Perturbative vs Non-Perturbative Renormalization: the case of the quark mass

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Motivation

Goal for nowadays LGT tasks can be even percent accuracy: this is the era of precision LGT computations!

The quark mass values changed quite a lot over the years: many groups involved, many regularizations adopted, many systematic effects to control.

Much to do with Renormalizations constants, but do you trust green/red flags?

To which extent Perturbative vs non-Perturbative is the real issue?

Now, there are obvious concerns with PT, but renormalization systematics is rich for both PT and non-PT!

1. truncation errors (PT)
2. (almost always) chiral extrapolations
3. (always) continuum extrapolation
4. (often) finite size effects
5. n_f ...

Our project (and a little disclaimer)

Keep in mind:

- No theoretical obstacle for Z_s computation (only logs)
- In principle proof of multiplicative renormalization is PT

(NSPT) Z's to 3 loops; many regularizations and good control of systematics

- Wilson/Wilson (various n_f) to 3/4 loops
- $n_f = 2$ TLSymanzick/Wilson (this work) to 3 loop
- Staggered computations started

We will not give numbers (they're on their way) but discuss systematics.

References:

Di Renzo, Miccio, Scorzato, Torrero Eur.Phys.J.C51(2007)645;

Di Renzo, Ilgenfritz, Perlt, Schiller, Torrero Nucl.Phys.B831(2010)262.

1a We can go to high loops (NSPT)

In the Stochastic Quantization framework

$$\frac{\partial}{\partial t}\phi_\eta(x,t) = -\frac{\delta S[\phi]}{\delta \phi_\eta(x,t)} + \eta(x,t).$$

$$\lim_{t\to\infty} \langle \phi(x_1,t)\ldots\phi(x_n,t)\rangle_\eta = \langle \phi(x_1)\ldots\phi(x_n)\rangle.$$

we expand the solution to Langevin equation

$$\phi_\eta(x,t) = \phi_\eta^{(0)}(x,t) + \sum_{n>0} g^n \phi_\eta^{(n)}(x,t)$$

and compute observables order by order

$$O\left[\sum_n g^n \phi_\eta^{(n)}(x,t)\right] = \sum_n g^n O^{(n)}(x,t).$$

Something like a perturbative MonteCarlo

1b RI'-MOM scheme is our working ground

We compute quark bilinears bracketted in fixed momentum states and amputate them to Γ functions

$$\int dx \, \langle p | \, \overline{\psi}(x)\Gamma\psi(x) \, | p \rangle \; = \; G_\Gamma(p) \qquad\qquad G_\Gamma(p) \;\; \rightarrow \;\; \Gamma_\Gamma(p)$$

We project on tree-level structures

$$O_\Gamma(p) = Tr\left(\hat{P}_{O_\Gamma}\Gamma_\Gamma(p)\right).$$

We define the field renormalization

$$Z_q(\mu, g) = -i\,\frac{1}{12}\,\frac{Tr(\not{p}S^{-1}(p))}{p^2}$$

and finally define renormalization constants

$$Z_{O_\Gamma}(\mu, g) Z_q^{-1}(\mu, g) O_\Gamma(p)|_{p^2=\mu^2} = 1$$

Much is known in this scheme! 3 loops (J. Gracey)

1c We can take anomalous dimensions for free (an example)

$$Z_q(\hat{\mu}) = 1 + \sum_{n>0} d_n \alpha_0^n + F(\hat{\mu}) \qquad d_n = \sum_{i=0}^{n} d_n^{(i)} L^i$$

$$\psi_0 = Z_q(\mu)^{-1/2} \psi_R(\mu) \qquad Z_q(\mu)^{-1/2} = 1 + \sum_{n>0} c_n \alpha^n.$$

