Heavy-light current-current correlators

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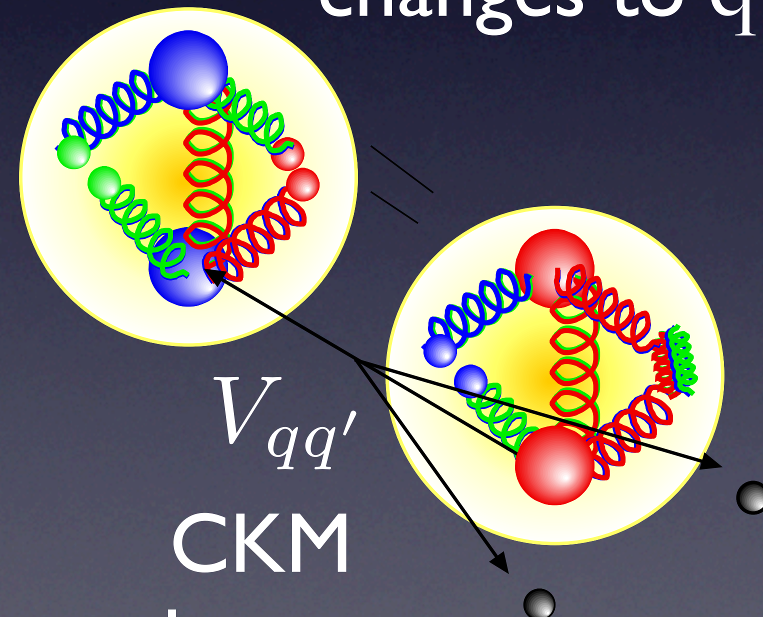
Lattice 2010, Villasimius, Sardinia

Motivation

- want to study semileptonic and leptonic processes like $B\rightarrow\pi l\upsilon$, $B\rightarrow l\upsilon$
- b→light: $V_{\mathrm{ub}},\ f_B$
  b→charm: $V_{\mathrm{cb}},\ f_{B_c}$
  b→strange: f_{B_s}
- need renormalization factor Z

q emits W and changes to q'

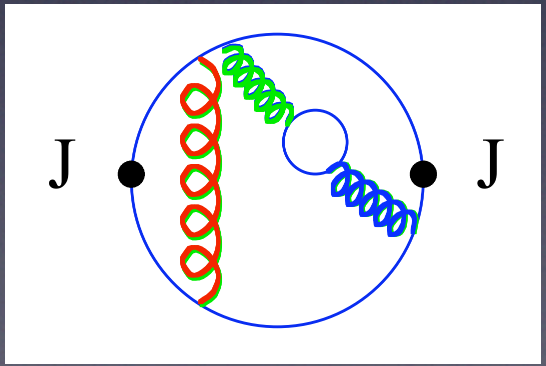


$V_{qq'}$

CKM element

Current-current correlator method

- Match time moments of meson correlators to energy-derivative moments at $q^2=0$ of polarization functions Π calculated in continuum QCD perturbation theory to high order
- Can determine quark masses and α_s (has been done using heavy-heavy JJ correlators, very accurate results)



Time moments

- Current-current correlators

$$G(t) = a^6 \sum_{\vec{x}} (am_q)^2 \langle 0 | j_5(\vec{x}, t) j_5(0,0) | 0 \rangle$$

- Moments

$$G_n = \sum_t (t/a)^n G(t)$$

Reduced moments

- Divide each moment by the tree value to get smaller errors

$$R_4^{\mathrm{latt}} = \frac{G_4}{G_4^{(0)}},$$

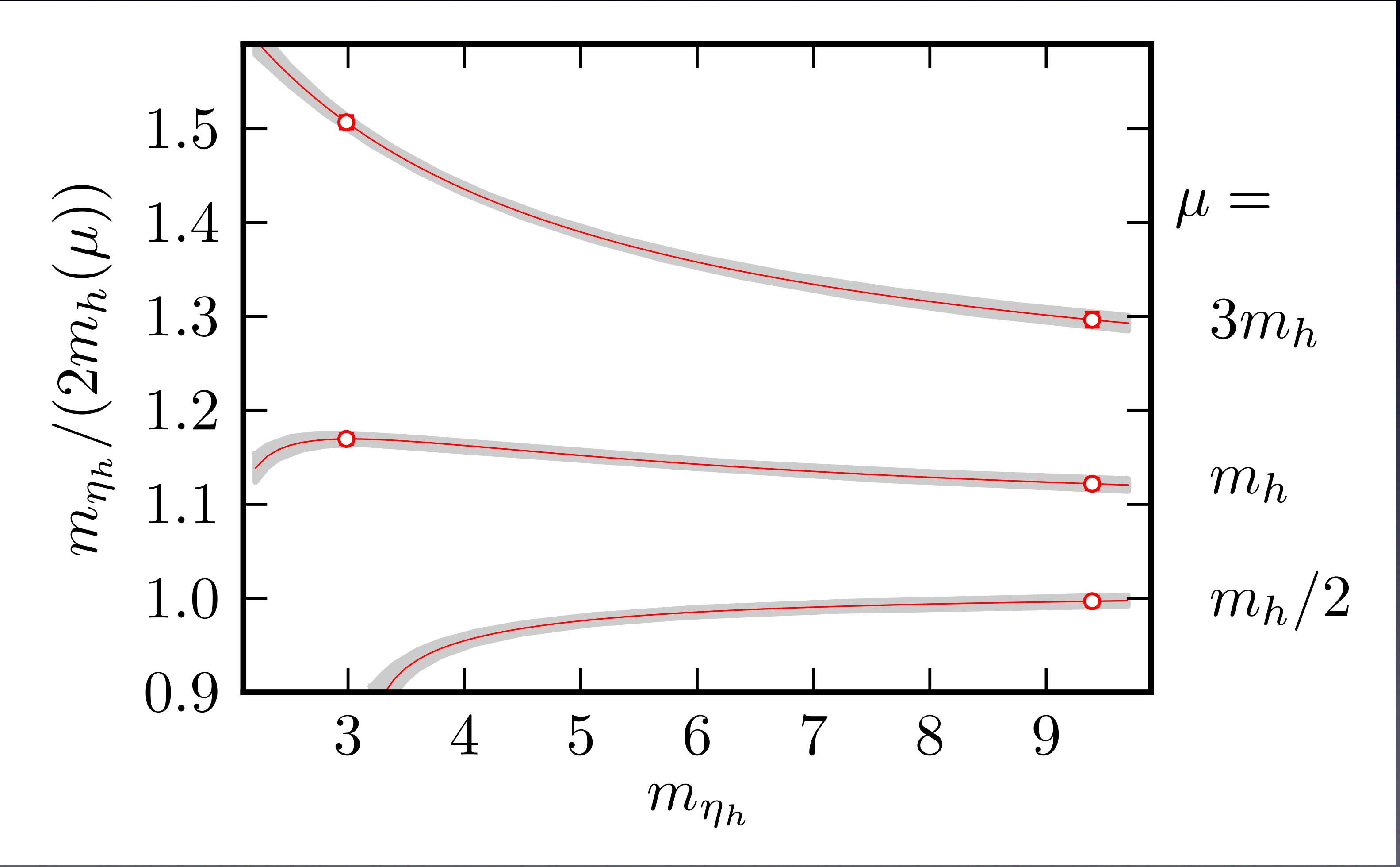
$$R_n^{\mathrm{latt}} = \frac{am_{\eta_h}}{2am_h}\left(\frac{G_n}{G_n^{(0)}}\right)^{\frac{1}{n-4}} \qquad \text{for } n \geq 6$$

