Effective Polyakov-loop theory for pure Yang-Mills from strong coupling expansion: numerical aspects and conclusions

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on Lattice Field Theory

Outline

1. The setting
   - The starting point
   - Analytical aspects
2. Monte Carlo implementation
   - “Sign” problems
   - Getting to the numbers
   - Another coupling in the effective theory
3. Results & Outlook
   - Results
   - Future developments

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General strategy

- Start from a reliable 3D effective theory with scalars;
- find its critical points;
- with N_τ-dependent (analytically found) maps translate this back to an array of critical $\beta_c(N_\tau)$;
- verify the predictiveness against known (3+1)D gauge simulations.

The ideal "$SU(3)$" effective theory

- Goal: the critical point $\lambda_{1,c}$ for the NN effective theory:

$$Z = \Big(\prod_x \int \mathrm{d}L_x e^{V_x}\Big) \prod_{<i,j>} (1 + 2\lambda_1 \mathfrak{Re} L_i L_j^*) \; ;$$

  $L_x = \mathrm{Tr}\, W_{\mathbf{x}}$ lives on *points* of the 3D lattice; $|L| \leq 3$.

- The N_τ-dependence is hidden in the maps $\lambda_{1(N_\tau)}(u(\beta))$.
- Useful to think of it as $Z = \sum_{\mathrm{config.}} e^{-S_{\mathrm{eff}}}$:

$$S_{\mathrm{eff}} = - \sum_{<i,j>} \log(1 + 2\lambda_1 \mathfrak{Re} L_i L_j^*) - \sum_x V_x \; .$$

- Reference case: the $SU(2)$ version, with $2\lambda_1 \mathfrak{Re} L_i L_j^* \mapsto \lambda_1 L_i L_j$, is more under control.

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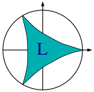
Group parametrisation

- The potential encodes the Haar measure:

$$V_x = \frac{1}{2}\log(27 - 18|L|^2 + 8\mathfrak{Re}L^3 - |L|^4)\ .$$

- Parametrisation: $L = e^{i\theta} + e^{i\phi} + e^{-i(\theta+\phi)}$, so that

$$\int \mathrm{d}_g L_x = \int_{-\pi}^{+\pi} \mathrm{d}\phi_x \int_{-\pi}^{+\pi} \mathrm{d}\theta_x e^{V_x}\ .$$



- In the case of $SU(2)$, it is $-2 \leq L \leq +2$, and simply

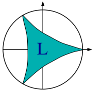
$$\int \mathrm{d}_g L_x = \int_{-2}^{+2} \mathrm{d}L_x e^{V_x}\ , \quad V_x = \frac{1}{2}\log(4 - L_x^2)\ .$$

Outline

Virtues and Vices of the S_{eff}

Metropolis accept/reject update for:

$$S_{\mathrm{eff}} = -\sum_{<i,j>} \log(1 + 2\lambda_1 \mathfrak{Re} L_i L_j^*) - \sum_x V_x$$



- The appearance of a “sign problem” makes it difficult!
- In $SU(3)$ the above S_{eff} cannot be implemented;
- the problem gets milder for $SU(2)$.

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Solution

- Way out: Taylor-expand the log in S_{eff} to some order M:

$$S_{\text{eff}}^{(M)} = -\sum_x V_x - \left(+2q - 2q^2 + \frac{8}{3}q^3 - 4q^4 + \ldots + \#q^M \right)$$

with $q \equiv \lambda_1 \mathfrak{Re} L_i L_j^*$.

- A family of problem-free models $\Rightarrow$ a family of $\lambda_{1,c}^{(M)}$.
- Convergence for $M \to \infty$?

Practical Aspects - I

Finding the critical point:

- Collect the pseudo-critical point $\lambda_{1,c}(N_s)$ for a variety of system volumes N_s^3;
- Perform a scaling analysis to the thermodynamic limit:

$$\lambda_{1,c}(N_s) = \lambda_{1,c} + bN_s^{-1/\nu} \ ;$$

$\nu = 1/3$ for the 1st order $SU(3)$, ν_{Ising3D} for $SU(2)$.