After differentiating with respect to $\log \mu$

$$0 = \sum_{n>0} \sum_{i=1}^{n} \left[i c_n^{(i)} L^{i-1} \alpha^n + n c_n^{(i)} L^i \alpha^{n-1} 2\beta \right] \psi_R + Z_q^{-1/2} \gamma_q \psi_R$$

we can collect orders in α and logs to get the $c_n^{(i)}$:

$$\begin{aligned} c_1^{(1)} &= \gamma_q^{(1)} \\ c_2^{(1)} &= \gamma_q^{(2)} + c_1^{(0)} \left(\gamma_q^{(1)} + 2\beta_0 \right) \\ &\dots \end{aligned}$$

In Landau gauge $\gamma_q^{(1)} = 0$ and $Z_q(\hat{\mu})$ is

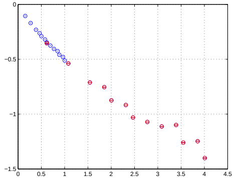
$$\begin{aligned} Z_q(\hat{\mu}) &= 1 + Z_q^{(1)} \alpha_0 + \left[Z_q^{(2)} - 2\gamma_q^{(2)} L \right] \alpha_0^2 + \\ &+ \left[Z_q^{(3)} - \left(4\gamma_q^{(2)} K_1 + 2\gamma_q^{(3)} + 2\gamma_q^{(2)} Z_q^{(1)} \right) L + 4\beta_0 \gamma_q^{(2)} L^2 \right] \alpha_0^3 \end{aligned}$$

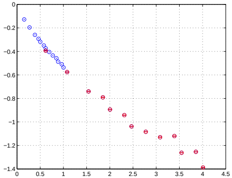
In the (Wilson) quark self-energy there is a counterterm (critical mass)

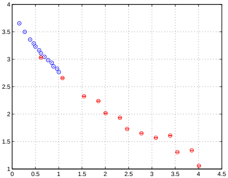
$$
\begin{aligned}
a\Gamma_2(\hat{p}, \hat{m}_{cr}, \beta^{-1}) &= aS(\hat{p}, \hat{m}_{cr}, \beta^{-1})^{-1} \\
&= i\hat{\not{p}} + \hat{m}_W(\hat{p}) - \hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1})
\end{aligned}
$$

$$
\hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1}) = \hat{\Sigma}_c(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_V(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_o(\hat{p}, \hat{m}_{cr}, \beta^{-1})
$$

which we plug in order by order (data from 32^4 and 16^4 lattices)







3 Continuum limit, i.e. $a \longrightarrow 0$

As an example, let's go back to the quark self-energy and look for the field renormalization.

(In our notation $\hat{p} = pa$)

$$\hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1}) = \hat{\Sigma}_c(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_V(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_o(\hat{p}, \hat{m}_{cr}, \beta^{-1})$$

Let's H4-Taylor expand it

$$\hat{\Sigma}_V = i \sum_\mu \gamma_\mu \hat{p}_\mu \left(\hat{\Sigma}_V^{(0)} + \hat{p}_\mu^2 \hat{\Sigma}_V^{(1)} + \hat{p}_\mu^4 \hat{\Sigma}_V^{(2)} + \dots \right)$$

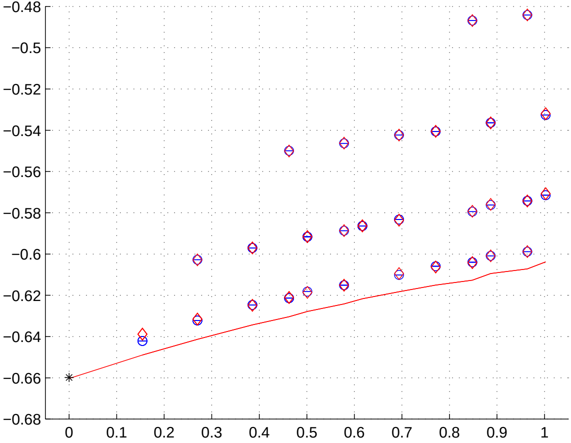
$\Sigma^{(n)}$ are also H4-Taylor expanded

$$\hat{\Sigma}_V^{(n)} = \alpha_1^{(n)} 1 + \alpha_2^{(n)} \sum_\nu \hat{p}_\nu^2 + \alpha_3^{(n)} \sum_\nu \hat{p}_\nu^4 + \alpha_4^{(n)} \sum_{\nu \neq \rho} \hat{p}_\nu^2 \hat{p}_\rho^2 + \mathcal{O}(a^6)$$

The only term surviving the $a \rightarrow 0$ limit is $\alpha_1^{(0)}$.