Continuum R_n

$$R_4^{\mathrm{cont}} = \frac{g_4}{g_4^{(0)}}, \; R_n^{\mathrm{cont}} = \frac{m_{\eta_h}}{2m_h(\mu)} \frac{g_n}{g_n^{(0)}}$$

- the g_n are perturbative series in $\alpha_s(\mu)$, known for the heavy-heavy case through $\alpha_s^3(\mu)$ and for heavy-light through $\alpha_s^2(\mu)$
- the mass m_h is the heavy quark mass in the MSbar scheme at the scale μ

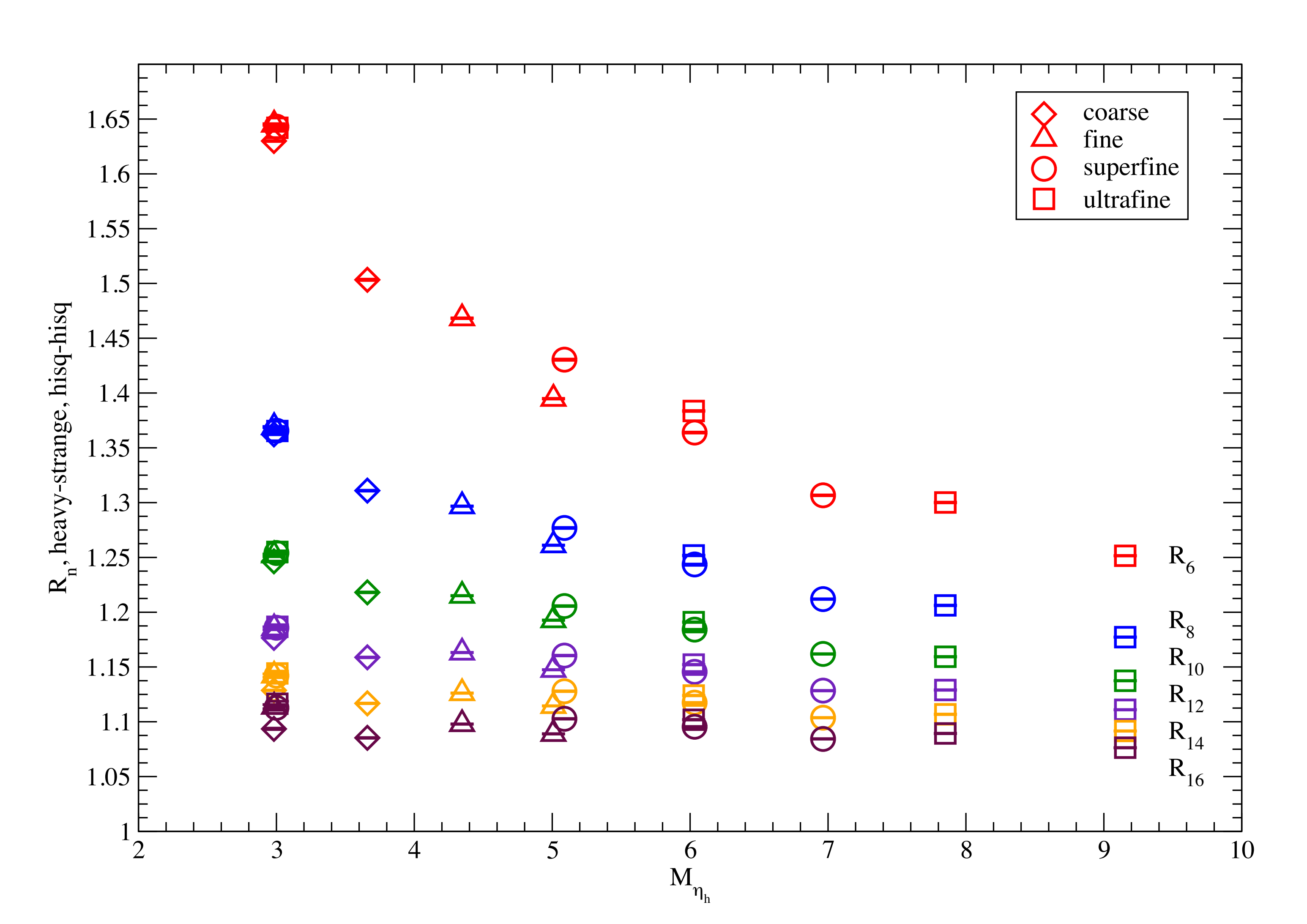
Mass ratio from heavy-heavy JJ correlators