Typical sizes range from $N_s = 6$ to 16; time needed is of order **a few days** on an ordinary PC.

Practical Aspects - II

- A "safe" observable is the modulus $|L|$, well-defined independently of S_{eff} (unlike the energy).
- One can construct the Binder cumulant for $|L|$:

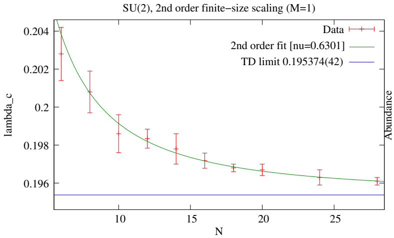
$$B(|L|) = 1 - \frac{\langle |L|^4 \rangle}{3\langle |L|^2 \rangle^2} \quad \rightarrow \quad \lambda_{1,c}(N_s) \text{ is the minimum },$$

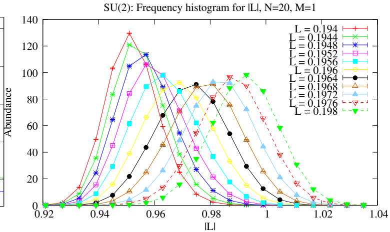
- or alternatively the associated susceptibility:

$$\chi(|L|) = \left\langle \left(|L| - \langle |L| \rangle \right)^2 \right\rangle \quad \rightarrow \quad \lambda_{1,c}(N_s) \text{ is the maximum.}$$

Order of the transition for $SU(2)$

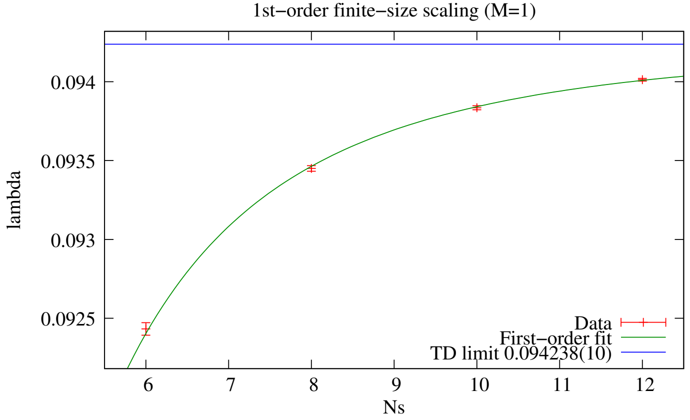
In this model the transition is **second order**, with the 3D Ising critical scaling and no double-peak histograms.





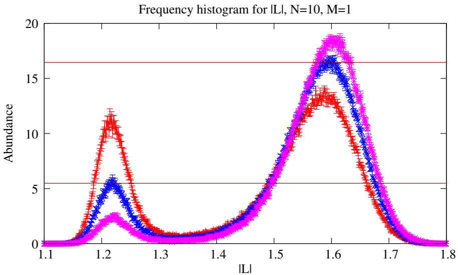
Order of the transition for $SU(3)$ - I

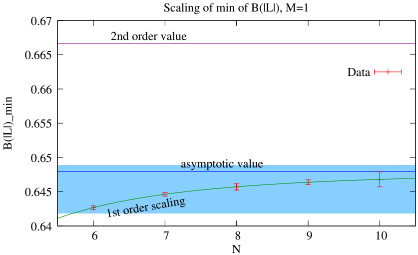
The finite-size analysis confirms a **first-order** transition.