Continuum limit at work

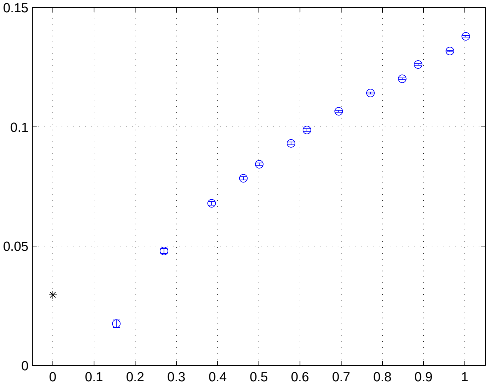
Here is the field renormalization at 1 loop (from the self-energy) on a 32^4 lattice



Something can still go wrong

So, let's finally look for the quark mass renormalization (Z_s actually; 32^4)
In this case a log has been subtracted

$$Z_q^{(1)} - Z_s^{(1)} = O_s^{(1)} - \gamma_s^{(1)} L$$

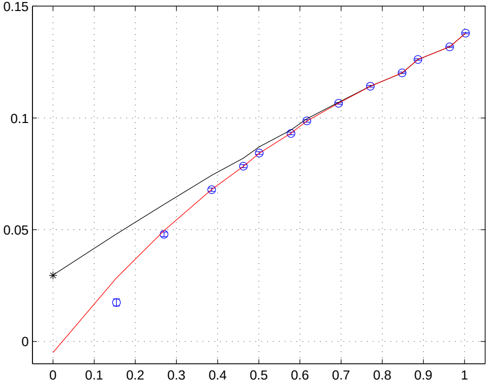


At this stage, don't trust IR...

IR can't be trusted (in particular when an anomalous dimension is around).

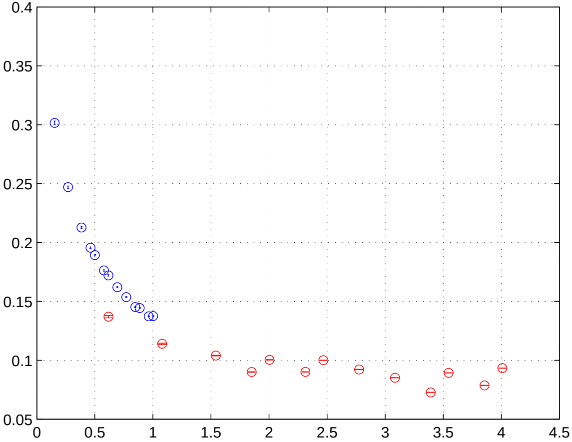
Landau gauge for field renormalization did not need a subtraction, while now (remember)

$$Z_q^{(1)} - Z_s^{(1)} = O_s^{(1)} - \gamma_s^{(1)} L$$



It's a finite size effect!

... as can be seen by inspecting O_s (the un-log-subtracted observable) on different lattice sizes (32^4 and 16^4)



4 Taming finite size: get to $L \to \infty$

On dimensional grounds we expect (take once again $\Sigma^{(n)}$) pL effects

$$\begin{aligned}\hat{\Sigma}_V^{(n)}(\hat{p}, pL) &= \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \left(\hat{\Sigma}_V^{(n)}(\hat{p}, pL) - \hat{\Sigma}_V^{(n)}(\hat{p}, \infty)\right) \\ &= \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL)\end{aligned}$$

so that a better expansion to fit is

$$\begin{aligned}\hat{\Sigma}_V^{(n)}(\hat{p}, pL) &= \alpha_1^{(n)} 1 + \alpha_2^{(n)} \sum_\nu \hat{p}_\nu^2 + \alpha_3^{(n)} \sum_\nu \hat{p}_\nu^4 + \\ &\quad + \alpha_4^{(n)} \left(\sum_\nu \hat{p}_\nu^2\right)^2 + \Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL) + \ldots\end{aligned}$$