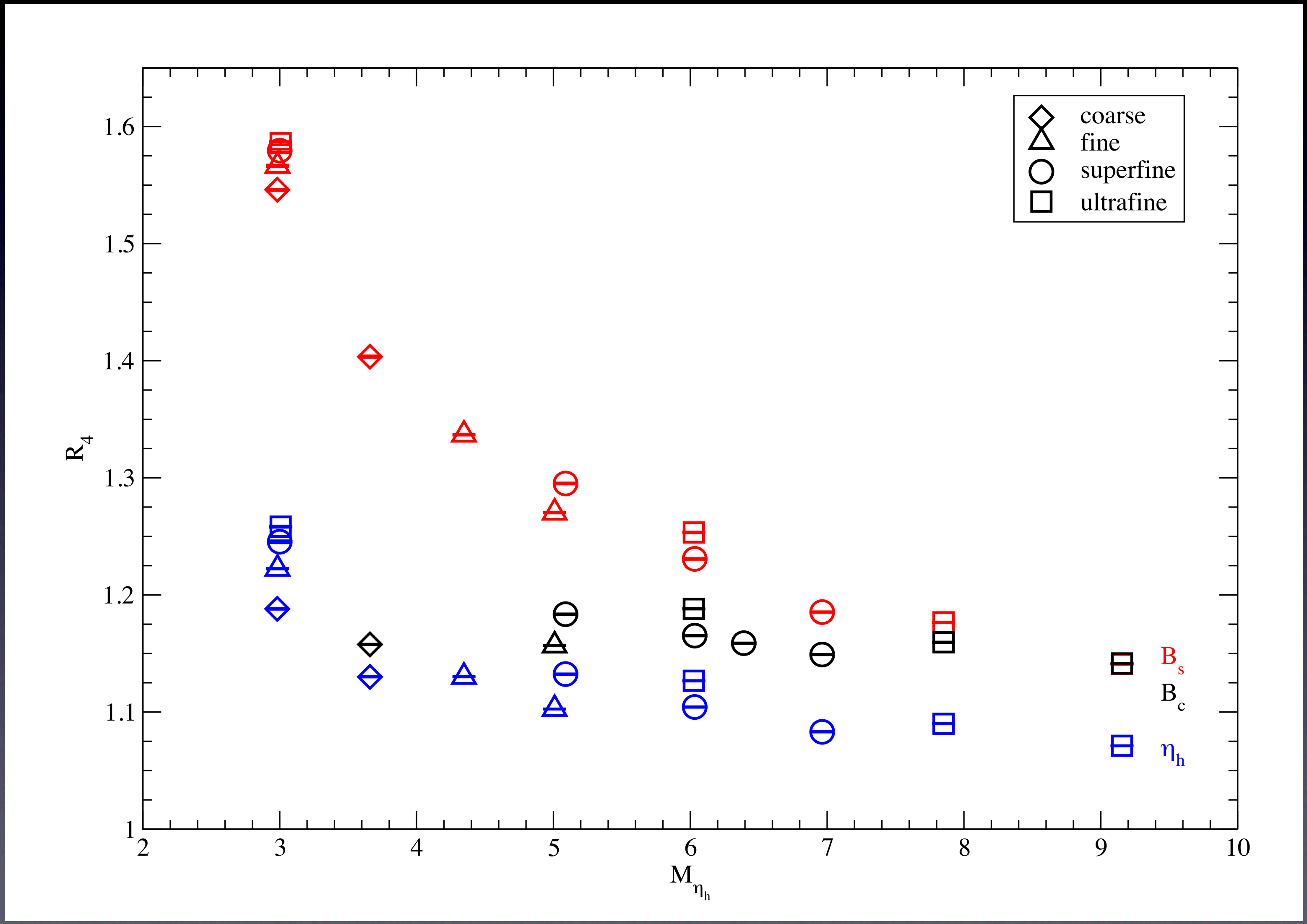
Heavy-light JJ correlators

- Compare lattice calculations to continuum perturbation theory through α_s^2
- Test method in the heavy-light case - could e.g. extract quark masses and compare to heavy-heavy results
- Using two different actions for the heavy quark, HISQ and NRQCD, extract Z factors (HISQ pseudoscalar current absolutely normalized, no Z factor → get Z for NRQCD)

