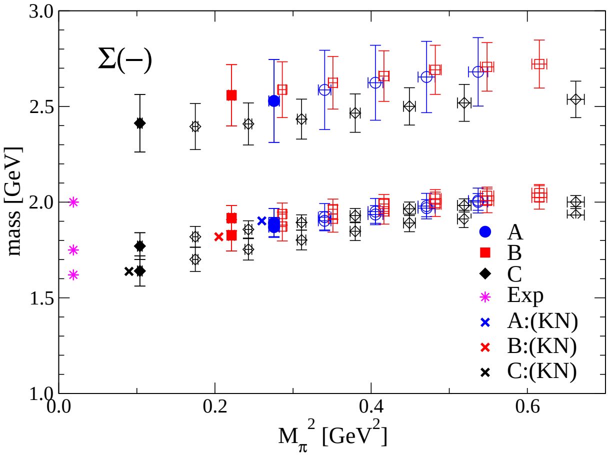
Setting the strange quark mass parameter



- Neglect strange sea quarks.
- Strange hadron spectrum is accessed using partially quenched strange quarks.
- Strange quark mass parameter is set by identification of the partially quenched $\Delta(+)$ with the $\Omega(1672)$.
- Confirmed by $\phi(1020)$, $\Sigma(1190)$ and $\Xi(1320)$.

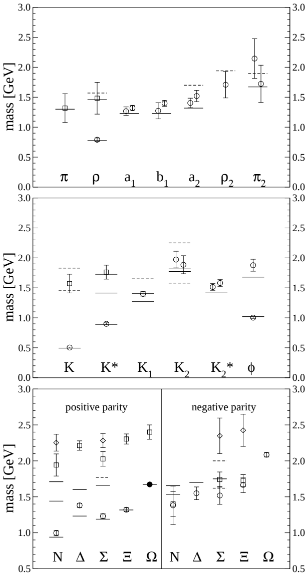


- $K_0^*(800)$ is a broad resonance, not confirmed so far.
- Situation similar to the 0^{++} (a_0) channel.
- Results compatible with both, $K_0^*(800)$ and the S wave πK.



- Situation similar to Nucleon negative parity.
- Masses suggest a level crossing of the $\Sigma(-)$ ground state with the *S* wave *KN*, but eigenvectors contradict this picture.

- Hadron spectroscopy with 2 flavors of dynamical Chirally Improved quarks.
- Pion masses of 320, 470 and 520 MeV.
- Several ground states reproduced with fairly high precision.
- Various radial excitations found.
- Appearance of scattering states discussed, considering partially quenched data and eigenvectors.
- Clear statement about particle content difficult.
- Excited baryons suffer from finite volume effects.



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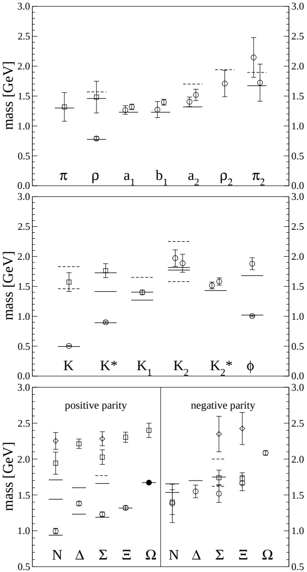
- Larger volume, additional ensembles and enlarged statistics will be investigated in future.