In first approximation

$$\Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL) \sim \Delta\hat{\Sigma}_V^{(n)}(pL)$$

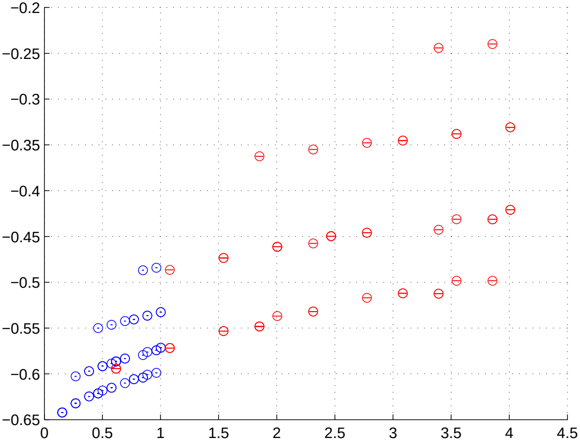
But

$$p_\mu L = \frac{2\pi n_\mu}{L} L = 2\pi n_\mu!$$

i.e. same correction on different lattice sizes for the same $\{n_1, n_2, n_3, n_4\}$.

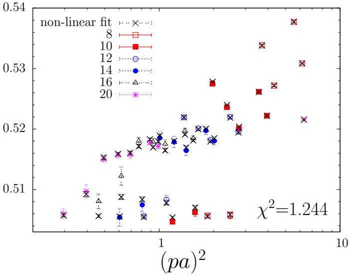
Let's gain some insight

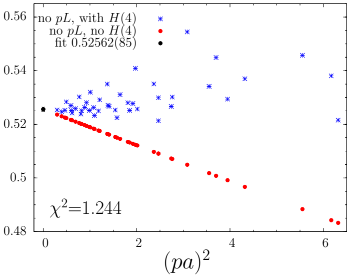
Go back to 1 loop field renormalization, both on 32^4 and 16^4



The method has already been successful

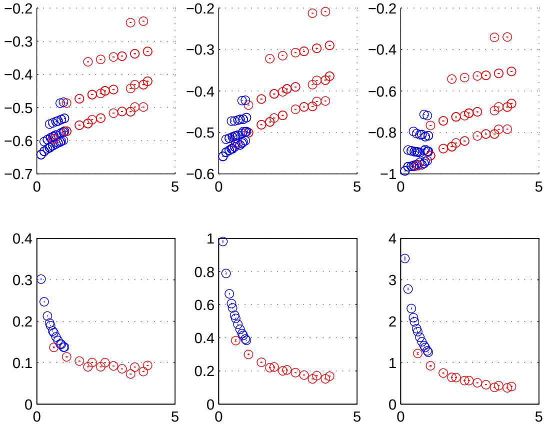
It has been working pretty well in the case of the gluon and ghot propagators (in collaboration with M. Ilgenfritz, H. Perlt, A. Schiller, C. Torrero; see Torrero's talk)





So, where is the quark mass renormalization?

Well, here it is! We are still collecting statistics for intermediate and small lattices. We are not going to quote numbers, but a substantial fractions of data is already available.



Conclusions

We have an ambitious program: computations of renormalization constants to 3 loops keeping all the systematics under a very good control

1. truncation errors
2. chiral limit
3. continuum imit
4. infinite volume limit
5. n_f dependence easy to control! (move to $n_f = 4$)

We will (hopefully soon)

- release results for (log-divergent) Wilson/Wilson Z's
- release results for TLSymanzick/Wilson Z's
- investigate staggered actions

The real issue is not PT vs Non-PT; the real issue is how to best control systematics.

We hope we can reduce systematics and bridge various approaches