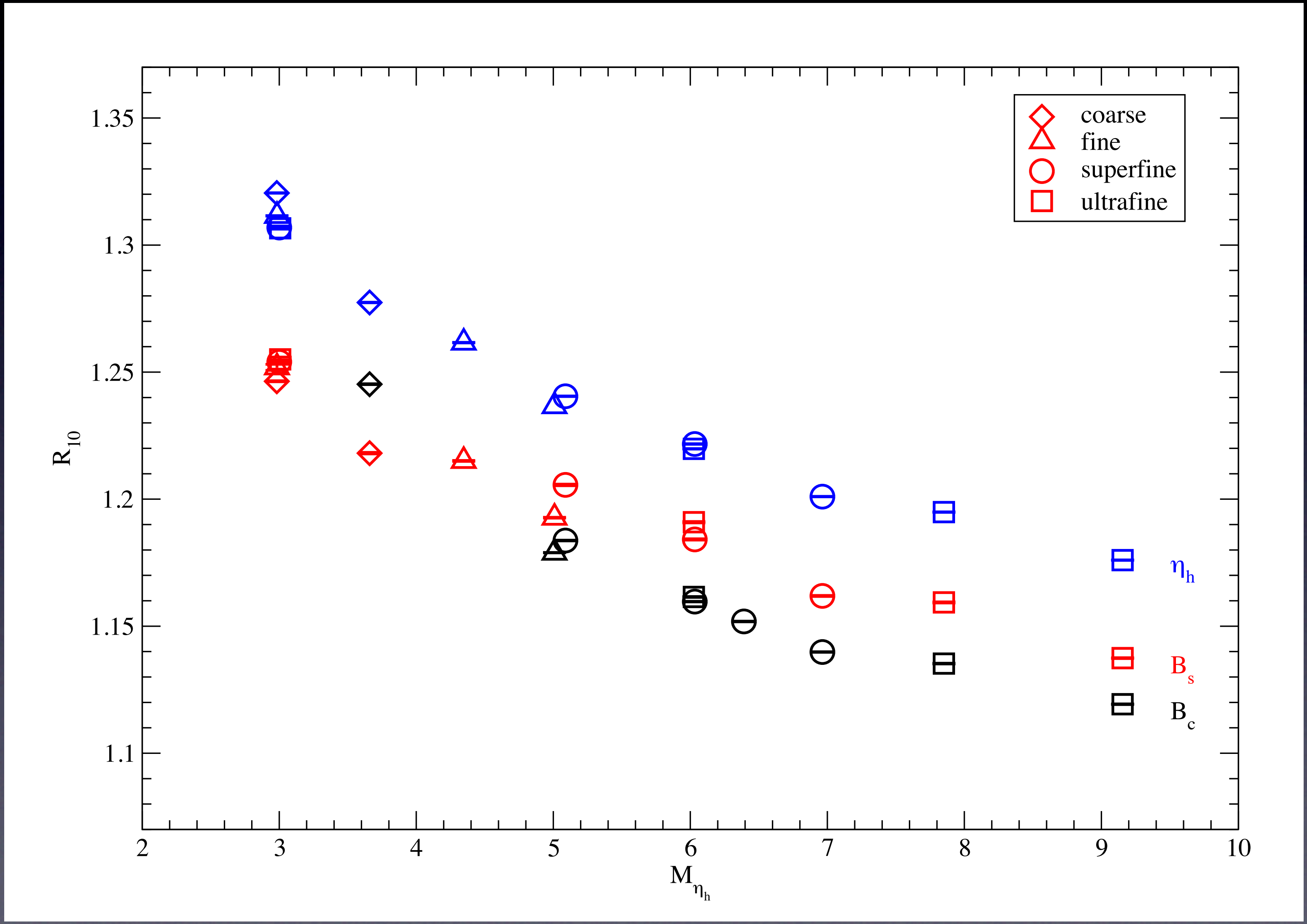
Reduced moments R_n



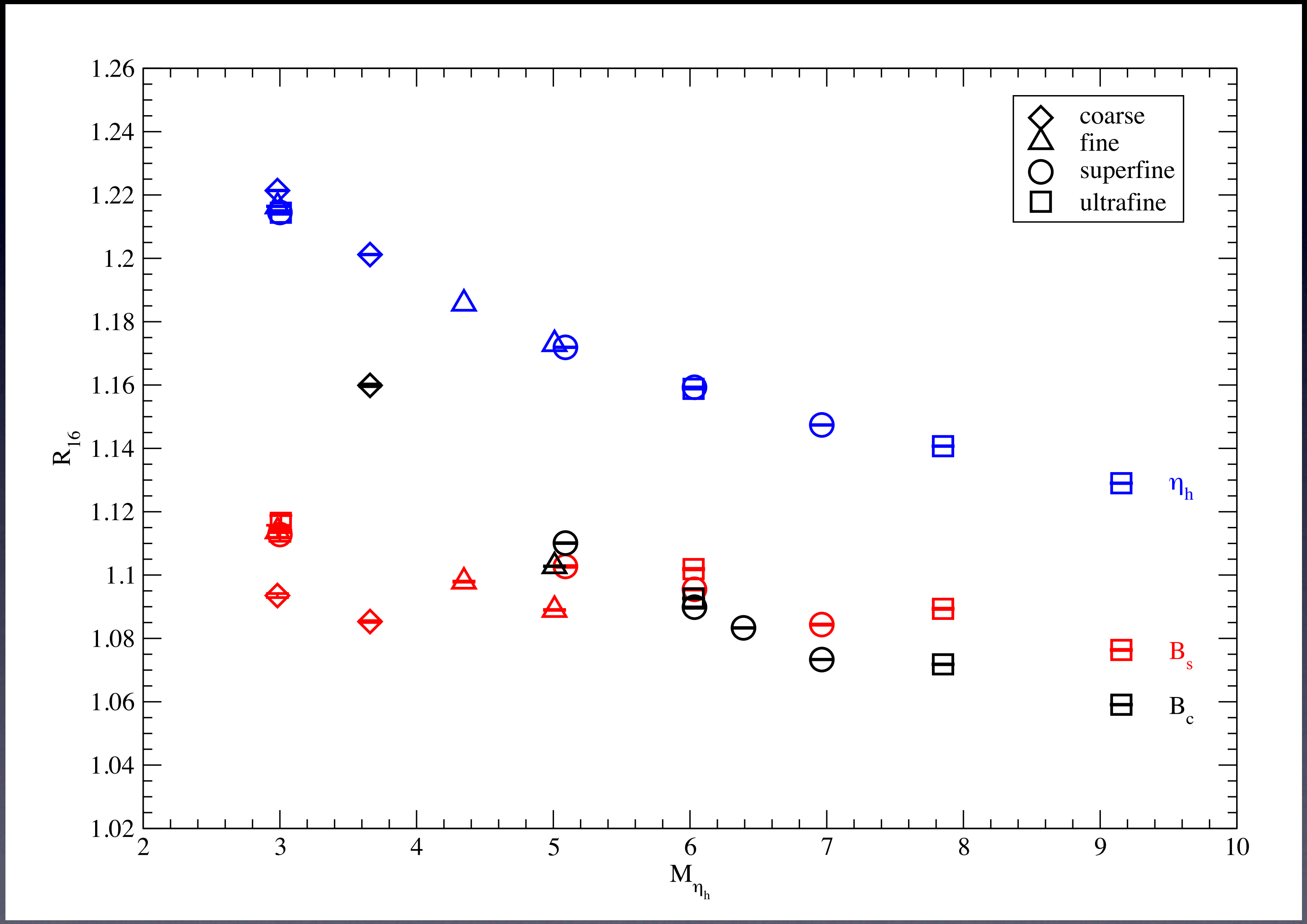
Comparing B_s, B_c, η_h - R_4



Comparing B_s, B_c, η_h - R_{10}



Comparing B_s, B_c, η_h - R_{16}

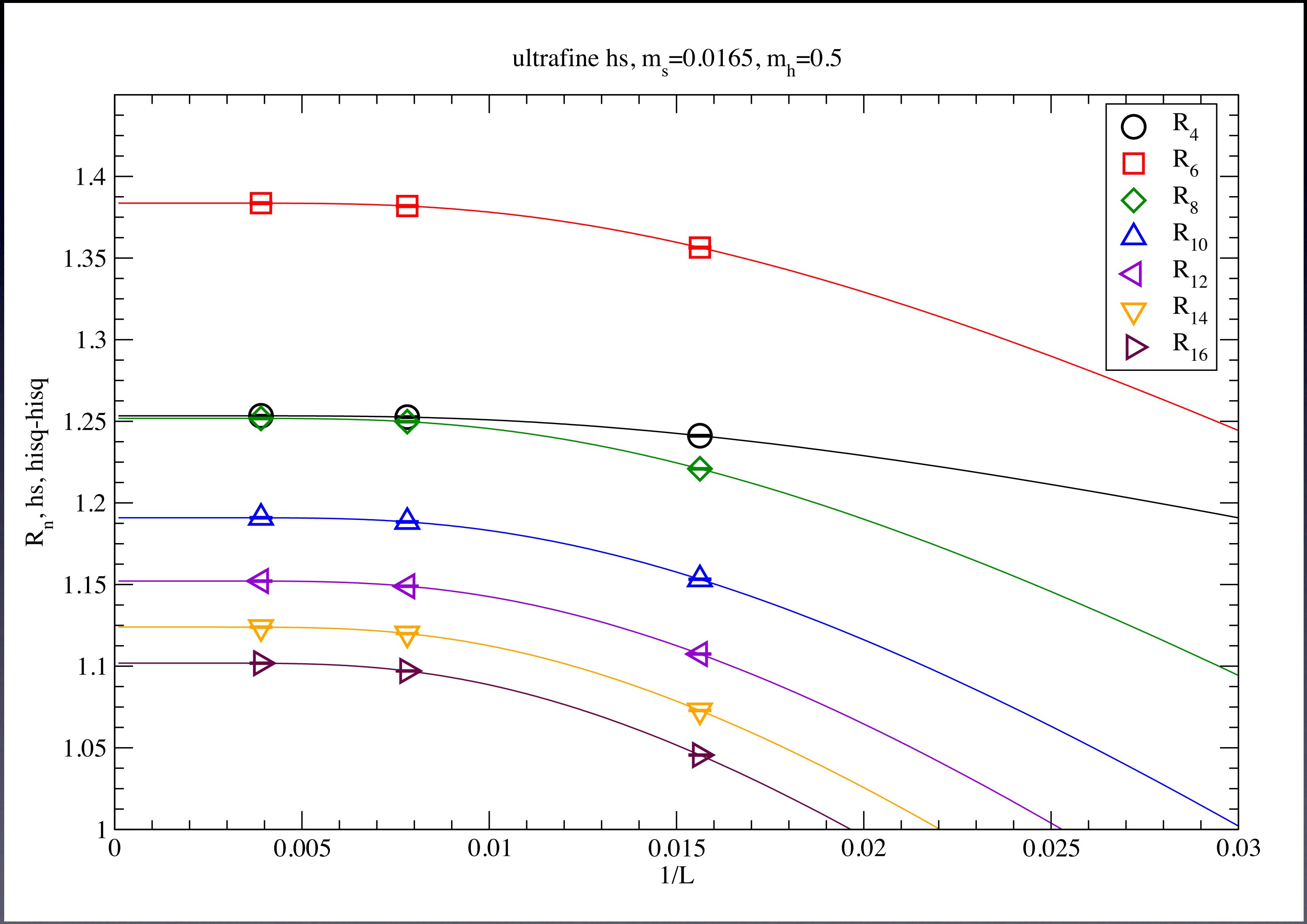


Challenges

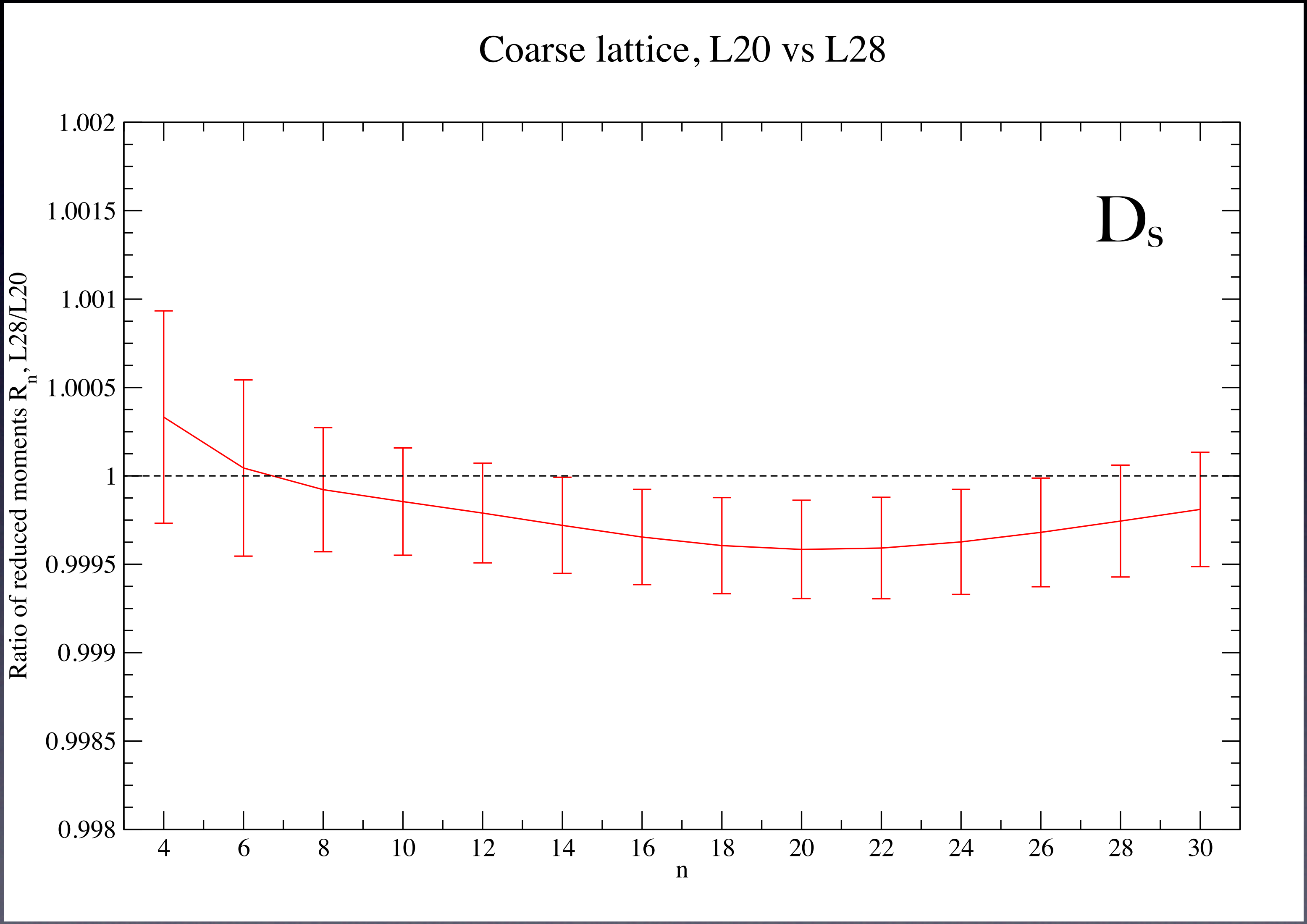
- Tree level (free) moments depend on volume (artifact of the free case only) - need infinite volume calculation