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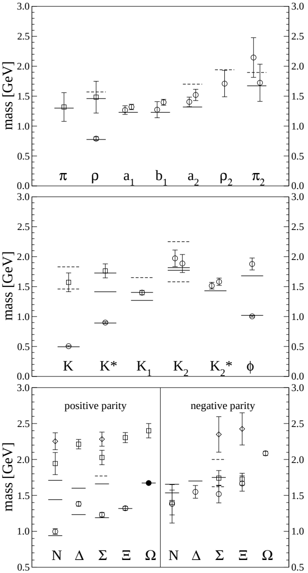
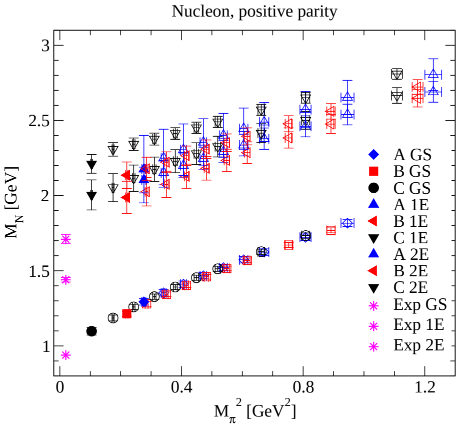
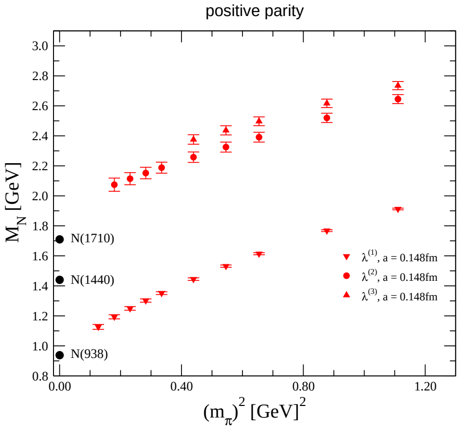
THANK YOU!



Comparison to quenched

- Compatible within error bars:
  - Mesons ground states:
    $K(490)$, $K^*(890)$
  - Mesons excitations:
    Quenched results show discretization effects (lighter masses at $a = 0.12$ fm than $a = 0.15$ fm)
    $\pi(1300)$, $\rho(1450)$, $K(1460)$, $K^*(1460)$ (quenched are lighter when $a = 0.12$ fm than $a = 0.15$ fm)
  - All baryons.
- Dynamical lighter:
  - $\rho(770)$, $a_1(1260)$, $b_1(1235)$
- However, often the quenched results do not go below pion masses of 600 MeV. Dynamical effects are weak there. E.g. the negative parity baryon results agree there, but the dynamical bend down for small pion masses, undershooting the experimental values in the chiral limit.

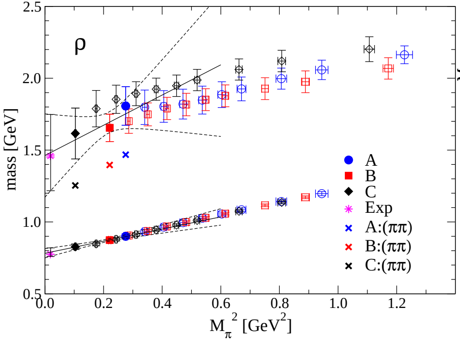




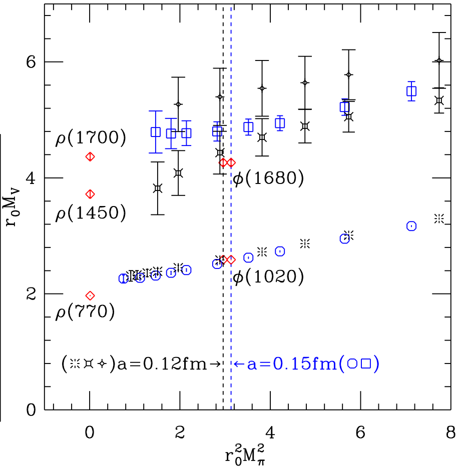
Two excitations, but too high up.
Roper: For confirmation probably larger volume and smaller quark masses needed.

Quenched results (Burch et al., PRD 74 (2006) 014504)