Order of the transition for $SU(3)$ - II

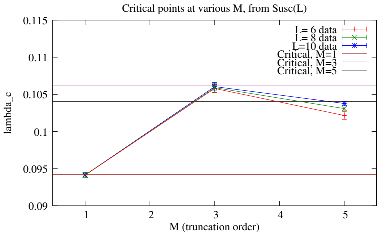
Also the histogram for $|L|$ supports a first-order transition:

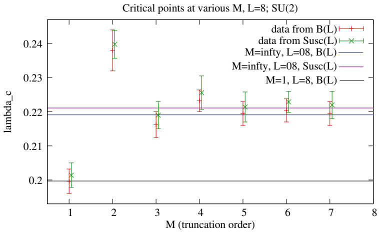




Truncations and Convergence

- Larger $M \Rightarrow$ larger finite-size effects.
- $\lambda_{1,c}^{(M)}$ "stabilises" after $M \simeq 3$ (as in the $SU(2)$ model).
- Compared to other systematic errors, using $\lambda_{1,c}^{(3)}$ is safe.





L-neighbours interaction

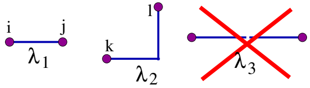
Now we turn on a second coupling between "L-shaped" next-to-nearest neighbours:

$$Z = \Big(\prod_x \int \mathrm{d}L_x\Big) \prod_{<i,j>} (1 + 2\lambda_1 \mathfrak{Re} L_i L_j^*) \prod_{[k,l]} (1 + 2\lambda_2 \mathfrak{Re} L_k L_l^*) \ .$$

$\lambda_1 \sim u^{N_\tau} \exp(u^4 + \ldots)$;

$\lambda_2 \sim u^{2N_\tau + 2}(1 + \ldots)$;

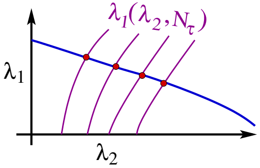
$\lambda_3 \sim u^{2N_\tau + 6}$.

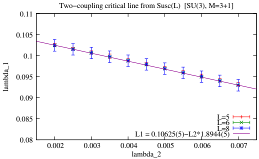


In expanding the log in S_{eff}, keep terms up to $(\lambda_1)^3$ and $(\lambda_2)^1$.

Properties of the two-coupling model

In the (λ_1, λ_2) plane there is now a *critical line*:





A linear relation is sufficient, $\lambda_{1,c} = a + b\lambda_2$, with:

$$a = 0.10625(5)\ ; \quad b = -1.8944(5)$$

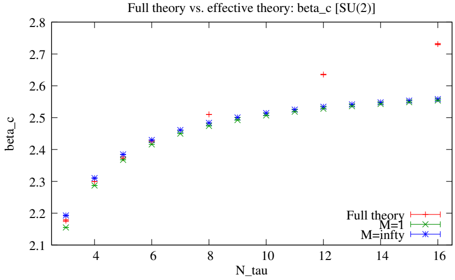
The map back to $\beta_c(N_\tau)$ starts from the intersection

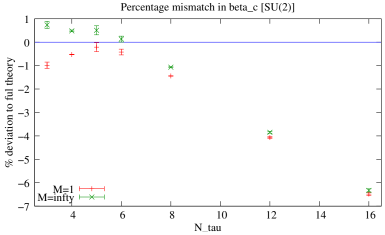
$$\lambda_{1,c}(\lambda_2) = \lambda_1(\lambda_2, N_\tau) \quad .$$

Outline

Predictiveness of the $SU(2)$ model

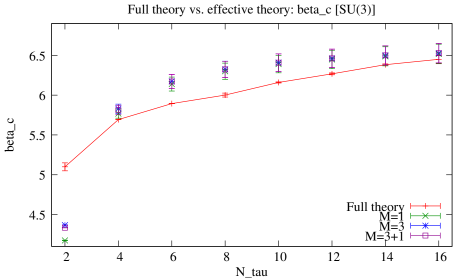
Comparison with the Monte Carlo known β_c values for the (3+1)D theory:
similar outcome from $\lambda_{1,c}^{(1)} = 0.195374$ and $\lambda_{1,c}^{(\infty)} = 0.21423$:
good within few %.