    - Calculate free moments using different volumes, L^3, and fit with

$$A_0 + A_1 \frac{e^{-A_2 L}}{L}$$

Volume dependence: free



Volume dependence: interacting



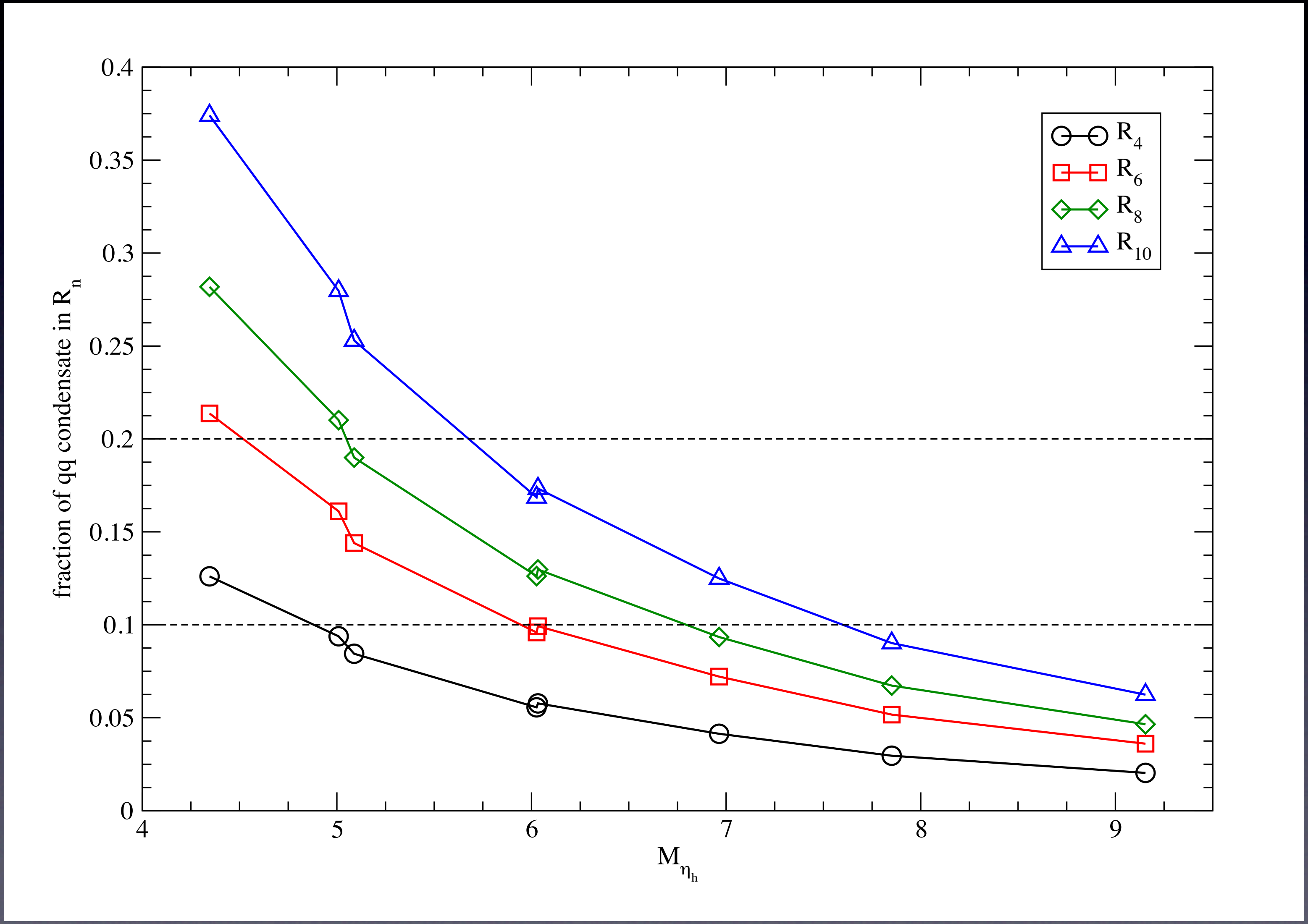
More challenges

- Quark condensate appears at tree level

$$\frac{4\pi^2}{3}\,\frac{(n-1)(n-2)(n-3)\left[-\frac{m_h}{m_l}+\frac{n}{2}\right]}{1+\frac{(n-3)m_l}{m_h}}\,\frac{\langle m_l\psi\bar{\psi}\rangle}{m_h^4}$$