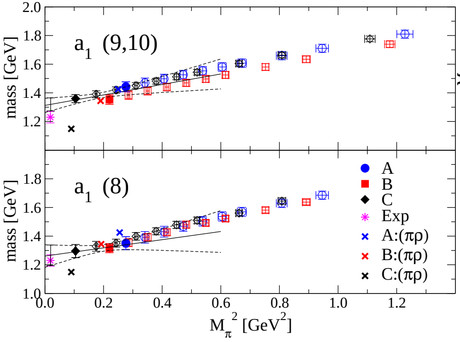
Results: ρ



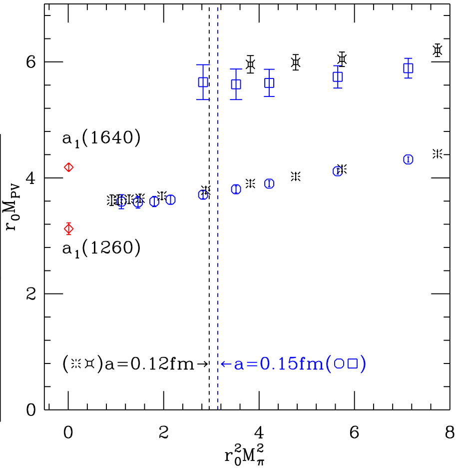
ρ
mass [GeV]
M_π^2 [GeV^2]
A
B
C
Exp
A:(ππ)
B:(ππ)
C:(ππ)



ρ(1700)
ρ(1450)
ρ(770)
φ(1680)
φ(1020)
a=0.12fm→
←a=0.15fm
r_0M_V
r_0^2M_π^2

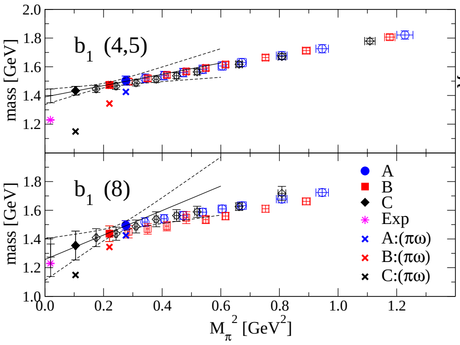


mass [GeV]
a1 (9,10)
a1 (8)
A
B
C
Exp
A:(πρ)
B:(πρ)
C:(πρ)
2.0
1.8
1.6
1.4
1.2
1.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Mπ2 [GeV2]

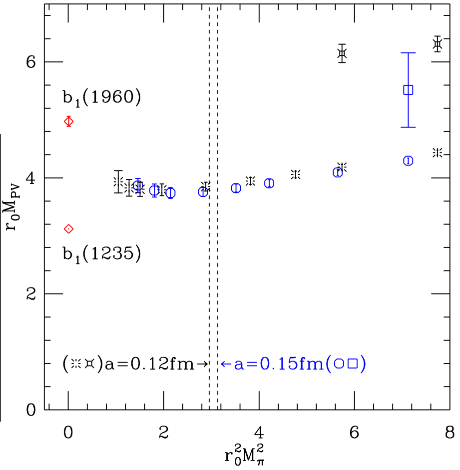


r0MPV
a1(1640)
a1(1260)
(⁜¤)a=0.12fm→
←a=0.15fm(○□)
0
2
4
6
8
r0²Mπ²

Results: b_1

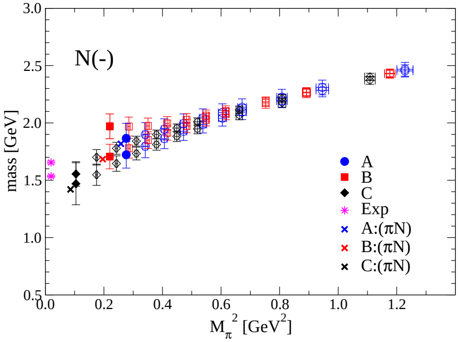


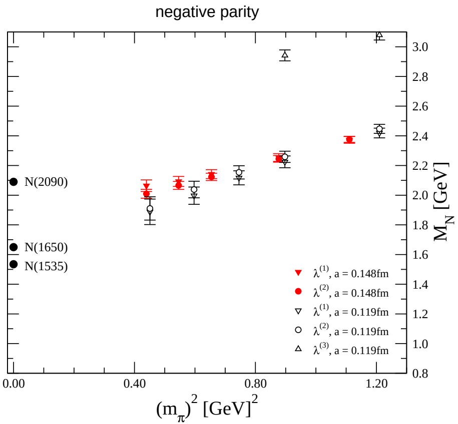
b1 (4,5)
b1 (8)
mass [GeV]
A
B
C
Exp
A:(πω)
B:(πω)
C:(πω)
M_π^2 [GeV^2]



b1(1960)
b1(1235)
r0MPV
r0^2Mπ^2
a=0.12fm→
←a=0.15fm

Results: Nucleon neg. parity





Results: Sigma neg. parity