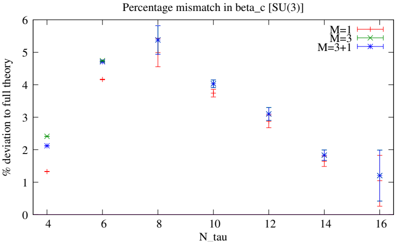




The situation for $SU(3)$

- Less accurate than $SU(2)$, but still within $\sim 5\%$.
- dominant errors from truncation of the series $\lambda_1(u)$.
- Truncation of S_{eff} in M is not so important.





Outlook

Conclusions

- This simple, light model can be simulated quickly on a PC.
- Reproducing β_c of the (3+1)-D gauge theory within 5%.
- A single experiment gives the whole series of $\beta_c(N_\tau)$.
- (All of this without 4D matrix-based simulation!)

Future developments

- Investigate the spectrum from $\langle L(0)L^*(x)\rangle$ correlators.
- Introduce fermions.

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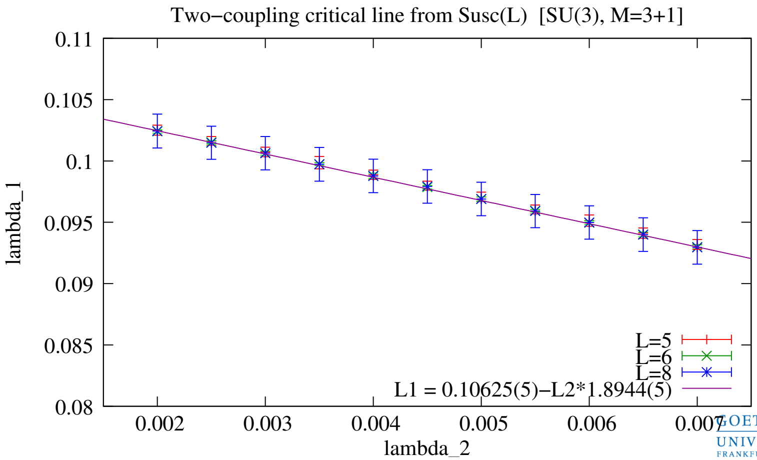
End of presentation!

From now on: misc. plots, backup slides, *hic sunt leones*!

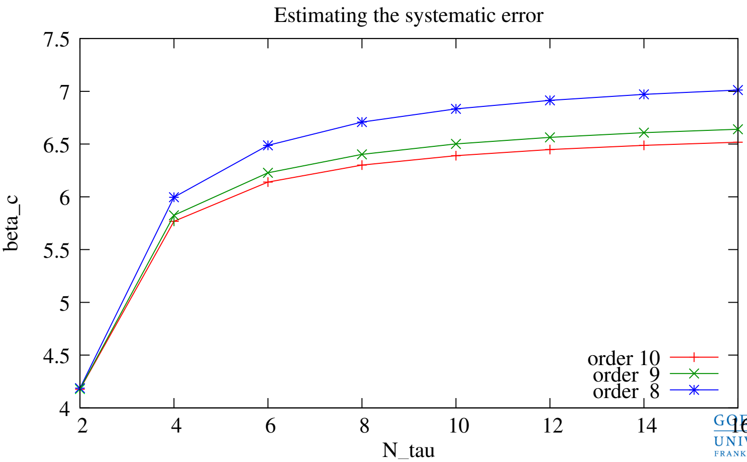
Two-coupling critical line, numerical determination

From $\chi(L)$:



Estimate of the systematic errors

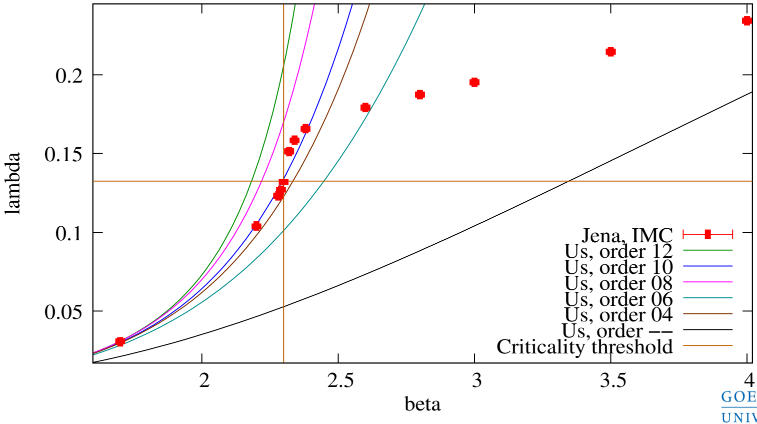
Compare the β_c from different truncations of $\lambda_{1(N_\tau)}(u)$:



Jena Inverse Monte Carlo vs. strong-coupling for λ_1

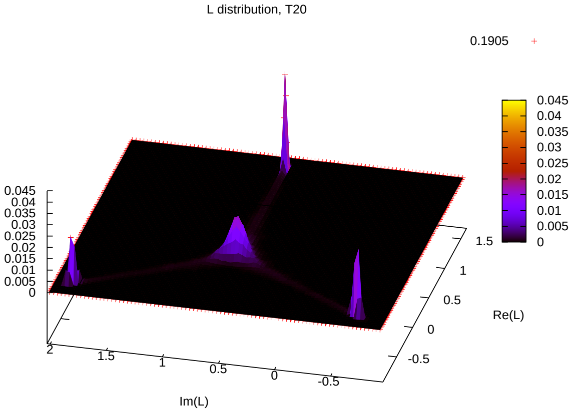
In the $SU(2)$ case (arxiv:hep-lat/0502013):

Ntau=4, Jena IMC versus our series expansion for lambda(beta)



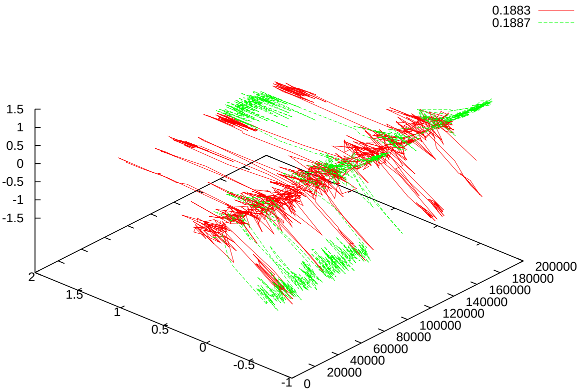
L distribution at criticality

Peaks at center elements and at zero, first-order.



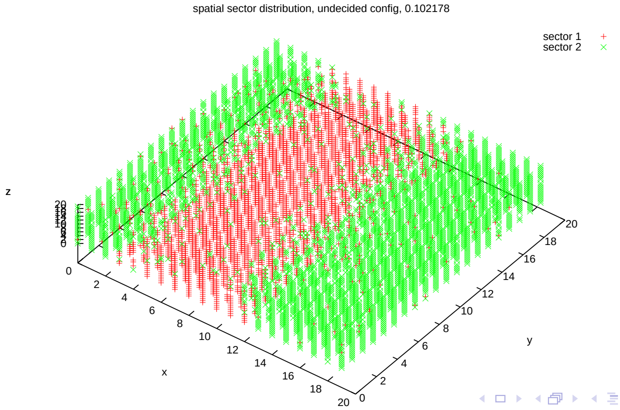
Evidence for tunnelling

Time trajectory for L on single evolutions, first-order.



Metastable states

Metastable interface between broken-symmetry domains; call for cluster update.



The end, really

This is really the last slide, thank you.