  - not present in the heavy-heavy case, but sizeable in the heavy-light → have to understand α_s corrections
  - gluon condensate much smaller

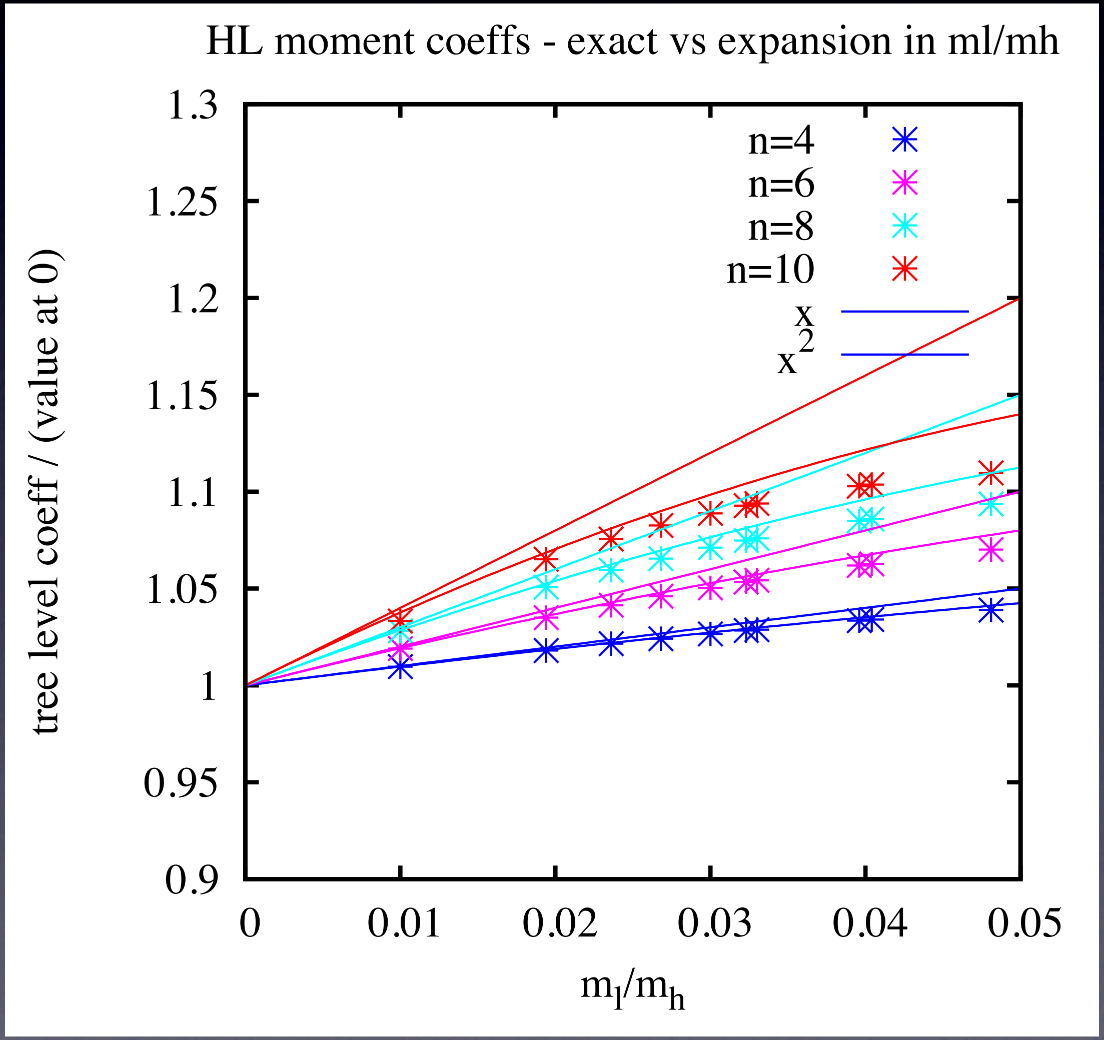
Quark condensate



More challenges

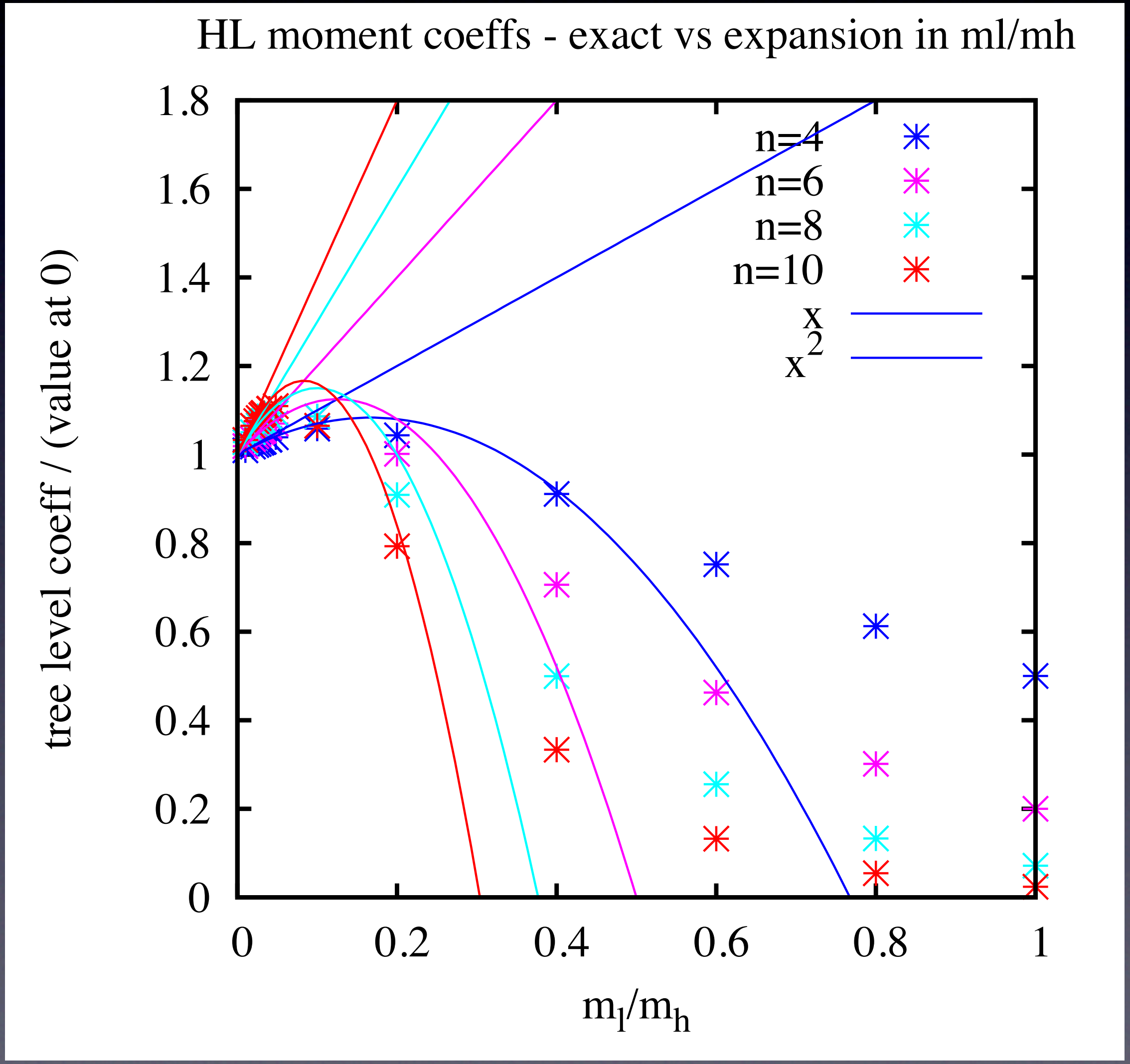
- Perturbation theory with m_q=0 not sufficient, as $m_l(\mu)/m_h(\mu)$ corrections become important for B_c: $m_c(\mu)/m_b(\mu)\approx 0.22$ is not that small

Mass dependence



- Corrections to continuum perturbation theory coefficients
- At small values of m_l the m_l/m_h expansion is good. This is the region relevant for B_s.

Mass dependence



At large values of m_l the expansion in powers of m_l/m_h is not good enough.

Fitting the data

- We fit the lattice data with

$$R_n^{\rm fit} = \left(\frac{m_{\eta_h}}{2m_h(\mu)}\right)\left(1 + c_1\alpha_s^2(\mu) + c_2\alpha_s^3(\mu) + c_3\alpha_s^4(\mu) + \ldots + q\bar{q}\ \text{cond.}\right)\left(1 + b_1(am_h(\mu))^2 + b_2(am_h(\mu))^4 + b_2(am_h(\mu))^6 + b_4a^2 + b_5a^4\right)$$

and choose the scale $\mu = m_h$.

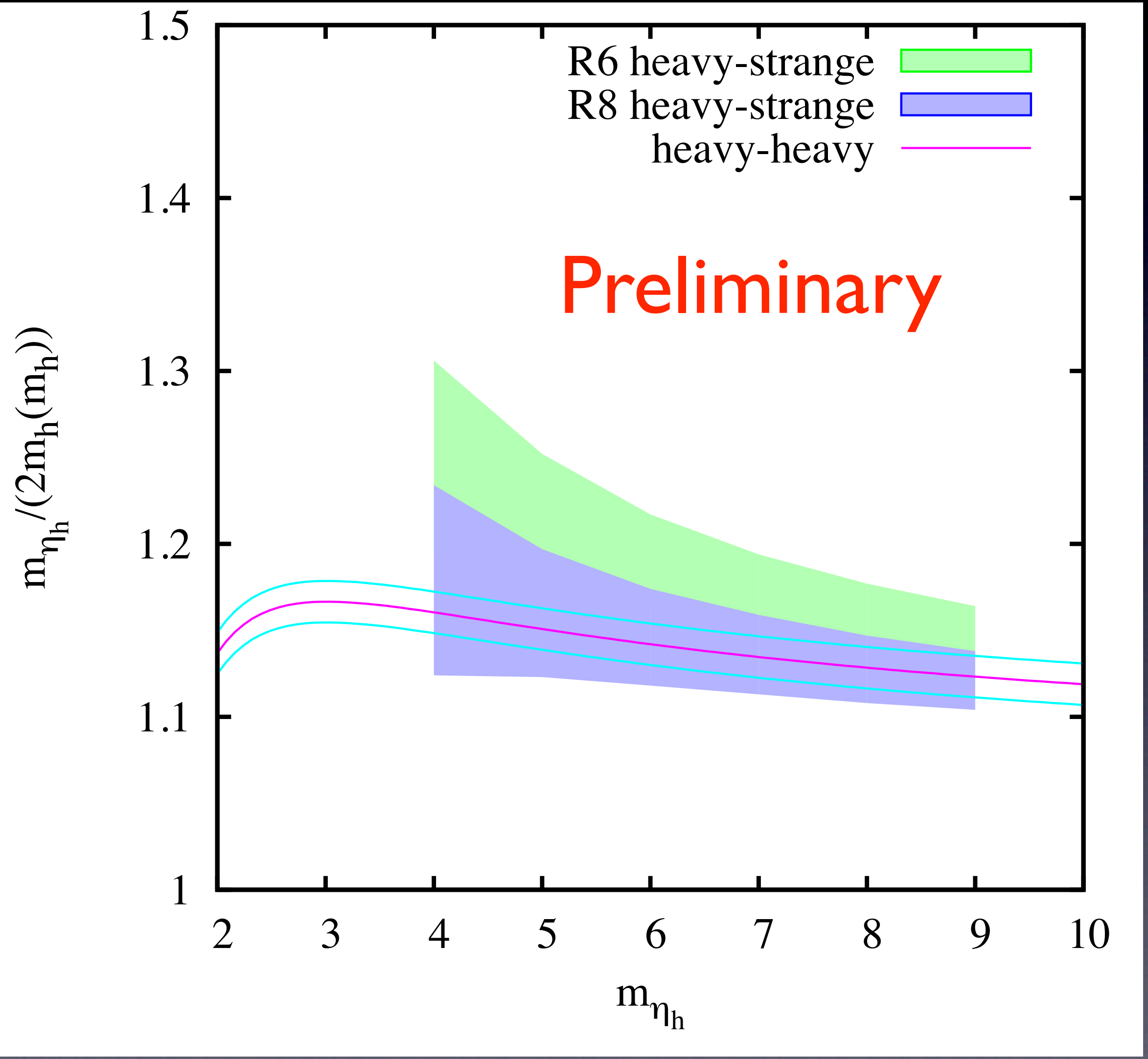
Fitting the data, contd.

- We take the $q\bar{q}$ condensate value to be

  $$\langle m_s s\bar{s}\rangle = (0.2\mathrm{GeV})^4$$

  from the Gell-Mann - Oakes - Renner relation allowing the $s\bar{s}$ condensate to be 0.7 times the light quark condensate
- Also allow for the presence of higher order condensate terms estimating them with powers of the leading condensate

Mass ratio



Extracting the ratio

$$\frac{m_{\eta_h}}{2m_h(m_h)}$$

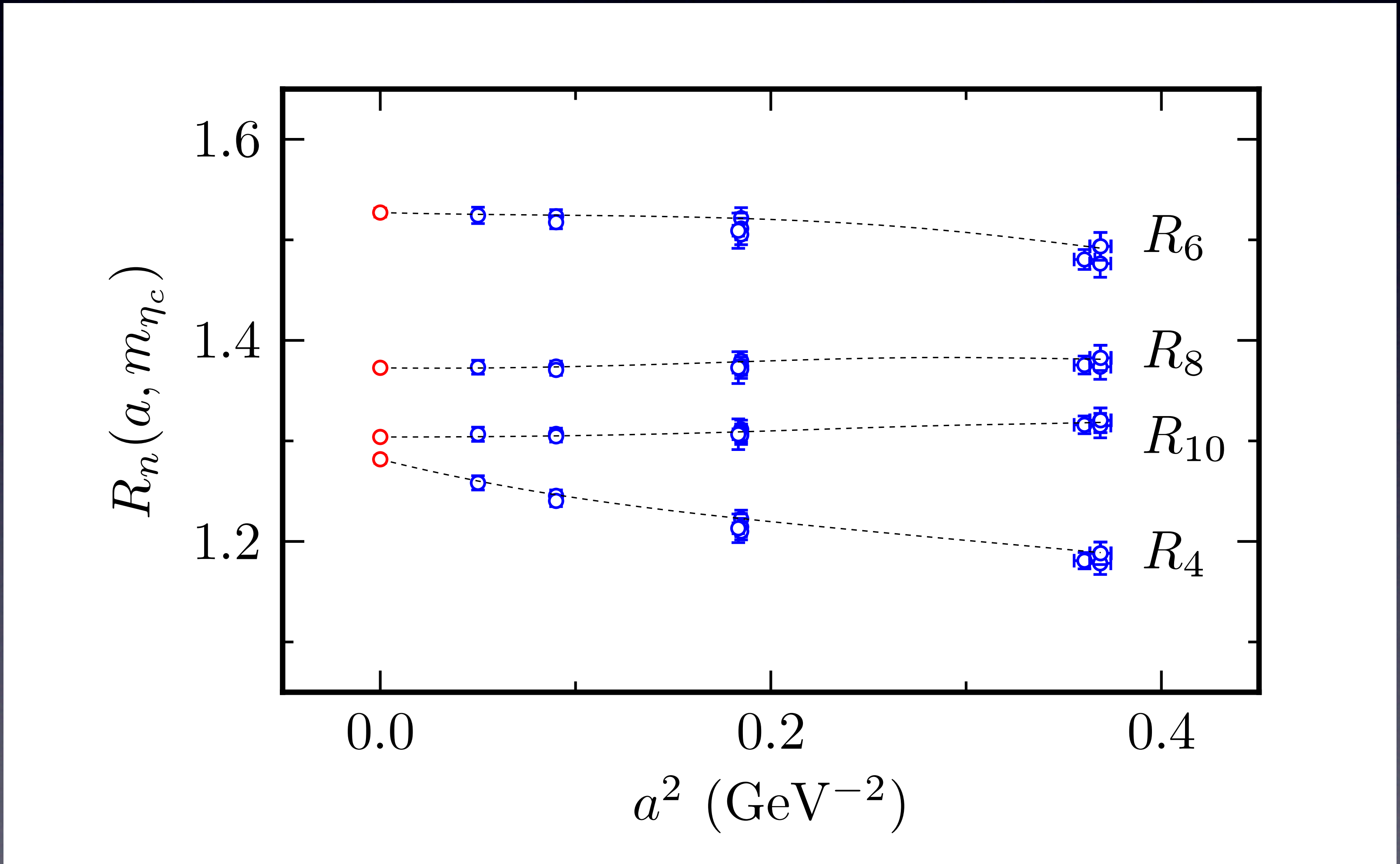
in the heavy-strange case. Note that this is a physical quantity!

Conclusions

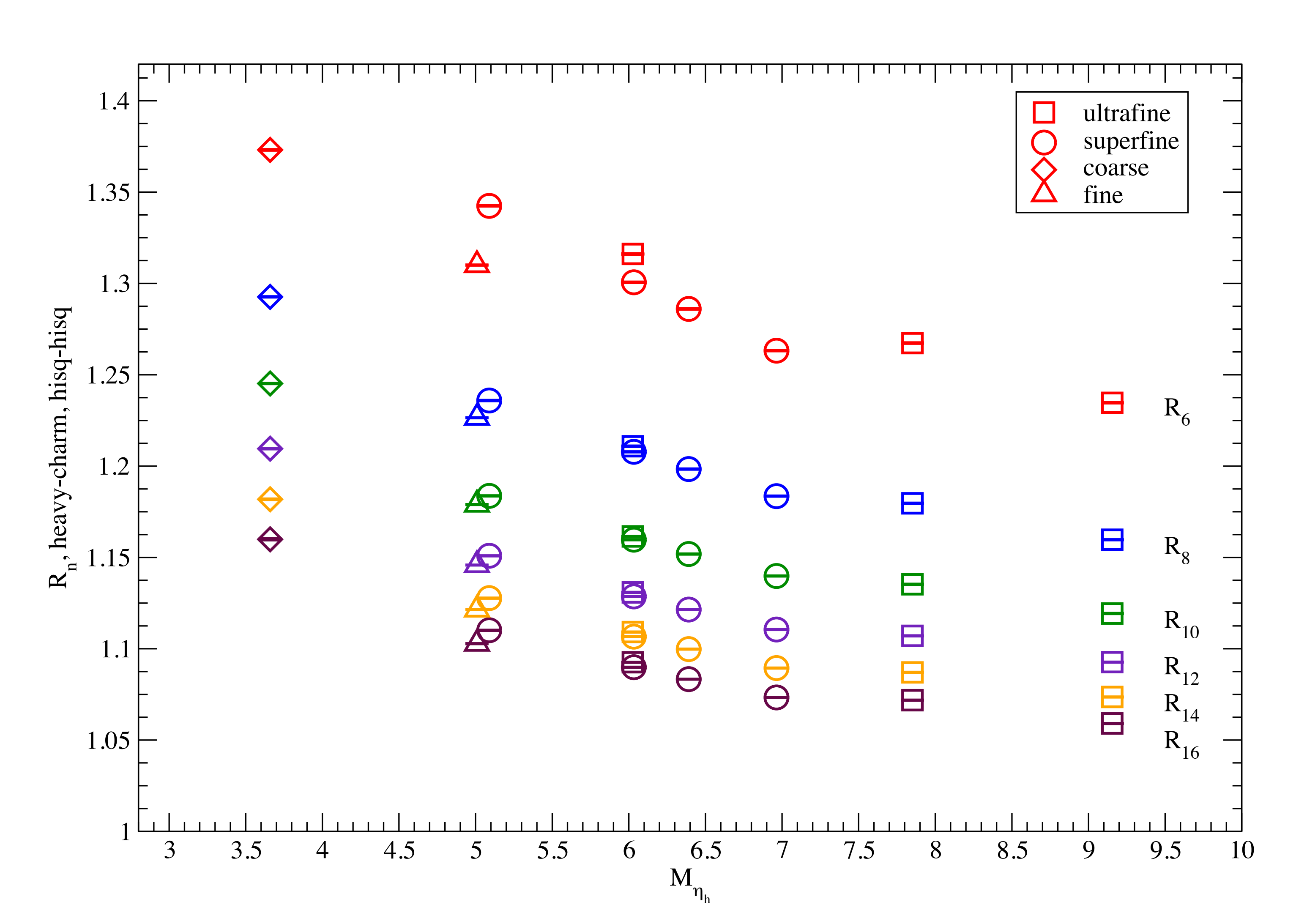
- We are extending the use of JJ correlator method, earlier used to study heavy-heavy systems, to heavy-light systems
- The full analysis of heavy-light data is in progress - JJ correlator method works well
- The quark condensate sets some limitations - can not use high moments in the B_s case. In B_c there is no condensate contribution.
- Want to extract Z for NRQCD - the challenge is to control relativistic corrections

Thank you!

Continuum extrapolation: charm-charm



R_n - heavy-charm



R_n - heavy-heavy